ILOC
SEM-VIII

Finance Management
Module 2
(2021-22)

Class
D17A & D18

Concepts of Returns and Risks

Risk is present in virtually every decision. Assessing risks and incorporating them in the final decision is an integral part of financial analysis. Note that,

- Returns and risks are highly correlated in investing. Increased potential returns on investment usually implies increased risk. So the decisions are not taken to eliminate or avoid the risk, but often in the financial analysis, it is assessed that whether the risk is worth to be taken.
- In a well-ordered market there is a linear relationship between market risk and expected return.

In financial systems, assets are expected to generate cash flows and hence the riskiness of a financial asset is measured in terms of the riskiness of its cash flows. Furthermore, The riskiness of an asset may be measured on a stand-alone basis or in a portfolio context. An asset may be very risky if held by itself but maybe much less risky when it is part of a large portfolio.

In the context of a portfolio, the risk of an asset is divided in to two parts: diversifiable risk and market risk.

- **Diversifiable risk** arises from company-specific (or sector specific) factors and hence one can do away with it through diversification. It allows investors to reduce the overall risk associated with their portfolio, however that may result into limited potential returns.
- **Market risk**, arises from general market movements and hence can not be diversified away.

For example

Making investments in only one of the securities in one of the sectors may generate superior returns if that sector significantly outperforms the overall market.

But should the sector decline then you may experience lower returns than could have been achieved with a broadly diversified portfolio.

For a diversified investor what matters is the market risk and not the diversifiable risk.

Historical Returns and Risks

Historical returns helps you to estimate the distribution of returns expected in future.

Computation of Historical Returns over a period

Suppose that, P_B = Price of the investment at the beginning. P_E = Price of the investment in the end C = Cash Payment Received during the period. So % return R on the investment can be given by,

$$R = \frac{C + (P_E - P_B)}{P_B} \times 100$$

Note that, $P_B > 0$, $C \geq 0$ and P_E can be 0, *Positive* or *Negative*. So, return R can be 0, *Positive* or *Negative*.

For example

Suppose that, an investor bought 100 equity shares at the price of ₹ 600 per share. After one year the price of an equity share was ₹ 684 and he received dividend of ₹ 300 on his investment.

So his return on Investment can be calculated as follows. P_B=₹ 600×100=₹ 60000, P_E=₹ 684×100=₹ 68400 and C=₹ 300
So % return R on the investment is given by,

$$\begin{aligned} R &= \frac{C + (P_E - P_B)}{P_B} \times 100 \\ &= \frac{300 + (68400 - 60000)}{60000} \times 100 \\ &= \frac{8700}{60000} \times 100 = \mathbf{14.5\ \%} \end{aligned}$$

Note that,(300/60000) × 100 = 0.5% is current yield and (8400/60000) × 100 = 14% is capital gain/loss yield.

Average Annual Returns

There are two commonly used ways to calculating the average annual return of an investment one is an arithmetic mean and another is geometric mean that calculates annual compounded growth.

Arithmetic Mean

An arithmetic mean is simple average of annual realized returns, given by

$$\bar{R} = \frac{\sum_{i=1}^{n} R_i}{n}$$

Where, R_i is annual return for the i^{th} year and n is period of investment in years.

For example

Suppose that for an investment of ₹ 60000 for 5 years has annual returns on each years as $R = \{10.5\%, 6.5\%, -3.4\%, 7.5\%, 11.6\%\}$. So the simple average returns can be obtained as

$$\bar{R} = \frac{10.5 + 6.5 - 3.4 + 7.5 + 11.6}{5} = 6.54\%$$

So average yield on the investment is 6.54% or ₹ 3924 per year.

Compounded Annual Growth Rate (CAGR)

To calculate the Compound Average Growth Rate (CAGR) over a period of time, the geometric mean is used.

Geometric Mean

An geometric mean for the realized returns can be given by,

$$GM = \left(\prod_{i=1}^{n} (1 + R_i) \right)^{1/n} - 1$$

Where, R_i is annual return (normalized to 1) for the i^{th} year and n is period of investment in years.

