Linear Programming Problems (L.P.P)

* Simplex Method *

(1) **Maximize** : $Z = x_1 + 4x_2$
Subject to : $2x_1 + x_2 \leq 3$
$3x_1 + 5x_2 \leq 9$
$x_1 + 3x_2 \leq 5$
$x_1, x_2 \geq 0$ **(M.U. D2010)**

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Sol: The standard form of LPP is

Maximize $Z = x_1 + 4x_2 + 0S_1 + 0S_2 + 0S_3$

subject to $2x_1 + x_2 + S_1 = 3$

$3x_1 + 5x_2 + S_2 = 9$

$x_1 + 3x_2 + S_3 = 5$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

	$C_j \rightarrow$	1	4	0	0	0		
C_B	x_B	x_1	x_2	S_1	S_2	S_3	b	$\theta = \frac{b}{\text{key col.}}$
0	S_1	2	1	1	0	0	3	$3/1 = 3$
0	S_2	3	5	0	1	0	9	$9/5 = 1.8$
0	S_3	1	3	0	0	1	5	$5/3 = 1.66 \leftarrow$
	Z_j	0	0	0	0	0	0	
	$C_j - Z_j$	1	4 ↑	0	0	0		
0	S_1	$5/3$	0	1	0	$-1/3$	$4/3$	
0	S_2	$4/3$	0	0	1	$-5/3$	$2/3$	
4	x_2	$\frac{1}{3}$	1	0	0	$1/3$	$5/3$	
	Z_j	$4/3$	4	0	0	$4/3$	$20/3$	
	$C_j - Z_j$	$-1/3$	0	0	0	$-4/3$		

$R_3(N) = \frac{R_3(0)}{3}$

$R_1(N) = R_1(0) - 1R_3(N)$

$R_2(N) = R_2(0) - 5R_3(N)$

$C_j - Z_j \leq 0 \ \forall j$ $\quad Z_{max} = \frac{20}{3}$ at $x_1 = 0, x_2 = \frac{5}{3}$

(2)

(2) **Maximize** : $Z = 3x_1 + 2x_2$
Subject to : $3x_1 + 2x_2 \leq 18$
$0 \leq x_1 \leq 4$
$0 \leq x_2 \leq 6$
$x_1, x_2 \geq 0$ **(M.U. 2001)**

The standard form of LPP is

Max. $Z = 3x_1 + 2x_2 + 0S_1 + 0S_2 + 0S_3$

subject to $3x_1 + 2x_2 + S_1 = 18$

$x_1 + S_2 = 4$

$x_2 + S_3 = 6$

$x_1, x_2, S_1, S_2, S_3 \geq 0$

	$C_j \rightarrow$	3	2	0	0	0		
C_B	x_B	x_1	x_2	S_1	S_2	S_3	b	$\theta = \frac{b}{\text{key col.}}$
0	S_1	3	2	1	0	0	18	$\frac{18}{3} = 6$
0	S_2	1	0	0	1	0	4	$\frac{4}{1} = 4 \leftarrow$
0	S_3	0	1	0	0	1	6	$\frac{6}{0} = \infty$

C_B	x_B	x_1	x_2	s_1	s_2	s_3	b	θ
0	s_1	3	2	1	0	0	18	$\frac{18}{3}=6$
0	s_2	1	0	0	1	0	4	$\frac{4}{1}=4 \leftarrow$
0	s_3	0	1	0	0	1	6	$\frac{6}{0}=\infty$
	z_j	0	0	0	0	0	0	
	c_j-z_j	3 ↑	2	0	0	0		
0	s_1	0	2	1	−3	0	6	$\frac{6}{2}=3 \leftarrow$
3	x_1	1	0	0	1	0	4	$\frac{4}{0}=\infty$
0	s_3	0	1	0	0	1	6	$\frac{6}{1}=6$
	z_j	3	0	0	3	0	12	
	c_j-z_j	0	2 ↑	0	−3	0		
2	x_2	0	1	$\frac{1}{2}$	$-\frac{3}{2}$	0	3	
3	x_1	1	0	0	1	0	4	
0	s_3	0	0	$-\frac{1}{2}$	$\frac{3}{2}$	1	3	
	z_j	3	2	1	0	0	18	
	c_j-z_j	0	0	−1	0	0		

