

Assignment 5

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Outline

1 Problem

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Problem Statement

We denote by x_m a random variable equal to the number of tosses of a coin until heads shows for the m th time. Show that if $P[h] = P$. then $E_{x_m} = m/p$

Solution

As we know

$$1 + x + x^2 + \dots + x^n = \frac{1}{1-x} \quad (1)$$

for

$$-1 < x < 1$$

Differentiating on both side with respect to x we get

$$1 + 2x + 3x^2 + 4x^3 + \dots + (n-1)x^{n-1} = \frac{1}{(1-x)^2} \quad (2)$$

The random variable x_1 equals the number of tosses until head shows for the time,

Hence, x_1 takes the value 1,2..... with $P(x_1=k)=pq^{(k-1)}$.

Hence,

$$E(x_1) = \sum_{k=1}^{\infty} kP(x_1 = k) \quad (3)$$

$$E(x_1) = \sum_{k=1}^{\infty} kpq^{(k-1)} \quad (4)$$

from equation 1 we can say

$$E(x_1) = \frac{p}{(1-q)^2} \quad (5)$$

as we know that $p+q=1$

$$E(x_1) = \frac{1}{p} \quad (6)$$

Starting the count after the first head shows , we conclude that the random variable x_2-x_1 has the same statistics to x_1
Hence,

$$E(X_2 - X_1) = E(x_1) \quad (7)$$

$$E(x_2) = 2E(x_1) = \frac{2}{p} \quad (8)$$

From induction

$$E(X_n - X_{(n-1)}) = E(x_1) \quad (9)$$

$$E(x_n) = E_{X_{(n-1)}} + E(x_1) = \frac{n-1}{p} + \frac{1}{p} = \frac{n}{p} \quad (10)$$

So

$$E(x_n) = \frac{n}{p} \quad (11)$$