Chapter 3

Traditional Symmetric-Key Ciphers

Chapter 3

Objectives

- To define the terms and the concepts of symmetric key ciphers
- To emphasize the two categories of traditional ciphers: substitution and transposition ciphers
- To describe the categories of cryptanalysis used to break the symmetric ciphers
- To introduce the concepts of the stream ciphers and block ciphers
- To discuss some very dominant ciphers used in the past, such as the Enigma machine

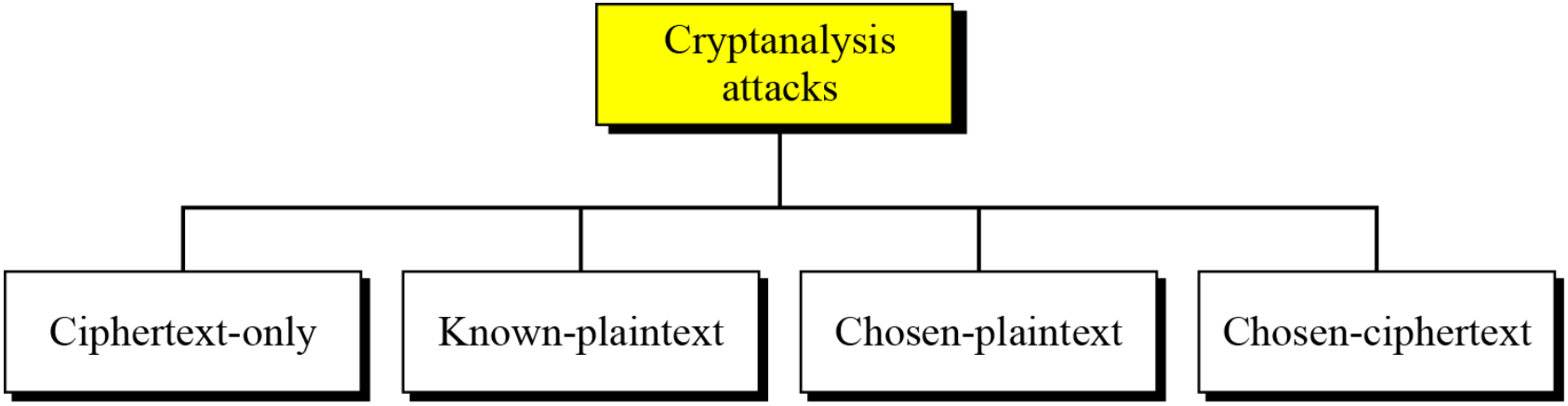
3.1.1 Kerckhoff's Principle

Based on Kerckhoff's principle, one should always assume that the adversary, Eve, knows the encryption/decryption algorithm. The resistance of the cipher to attack must be based only on the secrecy of the key.

3.1.2 Cryptanalysis

As cryptography is the science and art of creating secret codes, cryptanalysis is the science and art of breaking those codes.

