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Q1) Compare raster scan and random scan display system.

| Random scan | Raster Scan |
|---|---|
| ① Random scan operate by directing the election be on to only those parts of screen where the picture is to be drawn. | ① The election beam starts at top left corner of the screen and moves horizontally to the right. |
| ② Requires less memory | ② Requires more memory |
| ③ Requires intelligent electron beam | ③ No such requirement. |
| ④ Pen plotters and direct storage view tubes (DVST) devices are used | ④ Cathode ray tubes (CRT) are used. |

Q2) Applications of CG.

① Computer art :- Using CG we can create fine and commercial art which involve animation packages, point packages

② Computer aided drawing :- Designing of building automobiles, aircrafts is done with the help of CAD.

③ Presentation Graphics :- For the presentations of reports ~~of~~ or summarising the financial, statistical mathematical, scientific, economic data for reasearch paper.

4. Image processing - various kinds of processing or images requires editing in order to be used in different place.

5. Graphical user interface -
the use of pictures, images, icons, popup menus, graphical edge offs helps in creating a user friendly environment where ~~too~~ working is easy and pleasant.

Qs. Derive & write Bresenham's line drawing algorithm.

1. Derivation - Consider P as height of pixel or error our condition is whether P≥0.5 or not if P≥0.5.

Converting in integer,

$2P-1>0$ , $P = P +$ slope or $P = P + slope - 1$

Here slope of py

To ~~elim~~ eliminate the tradition part.
(Multiplying both sides by Dx)

$2PDx - Dx > 0$

Now, let $G = Dx > 0$.
Test will be $G \geq 0$.

Now G can be used to decide which row or colum to select

$$G = 2PDx - Dx$$

$$G + Dx = 2PDx$$

$$P = \frac{G + Dx}{2Px}.$$

If $P = p + \text{slope}.$

then, $\frac{G+Dx}{2Dx} = \frac{G + Dx}{2Dx} + \frac{Dy}{Dx}$

$$G = G + 2Dy.$$

Similarly if $P = P + \text{slope} - 1$

$$G = 2Dx\left(\frac{Dy}{Dx}\right) - Dx.$$

$$G = 2Dy - Dx.$$

Algorithm:

1) Accept 2 endpoints from user.
2) Plot the point $(x_0\ y_0)$
3) Calculate all constants from endpoint such as $Dx$, $Dy$, $2Py$, $2Dy - 2Dx$ and $G$.
4) For each column increment $y$ and decide $y$ by checking $G \leq 0$ condition.
   if true, then $y+1$ and add $(2Dy - 2Dx)$ to current value of $G$.
5) Repeat step 4 till $Dx$ times for slope cases just interchange poles of $x$ and $y$.

Q4) Derive and write midpoint ellipsed drawing algorithm

① Derivation :-

Eqn of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

$r_y^2 x^2 + r^2_x y^2 - r^2_x r^2_y = 0.$

for region-1

Starting point $(0, r_y)$

slope $(0-1)$

$x$ next $= xR + 1$

$y$ next $= yR$

$yR - 1$

Therefore,

co-ordinates :- $(xR+1, yR) : (xR+1, yR-1)$

midpoint $(xR+1, yR - 1/2)$

$PR = r^2_y (xR+1)^2 + r^2_x (yR - 1/2)^2 - r^2_x r^2_y$ — ②

$p1\ next = r^2_y (xR+1+1)^2 + r_x^2 (y\,next - 1/2)^2 - r^2_x r^2_y$ — ③

$p1\ next - p_1R = r^2_y (xR + 1 + 1)^2 + r^2_x (y\,next - 1/2)^2$
$- r_x^2 r_y^2 - [r^2_y (xR+1)^2 + r^2_x (yR - 1/2)^2$
$- r^2_x r^2_y].$

Calculating initial decision parameter (subs $(0, r_y)$ ($n_{eq}$ 2))

$$P_1 = r^2_y - r_x^2 r^2_y + \frac{r_x^2}{4}$$

for region 2:- Slopes -1

$$y_{next} = y_R - 1$$
$$x_{next} = x_R$$
$$= x_R + 1$$

Coordinates - $(x_R, y_R)$ ; $(x_R + 1, y_R - 1)$
Midpoints - $(x_R + 1/2, y_R - 1)$.

$$P^2_R = r^2_y (x_R + 1/2)^2 + r^2_x (y_R - 1)^2 - r_x^2 r^2_y \quad \text{— (4)}$$

$$P^2_{next} = r^2_y (x_{next} - 1/2)^2 + r^2_x (y_R - 1 - 1)^2 - r^2_x r^2_y \quad \text{— (5)}$$

$$P^2_{next} - P^2_R = r^2_y (x^2_{next} + x_{next} - x^2_R - x_R + r^2_x [1 - 2y_{next}]$$

ii) Algorithm:-

Ellipse $(r_x, r_y)$

$x = 0$, $y = r_y$.