For example

Suppose that for an investment of ₹ 60000 for 5 years has annual returns on each years as $R = \{10.5\%, 6.5\%, -3.4\%, 7.5\%, 11.6\%\}$. So the CAGR is given by

$$CAGR = (1.105 \times 1.065 \times 0.966 \times 1.075 \times 1.116)^{1/5} - 1 = 0.064$$

So CAGR on the investment is 6.4% or ₹ 3840 per year.

Variance of returns

Suppose you are analyzing the return of an equity stock over a period of time. In addition to the arithmetic mean return, you would also like to know the variability of returns. The variance is a measure of variability. It is calculated by taking the *average of squared deviations* from the mean.

Varaince

Let R be the return from the investment, $\bar{R}$ be the arithmetic mean and σ be standard deviation, then variance is given by

$$\sigma^2 = \frac{\sum_{i=1}^{n}(R_i - \bar{R})^2}{n-1}$$

For example

Suppose that for an investment of ₹ 60000 for 5 years has annual returns on each years as $R = \{10.5\%, 6.5\%, -3.4\%, 7.5\%, 11.6\%\}$. So the simple average returns can be obtained as

$$\bar{R} = \frac{10.5 + 6.5 - 3.4 + 7.5 + 11.6}{5} = 6.54\%$$

So average yield on the investment is 6.54% or ₹ 3924 per year. Furthermore, Here period $n = 5$ years. So Variance is given by,

$$\begin{aligned}\sigma^2 =& \frac{\sum_{i=1}^{n}(R_i - \bar{R})^2}{n-1} \\ =& \frac{(10.5-6.54)^2 + (6.5-6.54)^2 + (-3.4-6.54)^2 + (7.5-6.54)^2 + (11.6-6.54)^2}{4} \\ =& \frac{141.012}{4} = \mathbf{35.253}\end{aligned}$$

So the Variance is 35.253 and standard deviation is $\sigma = \sqrt{35.253} = 5.9374$.

Expected Return and Risk of the Single Asset

So far we have discussed about past returns.Now we discuss about prospective returns. When you have invested in an asset, return on the investment can be any value positive, negative or zero return. Likelihood of return therefore vary from value to value. So one should think in terms of a probability distribution. The probability of an event represents the likelihood of its occurrence. For instance, suppose that you think that there is a 4:1 chance that the market price of an asset will increase.

So probability distribution can be given by,

Outcome	Probability
Stock Price will rise	80%
Stock Price will not rise	20%

Expected Rate of Return

It is calculated by taking the average of the probability distribution of all possible returns. Suppose that there are n possible returns (outcomes) R_i, $i = 1, 2, \cdots, n$, then expected rate of return can be given by

$$E(R) = \sum_{i=1}^{n} p_i R_i$$

Where, p_i is probability of occurrence of R_i return.

For example, if a stock has a 50% probability of providing a 10% rate of return, a 30% probability of providing a 15% rate of return, and a 20% probability of providing a 20% rate of return.

So the expected rate of return is,

$$E(R) = \sum_{i=1}^{n} p_i R_i = 0.5 \times 10 + 0.3 \times 15 + 0.2 \times 20 = \mathbf{13.5\%}$$

Standard Deviation of Expected Returns

Note that, risk refers to the dispersion of a variable and it is commonly measured by the variance or the standard deviation.

The variance of a probability distribution is the *sum of the squares of the deviations of actual returns from the expected return* weighted by the associated probabilities. So the variance is given by,

$$\sigma^2 = \sum_{i=1}^{n} p_i (R_i - E(R))^2 \tag{1}$$

Where, R_i is i^{th} possible outcome, $E(R)$ is expected return and p_i is the probability of the i^{th} possible income.

For example

If a stock has a 50% probability of providing a 10% rate of return, a 30% probability of providing a 15% rate of return, and a 20% probability of providing a 20% rate of return.