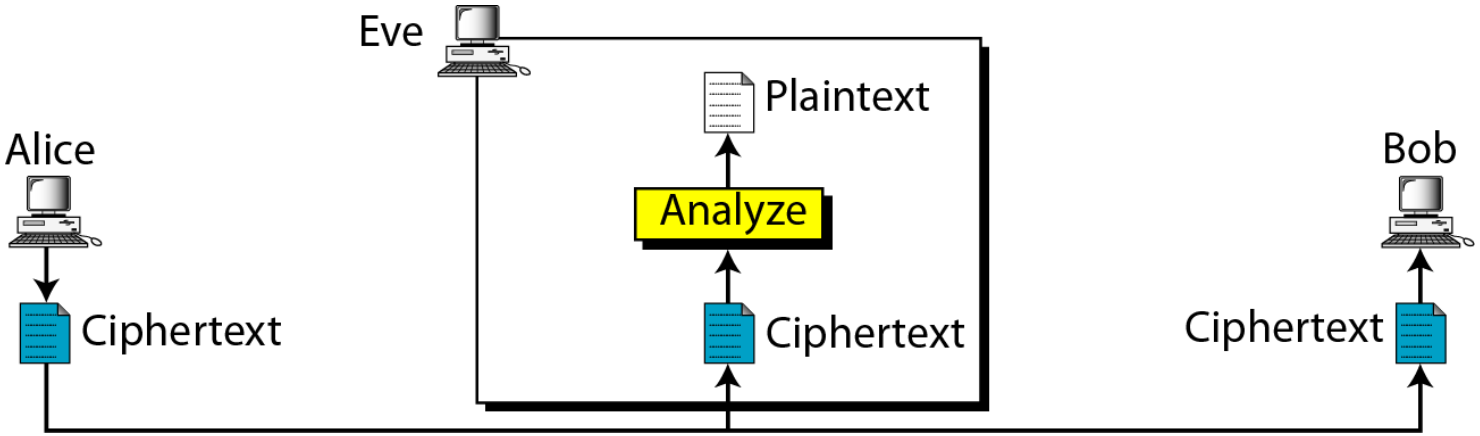
Figure 3.3 *Cryptanalysis attacks*



3.1.2 Continued

Ciphertext-Only Attack

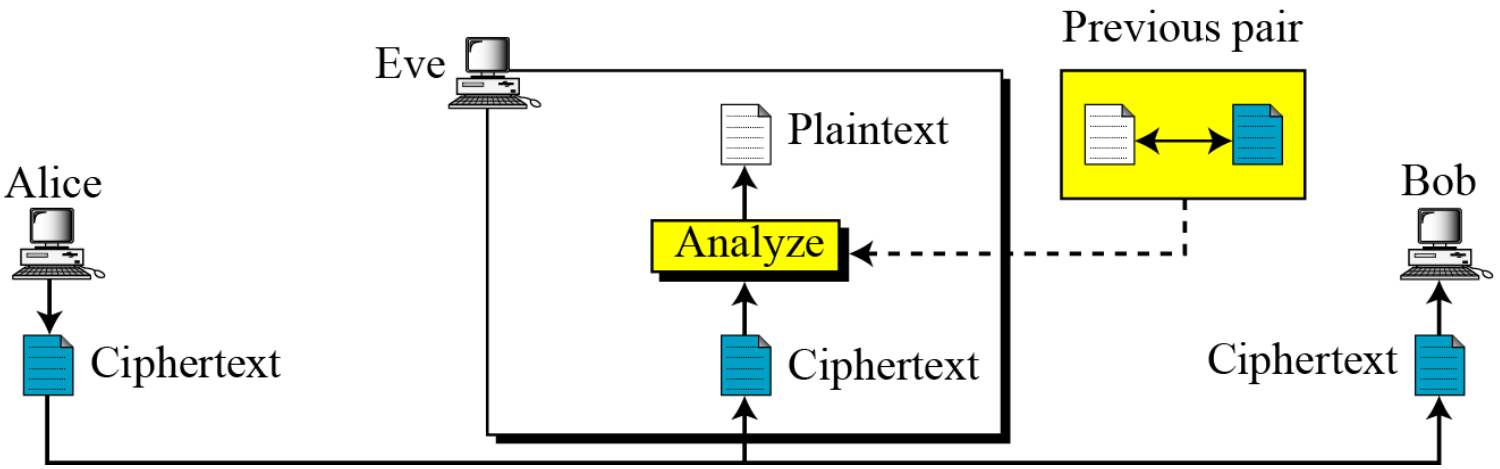
Figure 3.4 *Ciphertext-only attack*



3.1.2 Continued

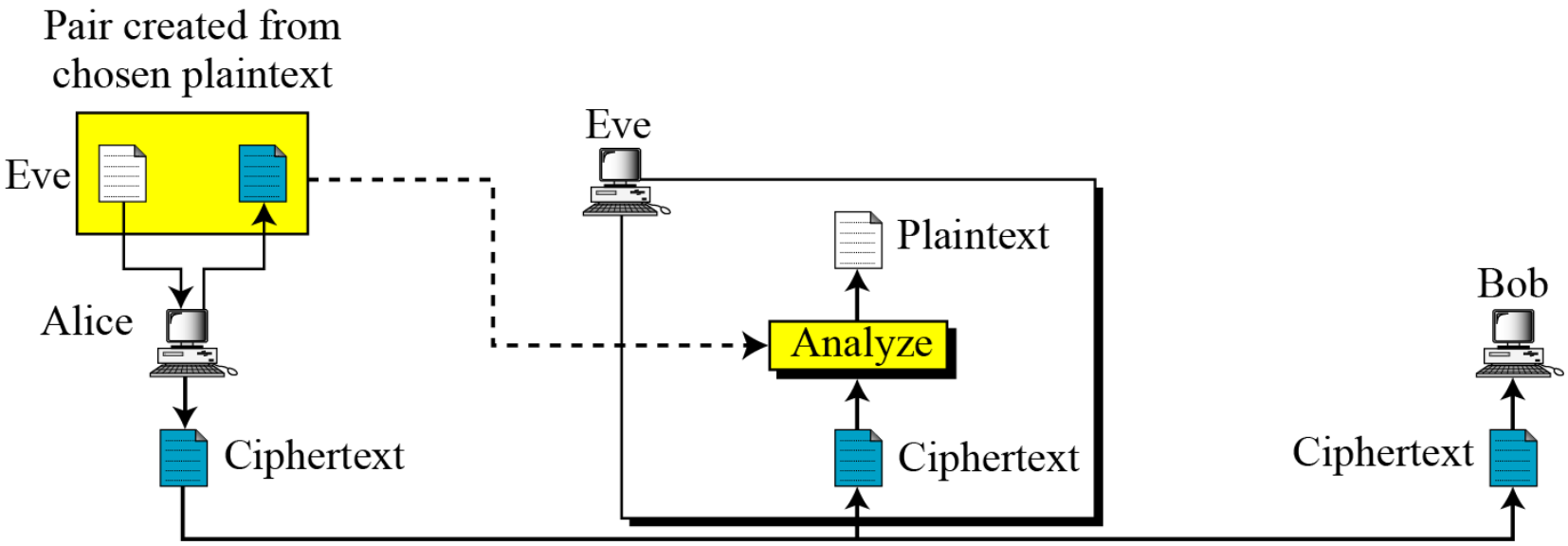
Known-Plaintext Attack

Figure 3.5 *Known-plaintext attack*



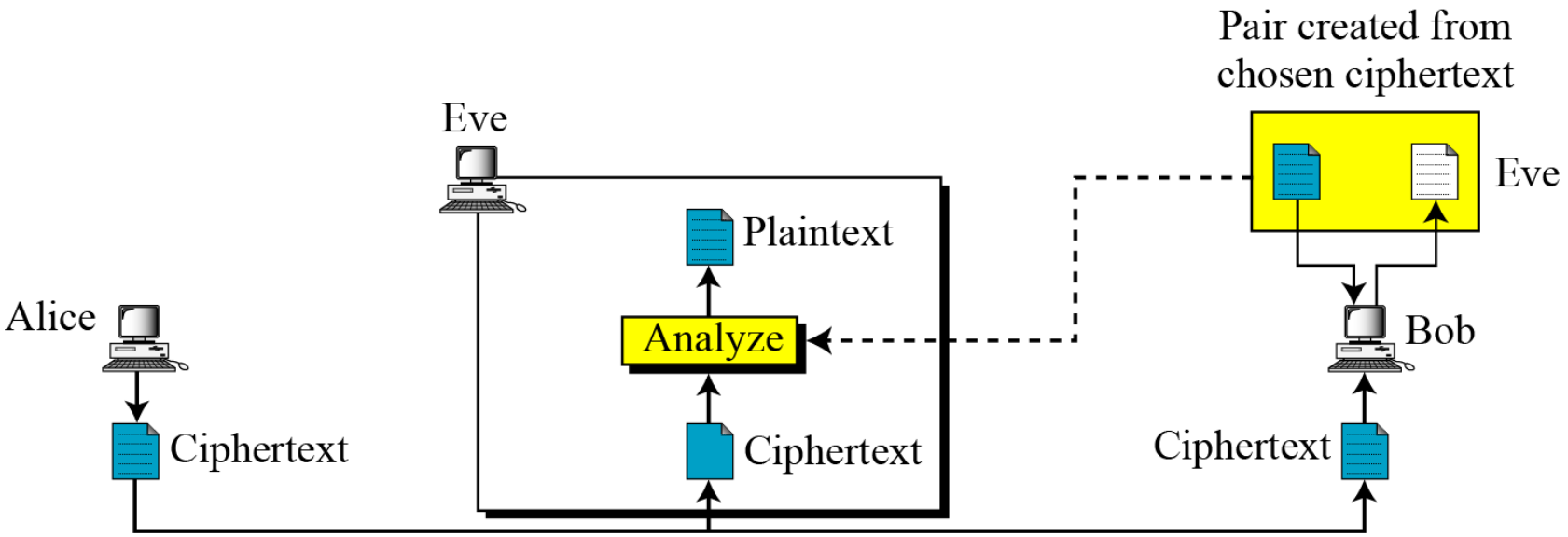
Chosen-Plaintext Attack

Figure 3.6 *Chosen-plaintext attack*



Chosen-Ciphertext Attack

Figure 3.7 *Chosen-ciphertext attack*

