SEAT NO: A13A047

ROLL NO: 47

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SEMESTER: III

SUBJECT: DSGT

PAGE NO: 1/16

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Q1.

1. Option C: 16.

2. Option D: $\sim q \wedge p$.

3. Option C: Transitive.

4. Option B: $(p \wedge q) \rightarrow p$.

5. Option B: No two edges of the graph intersect

6. Option C: Closure, Associative, Identity, Inverse.

7. Option B: $\{(1,2),(2,3),(3,4),(5,4),(1,3),(1,4),(2,4)\}$

8. Option D: Monoid

9. Option C: 55

10. Option A: 0

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Q2. A. (i)

→ Let $A = \{1, 2, 3, 4, 5, 6, 7\}$

and $R = \{(a,b) \mid a-b \text{ is divisible by } 3\}$ in A.

Since $a-a=0$ is divisible by 3 ~~for~~ and $a \in A$,
hence R is Reflexive relation.

If $a, b \in A$ and $(a,b) \in R$,
then $a-b$ is divisible by 3.
It follows that $b-a = -(a-b)$ is also divisible by 3, hence $(b,a) \in R$.

We conclude that the relation R is symmetric.

If $a, b, c \in A$ and $(a,b) \in A$, $(b,c) \in R$,
then $a-b$ and $b-c$ are divisible by 3.

$a-c = (a-b) + (b-c)$ is also divisible by 3,
and hence, $(a,c) \in R$.
Thus, we conclude that the given relation R is transitive, symmetric and reflexive.

$\therefore$ Given relation R is an equivalence relation on A

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Q2. A. (iii)

→ Given : $(x \land y) \lor (\sim x \land y)$

$= [x \lor (\sim x \land y)] \land [y \lor (\sim x \land y)]$

— (by distributive law)

$= [(x \lor \sim x) \land (x \lor y)] \land [(y \lor \sim x) \land (y \lor y)]$

— by distributive law.

$= [T \land (x \lor y)] \land [(y \lor \sim x) \land y]$

— by inverse law and idempotent law.

$= (x \lor y) \land (y \lor \sim x) \land y$

— by identity law.

$\therefore (x \lor y) \land (y \lor \sim x) \land y$ is the required Conjunctive Normal form.

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Q2. B. (ii)

→

To prove that a function is injective (one-to-one) we must show that if $f(x_1) = f(x_2)$ then $x_1 = x_2$:

Let us take two elements x_1 & x_2 in R.

& let $f(x_1) = f(x_2)$

$\therefore f(x_1) = f(x_2) \quad \therefore x_1^2 = x_2^2$

By taking square roots, $\pm x_1 = \pm x_2$.

but $f: N \rightarrow N$

$\therefore$ negative values will not be considered.

$\therefore x_1 = x_2$.

Hence $f: N \rightarrow N$, $f(x) = x^2$ is Injective

Now, to prove that a function is surjective (onto) we have to show that for every element $y = f(x)$ in y, we have at least one corresponding element of x in x.

Let, $y = f(x) \quad \therefore y = x^2 \quad \therefore x = \sqrt{y}$ (let $y = z \in N$)

but there does not exist any 'x'.

i.e $x^2 = z$ thus, function is not surjective.

Hence, $f: N \rightarrow N$ given by $f(x) = x^2$, is injective but not surjective.

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Q3. A. (ii)

i] $\{1, 1, 1, 1, \ldots\ldots\}$

If $\{a_n\} = \{a_0, a_1, a_2, \ldots\}$ is a sequence of real numbers and x is a real variable then ordinary generating function of the sequence is infinite sum.

$$g(x) = \sum_{n=1}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

for sequence $\{a_n\} = \{1, 1, 1, 1, \ldots\}$

$g(x) = 1 + 1x + 1x^2 + 1x^3 + \ldots$ which is a G.P with $a = 1$ & with common ratio $(r) = x$.

$\therefore$ In G.P, sum of infinite series $= \frac{a}{1-r}$

$$\boxed{\text{i] } \therefore \text{ The generating function } g(x) = \frac{1}{1-x}.}$$

ii] $\{1, 2, 3, 4, \ldots\}$

(Similarly from i)

for sequence $\{a_n\} = \{1, 2, 3, 4, \ldots\}$

$g(x) = 1 + 2x + 3x^2 + 4x^3 + \ldots$

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Q3. A. (ii)

contd. $g(x) = (1-x)^{-2}$

$= 1/(1-x)^2$

$\boxed{\text{ii] } \therefore \text{ The generating function } g(x) = \frac{1}{(1-x)^2}}$

Q3. A. (iii)

→ To prove: $1+3+5+\ldots+(2k-1) = k^2$

Solution: for $k=1$,

$L.H.S = 1$, $R.H.S = 1$.

