SEAT No. A13A047.

NAME: YASH SARANG.

ROLL No. 47.

SEMESTER: III.

SUBJECT:

EM-III

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Jash Sarang. Sarangyash

Q1.

① Laplace transform of $e^{-5t}(t^2 + \sin 2t)$ is

ⓐ Option A: $\frac{2}{(s+5)^2} + \frac{2}{(s+5)^2 + 2^2}$

② If $L\{F(t)\} = \frac{3s}{s^2+1}$, then $L\{F(2t)\}$ at $s=1$, is

ⓐ Option A: $3/5$

③ Inverse Laplace transform of $\frac{1}{s^2+4}$ is

ⓐ Option A: $\int_0^t \cos 2u \, du$

④ Inverse Laplace transform of $f(s) = \frac{6e^{-5s}}{(s+2)^4}$ is

ⓒ Option C: $f(t) = \begin{cases} 0 & 0 < t < 5 \\ e^{-2t} t^5 & t > 5. \end{cases}$

⑤ If $f(z) = u(x,y) + iv(x,y)$ is analytic then $f'(z)$ is equal to

ⓑ Option B: $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

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Q1.

(6) The value of 'm' so that $2x - x^2 + my^2$ is ~~two~~ harmonic, is

(c) Option C: 1

(7) The value of coefficient of correlation lies between

(d) Option D: -1 to 1

(8) The rank correlation of the following data is

(c) Option C: 1

(9) Expansion of Fourier series of $f(x) = x$ in $(-1, 1)$ is

(d) Option D: $f(x) = \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \sin n\pi x$.

(10) What would be the expectation of the number of failures........ failure q.

(b) Option B: q/p

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Q2. (F)

→ Let, 'A' be an event : item produced by machine A.

'B' be the event : item produced by machine B.

'C' be the event : item produced by machine C.

'D' be the event : item produced is defective.

Then, $P(A) = \frac{1}{2}$, $P(B) = P(C) = \frac{1}{4}$.

Now,

$$P(D/A) = 0.03 = P(D/B)$$

& $$P(D/C) = 0.05.$$

∴ By theorem of total probability,

$$P(D) = P(A).P(D/A) + P(B).P(D/B) + P(C).P(D/C)$$

$$= 0.03 \times \frac{1}{2} + 0.03 \times \frac{1}{4} + 0.05 \times \frac{1}{4}$$

$$= 0.015 + \cancel{0.015}\ 0.0075 + 0.0125.$$

$$= 0.015 + 0.02$$

$$\boxed{\therefore P(D) = 0.035}$$

∴ The probability for the chosen bolt to be defective is 0.035 i.e. 35/1000.

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Q2. (c)

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2}(\pi - x)\,dx$$

$$= \frac{1}{4\pi}\left(\pi x - \frac{x^2}{2}\right)_0^{2\pi} = \frac{1}{4\pi}\left(2\pi^2 - 2\pi^2\right)$$

$$\boxed{\therefore a_0 = 0}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} \frac{1}{2}(\pi - x).\cos x\,dx$$

$$= \frac{1}{2\pi}\int_0^{2\pi} (\pi\cos nx - x\cos nx).\,dx$$

$$= \frac{1}{2\pi}\left\{\pi\left[\frac{\sin nx}{n}\right]_0^{2\pi} - \left[x\left(\frac{\sin nx}{n}\right) - \left(\frac{-\cos nx}{n^2}\right)\right]_0^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{[0] - \left[0 + \left(\frac{1}{n^2} - \frac{1}{n^2}\right)\right]\right\}$$

$$\boxed{\therefore a_n = 0}$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} \frac{1}{2}(\pi - x)\sin nx.\,dx = \frac{1}{2\pi}\int (\pi\sin nx - x\sin nx)\,dx$$

$$= \frac{1}{2\pi}\left\{\pi\left(\frac{-\cos nx}{n}\right)_0^{2\pi} - \left[x\left(\frac{-\cos nx}{n}\right) - \left(\frac{-\sin nx}{n^2}\right)\right]_0^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{-\pi}{n}[1-1] - \left[\left(\frac{-2\pi}{n} - 0\right) - 0\right]\right\} = \frac{1}{n}$$

$$\boxed{\therefore b_n = 1/n}$$

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Q 2. (c)

$$\therefore f(x) = \sum_{1}^{\infty} \frac{1}{n} \sin nx$$

$$= \frac{1}{1} \sin x + \frac{1}{2} \sin 2x + \ldots\ldots$$

put $x = \pi/2$.