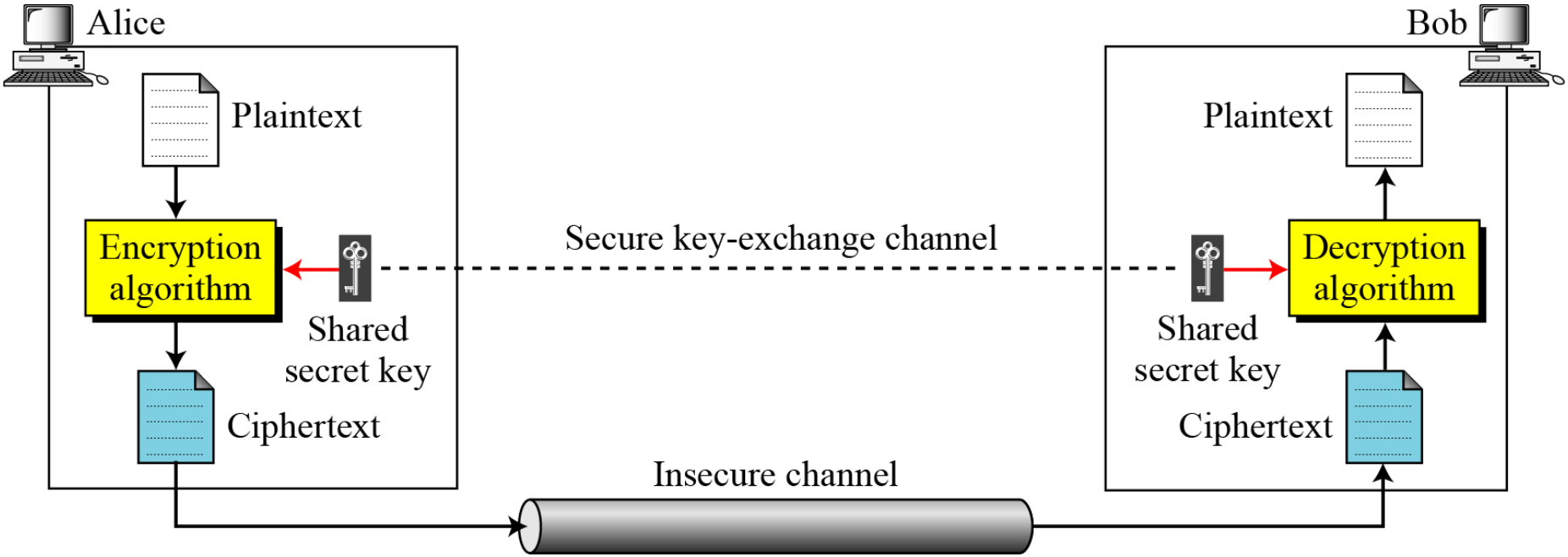


3-1 INTRODUCTION

Figure 3.1 shows the general idea behind a symmetric-key cipher. The original message from Alice to Bob is called plaintext; the message that is sent through the channel is called the ciphertext. To create the ciphertext from the plaintext, Alice uses an encryption algorithm and a shared secret key. To create the plaintext from ciphertext, Bob uses a decryption algorithm and the same secret key.

