47_D6AD_YASH SARANG_EM4_ASSIGNMENT 4

Q2 →

$$f(k) = 3^{|k+1|} = \begin{cases} 3^{-(k+1)} & k+1 < 0, \; k \le -1 \\ 3^{k+1} & k+1 \ge 0, \; k \ge -1. \end{cases}$$

$$Z\{f(k)\} = \sum_{k=-\infty}^{\infty} f(k).\, z^{-k}$$

$$= \sum_{k=-\infty}^{-1} 3^{-(k+1)} .\, z^{-k} + \sum_{k=0}^{\infty} 3^{k+1} .\, z^{-k}$$

$$= \sum_{k=1}^{\infty} 3^{k-1} .\, z^{k} + \sum_{k=0}^{\infty} 3 \times \left(\frac{3}{z}\right)^{k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{3}{z}\right)^{k} \times \frac{1}{3^2} + 3 \sum_{k=0}^{\infty} \left(\frac{3}{z}\right)^{k}$$

$$= \frac{1}{9} \times \frac{1}{1-3z} + 3 \times \frac{1}{1-3/z} = \frac{1/9}{1-3z} + \frac{3z}{z-3}.$$

for $|3z| < 1$ ie $|z| < 1/3$ & $\left|\frac{3}{z}\right| < 1$ ie $|z| > 3$.

$$= \sum_{k=1}^{\infty} 3^{k-1} .\, z^{k} + \sum_{k=0}^{\infty} \left(\frac{3}{z}\right)^{k} \times 3.$$

$$= \sum_{k=0}^{\infty} (3z)^{k} \times \frac{1}{9} + 3 \, \frac{1}{1-3/z}$$

$$= \frac{1}{9} \times \frac{1}{1-3z} + \frac{3z}{z-3}.$$

for $|3z| < 1$, $\therefore |z| < 1/3$ & $|3/z| < 1$ & $|z| > 3$.

$\therefore$ for $|z| < 1/3$ & $|z| > 3$,

$$Z\{f(k)\} = \frac{1/9}{1-3z} + \frac{3z}{z-3}.$$

Q7 →

$$f(k) = \sin(3k+5), \quad k \geq 0.$$

$$\sin\theta = \frac{e^{i\theta} - e^{-i\theta}}{2i}, \quad \sin(3k+5) = \frac{e^{i(3k+5)} - e^{-i(3k+5)}}{2i}$$

$$Z\{f(k)\} = \sum_{k=-\infty}^{\infty} \left\{ \frac{e^{i3k+i5}}{2i} - \frac{e^{-i3k-i5}}{2i} \right\} \cdot z^{-k}$$

$$= \sum_{k=0}^{\infty} \left(\frac{e^{i3}}{z}\right)^{k} \times \frac{e^{i5}}{2i} - \sum_{k=0}^{\infty} \left(e^{i3} \cdot z\right)^{-k} \times \frac{1}{2i e^{i5}}$$

$$= \frac{e^{i5}}{2i} \times \frac{1}{1 - e^{i3}/z} - \frac{e^{-i5}}{2i} \times \frac{1}{1 - 1/e^{i3} z}$$

$$= \frac{e^{5i}}{2i} \times \frac{z}{z - e^{i3}} - \frac{e^{-5i}}{2i} \times \frac{z \cdot e^{3i}}{z \cdot e^{3i} - 1}$$

for $\left|\frac{e^{3i}}{z}\right| < 1$ ie $|z| > |e^{3i}|$ & $|z \cdot e^{3i}| < 1$

i.e $|z| < |1/e^{3i}|$

$\therefore$ for $|z| > 1$ & $|z| < 1$.

$$Z\{f(k)\} = \left[\frac{e^{5i} \cdot z}{z - e^{3i}} - \frac{z \cdot e^{3/5 i}}{z \cdot e^{3i} - 1} \right] \frac{1}{2i}.$$

Q9) →

$$f(k) = \frac{k}{k-1}, \quad k \geq 1.$$

$$Z\{f(k)\} = Z\left\{\frac{Z\{k-1\}}{k-1}\right\}$$

$$Z\{k-1\} = \cancel{\sum_{k=1}^{\infty}\left(\frac{k-1}{z}\right)^{k}} = \cancel{\sum_{k=0}^{\infty}\left(\frac{k}{z}\right)^{k+1}} = \cancel{\frac{k}{z} \times \frac{1}{1-k/z}}$$

$$= \cancel{\frac{k}{z} \times \frac{z}{z-k}} = \cancel{\frac{k}{z-k}} = \sum_{k=1}^{\infty}(k-1) \times z^{-k}$$

$$= \sum_{k=0}^{\infty} k\left(\frac{\cancel{k}}{z}\right)^{k+1} = 0 + \sum_{k=1}^{\infty}\left(\frac{\cancel{k}}{z}\right)^{k+1} k$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{z}\right)^{k+2} \times (k+1).$$

$$= \frac{1}{z^2} \times \left[\frac{1}{(1-z)^2}\right]$$

$$= \frac{1}{z^2(1-z)^2}$$

$$Z\left\{\frac{Z\{k-1\}}{k-1}\right\} = -\int \frac{1}{z} \times \frac{1}{z^2(1-z)^2} \, dz.$$

$$=$$

4/6

(Q11) $F(z) = \frac{z+1}{(z-2)^2}$, i) $|z|>2$, ii) $|z|<2$.