So the expected rate of return is,

$$E(R) = \sum_{i=1}^{n} p_i R_i = 0.5 \times 10 + 0.3 \times 15 + 0.2 \times 20 = \mathbf{13.5\%}$$

So variance of expected returns is given by,

$$\begin{aligned}\sigma^2 =& \sum_{i=1}^{n} p_i (R_i - E(R))^2 \\ =& 0.5 \times (10 - 13.5)^2 + 0.3 \times (15 - 13.5)^+ 0.2 \times (20 - 13.5)^2 = \mathbf{15.25}\end{aligned}$$

and standard deviation is $\sigma = \sqrt{15.25} = 3.9051\%$.

Features of Standard Deviation

- Because $R_i - E(R)$ is squared, therefore farther the possible value of R_i higher it impacts on standard deviation.
- As squared difference $(R_i - E(R))^2$ is multiplied by associated probability p_i, therefore lesser the probability of occurrence of particular R_i smaller is the effect on standard deviation.
- As standard deviation and expected value are measured in the same units and hence the two can be directly compared.

Rationale for Standard Deviation

Why is standard deviation employed commonly in finance as a measure of risk?

If a variable is normally distributed, its mean and standard deviation contain all the information about its probability distribution.

- Note that, the normal distribution is a continuous probability distribution used most commonly in finance. Typically, It is given by

$$PDF = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}} \left(\frac{R_i - E(R)}{\sigma} \right)^2$$

- It has bell shape characteristics as given bellow,
- If the utility of money is represented by a quadratic function (a function commonly suggested to represent diminishing utility of wealth),then the expected utility is a function of mean and standard deviation.
- Standard deviation is analytically more easily tractable.
- Typically, it can be characterized as in Fig. .

Risk Aversion and Required Returns

Suppose that you are given a choice to take one of the two boxes among them one is empty and another contains ₹ 10000. So expected return is ₹ 5000. If you are offered to forfeit the option to choose a box at cost of ₹ 3000. Let us assume you deny it and want to take a risk for additional ₹ 2000. Then you are offered ₹ 3500 for the same. Now you

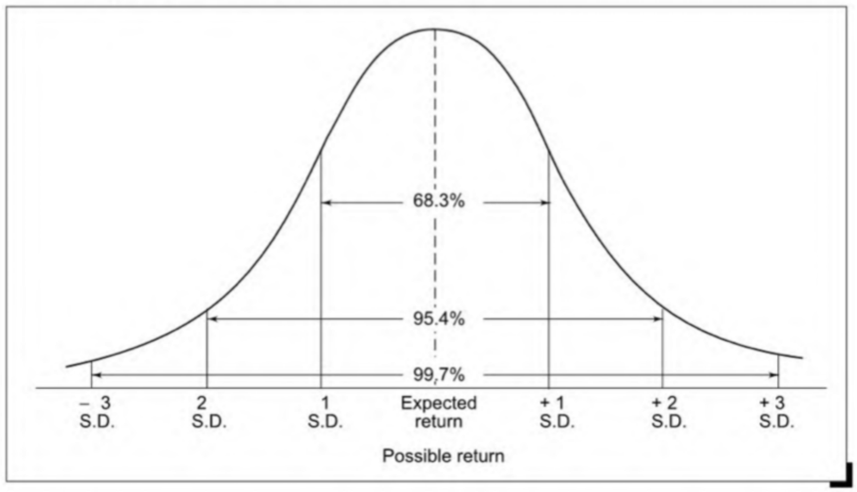


Figure 1: Typical normal distribution of possible returns

may accept the offer as, it is certain that you are getting ₹ 3500. Thus certain returns, "amounts offered ₹ 3500 is a "Certainty Equivalent" which is less than the risky expected value ₹ 5000.

- If the certainty equivalent of a person is less than the expected value, then he is a **risk-averse** person.
- If the certainty equivalent of a person is equal to the expected value, then he is a **risk-averse** person.
- If the certainty equivalent of a person is greater than the expected value, then he is a **risk-loving** person.

0.1 Standard deviation and Risk

Risk and return go hand in hand.This indeed is a well-established empirical fact,particularly over long periods of time.