Topics discussed in this section:

Figure 3.1 *General idea of symmetric-key cipher*



3.1 Continued

If P is the plaintext, C is the ciphertext, and K is the key,

Encryption: $C = E_k(P)$ Decryption: $P = D_k(C)$

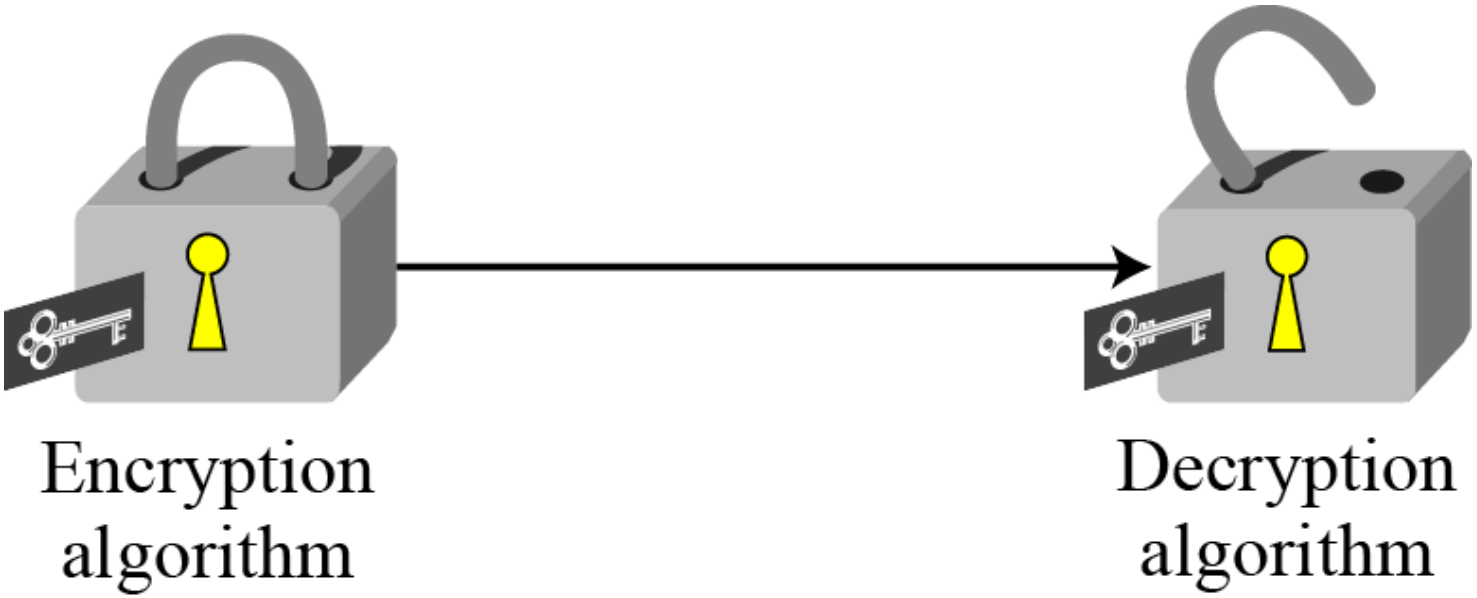
In which, $D_k(E_k(x)) = E_k(D_k(x)) = x$

We assume that Bob creates P_1; we prove that $P_1 = P$:

Alice: $C = E_k(P)$

Bob: $P_1 = D_k(C) = D_k(E_k(P)) = P$

Figure 3.2 *Locking and unlocking with the same key*



3-2 SUBSTITUTION CIPHERS

A substitution cipher replaces one symbol with another. Substitution ciphers can be categorized as either monoalphabetic ciphers or polyalphabetic ciphers.

Note

A substitution cipher replaces one symbol with another.

Topics discussed in this section:

3.2.1 Monoalphabetic Ciphers

Note

In monoalphabetic substitution, the relationship between a symbol in the plaintext to a symbol in the ciphertext is always one-to-one.

3.2.1 Continued

Example 3.1

The following shows a plaintext and its corresponding ciphertext. The cipher is probably monoalphabetic because both *l*'s (els) are encrypted as *O*'s.

Plaintext: hello **Ciphertext:** KHOOR

Example 3.2

The following shows a plaintext and its corresponding ciphertext. The cipher is not monoalphabetic because each *l* (el) is encrypted by a different character.

Plaintext: ***hello*** *Ciphertext:* ***ABNZF***

3.2.1 Continued

Additive Cipher

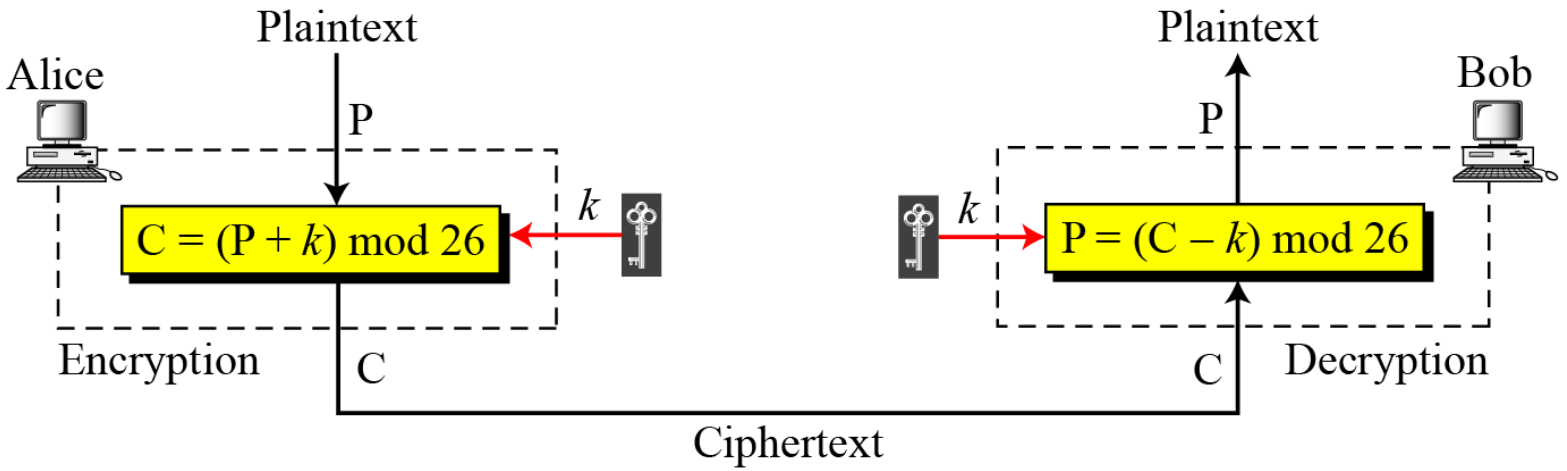
The simplest monoalphabetic cipher is the additive cipher. This cipher is sometimes called a shift cipher and sometimes a Caesar cipher, but the term additive cipher better reveals its mathematical nature.