$P_1 = r^2_y - r^2_x r_y + r_x^2/4$.

$dx = 2r^2_y x$

$dy = 2r^2_x y$.

while $(dx < dy)$

put pixel $(x, y)$

if $p1(d)$

$x = x + 1$

else $x = x + 1$.

$y = y - 1$.

$dx = 2r^2_y \, xy$.

$dx = 2r^2y\, u$ $\qquad\qquad$ $dy = 2r^2x\, y$

$P_1 = P_c + dx + r^2y$ $\qquad\qquad$ $P_1 = P_c + dx - dy + r^2y$.

while $(dx <= dy)$

$P2 = r^2y\,(u+1/2)^2 + r^2x\,(y-1)^2 - r^2x\,(y-1)^2 - r^2x\, r^2y$.

if $(y>0)$

putpixel $(x, y)$

if $(P2 > 0)$

$y = y-1$

$dy = 2r^2xy$

$P2 = P2 - dy + r^2x$

else

$x = x+1$.

$y = y-1$

$dy = 2r^2xy$

$dx = 2r^2yx$

$P2 = P2 + dx - dy + rx^2$

Q5) Write a short note on scanline polygon filling algorithm.

→ This algorithm works by intersecting scanline with polygon edges and fills the polygon between pair of intersection.

Scan line fill algorithm is defined at geometric level i.e. coordinate edges, vertices. It start with first scanline and proceeds a line by line to the last scan line.

(1) Edge buckets.

It contains an edge information. The entries of edge booked vary according to data structures used.

2. Edge table
It consists of several edges just holds all the edges that compose the figure according to the data.

3. ~~Act~~ Actual ~~dat~~ list.
It maintains constant edges being used to fill in polygon.

4. Next coordinates $[(xR+1), (yR+1)]$

$$xR+1 = \frac{xR+1}{m}.$$

Q6) 2D Transformation.

* Translation
let $T = \begin{bmatrix} x \\ y \end{bmatrix}$ and $T' \begin{bmatrix} x' \\ y' \end{bmatrix}$

for a vector $t = \begin{bmatrix} tx \\ ty \end{bmatrix}$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} tx \\ ty \end{bmatrix}$$

* Rotation.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} - \begin{bmatrix} x \\ y \end{bmatrix}$$

* Scaling
Let Sx Sy be the scaling factors.

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} Sx & 0 \\ 0 & Sy \end{bmatrix}$$

* Shearing $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & \tan\beta \\ \tan\alpha & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

(2) Homogenous coordinate system.

Translation → Mtr $\begin{bmatrix} 1 & 0 & tx \\ 0 & 1 & ty \\ 0 & 0 & 1 \end{bmatrix}$

Rotation MRD $= \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$

scaling MR $= \begin{bmatrix} Sx & 0 & 0 \\ 0 & Sy & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Shearing MSR $= \begin{bmatrix} 1 & \tan\beta & 0 \\ \tan\alpha & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

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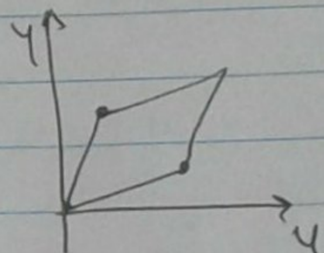
Q7 Derive homogenous transformation matrix for rotation about arbitrary point.

① Assume that we have to rotate a point $P_1$ wrt $(x_m, y_m)$ then we have to perform 3 steps. let we have to translate the $(x_m, y_m)$ to origin.

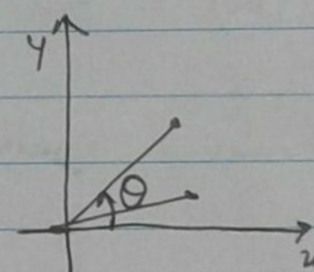
$$T_1 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_m & -y_m & 1 \end{vmatrix}$$




Here $t_x = -x_m$
$t_y = -y_m$.

② Then we have to rotate it in clockwise or anticlockwise by angle $\theta$.

$$R = \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix}$$




③ Then we have to translate the point back to original position.

$$T_2 = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_m & y_m & 1 \end{vmatrix}$$

let us form a combined matrix
$= T_1 * R_1 * T_2$.

$$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -x_m & -y_m & 1 \end{vmatrix} \cdot \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} \cdot \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ x_m & y_m & 1 \end{vmatrix}$$

$$= \begin{vmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ -x_m\cos\theta + y_m\sin\theta + x_m & -x_m\sin\theta - y_m\cos\theta + y_m & 1 \end{vmatrix}$$

This matrix is the overall transformation matrix for rotation about an arbitrary point $(x_m, y_m)$ by angle $\theta$ in anticlockwise direction.

Q8. Let the arbitrary line be $y = mx + c$.
slope $= m$, $y$-intercept $= c$,
we can relate slope to angle $\theta$ by
$\theta = \tan^{-1}(\theta_m)$.