$R_2(N) = \frac{R_2(O)}{1}$

$R_1(N) = R_1(O) - 3R_2(N)$

$R_3(N) = R_3(O) - 0R_2(N)$

$R_1(N) = \frac{R_1(O)}{2}$

$R_2(N) = R_2(O) - 0\,R_1(N)$

$R_3(N) = R_3(O) - 1\,R_1(N)$

$\therefore$ $c_j - z_j \le 0 \ \forall j$ $Z_{max} = 18$ at $x_1 = 4, x_2 = 3$

(3) **Maximize** : $Z = 3x_1 + 2x_2 + 5x_3$
Subject to : $x_1 + 2x_2 + x_3 \le 430$
$3x_1 + 2x_3 \le 460$
$x_1 + 4x_2 \le 420$
$x_1, x_2, x_3 \ge 0$ **(M.U. 96,2004,06,11)**

Sol: The standard form is Max. $Z = 3x_1 + 2x_2 + 5x_3 + 0s_1 + 0s_2 + 0s_3$
subject to
$x_1 + 2x_2 + x_3 + s_1 = 430$
$3x_1 + 2x_3 + s_2 = 460$
$x_1 + 4x_2 + s_3 = 420$
$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$

	$c_j \rightarrow$	3	2	5	0	0	0		
C_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3	b	$\theta = \frac{b}{\text{key col.}}$
0	s_1	1	2	1	1	0	0	430	$430/1 = 430$
0	s_2	3	0	2	0	1	0	460	$460/2 = 230 \leftarrow$
0	s_3	1	4	0	0	0	1	420	$420/0 = \infty$
	z_j	0	0	0	0	0	0	0	
	c_j-z_j	3	2	5 ↑	0	0	0		
0	s_1	$-1/2$	2	0	1	$-1/2$	0	200	$\frac{200}{2} = 100 \leftarrow$
5	x_3	$3/2$	0	1	0	$1/2$	0	230	$\frac{230}{0} = \infty$
0	s_3	1	4	0	0	0	1	420	$\frac{420}{4} = 105$
	z_j	$15/2$	0	5	0	$5/2$	0	1150	
	c_j-z_j	$-9/2$	2 ↑	0	0	$-5/2$	0		
2	x_2	$-1/4$	1	0	$1/2$	$-1/4$	0	100	
5	x_3	$3/2$	0	1	0	$1/2$	0	230	
0	s_3	2	0	0	−2	1	1	20	
	z_j	7	2	5	1	2	0	1350	
	c_j-z_j	−4	0	0	−1	−2	0		

$R_2(N) = \frac{R_2(O)}{2}$

$R_1(N) = R_1(O) - 1R_2(N)$

$R_3(N) = R_3(O) - 0\,R_2(N)$

$R_1(N) = \frac{R_1(O)}{2}$

$R_2(N) = R_2(O) - 0R_1(N)$

$R_3(N) = R_3(O) - 4R_1(N)$

$\therefore$ $c_j - z_j \le 0 \ \forall j$ $Z_{max} = 1350$ at $x_1 = 0, x_2 = 100$
$x_3 = 230$

Artificial Variable Technique / Big Method / Penalty Method / Charne's Method

Simplex	Big Method
max/min. Z	max./min. Z
Subject to	Subject to
$a_{11}x_1 + a_{12}x_2 \le b_1$	$a_{11}x_1 + a_{12}x_2 \geq b_1$
$\vdots$	$a_{21}x_1 + a_{22}x_2 = b_2$
$a_{n1}x_1 + a_{n2}x_2 \le b_2$	$a_{n1}x_1 + a_{n2}x_2 \le b_3$
$x_1, x_2, \geq 0$	$x_1, x_2 \geq 0$
① All $b_i \geq 0$	① All $b_i \geq 0$
② All constraints are '$\le$' type	② At least one constraint is of type '$\geq$' or '$=$'