0.1.1 For example

Suppose that an equity has expected return of $E(R) = 15\%$ each year with standard deviation of $\sigma = 30\%$. Assume that there are two equally possible outcomes each year, $E(R) \pm \sigma$. i.e 45% and -15%.

Clearly $AM = (45 - 15)/2 = 15\%$ and

$$GM = \left(\prod (1 + R_i)^{1/2} - 1\right) \times 100 = ([(1.45)(0.85)]^{1/2} - 1) \times 100 = 11\%$$

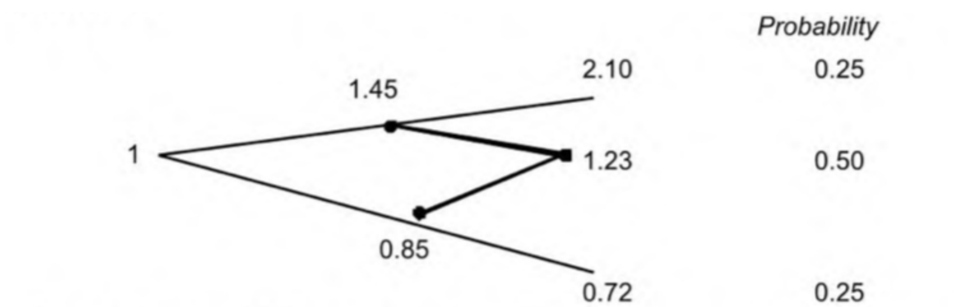
So compounded value for ₹ 1 is year 1 : 1.45 or 0.85

year 2: $1.45 - 0.45 \times 1.45 = 2.10$ or $0.85 - 0.15 \times 0.85 = 0.72$

While median value (Geometric mean) is given by $(1.11)^2 = 1.23$. (1.11*0.11+0.11)

For two years probability is 0.25 for each 45% and -15% returns, So median value has p=50%. However, expected value of all possible returns is, $E(R) = (0.25 \times 2.10) + (0.50 \times 1.23) + (0.25 \times 0.72) = 1.32$

Note that, $\sqrt{1.32} = 1.15$. This means that the expected value of the terminal wealth is obtained by compounding up the arithmetic mean,not the geometric mean.



Risk and Return of Portfolio

The expected return on a portfolio is simply the weighted average of the expected returns on the assets comprising the portfolio.

So in general, we can write expected return on multiple security portfolio as,

$$E(R_p) = \sum_{i=1}^{n} w_i E(R_i)$$

Where, w_i is the proportion of portfolio invested in i^{th} security and $E(R_i)$ is the expected return on it.

Return of Portfolio comprising Two Securities

Suppose that, a portfolio comprises two stocks with the expected returns $E(R_1)$ and $E(R_2)$ respectively, then effective expected return on the portfolio can be given by

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2)$$

Where w_1 is proportion of first stock in the portfolio and w_1 is proportion of first stock in the portfolio i.e. $w_2 = 1 - w_1$.

For example

Let Stock A and B has expected returns of 13% and 16% respectively. Portfolio consists of 45% **Stock A** and 55% of **Stock B**. Then expected return on the portfolio is given by,