$$\therefore \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$$

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Q2. (D)

→ $f(z) = (ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2) + i(4x^3y - exy^3 + 4xy)$. [Given]

we have,
$$f(z) = u + iv$$
where
$$u = ax^4 + bx^2y^2 + cy^4 + dx^2 - 2y^2.$$
$$v = 4x^3y - exy^3 + 4xy.$$

$$\therefore \frac{du}{dx} = u_x = 4ax^3 + 2bxy^2 + 2dx$$

$$\frac{du}{dy} = u_y = 2bx^2y + 4cy^3 - 4y.$$

Similarly
$$\frac{dv}{dx} = v_x = 12x^2y - ey^3 + 4y.$$

$$\frac{dv}{dy} = v_y = 4x^3 - 3exy^2 + 4x.$$

Since $f(z)$ is analytic, (Given)
Cauchy - Riemann's equations ~~a~~ would satisfy.

i.e $u_x = v_y$ & $u_y = -v_x$.

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Q2. D

$\therefore \quad 4ax^3 + 2bxy^2 + 2dx = 4x^3 - 3exy^2 + 4x.$

by comparing the coefficients we get

$$a = 1, \; d = 2, \quad b = -3e/2.$$

Similarly, from

$$2bx^2y + 4cy^3 - 4y = -12x^2y + ey^3 - 4y$$

we get $\quad b = -6, \quad c = e/4,$

$b = -3e/2 = -6 \quad , \quad \therefore \; e = 4. = 4c$

$\therefore \; c = 1.$

$\therefore \; a = 1, \; b = -6, \; c = 1, \; d = 2, \; e = 4$ are the values of the constants.

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Q2 (A)

→ $f(t) = \sqrt{1-\sin 2t}$

$= \left(\sin^2 t + \cos^2 t - 2\sin t \cos t\right)^{1/2}$

$= [\cos t - \sin t]$

$2\sqrt{t}\,\sin 2t = L\{\cos t - \sin t\}$

$= \frac{s}{s^2+1} - \frac{1}{s^2+1}$

$= \frac{s-1}{s^2+1}$

$L\{t\sqrt{1-\sin 2t}\} = \frac{-d}{ds}\, \frac{s-1}{s^2+1}$

$= -\left[\frac{(s^2+1) - (s-1)\,2s}{(s^2+1)^2}\right]$

$= \left[\frac{s^2+1 - 2s^2 + 2s}{(s^2+1)^2}\right]$

$= -\left[\frac{-s^2 + 2s + 1}{(s^2+1)^2}\right]$

$\therefore L\{t\sqrt{1-\sin 2t}\} = \frac{s^2-2s-1}{(s^2+1)^2}$

Put $s = 3$, $= 9-6-1 / 10^2 = 2/100 = 1/50$

$= 1/50.$

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Q. No. 10/22

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Q3. → (F)

Given: for a r.v x, the p.d.f is

x	-2	-1	0	1	2	3
P	0.1	K	0.1	2K	0.2	3K.

Solution:

We know, for a p.d.f

$\sum P_i = 1.$

$\therefore 0.1 + K + 0.1 + 2K + 0.2 + 3K = 1$

$\therefore 6K = 1 - 0.4$

$\boxed{\therefore K = 0.1}$

for mean, Mean $E(x) = \frac{\sum_{i=1}^{n} P_i x_i}{n}$

$= \frac{0.1\times(-2) + 0.1\times(-1) + 0.2\times1 + 0.2\times2 + 0.3\times3}{6}$