Big M method

① If the constraint is of '$\geq$' type we substract S_1 and add artificial variable A_1. In the objective function, we assign big penalty for this artificial variable i.e. MA_1 is substracted from objective function

$$2x_1 + 3x_2 \geq 5$$

$$2x_1 + 3x_2 - S_1 + A_1 = 5$$

In objective function add $-MA_1$

② If constraint is of '$=$' type then just add artificial variable and substract MA_1 in objective function

eg. $$2x_1 + 3x_2 = 5$$

$$2x_1 + 3x_2 + A_1 = 5$$

In objective function substract MA_1

Using Penalty OR Big M Method Solve the followi

(1) **Maximize** : $Z = 3x_1 - x_2$
Subject to : $2x_1 + x_2 \geq 2$

$x_1 + 3x_2 \leq 3$

$x_2 \leq 4$

$x_1, x_2 \geq 0$ **(M.U. M2001,D2009)**

Sol: The standard form is

$$\text{Max. } Z = 3x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 - MA_1$$

subject to

$$2x_1 + x_2 - s_1 + A_1 = 2$$
$$x_1 + 3x_2 + s_2 = 3$$
$$x_2 + s_3 = 4$$
$$x_1, x_2, s_1, s_2, s_3, A_1 \geq 0$$

	$C_j \rightarrow$	3	-1	0	0	0	$-M$			
C_B	x_B	x_1	x_2	s_1	s_2	s_3	A_1	b	$\theta = \frac{b}{\text{key col.}}$	
$-M$	A_1	2	1	-1	0	0	1	2	$2/2 = 1 \leftarrow$	
0	s_2	1	3	0	1	0	0	3	$3/1 = 3$	
0	s_3	0	1	0	0	1	0	4	$4/0 = \infty$	
	Z_j	$-2M$	$-M$	M	0	0	$-M$			
	$C_j - Z_j$	$3+2M \uparrow$	$-1+M$	$-M$	0	0	0			
3	x_1	1	$1/2$	$-1/2$	0	0	$1/2$	1	$\frac{1}{-1/2} = -2$	$R_1(N) = \frac{R_1(0)}{2}$
0	s_2	0	$5/2$	$1/2$	1	0	$-1/2$	2	$\frac{2}{1/2} = 4 \leftarrow$	$R_2(N) = R_2(0) - 1R_1(N)$
0	s_3	0	1	0	0	1	0	4	$4/0 = \infty$	$R_3(N) = R_3(0) - 0R_1(N)$
	Z_j	3	$3/2$	$-3/2$	0	0	$3/2$	3		
	$C_j - Z_j$	0	$-5/2$	$3/2 \uparrow$	0	0	$-M-3/2$			
3	x_1	1	3	0	1	0	0	3		$R_2(N) = \frac{R_2(0)}{1/2}$
0	s_1	0	5	1	2	0	-1	4		$R_1(N) = R_1(0) + \frac{1}{2} R_2(N)$
0	s_3	0	1	0	0	1	0	4		$R_3(N) = R_3(0) + 0R_2(N)$
	Z_j	3	9	0	3	0	0	9		
	$C_j - Z_j$	0	-10	0	-3	0	$-M$			

$\because C_j - Z_j \leq 0 \ \forall j$, $Z_{max} = 9$ at $x_1 = 3, x_2 = 0$

(2) **Maximize** : $Z = 6x_1 + 4x_2$

Subject to : $2x_1 + 3x_2 \leq 30$

$3x_1 + 2x_2 \leq 24$

$x_1 + x_2 \geq 3$

$x_1, x_2 \geq 0$

(5) **Minimize** : $Z = 2x_1 + x_2 + 3x_3$

Subject to : $x_1 + x_2 + 2x_3 \leq 5$

$2x_1 + 3x_2 + 4x_3 = 12$

$x_1, x_2, x_3 \geq 0$ **(M.U. 2007)**

$x_1 + x_2 + 2x_3 + S_1 = 5$

$2x_1 + 3x_2 + 4x_3 + A_1 = 12$

$x_1, x_2, x_3, S_1, A_1 \geq 0$

* Dual simplex Method *

Simplex	Big Method	Dual simplex
① max.	① max.	① max.
② All constraints $\leq$	② Atleast one constraint '$\geq$' or '$=$'	② All constraint '$\leq$' type
③ All $b_i \geq 0$	③ All $b_i \geq 0$	③ Atleast one $b_i \leq 0$
④ only slack variables	④ slack, surplus & Artificial variables are used	④ only slack variables are used

① Use the dual simplex method to solve the foll. L.P.P.

Maximise $Z = -3x_1 - 2x_2$

subject to
$$x_1 + x_2 \geq 1$$
$$x_1 + x_2 \leq 7$$
$$x_1 + 2x_2 \leq 10$$
$$x_2 \leq 3 \quad , x_1, x_2 \geq 0$$

Sol:

Simplex / Big method	Dual simplex
key col.	key Row
Ratio	Ratio
key Row	key col.

