ML

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37.

Assignment 1

Q1.

$$A = \begin{matrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{matrix}$$

$A - I\lambda = 0$

$$\begin{bmatrix} -9-\lambda & 4 & 4 \\ -8 & 3-\lambda & 4 \\ -16 & 8 & 7-\lambda \end{bmatrix} = 0$$

$= -\lambda^3 + \lambda^2 + 5\lambda + 3 = 0$

$\therefore (\lambda+1)(\lambda+1)(\lambda-3) = 0$

$\therefore \lambda = -1$

$\lambda = +3$

For $\lambda = -1$

$$\begin{bmatrix} -8 & 4 & 4 \\ -8 & +4 & 4 \\ -16 & 8 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\begin{bmatrix} -2 & 1 & 1 \\ -2 & 1 & 1 \\ -4 & 2 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\therefore \quad v_1 = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$$v_2 = \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}$$

$\lambda = 3$

$$\begin{bmatrix} +2 & 4 & 4 \\ 8 & 0 & 4 \\ -16 & 8 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\therefore \quad v_3 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$$

$\therefore$ Vectors are $\begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1/2 \\ 1/2 \\ 1 \end{bmatrix}$

B)

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{bmatrix}$$

$A - \lambda I = 0$

$$\begin{bmatrix} 3-\lambda & 1 & 1 \\ 1 & 3-\lambda & -1 \\ 1 & -1 & 3-\lambda \end{bmatrix}$$

$(\cancel{3-\lambda})$

$\lambda^3 + 9\lambda^2 - 24\lambda + 16 = 0$

$\lambda_1 = 1$

$\lambda_2 = 4$

$\therefore$ For $\lambda_1 = 4$

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$2x + y + z = 0$

$x + 2y - z = 0$

$x - y + 2z = 0$

$\therefore x = -2$

$y = 2$

$\therefore v_1 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$

$\lambda_2 = 4$

$$\begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & -1 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$-x + y + z = 0$$
$$x - y - z = 0$$
$$x - y - z = 0$$

$\therefore \quad x = y + z$

$\therefore \quad v_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$v_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ Eigen vectors are

$$\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Q2. Diagonalise

$$A = \begin{bmatrix} 2 & 0 & 2 \\ -1 & 2 & 1 \\ 0 & 1 & 4 \end{bmatrix}$$

$$A - \lambda I = 0$$

$$\therefore \begin{bmatrix} 2-\lambda & 0 & 2 \\ -1 & 2-\lambda & 1 \\ 0 & 1 & 4-\lambda \end{bmatrix} = 0$$

$$2-\lambda\left[(2-\lambda)(4-\lambda)-1\right] - 0 + 2\left[-1-0\right]$$

$$2-\lambda\left[8-6\lambda+\lambda^2-1\right] - 2 = 0$$

$$2-\lambda\left[7-6\lambda+\lambda^2\right] - 2 = 0$$

$$14 - 12\lambda + 2\lambda^2 - 7\lambda + 6\lambda^2 - \lambda^3 - 2 = 0$$

$$-\lambda^3 + 8\lambda^2 - 19\lambda + 12 = 0$$

$$\lambda^3 - 8\lambda^2 + 19\lambda - 12 = 0$$

$$\lambda_1 = 1, \quad \lambda_2 = 3, \quad \lambda_3 = 4$$

For λ_1

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$\therefore$ $x + 2z = 0$

$-x + y + z = 0$

$y + 3z = 0$

$x = -2z$, $y = -3z$

$\therefore$ $X_1 = \begin{bmatrix} -2 \\ -3 \\ 1 \end{bmatrix}$

For $\lambda_2 = 3$

$$\begin{bmatrix} -1 & 0 & 2 \\ -1 & -1 & 1 \\ 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$-x + 2z = 0$

$-x - y + z = 0$

$y + z = 0$

$\therefore$ $x = 2z$

$z = x + y$

$z = -y$

$\therefore$ $X_2 = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$

for $\lambda_3 = 4$

$$\begin{bmatrix} -2 & 0 & 2 \\ -1 & -2 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-2x + 2z = 0$$

$$y = 0$$

$$\therefore x_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} -2 & 2 & 1 \\ -3 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Since all eigen values are distinct, matrix is diagonalizable.

$$\therefore D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

Q3. Find SVD

$$A = \begin{matrix} 3 & 3 & 2 \\ 2 & 3 & -2 \end{matrix}$$

Find $A \cdot A^T$ & its eigen values

$$\begin{bmatrix} 3 & 3 & 2 \\ 2 & 3 & -2 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 3 & 3 \\ 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 22 & 11 \\ 11 & 22 \end{bmatrix}$$

$\therefore$ Divide matrix by 11

$$A \cdot A^T = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 1 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)[2-\lambda] - 1 = 0$$

$$4 - 4\lambda + \lambda^2 - 1 = 0$$

$$\lambda^2 - 4\lambda + 3 = 0$$

$$\therefore (\lambda - 1)(\lambda - 3) = 0$$

$$\lambda = 1$$

$$\lambda = 3$$

For $\lambda_1 = 1$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$x_1 + x_2 = 0$
$x_1 + x_2 = 0$

$$\begin{bmatrix} -1 \\ +1 \end{bmatrix}$$

Normalised eigenvector is $\begin{bmatrix} -1/\sqrt{2} \\ +1/\sqrt{2} \end{bmatrix}$

For $\lambda_2 = 3$

$$\begin{bmatrix} -1 & +1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-x_1 + x_2 = 0$ $\qquad$ $x_1 = x_2$
$+x_1 - x_2 = 0$

$\therefore$ $\begin{bmatrix} +1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$

$\therefore$ $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

For $A^T A$

$$\begin{bmatrix} -12 & 12 & 2 \\ 12 & -12 & -2 \\ 2 & 2 & -17 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = 0$$

$$\therefore \quad x_1 = \begin{bmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \\ 0 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 1/\sqrt{18} \\ -1/\sqrt{18} \\ 4/\sqrt{18} \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 2/3 \\ -2/3 \\ -1/3 \end{bmatrix}$$

$$\therefore \quad W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \end{bmatrix}$$

$$V = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{18} & 2/3 \\ 1/\sqrt{2} & -1/\sqrt{18} & -2/3 \\ 0 & 4/\sqrt{18} & 1/3 \end{bmatrix}$$

$$U = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$$

$$\therefore \quad A = UWV^T$$

Q4.

$$A = \begin{bmatrix} 25 & 15 & -5 \\ 15 & 18 & 0 \\ -5 & 0 & 11 \end{bmatrix}$$

$$A - \lambda I = 0$$

$$\begin{bmatrix} 25-\lambda & 15 & -5 \\ 15 & 18-\lambda & 0 \\ -5 & 0 & 11-\lambda \end{bmatrix} \neq 0$$

lets eliminate pivots

$$R_2 \leftarrow R_2 - 0.6 \times R_1$$

$$A = \begin{bmatrix} 25 & 15 & -5 \\ 0 & 9 & 3 \\ -5 & 0 & 11 \end{bmatrix}$$

$$R_3 \leftarrow R_3 + 0.2 \times R_1$$

$$A = \begin{bmatrix} 25 & 15 & -5 \\ 0 & 9 & 3 \\ 0 & 3 & 10 \end{bmatrix}$$

$$R_3 \leftarrow R_3 - 0.33 \times R_2$$

$$A = \begin{bmatrix} 25 & 15 & -5 \\ 0 & 9 & 3 \\ 0 & 0 & 9 \end{bmatrix}$$

Pivots are positive, so matrix is positive definite

Pivots 25, 9, 9.