1/15

47_ DGAD_ YASH SARANG_ EM4_ ASSIGNMENT 3

Q.3)

Probability of a bolt manufactured by machine A = 25/100
" B = 35/100
" C = 40/100

Probability that the bolt is defective for A = 5/100
" B = 4/100
" C = 2/100

∴ Probability that a bolt was manufactured by A and was defective $= \frac{25}{100} \times \frac{5}{100}$

" $B = \frac{35}{100} \times \frac{4}{100}$

" $C = \frac{40}{100} \times \frac{2}{100}$

∴ Probability that a manufactured bolt is defective $= \frac{25\times5 + 35\times4 + 80}{100\times100}$

$= \frac{345}{100\times100}$

a) ∴ For a defective bolt,

P(Bolt manufactured from machine A) = $\frac{P(\text{Bolt manufactured by A}) \cap P(\text{Bolt is defective by A})}{P(\text{Bolt is defective})}$

$= \frac{25/\cancel{100} \times 5/100}{\cancel{69}345/\cancel{100\times100}} = \frac{25}{69} = 0.3623.$

Similarly

b) P(Bolt manufactured by B) = 35×4/345 = 0.4058.

c) P(Bolt manufactured by C) = 40×2/345 = 0.2319.

Sarangaprash

8)

i) $P(x \leq 150) = \int_{-\infty}^{150} f(x).\,dx = \int_{100}^{150} (100/x^2).\,dx$

$= \left[\frac{-100}{x}\right]_{100}^{150}$

$= \frac{-100}{150} - \left(\frac{-100}{100}\right) = 1 - \frac{2}{3}$

$\boxed{P(x \leq 150) = 1/3}$

ii) $P(x \geq 150) = 1 - P(x \leq 150)$

$= 1 - \frac{1}{3}$

$\boxed{P(x > 150) = \frac{2}{3}}$

i) Probability that all of three such tubes in a given radio will have to be replaced during the first 150 hours of operation is 0.33.

ii) Probability that none of three original tubes will have to be replaced during first 150 hours of operation is 0.67.

Sorangpat

11)
→ $f(x) = kx^2 e^{-x}, \quad x \geq 0.$ is a p.d.f.

$\therefore \int_{-\infty}^{\infty} f(x) = 1. = \int_0^{\infty} kx^2 e^{-x} + \int_{-\infty}^{0} 0.dx$

$\therefore$

$\therefore \int_{-\infty}^{\infty} f(x) = 1 = k\int_0^{\infty} e^{-x} x^2.$

$= k\left[\cancel{e^{-x}\int x^2.dx - \int\left(\frac{d e^{-x}}{dx}\right)\left(\int x^2.dx\right).dx}\right]_0^{\infty}$

$\cancel{= k\left[\frac{e^{-x}x^3}{3} + \int \frac{e^{-x}x^3}{3}.dx\right]}$

we know, $\int_0^{\infty} e^{-x} x^n dx = n!$

$\therefore 1 = k \times 2!, \quad \therefore k = 1/2.$

now, Mean $= E(x) = \int_{-\infty}^{\infty} x.f(x).dx = \frac{1}{2} \times \int_0^{\infty} x^3 e^{-x}.dx$

$= \frac{1}{2} \times 3! = 3.$

Similarly, Variance $= E(x^2) = \int_{-\infty}^{\infty} x^2.f(x).dx = \frac{1}{2} \times 4! = 12.$

$\therefore$ The mean and variance of the given pdf is 3 and 12, respectively.

Sorangyapdi

13)

→ $E(x_1) = 5,$ $E(x_2) = -2,$ x_1 & x_2 are independent.

$E(x_1^2) = 5.$ $E(x_2^2) = 3,$

i) ∴ $E(x_1 + x_2) = E(x_1) + E(x_2)$

$= 3.$

ii) $E(x_1 - x_2) = 7.$

iii) $E(2x_1 + 3x_2 - 5) = (2 \times 5 + 3(-2) + 5)$

$= 9.$

iv) $Var(x_1 + x_2) = E(x_1^2) + E(x_2^2)$

$= 8.$

v) $Var(x_1 - x_2) = 2.$

vi) $Var(2x_1 + 3x_2 - 5) = 2 \times 5 + 3(3) - 5$

$= 14.$

Sarangyash

16)

→ x: No. of defective pins in a box of 100 pins.

$x \sim B(n, p)$ where $n = 100$, $p = 5/100$

$n \geq 20$,

Hence, $x \sim P(\lambda)$ and $\lambda = np = 5$.

$\therefore P(\text{Guarantee fails}) = P(x > 10)$

$= 1 - P(x \leq 10)$

$= 1 - \sum_{x=0}^{10} e^{-\lambda} \frac{(\lambda)^x}{x!}$

$= 1 - 0.9863$

$= 0.0137$

$\therefore P(\text{Guarantee fails}) = 0.0137 = \underline{\underline{1.37\%}}$

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