Maximise $Z = -3x_1 - 2x_2$

subject
$$-x_1 - x_2 \leq -1$$
$$x_1 + x_2 \leq 7$$
$$x_1 + 2x_2 \leq 10$$
$$x_2 \leq 3 \quad , x_1, x_2 \geq 0$$

The std. form of LPP is

max. $Z = -3x_1 - 2x_2 + 0S_1 + 0S_2 + 0S_3 + 0S_4$

subject to
$$-x_1 - x_2 + S_1 = -1$$
$$x_1 + x_2 + S_2 = 7$$
$$x_1 + 2x_2 + S_3 = 10$$
$$x_2 + S_4 = 3$$
$$x_1, x_2, S_1, S_2, S_3, S_4 \geq 0$$

	$C_j \rightarrow$	-3	-2	0	0	0	0	
C_B	x_B	x_1	x_2	S_1	S_2	S_3	S_4	b

	$C_j \rightarrow$	-3	-2	0	0	0	0	
C_B	x_B	x_1	x_2	s_1	s_2	s_3	s_4	b
0	s_1	$\boxed{-1}$	$\boxed{-1}$	1	0	0	0	-1 ←
0	s_2	1	1	0	1	0	0	7
0	s_3	1	2	0	0	1	0	10
0	s_4	0	1	0	0	0	1	3
	z_j	0	0	0	0	0	0	
	$c_j - z_j$	-3	-2	0	0	0	0	
$\theta = \frac{c_j - z_j}{\text{Key Row}}$		$\frac{-3}{-1}=3$	$\frac{-2}{-1}=2$ ↑	$\frac{0}{1}=0$	$\frac{0}{0}$	$\frac{0}{0}$	$\frac{1}{0}$	
-2	x_2	1	1	-1	0	0	0	1
0	s_2	0	0	1	1	0	0	6
0	s_3	-1	0	2	0	1	0	8
0	s_4	-1	0	1	0	0	1	2
	z_j	-2	-2	2	0	0	0	-2
	$c_j - z_j$	-1	0	-2	0	0	0	

$R_1(N) = \frac{R_1(O)}{-1}$

$R_2(N) = R_2(O) - R_1(N)$

$R_3(N) = R_3(O) - 2R_1(N)$

$R_4(N) = R_4(N) - R_1(N)$

$\because \forall\, c_j - z_j \le 0$ & $\forall\, b_i \ge 0$ $\therefore Z_{max} = -2$ at $x_2 = 1, x_1 = 0$

② Use Dual simplex method to solve

Minimize $Z = 2x_1 + 2x_2 + 4x_3$

subject to

$$2x_1 + 3x_2 + 5x_3 \ge 2$$
$$3x_1 + x_2 + 7x_3 \le 3$$
$$x_1 + 4x_2 + 6x_3 \le 5$$
$$x_1, x_2, x_3 \ge 0$$

Sol: Maximize $Z' = -2x_1 - 2x_2 - 4x_3$

subject to

$$-2x_1 - 3x_2 - 5x_3 \le -2$$
$$3x_1 + x_2 + 7x_3 \le 3$$
$$x_1 + 4x_2 + 6x_3 \le 5$$
$$x_1, x_2, x_3 \ge 0$$

The standard form is

Max. $Z' = -2x_1 - 2x_2 - 4x_3 + 0s_1 + 0s_2 + 0s_3$

subject to

$$-2x_1 - 3x_2 - 5x_3 + s_1 = -2$$
$$3x_1 + x_2 + 7x_3 + s_2 = 3$$
$$x_1 + 4x_2 + 6x_3 + s_3 = 5$$
$$x_1, x_2, x_3, s_1, s_2, s_3 \ge 0$$