Figure 3.8 *Plaintext and ciphertext in Z_{26}*

Plaintext →	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext →	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
Value →	00	01	02	03	04	05	06	07	08	09	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

3.2.1 Continued

Figure 3.9 *Additive cipher*



Note

When the cipher is additive, the plaintext, ciphertext, and key are integers in Z_{26}.

3.2.1 Continued

Example 3.3

Use the additive cipher with key = 15 to encrypt the message "hello".

Solution

We apply the encryption algorithm to the plaintext, character by character:

Plaintext: h → 07	Encryption: (07 + 15) mod 26	Ciphertext: 22 → W
Plaintext: e → 04	Encryption: (04 + 15) mod 26	Ciphertext: 19 → T
Plaintext: l → 11	Encryption: (11 + 15) mod 26	Ciphertext: 00 → A
Plaintext: l → 11	Encryption: (11 + 15) mod 26	Ciphertext: 00 → A
Plaintext: o → 14	Encryption: (14 + 15) mod 26	Ciphertext: 03 → D

3.2.1 Continued

Example 3.4

Use the additive cipher with key = 15 to decrypt the message "WTAAD".

Solution

We apply the decryption algorithm to the plaintext character by character:

Ciphertext: W → 22	Decryption: (22 – 15) mod 26	Plaintext: 07 → h
Ciphertext: T → 19	Decryption: (19 – 15) mod 26	Plaintext: 04 → e
Ciphertext: A → 00	Decryption: (00 – 15) mod 26	Plaintext: 11 → l
Ciphertext: A → 00	Decryption: (00 – 15) mod 26	Plaintext: 11 → l
Ciphertext: D → 03	Decryption: (03 – 15) mod 26	Plaintext: 14 → o

3.2.1 Continued

Shift Cipher and Caesar Cipher

Historically, additive ciphers are called shift ciphers. Julius Caesar used an additive cipher to communicate with his officers. For this reason, additive ciphers are sometimes referred to as the Caesar cipher. Caesar used a key of 3 for his communications.

Note

Additive ciphers are sometimes referred to as shift ciphers or Caesar cipher.

3.2.1 Continued

Example 3.5

Eve has intercepted the ciphertext "UVACLYFZLJBYL". Show how she can use a brute-force attack to break the cipher.

Solution

Eve tries keys from 1 to 7. With a key of 7, the plaintext is "not very secure", which makes sense.

Ciphertext: UVACLYFZLJBYL

K = 1	→	**Plaintext:** tuzbkxeykiaxk
K = 2	→	**Plaintext:** styajwdxjhzwj
K = 3	→	**Plaintext:** rsxzivcwigyvi
K = 4	→	**Plaintext:** qrwyhubvhfxuh
K = 5	→	**Plaintext:** pqvxgtaugewtg
K = 6	→	**Plaintext:** opuwfsztfdvsf
K = 7	→	**Plaintext:** notverysecure

3.2.1 Continued

Table 3.1 *Frequency of characters in English*

Letter	*Frequency*	*Letter*	*Frequency*	*Letter*	*Frequency*	*Letter*	*Frequency*
E	12.7	H	6.1	W	2.3	K	0.08
T	9.1	R	6.0	F	2.2	J	0.02
A	8.2	D	4.3	G	2.0	Q	0.01
O	7.5	L	4.0	Y	2.0	X	0.01
I	7.0	C	2.8	P	1.9	Z	0.01
N	6.7	U	2.8	B	1.5		
S	6.3	M	2.4	V	1.0		

Table 3.2 *Frequency of diagrams and trigrams*

Digram	TH, HE, IN, ER, AN, RE, ED, ON, ES, ST, EN, AT, TO, NT, HA, ND, OU, EA, NG, AS, OR, TI, IS, ET, IT, AR, TE, SE, HI, OF
Trigram	THE, ING, AND, HER, ERE, ENT, THA, NTH, WAS, ETH, FOR, DTH

3.2.1 Continued

Example 3.6

Eve has intercepted the following ciphertext. Using a statistical attack, find the plaintext.