$$E(R_p) = 0.45 \times 13 + 0.55 \times 16 = 14.65\%$$

Measurement of Market Risk

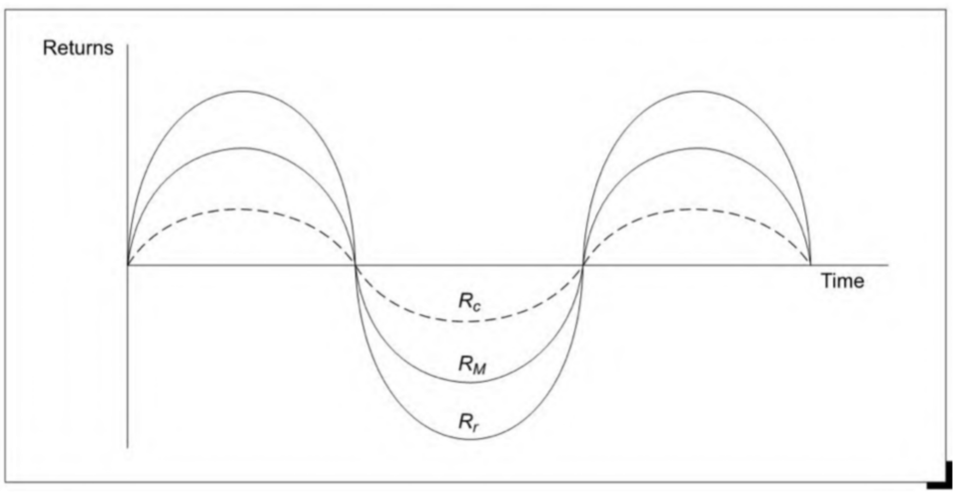
As a risk on individual stock can diversifiable by changing the its proportion in portfolio. So its important to see market risk though the portfolio is well-diversified.

The market risk of a security reflects its sensitivity to the market movements. Generally, every security has different sensitivity to market movements. The sensitivity of a security to market movements is called as β.

By definition β for a Market Portfolio is 1. So if a particular security has $\beta > 1$ is more sensitive to the fluctuations in market portfolio while $\beta < 1$ is less sensitive. Clearly, $\beta = 1$ is insensitive to the market fluctuations.

Standard deviation (σ) and Risk (β)

Figure shows, the returns on the market portfolio R_M over a time, along with returns on two other securities, a risky security,whose return is denoted by R_r and a conservative security,whose return is denoted byR_c.It is evident that R_r is more volatile than R_M where as R_c is less volatile than R_M. So β for risky stock is higher than conservative.



Calculation of β

β of the security can be calculated using simple linear regression as follows,

$$R_{jt} = \alpha_j + \beta_j R_{Mt} + e_j$$

Where,

R_{jt} is return of the j^{th} security over a period t.

α_j is intercept term (It's value at which regression line intercepts y axis). β_j is regression coefficient (slope of the regression line).

R_{Mt} is a market returns over a period t, and

e_j is random error.

Note that, covariance between return R_{jt} on j^{th} security and return R_{Mt} on market portfolio can be given by,

$$\begin{aligned} cov(R_j, R_M) &= \sum_{t=1}^{n}(R_{jt} - \bar{R}_j)(R_{Mt} - \bar{R}_M)/(n-1) \\ &= \rho_{jM}\sigma_j\sigma_M \end{aligned}$$

Where, ρ_{jM} is correlation coefficient between R_{jt} and R_{Mt}. σ_j and σ_M are SD in j^{th} security and market portfolio.

β reflects the slope of regression, so we can write as the ratio of covariance between return R_{jt} and R_{Mt} to variance of return on market R_{Mt}. i.e

$$\beta_j = \frac{cov(R_j, R_M)}{\sigma_M^2} = \frac{\rho_{jM}\sigma_j}{\sigma_M}$$

For example

Consider a security and Market has returns for the period of 5 years on YOY basis as in Table 1. Find β_j and α_j.

Year	Return on security (%)	Return on Portfolio (%)
1	10	12
2	7	4
3	-3	-5
4	6	8
5	11	10

Table 1:

Solution

Covariance between R_{jt} and R_{Mt} is given by,

$$cov(R_j, R_M) = \sum_{t=1}^{n}(R_{jt} - \bar{R}_j)(R_{Mt} - \bar{R}_M)/(n-1)$$

Here,

$$\begin{aligned} \bar{R}_j &= \frac{10+7-3+6+11}{5} = \frac{31}{5} = 6.2\% \\ \bar{R}_M &= \frac{12+4-5+8+10}{5} = \frac{29}{5} = 5.8\% \end{aligned}$$

Now, $R_{j1} - \bar{R}_j = 3.8$, $R_{j2} - \bar{R}_j = 0.8$, $R_{j3} - \bar{R}_j = -9.2$, $R_{j4} - \bar{R}_j = -0.2$ and $R_{j5} - \bar{R}_j = 4.8$ and,

$R_{M1} - \bar{R}_M = 6.2$, $R_{M2} - \bar{R}_M = -1.8$, $R_{M3} - \bar{R}_M = -10.8$, $R_{M4} - \bar{R}_M = 2.2$ and $R_{M5} - \bar{R}_M = 4.2$.