$= \frac{-0.2 - 0.1 + 0.2 + 0.4 + 0.9}{6}$

$= \frac{1.2}{6} = 0.2$

$\boxed{\therefore \text{Mean } E(x) = 0.2}$

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$$E(x^2) = \frac{\sum_{i=1}^{n} P_i (x_i)^2}{n}$$

$$= \frac{0.1 \times (-2)^2 + 0.1(-1)^2 + 0.1(0)^2 \; 0.2 \times (1)^2 + 0.2(2)^2 + 0.3(3)^2}{6}$$

$$= \frac{0.4 + 0.1 + 0 + 0.2 + 0.8 + 2.7}{6}$$

$$= \frac{4.2}{6} = 0.7$$

$\therefore E(x^2) = 0.7$

for Variance $(\sigma^2) = E(x^2) - [E(x)]^2$

$= 0.7 - (0.2)^2$

$= 0.7 - 0.04$

$= 0.66$

$\boxed{\therefore \text{Variance } (\sigma^2) = 0.66}$

i) $K = 0.1$

ii) Mean $E(x) = 0.2$

Variance $(\sigma^2) = 0.66$

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Q 3. (A)

→ Independently, we have:

$L\{\sin 2t\} = \frac{2}{s^2+4}$ & $L\{\sin 3t\} = \frac{3}{s^2+9}$.

$\therefore L\{\sin 2t + \sin 3t\} = \int_0^\infty \left[\frac{2}{s^2+4} + \frac{3}{s^2+9}\right] . ds$

$= \left(\tan^{-1}(s/2)\right]_0^\infty + \left(\tan^{-1}(s/3)\right]_0^\infty$

$= \pi - \left[\tan^{-1} s/2 + \tan^{-1} s/3\right]$.

Let $\tan^{-1} s/2 = \alpha$, $\tan^{-1} s/3 = \beta$. $\therefore s/2 = \tan\alpha$

$s/3 = \tan\beta$.

$\tan(\alpha+\beta) = s/2 + s/3 \Big/ (1 - s^2/6)$

$\therefore \alpha+\beta = \tan^{-1}\left[\frac{s/2+s/3}{1-s^2/6}\right]$

$\therefore L\left\{\frac{\sin 2t + \sin 3t}{t}\right\} = \pi - \tan^{-1}\left(\frac{5s}{6-s^2}\right)$

Now put $s=1$, $\therefore \int_0^\infty e^{-1} \frac{(\sin 2t + \sin 3t)}{t} = \pi - \tan^{-1}(5/5)$

$= \pi - \pi/4$

$= 3\pi/4$.

$$\boxed{\therefore \int_0^\infty \frac{\sin 2t + \sin 3t}{t . e^t} = \frac{3\pi}{4}}$$

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Q3. (C)

→ We have $f(-x) = 1-(-x)^2$
$= 1 - x^2 = f(x)$

Hence, $f(x)$ is even.

Comparing $(-l, l)$ with $(-1, 1)$ we get $l = 1$.

Now, $a_0 = \frac{1}{l}\int_0^l f(x).dx$

$= \int_0^1 (1-x^2).dx = \left[x - \frac{x^3}{3}\right]_0^1 = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore a_0 = 2/3$

$a_n = \frac{2}{l}\int_0^l f(x).\cos\frac{n\pi x}{l}.dx$

$= 2\int_0^1 (1-x^2).\cos n\pi x.dx$

$= 2\left[(1-x^2)\left(\frac{\sin n\pi x}{n\pi}\right) - (-2x)\left(\frac{-\cos n\pi x}{n^2\pi^2}\right) + (-2)\left(-\frac{\sin n\pi x}{n^3\pi^3}\right)\right]_0^1$

$\therefore a_n = 2\left[-\frac{2\cos n\pi}{n^2\pi^2}\right] = \frac{-4(-1)^n}{n^2\pi^2}$

$f(x) = a_0 + a_n$

$$\boxed{\therefore f(x) = \frac{2}{3} - \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos n\pi x}$$

Q3. (E)

x	y	xy	x^2
65	67	4355	4225
66	68	4488	4356
67	65	4355	4489
68	72	4556	4624
69	72	4896	4761
70	69	4830	4900
72	71	5112	5184
67	68	4556	4489

$\Sigma x = 544$

$\Sigma y = 552$

$\Sigma xy = 37560$

$\Sigma x^2 = 37028$

$\Sigma y^2 = 381.32$

$\mathcal{E}(x) = \frac{544}{8} = 68$

$\mathcal{E}(y) = \frac{552}{8} = 69.$

$$b_{yx} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2} = \frac{192}{288} = \frac{2}{3}.$$