Translation matrix is $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Rotation matrix about $x$ axis.

$$R_x = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Relleation matrix about $x$-axis $M = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Inverse transformation matrices

$$R_2^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$T^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Final transformation matrix can be obtained as

$$RT = T \cdot R_2 \cdot M \cdot R_2^{-1} \cdot T^{-1}.$$

since $\tan\theta = m$, $\sin\theta = \frac{m}{\sqrt{m^2+1}}$

$$RT = \begin{pmatrix} \cos 2\theta & \sin 2\theta & 0 \\ \sin 2\theta & -\cos 2\theta & 0 \\ -\sin 2\theta & c\,[1+\cos 2\theta] & 1 \end{pmatrix}$$

By substituting values of $\sin\theta$ & $\cos\theta$.

$$R_4 = \begin{bmatrix} 1-m^2/m^2+1 & 2m/m^2+1 & 0 \\ 2m/m^2+1 & m^2-1/m^2+1 & 0 \\ 2m/m^2+1 & 2c/m^2+1 & 1 \end{bmatrix}$$

Q9) (a) First rotation by $45^\circ$ in anticlockwise.

$$R = \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Then scaling in both directions

$$S_x = S_y = 1/2 \cdot 3/2.$$

$$S - \begin{bmatrix} 3/2 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Final transformation matrix $= R \times S$.

$$= \begin{bmatrix} \cos 45 & \sin 45 & 0 \\ -\sin 45 & \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3/2 & 0 & 0 \\ 0 & 3/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3/2 \cos 45 & 3/2 \sin 45 & 0 \\ -3/2 \sin 45 & 3/2 \cos 45 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} R'/2\sqrt{2} & 3/2\sqrt{2} & 0 \\ -3/2\sqrt{2} & 3/2\sqrt{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$RXS = \begin{bmatrix} 1.0606 & 1.606 & 0 \\ -1.0606 & 1.606 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) First scaling in y-direction $S_y = 2$

$$S = \begin{bmatrix} 1 & 0 & 0 \\ a & 2 & a \\ 0 & 0 & 1 \end{bmatrix}$$

Shifting (translating) down by 1 unit.

$t_x = 0 \qquad t_y = -1$

$$T = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & a \\ 0 & -1 & 1 \end{bmatrix}$$

Then rotate by $30°$ in clockwise.

$$R = \begin{bmatrix} \cos 30 & -\sin 30 & 0 \\ \sin 30 & \cos 30° & a \\ 0 & 0 & 1 \end{bmatrix}$$

Final matrix $= S \times T \times R$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} \cos 30° & -\sin 30° & 0 \\ \sin 30° & \cos 30° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 0 \\ 2\sin 30^\circ & 2\cos 30^\circ & 0 \\ -\sin 30^\circ & -\cos 30^\circ & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1 & \sqrt{3} & 0 \\ -1/2 & \sqrt{3}/2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.6660 & -0.5 & 0 \\ 1 & 1.7320 & 0 \\ -0.5 & -0.8660 & 1 \end{bmatrix}$$

Q10) A (2,3), B (10,3), C (2,7), D (10,7)

First rotation about a point (8,8) by 30°

$\therefore$ Transformation matrix = (T)

$$T = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ & 0 \\ \sin 30^\circ & \cos 30^\circ & 0 \\ -8\cos 30^\circ - 8\sin 30^\circ + 8 & 8\sin 30^\circ - 8\cos 30^\circ + 8 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 & 0 \\ 1/2 & \sqrt{3}/2 & 0 \\ 4-4\sqrt{3} & 4+4\sqrt{3} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.866 & -0.5 & 0 \\ 0.5 & 0.866 & 0 \\ -2.928 & 10.928 & 1 \end{bmatrix}$$

Scaling $S_x = 2 \quad S_y = 3.$

$$S = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & a \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Object matrix} = O = \begin{bmatrix} 2 & 3 & 1 \\ 10 & 3 & 1 \\ 2 & 7 & 1 \\ 10 & 7 & 1 \end{bmatrix}$$

Final object matrix after transformations

$= O' = O \times T \times S.$

$$O' = \begin{bmatrix} 2 & 3 & 1 \\ 10 & 3 & 1 \\ 2 & 7 & 1 \\ 10 & 7 & 1 \end{bmatrix} \begin{bmatrix} 1.732 & -1.5 & 0 \\ 1 & 2.598 & 0 \\ -5.856 & 32.784 & 1 \end{bmatrix}$$

$$O' = \begin{bmatrix} 0.608 & 37.578 & 1 \\ 14.464 & 27.578 & 1 \\ 4.608 & 47.97 & 1 \\ 18.464 & 35.97 & 1 \end{bmatrix}$$

Final coordinates are

(0.608 , 37.578)
(14.464 , 27.578)
(4.608 , 47.97)
(18.464 , 35.97).

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