So covariance is

$$cov(R_j, R_M) = \frac{23.56 - 1.44 + 99.36 - 0.44 + 20.16}{4} = 35.3\%$$

and variance in market portfolio,

$$\begin{aligned}\sigma_M^2 &= \sum_{t=1}^{5} (R_{Mt} - \bar{R}_M)^2/(n-1) \\ &= \frac{38.44 + 3.24 + 116.64 + 4.84 + 17.64}{4} = 45.2\end{aligned}$$

So SD is $\sigma_M = 6.72\%$.

Hence, β_j is,

$$\beta_j = \frac{cov(R_j, R_M)}{\sigma_M^2} = \frac{35.3}{45.2} = 0.78$$

and α_j is,

$$\alpha_j = \bar{R}_j - \beta_j \bar{R}_M = 6.2 - 0.78 \times 5.8 = 1.68\%$$

Time value of Money

Money has time value. A rupee today is more valuable than a rupee a year hence. Reason for that is an inflation. So a rupee today represents a greater real purchasing power than a rupee a year hence. Suppose that, a capital is employed productively to generate positive returns. So an investment of one rupee today would grow to $(1 + r)$ a year hence if r is rate of return.

Most financial problems involve cash flows occurring at different points of time. Therefore, these cash flows have to be brought to the same point of time for purposes of comparison and aggregation.

Future value of amount

The process of investing money as well as reinvesting the interest earned thereon is called compounding. The future value or compounded value of an investment after n years when the interest rate is $r\%$ is given by

$$FV_n = PV(1 + r)^n$$

Where, FV_n is future value over the period n, PV present value and r is interest rate. Also, the term $(1 + r)^n$ is called as *future value interest factor.*

Fro example

Suppose you deposit ₹ 1,000 today in a bank which pays 10 percent interest compounded annually.Flow much will the deposit grow to after 8 years and12years?

Solution

The future value 8 years hence will be,

$$FV_n = PV(1+r)^n = 1,000(1+0.10)^8 = 1,000 \times 2.144 = ₹\ 2144$$

The future value 12 years hence will be,

$$FV_n = PV(1+r)^n = 1,000(1+0.10)^12 = 1,000 \times 3.138 = ₹\ 3138$$

Note that, if we compute the future value with simple interest rate, then we can write,

$$FV_n = PV(1+n \cdot r)$$

Doubling period

As the name suggests, doubling period is the period for which future value of the investment is doubled.

Indeed the doubling period can be obtained by the formula stated above in which n is unknown while all other terms are known. However, it involves logarithmic calculations. So there are two thumb rules that can approximately provide the value of doubling period.

- Rule of 72: doubling period, $n = 72/r$ where r is the rate of interest.
- Rule of 69: doubling period, $n = 0.35 + 69/r$ where r is the rate of interest.

For example

Suppose that an investment of ₹ 10000 has been done with the rate of interest of 8% and 12%. So the actual doubling period and by the rule of 72 and 69 are as follows.

rate of interest	actual n	n by rule of 72	n by rule of 69
8 %	9.0065	9	8.9750
12%	6.1163	6	6.10

Growth rate

It is possible to compute the growth rate of an investment if the terminal value of an investment over the specified period is known. It can be given by,

$$\begin{aligned} FV_n =& PV(1+g)^n \\ \Rightarrow g =& \sqrt[n]{FV_n/PV} - 1 \end{aligned}$$

Where, FV_n is future value over the period n, PV present value and r is interest rate. Also, the term $(1+r)^n$ is called as *future value interest factor.*

For example

Suppose that an investment of ₹150000 is grown to ₹350000 over the period of 6 years. What is the annual growth rate?