$\therefore L.H.S = R.H.S.$ $(k=1)$ —— (1)

Now, let us assume that

for $k=n$, $n \in N$ & $n \geq 1$.

$1+3+5+\ldots+(2n-1) = n^2$ —— (2)

when, $k=n+1$,

$1+3+5+\ldots+(2n-1)+(2n+1) = (n+1)^2$ —— (3)

L.H.S

$\therefore = n^2 + (2n+1)$ [from (2)] by assumption

$= (n+1)^2 = R.H.S.$

$\therefore L.H.S = R.H.S$ $(k=n+1)$ —— (4)

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Q 3. A. (iii)

cont.d. $\therefore \quad 1+3+5+7+\ldots + (2k-1) = k^2$

stands true for $k=1$ [from (1)]

for $k=n$ [assumption]

for $k=n+1$ [from (4)]

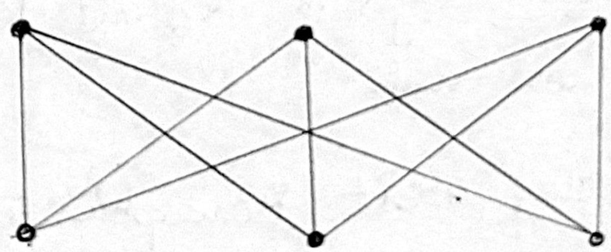
$\therefore$ By proof of mathematical induction, this statement is true for all integers greater than or equal to 1.

Q 3. B. i)

→ A graph can exist in different forms having the same number of vertices, edges and also the same edge connectivity.

Such graphs are called isomorphic graphs

$K_{3,3}$ graph:



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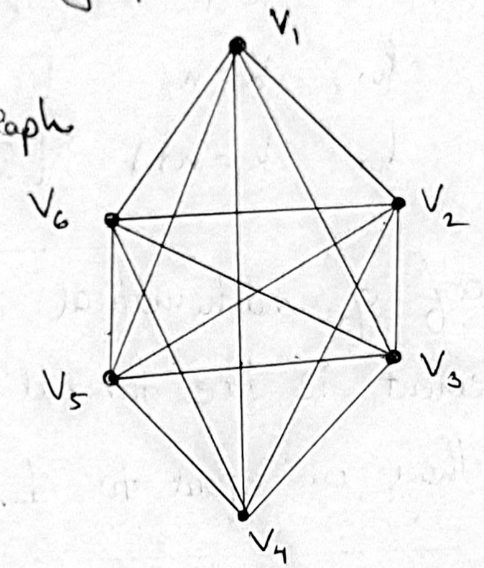
Q 3. B (ii)
cont.d.

K6 graph:

Complete graph (K_6)



for K_6 graph, number of vertices = 6.
degree of a vertex = 6-1 = 5.

$\therefore$ no. of edges $= \frac{6\times(6-1)}{2} = 3\times5 = 15.$

for $K_{3,3}$ graph, number of vertices = 6.
degree of a vertex = 3.

$\therefore$ no. of edges $= \frac{6\times3}{2} = 9.$

The no. of edges in K_6 and $K_{3,3}$ graphs are not equal.

Hence, they aren't isomorphic to each other.

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Q4. A. (ii)

→ Let A, B and C be the set of numbers between 1 and 500 that are divisible by 3, 5, and 7, respectively.

$n(A) = [500/3] = 166$

$n(B) = [500/5] = 100$

$n(C) = [500/7] = 71.$

Simply adding up all these numbers would not help us form our solution.

Note that numbers which are multiples of LCM of 3 and 5 are counted twice. So are multiples of 3 and 7, and 5 and 7.

Multiples of L.C.M of 3, 5 and 7 are counted thrice.

To make sure that each number is counted exactly once, The cardinality of union of sets A, B and C are calculated.

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Q4. A (ii)

contd. $n(A \cap B) = [500/15] = 33$

$n(A \cap C) = [500/21] = 23$

$n(B \cap C) = [500/35] = 14.$

$n(A \cap B \cap C) = [500/105] = 4.$

we know,

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - [n(A \cap B) + n(A \cap C) + n(B \cap C)] + n(A \cap B \cap C)$$

$$\therefore n(A \cup B \cup C) = 166 + 100 + 71 - [33 + 14 + 23] + 4$$

$$\boxed{n(A \cup B \cup C) = 271}$$

∴ There are 271 numbers between 1 and 500 that are divisible by either 3, 5 or 7.