$C_j \rightarrow$ -2 $\quad -2$ $\quad -4$ $\quad 0$ $\quad 0$ $\quad 0$

$x_1, x_2, x_3, s_1, s_2, s_3 \geq 0$

	$c_j \to$	-2	-2	-4	0	0	0	
C_B	x_B	x_1	x_2	x_3	s_1	s_2	s_3	b
0	s_1	-2	-3	-5	1	0	0	-2
0	s_2	3	1	7	0	1	0	3
0	s_3	1	4	6	0	0	1	5
	z_j	0	0	0	0	0	0	0
	$c_j - z_j$	-2	-2	-4	0	0	0	
$\theta = \frac{c_j - z_j}{\text{keyrow}}$		$\frac{-2}{-2}=1$	$\frac{-2}{-3}=2/3$	$\frac{-4}{-5}=0.8$	$\frac{0}{1}$	$\frac{0}{0}$	$\frac{0}{0}$	
-2	x_2	$2/3$	1	$5/3$	$-1/3$	0	0	$2/3$
0	s_2	$7/3$	0	$16/3$	$1/3$	1	0	$7/3$
0	s_3	$-5/3$	0	$-2/3$	$4/3$	0	1	$7/3$
	z_j	$-4/3$	-2	$-10/3$	$2/3$	0	0	$-4/3$
	$c_j - z_j$	$-2/3$	0	$-2/3$	$-2/3$	0	0	

$R_1(N) = \frac{R_1(0)}{-3}$

$R_2(N) = R_2(0) - R_1(N)$

$R_3(N) = R_3(0) - 4R_1(N)$

$\because$ $c_j - z_j \leq 0 \ \forall j$ & $b_i \geq 0 \ \forall i$ $\quad Z'_{max} = -4/3$ at $x_1 = 0, x_2 = 2/3, x_3 = 0$

$\therefore$ $Z_{min} = \frac{4}{3}$ at $x_1 = 0, x_2 = \frac{2}{3}, x_3 = 0$

H.W

(3) Solve using Dual simplex method

Minimise $Z = 3x_1 + 2x_2 + x_3 + 4x_4$

subject to
$$2x_1 + 4x_2 + 5x_3 + x_4 \geq 10$$
$$3x_1 - x_2 + 7x_3 - 2x_4 \geq 2$$
$$5x_1 + 2x_2 + x_3 + 6x_4 \geq 15$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Duality

Obtain the Dual of following L.P.P.

(1) Max. $Z = 5x_1 + 2x_2$

Subject to

$$3x_1 + 4x_2 \leq 5$$
$$x_1 + x_2 \geq -7$$
$$x_1, x_2 \geq 0$$

Sol:

Max. $Z = 5x_1 + 2x_2$

Subject to

$$3x_1 + 4x_2 \leq 5$$
$$-x_1 - x_2 \leq 7$$
$$x_1, x_2 \geq 0$$

$$[5, \ 2]$$

$$\begin{bmatrix} 3 & 4 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \leq \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 3 & -1 \\ 4 & -1 \end{bmatrix}$$

The Dual is given

min. $W = 5y_1 + 7y_2$

Subject to

$$3y_1 - y_2 \geq 5$$
$$4y_1 - y_2 \geq 2$$
$$y_1, y_2 \geq 0$$

(2) Obtain the dual of foll. L.P.P

Max. $Z = 2x_1 - x_2 + 4x_3$

Subject to

$$x_1 + 2x_2 - x_3 \leq 5$$
$$2x_1 - x_2 + x_3 \leq 6$$
$$x_1 + x_2 + 3x_3 \leq 10$$
$$4x_1 + x_3 \leq 12$$
$$x_1, x_2, x_3 \geq 0$$

$$[2 \ -1 \ 4]$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ 1 & 1 & 3 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \leq \begin{bmatrix} 5 \\ 6 \\ 10 \\ 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 2 & -1 & 1 & 0 \\ -1 & 1 & 3 & 1 \end{bmatrix}$$

Sol:

Min. $W = 5y_1 + 6y_2 + 10y_3 + 12y_4$

Subject to

$$y_1 + 2y_2 + y_3 + 4y_4 \geq 2$$
$$2y_1 - y_2 + y_3 + 0y_4 \geq -1$$
$$-y_1 + y_2 + 3y_3 + y_4 \geq 4$$
$$y_1, y_2, y_3, y_4 \geq 0$$

(3) Construct the Dual of foll. LPP.

min $Z = 3x_1 + 2x_2$

subject to

$$2x_1 + 7x_2 = 5$$
$$-x_1 + x_2 \geq 8$$
$$x_1, x_2 \geq 0$$

$2x_1 + 7x_2 \leq 5 \Rightarrow -2x_1 - 7x_2 \geq -5$

$2x_1 + 7x_2 \geq 5$ ✓

Sol:

Min $Z = 3x_1 + 2x_2$

subject to

$$-2x_1 - 7x_2 \geq -5$$

$$[3 \ \ 2]$$

$$\begin{bmatrix} -2 & -7 \\ \end{bmatrix} \quad \begin{bmatrix} -5 \\ 5 \end{bmatrix}$$

Sol: Min $Z = 3x_1 + 2x_2$

subject to
$$-2x_1 - 7x_2 \geq -5$$
$$2x_1 + 7x_2 \geq 5$$
$$-x_1 + x_2 \geq 8$$
$$x_1, x_2 \geq 0$$

$$\begin{bmatrix} 3 & 2 \end{bmatrix} \quad \begin{bmatrix} -2 & -7 \\ 2 & 7 \\ -1 & 1 \end{bmatrix} \quad \begin{bmatrix} -5 \\ 5 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 2 & -1 \\ -7 & 7 & 1 \end{bmatrix}$$

Max $W = -5y_1 + 5y_2 + 8y_3$

subject to
$$-2y_1 + 2y_2 - y_3 \leq 3$$
$$-7y_1 + 7y_2 + y_3 \leq 2$$
$$y_1, y_2, y_3 \geq 0$$

Let $y_4 = y_1 - y_2$

Max $W = -5y_4 + 8y_3$

subject to
$$-2y_4 - y_3 \leq 3$$
$$-7y_4 + y_3 \leq 2$$
$y_3 \geq 0$, y_4 is unrestricted.

① Using Duality solve the foll. L.P.P.

OR

Using Principle of Duality solve the foll. LPP.

Min $Z = 2x_1 + 4x_2 + 3x_3$

subject to
$$-x_1 + x_2 + x_3 \geq 2$$
$$2x_1 + x_2 + 0x_3 \geq 1, \quad x_1, x_2, x_3 \geq 0$$

Sol: The Dual is given by

Max. $W = 2y_1 + y_2$

subject to
$$-y_1 + 2y_2 \leq 2$$
$$y_1 + y_2 \leq 4$$
$$y_1 + 0y_2 \leq 3$$
$$y_1, y_2 \geq 0$$

The standard form is Max. $W = 2y_1 + y_2 + 0s_1 + 0s_2 + 0s_3$

subject to
$$-y_1 + 2y_2 + s_1 = 2$$
$$y_1 + y_2 + s_2 = 4$$
$$y_1 + 0y_2 + s_3 = 3$$
$$y_1, y_2, s_1, s_2, s_3 \geq 0$$

	$C_j \rightarrow$	2	1	0	0	0		
C_B	y_B	y_1	y_2	s_1	s_2	s_3	b	$\theta = \frac{b}{\text{key col.}}$
0	s_1	-1	2	1	0	0	2	$\frac{2}{-1} = -2$
0	s_2	1	1	0	1	0	4	$\frac{4}{1} = 4$
0	s_3	1	0	0	0	1	3	$\frac{3}{1} = 3 \leftarrow$
	z_j	0	0	0	0	0		
	$c_j - z_j$	2 $\uparrow$	1	0	0	0		
0	s_1	0	2	1	0	1	5	$5/2 = 2.5$
0	s_2	0	1	0	1	-1	1	$1/1 = 1 \leftarrow$
2	y_1	1	0	0	0	1	3	$3/0 = \infty$

0	s_1	0	2	1	0	1	5	$5/2 = 2.5$
0	s_2	0	1	0	1	-1	1	$1/1 = 1$ ←
2	y_1	1	0	0	0	1	3	$3/0 = \infty$
	Z_j	2	0	0	0	2	6	
	$C_j - Z_j$	0	1 ↑	0	0	-2		
0	s_1	0	0	1	-2	3	3	
1	y_2	0	1	0	1	-1	1	
2	y_1	1	0	0	0	1	3	
	Z_j	2	1	0	1	1	7	$R_2(N) = R_2(O) - 2R_2(N)$
	$C_j - Z_j$	0	0	0	-1	-1		

$\therefore\ C_j - Z_j \le 0\ \forall j$ $\cancel{Z}$max $= 7$ at $y_1 = 3,\ y_2 = 1$

$Z_{min} = 7$ at $x_1 = 0,\ x_2 = 1,\ x_3 = 1$