Solution

$$g = \sqrt[n]{FV_n/PV} - 1 = \sqrt[6]{350000/150000} - 1 = 1.1517 - 1 = 0.1517$$

So growth rate is 15.17 %.

Present Value of future amount

$$PV = \frac{FV_n}{(1+r)^n}$$

Where, r is discount rate and the term $1/(1+r)^n$ is called as *discounting factor*.

For example

What is the present value of ₹ 15000 receivable after 6 years if the discount rate is 8%?

Solution

$$PV = \frac{FV_n}{(1+r)^n} = \frac{15000}{(1+0.08)^6} = ₹\ 9452.5$$

Present value of cash flow stream

In financial analysis cash flow streams are not even. For example, the cash flow stream associated with a capital investment project or the dividend stream associated with an equity share are usually uneven. In such situation, the present value of the cash flow over the specified for known discount rate can be given by,

$$PV = \frac{A_1}{(1+r)} + \frac{A_2}{(1+r)^2} + \cdots + \frac{A_n}{(1+r)^n} = \sum_{t=1}^{n} \frac{A_t}{(1+r)^t}$$

For example

Suppose that following table shows the cash flow after every year. What is the present value if the discounting rate is 6%?

year	1	2	3	4	5
cash flow ₹	1000	1200	1250	1400	1550

Solution

$$PV = \frac{1000}{(1+0.06)} + \frac{1200}{(1+0.06)^2} + \frac{1250}{(1+0.06)^3} + \frac{1400}{(1+0.06)^4} + \frac{1500}{(1+0.06)^5} = ₹\ 5328.1$$

Future value of annuity

An annuity is a stream of constant cash flows (payments or receipts) occurring at regular intervals of time.For instance the premium payments of a life insurance.

> *Ordinary or deferred annuity:* When the cash flows occur at the end of each period,the annuity is called an ordinary annuity or a deferred annuity.
>
> *Annuity due:* When the cash flows occur at the beginning of each period, the annuity is called an annuity due.

In general future value of an annuity over the specified period can be given by,

$$FVA_n = \sum_{t=0}^{n-1} A(1+r)^t$$

To simplify it, multiply both the sides by $1+r$ and subtract the given expression of PV from it, we get,

$$\begin{aligned} FVA_n(1+r) - FVA_n =& \sum_{t=1}^{n} A(1+r)^t - \sum_{t=0}^{n-1} A(1+r)^t \\ \Rightarrow FVA_n \cdot r =& A(1+r)^n - A \\ \Rightarrow FVA_n =& A\left[\frac{(1+r)^n - 1}{r}\right] \end{aligned}$$

Where, FVA_n is the future value of annuity over the period n and r is the interest rate. The term $\frac{(1+r)^n - 1}{r}$ is the *future value interest factor* ($FVIFA_{r,n}$).

For example

Suppose that you are planning to deposit ₹ 50000 every year for 25 years in provident fund with assured interest rate of 8%. How much will be the funds accumulated in you account at the end of period?

Solution

Interest factor for 25 years with the rate 8% is,

$$FVIFA_{8,25} = \frac{(1+r)^n - 1}{r} = \frac{(1.08)^2 5 - 1}{0.08} = 73.11$$

So the future value of annuity is,

$$FVA_2 5 = A \cdot FVIFA_{8,25} = 50000 \times 73.11 = ₹\ 3655500$$

For example

Suppose that you want to buy a car of ₹ 500000 after 3 years. How much should you save every month if the rate of returns on savings is 6% every year?

Solution

Interest factor for 3years with the rate 6% is,

$$FVIFA_{8,25} = \frac{(1+r)^n - 1}{r} = \frac{(1.06)^3 - 1}{0.06} = 3.1836$$

So the future value of annuity is,

$$\begin{aligned} FVA_3 =& A \cdot FVIFA_{6,3} \\ \Rightarrow 500000 =& A \times 3.1836 \\ \Rightarrow A =& ₹\ 157050 \end{aligned}$$

So monthly saving should be $157050/12 =$ ₹ 13088