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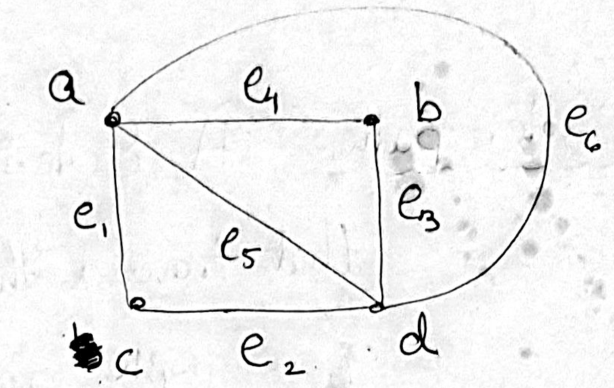
~~Q.14..~~

Q4.A.(iii)
cont.d

Graph G_{12}:



There exists an eulerian circuit in graph G_{12}.
It starts at vertex a, transverses each edge exactly once and ends at the same vertex 'a' at which it started.

Euler circuit: $a\,e_1\,c\,e_2\,d\,e_5\,a\,e_4\,b\,e_3\,d\,e_6\,a$.

The circuit started and ended at a.

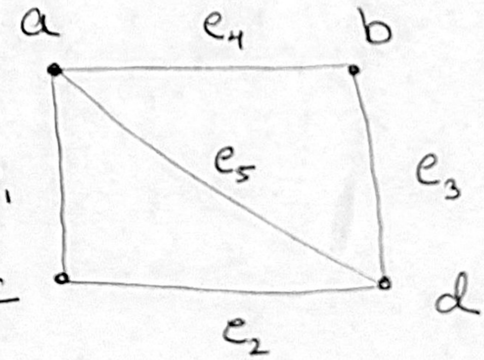
Q 4. A. (iii)

→ Euler path: A Eulerian path is a path that traverses through every edge of a graph exactly once.
It ~~strar~~ starts from one vertex but does not end on the same vertex.

Euler circuit: A Eulerian circuit is a circuit that traverses through every edge of a graph exactly once. It starts from a vertex and ends at the same vertex.

For example:

Graph G_1 :



In graph G_1, the euler path starts at a transverses through every edge exactly once and ends at vertex d.

Euler path : $a e_1 c e_2 d e_3 b e_4 a e_5 d$.

Q4. B.

(ii)

→ $A = \{1, 2, 3\}$

~~for~~ $R = \{(1,2), (2,3), (1,3)\}$

since $(1,2)$ & $(2,3)$ belong to R,
also $(1,3)$ belongs to R,
hence, R is transitive.
But, $(1,2) \in R$ and $(2,1) \notin R$
$\therefore$ R is not ~~reflexive~~ symmetric.

i] $\therefore$ $R = \{(1,2), (2,3), (3,1)\}$ is transitive but not symmetric on A.

Now, $R = \{(1,2), (2,1), (2,3), (3,2)\}$
since for $(1,2)$ & $(2,3)$, $(2,1)$ & $(3,2) \in R$
R is symmetric.
but, R is not transitive as $(1,2), (2,3) \in R$
but $(1,3) \notin R$.

ii] $\therefore$ $R = \{(1,2), (2,3), (2,1), (3,2)\}$ is symmetric but not transitive on A.

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Q4. B (ii)
Contd

for $R = \{(1,1), (2,2), (3,3)\}$

R is anti-symmetric since, $(1,1), (2,2)$ & $(3,3)$ $\in R$ and $\forall\, xRy,\ x = y$.

R is also symmetric as $(1,1), (2,2)$ & $(3,3)$ $\in R$ and thier symmetries are the same

$\boxed{\text{iii] } \therefore R = \{(1,1), (2,2), (3,3)\} \text{ is both symmetric and antisymmetric on A}}$

for $R = \{(1,2)\}$.

R is not anti symmetric since, $(1,2) \in R$ and $\forall\, xRy,\ x \neq y$.

R is also not symmetric since $(1,2) \in R$ but $(2,1) \notin R$.

$\boxed{\text{iv] } \therefore R = \{1,2\} \text{ is neither symmetric nor antisymmetric on A.}}$

Now, for $R = \{(1,2), (2,3), (1,3), (1,1), (2,2), (3,3), (2,1), (2,3), (1,3)\}$

R is transitive since $(1,2), (2,3) \in R$ & $(1,3) \in R$.

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Q 4. B (ii)

Contd.

R is also symmetric since
$(1,2), (2,3), (1,3) \in R$ and also
$(2,1), (3,2), (1,3) \in R$.

R is also reflexive since
$(1,1), (2,2), (3,3) \in R$.

$\therefore$ R is reflexive, symmetric & transitive.

v] Thus, $R = \{(1,2), (2,3), (1,3),$
$(2,1), (3,2), (1,3),$
$(1,1), (2,2), (3,3)\}$

is an equivalence relation on A.