XLILSYWIMWRSAJSVWEPIJSVJSYVQMPPMSRHSPPEVWMXMWASVX-LQSVILY-VVCFIJSVIXLIWIPPIVVIGIMZIWQSVISJJIVW

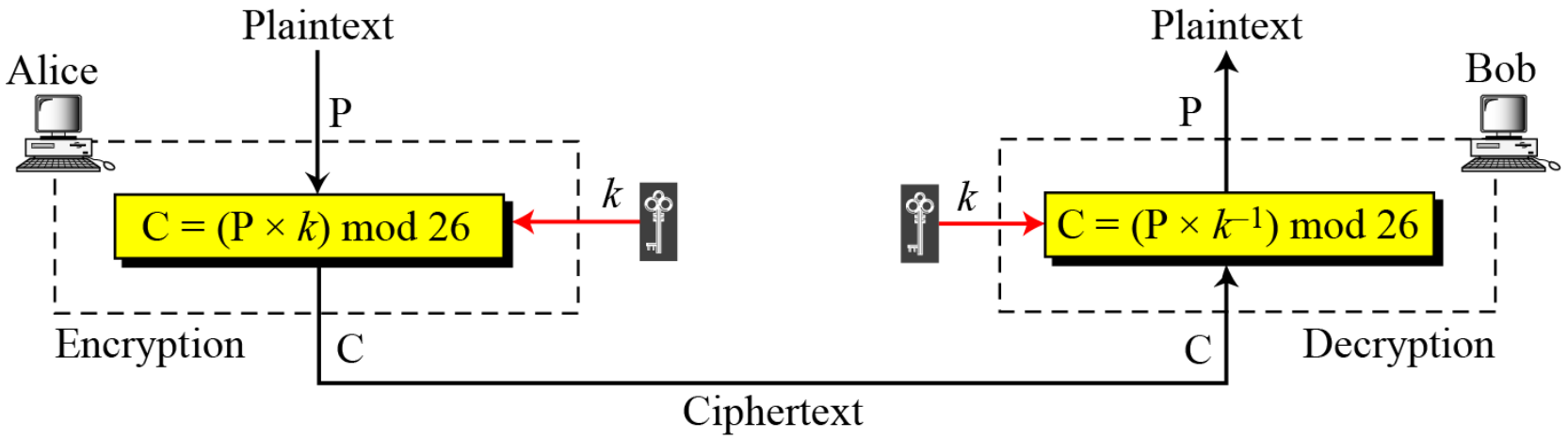
Solution

When Eve tabulates the frequency of letters in this ciphertext, she gets: I =14, V =13, S =12, and so on. The most common character is I with 14 occurrences. This means key = 4.

the house is now for sale for four million dollars it is worth more hurry before the seller receives more offers

Multiplicative Ciphers

Figure 3.10 *Multiplicative cipher*



Note

In a multiplicative cipher, the plaintext and ciphertext are integers in Z_{26}; the key is an integer in Z_{26}*.

3.2.1 Continued

Example 3.7

What is the key domain for any multiplicative cipher?

Solution

The key needs to be in Z_{26}^*. This set has only 12 members: 1, 3, 5, 7, 9, 11, 15, 17, 19, 21, 23, 25.

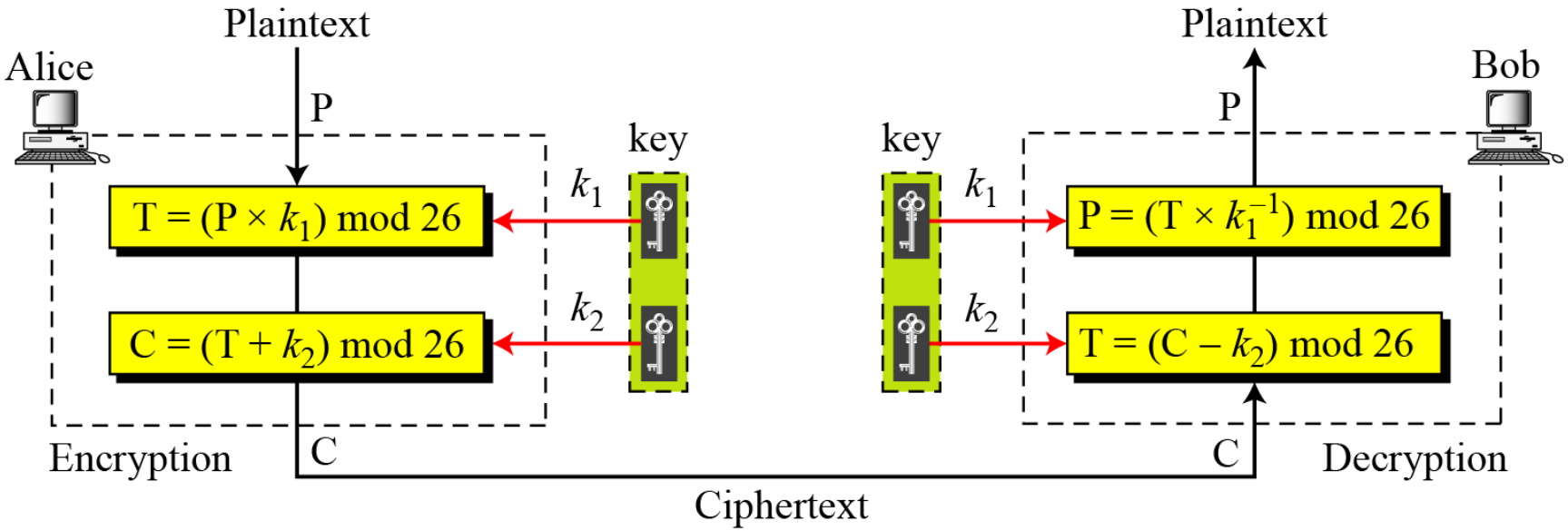
Example 3.8

We use a multiplicative cipher to encrypt the message "hello" with a key of 7. The ciphertext is "XCZZU".

Plaintext: h $\rightarrow$ 07	Encryption: $(07 \times 07) \bmod 26$	ciphertext: 23 $\rightarrow$ X
Plaintext: e $\rightarrow$ 04	Encryption: $(04 \times 07) \bmod 26$	ciphertext: 02 $\rightarrow$ C
Plaintext: l $\rightarrow$ 11	Encryption: $(11 \times 07) \bmod 26$	ciphertext: 25 $\rightarrow$ Z
Plaintext: l $\rightarrow$ 11	Encryption: $(11 \times 07) \bmod 26$	ciphertext: 25 $\rightarrow$ Z
Plaintext: o $\rightarrow$ 14	Encryption: $(14 \times 07) \bmod 26$	ciphertext: 20 $\rightarrow$ U

Affine Ciphers

Figure 3.11 *Affine cipher*



$$C = (P \times k_1 + k_2) \bmod 26 \qquad P = ((C - k_2) \times k_1^{-1}) \bmod 26$$

where k_1^{-1} is the multiplicative inverse of k_1 and $-k_2$ is the additive inverse of k_2

3.2.1 Continued

Example 3.09

The affine cipher uses a pair of keys in which the first key is from Z_{26}^* and the second is from Z_{26}. The size of the key domain is $26 \times 12 = 312$.