i) for $|z|>2$

$$F(z) = \frac{z+1}{z^2} \times \frac{1}{(1-2/z)^2} = \frac{z+1}{z^2} \sum_{k=0}^{\infty} (k+1) \times \left(\frac{2}{z}\right)^k$$

$$= \sum_{k=0}^{\infty} (k+1) \times \frac{2^k}{z^{k+1}} + \sum_{k=0}^{\infty} (k+1) \times \frac{2^k}{z^{k+2}}$$

for 1^{st} part, co.efficient of $z^{-(k+1)} = (k+1)\,2^k \quad k \geq 0$

$z^{-(k)} = k \cdot 2^{k-1} \quad k \geq 1.$

2^{nd} part, coefficient of $z^{-(k+2)} = (k+1) \cdot 2^k \quad k \geq 0$

$z^{-k} = (k-1) \cdot 2^{k-2}, \quad k \geq 2.$

$\therefore Z^{-1}\{F(z)\} = k \cdot 2^{k-1} + (k-1) \cdot 2^{k-2}$ for $k \geq 2$.

ii) for $|z|<2$,

$$F(z) = \frac{z+1}{2^2} \times \frac{1}{(1-z/2)^2} = \frac{z+1}{4} \times \sum_{k=0}^{\infty} (k+1) \times \left(\frac{z}{2}\right)^k$$

$$= \sum_{k=0}^{\infty} \frac{(k+1)}{2^{k+2}} \times z^{k+1} + \sum_{k=0}^{\infty} \frac{(k+1)}{2^{k+2}} \times z^k$$

for 1^{st} part, coefficient of $z^{k+1} = \frac{k+1}{2^{k+2}}, \quad k \geq 0$

$z^k = \frac{k}{2^{k+1}}, \quad k \geq 1.$

$z^{-k} = -k \times 2^{k-1}, \quad k \leq 1.$

for 2^{nd} part, co.eff of $z^k = (k+1)/2^k \quad k \geq 0.$

$z^{-k} = (-k+1) \times 2^k \quad k \leq 0.$

$\therefore Z^{-1}\{F(z)\} = -k \cdot 2^{k-1} + (1-k)\,2^k$ for $k \leq 1$.

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16)

→ $F(z) = \frac{2z-3}{z^2-3z-4} = \frac{2z-3}{(z-4)(z+1)}$. i) $|z| < 1$, ii) $1 < |z| < 4$
iii) $|z| > 4$.

$F(z) = \frac{2z-3}{(z-4)(z+1)} = \frac{A}{z-4} + \frac{B}{(z+1)}$.

for $z=4$, $2z-3 = A(5)$ | for $z=-1$, $-2-3 = B(-5)$
$5 = A(5)$ | $-5 = B(-5)$
$A=1$ | $B=1$.

$F(z) = \frac{2z-3}{(z-4)(z+1)} = \frac{1}{(z-4)} + \frac{1}{(z+1)}$.

i) for $|z|<1$

$\frac{1}{z-4} = \frac{1}{-4} \times \frac{1}{1-z/4} = \frac{1}{-4} \sum_{k=0}^{\infty} \left(\frac{z}{4}\right)^k$.

coefficient of $z^k = -4^{-(k+1)}$, $k \geq 0$.
coefficient of $z^{-k} = -4^{k-1}$, $k \leq 0$. ——— ①
for $|z|<4$

$\frac{1}{z+1} = \sum_{k=0}^{\infty} (-1)^k z^k$

∴ coefficient of $z^k = (-1)^k$, $k \geq 0$.
coefficient of $z^{-k} = (-1)^{-k}$, $k \leq 0$. ——— ②.
for $|z|<1$

∴ for $|z|<1$, from ①, ②

$Z^{-1}\{f(\cancel{z})\} = -4^{k-1} + (-1)^{-k}$, $k \leq 0$.

ii) for $1 < |z| < 4$,

$$\frac{1}{z+1} = \frac{1}{z} \times \frac{1}{1+1/z} = \frac{1}{z} \times \sum_{k=0}^{\infty} (-1)^k \times \left(\frac{1}{z}\right)^k = \sum_{k=0}^{\infty} \frac{(-1)^k}{z^{k+1}}$$

co.efficient of $z^{-(k+1)} = (-1)^k \qquad k \geq 0$

$z^{-k} = (-1)^{k-1} \qquad k \geq 1$ —— ③

for $1 < |z|$

$\therefore$ for $1 < |z| < 4$,

from ①, ③

$$Z^{-1}\{f(z)\} = \begin{cases} (-1)^{k-1} & k \geq 1 \\ -4^{k-1}. & k \leq 0. \end{cases}$$

iii) for $|z| > 4$,

$$\frac{1}{z-4} = \frac{1}{z} \times \frac{1}{1-4/z} = \frac{1}{z} \times \sum_{k=0}^{\infty} \left(\frac{4}{z}\right)^k = \sum_{k=0}^{\infty} \frac{4^k}{z^{k+1}}$$

coefficient of $z^{-k-1} = 4^k, \quad k \geq 0.$

$z^{-k} = 4^{k-1}, \quad k \geq 1$ —— ④

for $|4/z| < 1$, $|z| > 4$.

$\therefore$ for $|z| > 4$, from ③ and ④

$$Z^{-1}\{F(z)\} = (-1)^{k-1} + 4^{k-1}, \text{ for } k \geq 1.$$

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