$\therefore$ Regression line of y on x is

$$y - l_y = b_{yx}(x - l_x) \quad \text{i.e} \quad y - 69 = \frac{2}{3}(x - 68)$$

$\boxed{\therefore 2x - 3y + 71 = 0 \text{ is the regression line of } y \text{ on } x.}$

$$b_{xy} = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma y^2 - (\Sigma y)^2} = \frac{192}{352} = \frac{6}{11}.$$

$\therefore$ Regression line of x on y is.

$$x - l_x = b_{xy}(y - l_y) \quad \text{i.e} \quad x - 68 = \frac{6}{11}(y - 69)$$

$\boxed{\therefore x = 0.545y + 30.36}$ is the line of regression of x on y.

~~using the regression line of x on y~~

~~$x = 0.545y$~~

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Q4.

(A)

→

$$L\{\cos^2 t\} = L\left\{\frac{1}{2}(1+\cos 2t)\right\}$$

$$= \frac{1}{2} L\{1+\cos 2t\}$$

$$= \frac{1}{2}\left\{\frac{1}{s} + \frac{s}{s^2+4}\right\}$$

$$= F(s)$$

By F.S.T,

$$L\{e^{-at} F(t)\} = F(s+a)$$

$$\therefore L\{e^{-2t}\cos^2 t\} = F(s+2)$$

$$= \frac{1}{2}\left\{\frac{1}{s+2} + \frac{s+2}{(s+2)^2+4}\right\}$$

$$= F(s)$$

By Laplace Transform of integral

$$L\left\{\int_0^t f(u)\,du\right\} = \frac{1}{s} F(s) = \frac{1}{2s}\left[\frac{1}{s+2} + \frac{s+2}{(s+2)^2+4}\right]$$

$$= \frac{1}{2s}\left[\frac{(s+2)^2+4+(s+2)^2}{(s+2)\left[(s+2)^2+4\right]}\right]$$

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Q4. A

$$= \frac{1}{2s}\left[\frac{2(s+2)^2+4}{(s+2)\left[(s+2)^2+4\right]}\right]$$

$$\therefore L\left\{\int_0^t e^{-2t}\cos^2 t\right\} = \frac{1}{2s}\left[\frac{2(s+2)^2+4}{(s+2)\left[(s+2)^2+4\right]}\right]$$

Q4. E →

x	y	$x = x-\bar{x}$	x^2	xy
1	8	-4.667	21.78	-37.35
3	12	-2.667	7.11	-32.00
5	15	-0.667	0.44	-10
7	17	1.333	1.77	22.66
8	18	2.333	5.43	41.99
10	20	4.333	18.75	86.66
$\Sigma x = 34$	$\Sigma y = 90$	$\Sigma x = 0$	$\Sigma x^2 = 55.28$	$\Sigma xy = 71.96$

$n = 6$

$\bar{x} = 34/6$

$\bar{x} = 5.667.$

for the straight line $y = a + bx$

Normal eqn: $\cancel{\ }\ + b\Sigma x = \Sigma y$ — ①

$a\Sigma x + b\Sigma x^2 = \Sigma xy$ — ②

from ①,

$34a + 0 \times b = 90. \quad \therefore a = 90/34$

$\boxed{a = 2.647}$

from ②, $0 \times a + 55.28b = 71.96 \quad \therefore b = 71.96/55.28$

$\boxed{b = 1.302,}$

$\boxed{\therefore y = 2.647 + 1.302x \text{ is best fit straight line for our data}}$