Example 3.10

Use an affine cipher to encrypt the message "hello" with the key pair (7, 2).

P: h → 07	Encryption: $(07 \times 7 + 2) \bmod 26$	C: 25 → Z
P: e → 04	Encryption: $(04 \times 7 + 2) \bmod 26$	C: 04 → E
P: l → 11	Encryption: $(11 \times 7 + 2) \bmod 26$	C: 01 → B
P: l → 11	Encryption: $(11 \times 7 + 2) \bmod 26$	C: 01 → B
P: o → 14	Encryption: $(14 \times 7 + 2) \bmod 26$	C: 22 → W

Example 3.11

Use the affine cipher to decrypt the message "ZEBBW" with the key pair (7, 2) in modulus 26.

Solution

C: Z → 25	Decryption: $((25 - 2) \times 7^{-1}) \bmod 26$	P:07 → h
C: E → 04	Decryption: $((04 - 2) \times 7^{-1}) \bmod 26$	P:04 → e
C: B → 01	Decryption: $((01 - 2) \times 7^{-1}) \bmod 26$	P:11 → l
C: B → 01	Decryption: $((01 - 2) \times 7^{-1}) \bmod 26$	P:11 → l
C: W → 22	Decryption: $((22 - 2) \times 7^{-1}) \bmod 26$	P:14 → o

Example 3.12

The additive cipher is a special case of an affine cipher in which $k_1 = 1$. The multiplicative cipher is a special case of affine cipher in which $k_2 = 0$.

3.2.1 Continued

Monoalphabetic Substitution Cipher

Because additive, multiplicative, and affine ciphers have small key domains, they are very vulnerable to brute-force attack.

A better solution is to create a mapping between each plaintext character and the corresponding ciphertext character. Alice and Bob can agree on a table showing the mapping for each character.

Figure 3.12 *An example key for monoalphabetic substitution cipher*

Plaintext →	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
Ciphertext →	N	O	A	T	R	B	E	C	F	U	X	D	Q	G	Y	L	K	H	V	I	J	M	P	Z	S	W

Example 3.13

We can use the key in Figure 3.12 to encrypt the message

this message is easy to encrypt but hard to find the key

The ciphertext is

ICFVQRVVNEFVRNVSIYRGAHSLIOJICNHTIYBFGTICRXRS

3.2.2 Polyalphabetic Ciphers

In polyalphabetic substitution, each occurrence of a character may have a different substitute. The relationship between a character in the plaintext to a character in the ciphertext is one-to-many.

Autokey Cipher

$$P = P_1 P_2 P_3 \ldots \qquad C = C_1 C_2 C_3 \ldots \qquad k = (k_1, P_1, P_2, \ldots)$$

Encryption: $C_i = (P_i + k_i) \bmod 26$ Decryption: $P_i = (C_i - k_i) \bmod 26$

3.2.2 Continued

Example 3.14

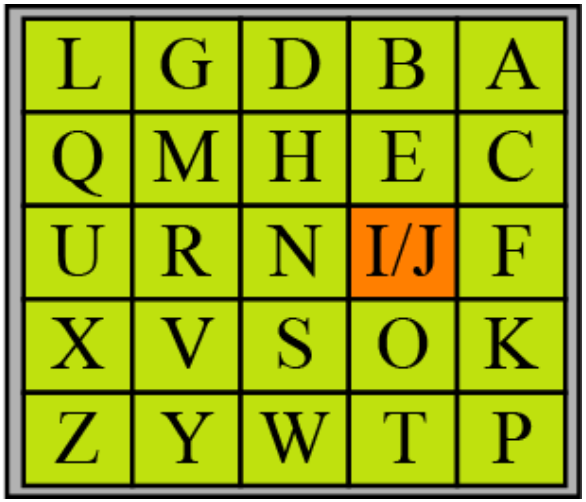
Assume that Alice and Bob agreed to use an autokey cipher with initial key value $k_1 = 12$. Now Alice wants to send Bob the message "Attack is today". Enciphering is done character by character.

Plaintext:	a	t	t	a	c	k	i	s	t	o	d	a	y
P's Values:	00	19	19	00	02	10	08	18	19	14	03	00	24
Key stream:	*12*	*00*	*19*	*19*	*00*	*02*	*10*	*08*	*18*	*19*	*14*	*03*	*00*
C's Values:	12	19	12	19	02	12	18	00	11	7	17	03	24
Ciphertext:	**M**	**T**	**M**	**T**	**C**	**M**	**S**	**A**	**L**	**H**	**R**	**D**	**Y**

Playfair Cipher

Figure 3.13 *An example of a secret key in the Playfair cipher*

Secret Key =



L	G	D	B	A
Q	M	H	E	C
U	R	N	I/J	F
X	V	S	O	K
Z	Y	W	T	P

Example 3.15

Let us encrypt the plaintext "hello" using the key in Figure 3.13.

he → EC lx → QZ lo → BX

Plaintext: hello Ciphertext: ECQZBX

Vigenere Cipher

$P = P_1P_2P_3 \ldots$ $C = C_1C_2C_3 \ldots$ $K = [(k_1, k_2, \ldots, k_m), (k_1, k_2, \ldots, k_m), \ldots]$

Encryption: $C_i = P_i + k_i$ Decryption: $P_i = C_i - k_i$

Example 3.16

We can encrypt the message "She is listening" using the 6-character keyword "PASCAL".

