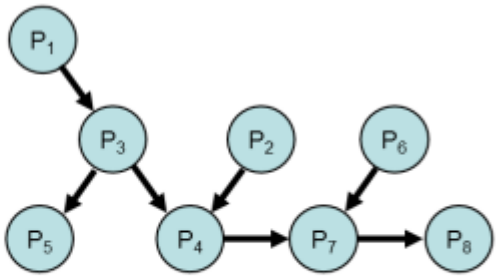
Advance Artificial Intelligence

Continuous Assessment: 3

Bayesian Network and Markov Chain

1. Having the network/graph shown in figure below, decide on the validity of following statements:



a) $P_1, P_5 \perp\!\!\!\perp P_6 | P_8$,

b) $P_2 \top\!\!\!\top P_6 | \varnothing$,

c) $P_1 \perp\!\!\!\perp P_2 | P_8$,

d) $P_1 \perp\!\!\!\perp P_2, P_5 | P_4$,

e) Markov equivalence class that contains the shown graph contains exactly three directed graphs.

2. Let us have an arbitrary set of (conditional) independence relationships among N variables that is associated with a joint probability distribution.

a) Can we always find a directed acyclic graph that perfectly maps this set (perfectly maps = preserves all the (conditional) independence relationships, it neither removes nor adds any)?

b) Can we always find an undirected graph that perfectly maps this set?

3. Consider the Markov chain with three states, S={1,2,3}}, that has the following transition matrix

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{3} & 0 & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}.$$

a. Draw the state transition diagram for this chain.
b. If we know $P(X_1 = 1) = P(X_1 = 2) = \frac{1}{4}$, find $P(X_1 = 3, X_2 = 2, X_3 = 1)$.