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Q4. (E)

for the straight line $y = a + bx$

Eqns for the Normal :

$$na + b\sum x = \sum y \quad \text{——} \ (1)$$

&

$$\sum x\, a + \sum x^2\, b = \sum xy \quad \text{——} \ (2)$$

from (1),

$$6a + b \times 0 = 90$$

$$\boxed{\therefore a = 15}$$

from (2),

$$0 \times a + 55.28 \times b = 71.96$$

$$b = \frac{71.96}{55.28}$$

$$\boxed{\therefore b = 1.302}$$

∴ The best fit line for our data $y = a + bx$ will be

$$\boxed{y = 15 + 1.302x.}$$

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Q4. F

$$\therefore E(x^2) = \frac{1}{2}\int_0^\infty x^4 e^{-x}\, dx$$

$$= \frac{1}{2}\left(\frac{4!}{1^{4+1}}\right) = \frac{4\times3\times\cancel{2}}{\cancel{2}}$$

$$\therefore E(x^2) = 12$$

We know,

$$\text{Variance } (\sigma^2) = E(x^2) - [E(x)]^2$$

$$= 12 - 3^2$$

$$= 12 - 9$$

$$= 3$$

$$\boxed{\therefore \text{Variance } (\sigma^2) = 3}$$

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Q4. → (F)

Given:

The probability function is

$$f(x) = kx^2 e^{-x} \quad \text{where } x>0, k>0.$$

Solution:

Since $f(x)$ is a p.d.f.

$$\int_{-\infty}^{\infty} f(x).dx = 1.$$

$$\therefore \int_{-\infty}^{0} f(x).dx + \int_{0}^{\infty} f(x).dx = 1.$$

$$\therefore 0 + \int_{0}^{\infty} kx^2.e^{-x} = 1$$

$$\therefore k \int_{0}^{\infty} x^2.e^{-x} = 1.$$

Using $\displaystyle\int_{0}^{\infty} x^n.e^{-ax}\,dx = \frac{n!}{a^{n+1}}$,

we get

$$k\left(\frac{2!}{1^{2+1}}\right) = 1$$

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Q 4. (F)

$$\therefore k = \frac{1}{2!}$$

$$\boxed{\therefore k = \frac{1}{2}}$$

for Mean,

$$E(x) = \int_{-\infty}^{\infty} x . f(x) . dx$$

$$= \frac{1}{2} \int_{0}^{\infty} x(x^2 . e^{-x}) dx .$$

$$= \frac{1}{2} \int_{0}^{\infty} x^3 e^{-x} . dx$$

$$= \frac{1}{2} \left(\frac{3!}{1^{3+1}} \right) = \frac{3 \times \cancel{2}}{\cancel{2}}$$

$$\boxed{\therefore \text{Mean } E(x) = 3}$$

Now, for Variance

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) . dx .$$

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Q4. E

→

x	y	xy	x^2
1	8	8	1
3	12	36	9
5	15	75	25
7	17	119	49
8	18	144	64
10	20	200	100
34	90	582	248
↓	↓	↓	↓
Σx	Σy	Σxy	Σx^2

Equation of the req straight line is $y = a + bx$

to find a & b, $\Sigma y = na + b\Sigma x$.

$90 = 6a + 34b$ —— ①

$\Sigma xy = a\Sigma x + b\Sigma x^2$, $582 = 34a + 248b$ —— ②

Solving ① & ②, we get $a = 7.63$ and $b = 1.30$.

$\therefore y = 7.63 + 1.30x$ is the best fit straight line eqⁿ for our data.

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Q4. D.

→ Given eqn $u = x^3y - xy^3$.

$\therefore \frac{du}{dx} = u_x = 3x^2y - y^3$ & $\frac{du}{dy} = u_y = x^3 - 3xy^2$.

$\therefore f'(z) = u_x + iv_x = u_x - iu_y$ [By Cauchy Riemann's equation]

$= (3x^2y - y^3) - i(x^3 - 3xy^2)$

By Milne-Thompson's method, we put $x = z, y = 0$.

$\therefore f'(z) = -iz^3$.

$\therefore f(z) = -\int iz^3 . dz = -i\frac{z^4}{4} + c$.

$= -\frac{i}{4}(x+iy)^4 + c$.

$= \frac{-i}{4}(x^4 + 4x^3yi - 6x^2y^2 - 4xy^3i + y^4) + c$

$\therefore$ Imaginary part $v = \frac{-1}{4}(x^4 - 6x^2y^2 + y^4)$

Hence, the required orthogonal trajectories are

$$\boxed{x^4 - 6x^2y^2 + y^4 = c'.}$$