Plaintext:	s	h	e	i	s	l	i	s	t	e	n	i	n	g
P's values:	18	07	04	08	18	11	08	18	19	04	13	08	13	06
Key stream:	***15***	***00***	***18***	***02***	***00***	***11***	***15***	***00***	***18***	***02***	***00***	***11***	***15***	***00***
C's values:	**07**	**07**	**22**	**10**	**18**	**22**	**23**	**18**	**11**	**6**	**13**	**19**	**02**	**06**
Ciphertext:	**H**	**H**	**W**	**K**	**S**	**W**	**X**	**S**	**L**	**G**	**N**	**T**	**C**	**G**

3.2.2 Continued

Example 3.16

Let us see how we can encrypt the message "She is listening" using the 6-character keyword "PASCAL". The initial key stream is (15, 0, 18, 2, 0, 11). The key stream is the repetition of this initial key stream (as many times as needed).

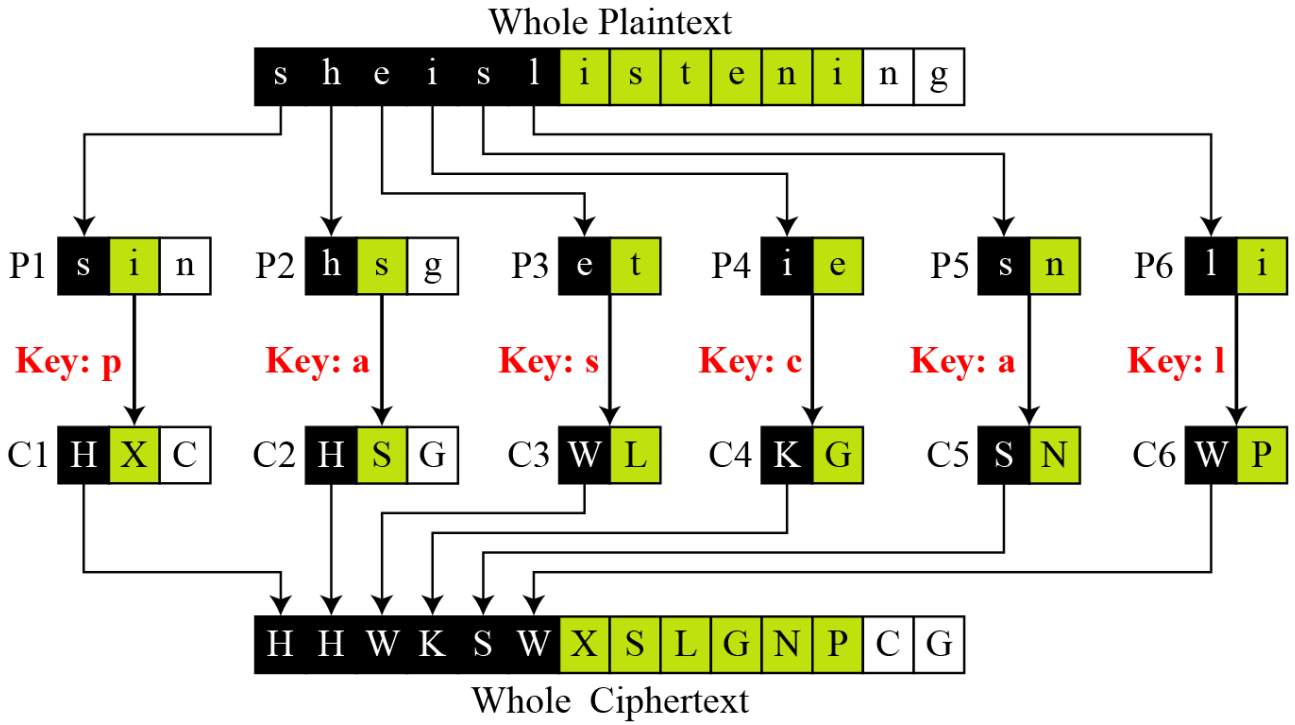
Plaintext:	s	h	e	i	s	l	i	s	t	e	n	i	n	g
P's values:	18	07	04	08	18	11	08	18	19	04	13	08	13	06
Key stream:	***15***	***00***	***18***	***02***	***00***	***11***	***15***	***00***	***18***	***02***	***00***	***11***	***15***	***00***
C's values:	**07**	**07**	**22**	**10**	**18**	**22**	**23**	**18**	**11**	**6**	**13**	**19**	**02**	**06**
Ciphertext:	**H**	**H**	**W**	**K**	**S**	**W**	**X**	**S**	**L**	**G**	**N**	**T**	**C**	**G**

3.2.2 Continued

Example 3.17

Vigenere cipher can be seen as combinations of m additive ciphers.

Figure 3.14 *A Vigenere cipher as a combination of m additive ciphers*



3.2.2 Continued

Example 3.18

Using Example 3.18, we can say that the additive cipher is a special case of Vigenere cipher in which $m = 1$.

Table 3.3

A Vigenere Tableau

	a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	v	v	w	x	y	z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Hill Cipher

Figure 3.15 *Key in the Hill cipher*

$$K = \begin{bmatrix} k_{11} & k_{12} & \cdots & k_{1m} \\ k_{21} & k_{22} & \cdots & k_{2m} \\ \vdots & \vdots & & \vdots \\ k_{m1} & k_{m2} & \cdots & k_{mm} \end{bmatrix}$$

$$C_1 = P_1 k_{11} + P_2 k_{21} + \cdots + P_m k_{m1}$$
$$C_2 = P_1 k_{12} + P_2 k_{22} + \cdots + P_m k_{m2}$$
$$\cdots$$
$$C_m = P_1 k_{1m} + P_2 k_{2m} + \cdots + P_m k_{mm}$$

Note

The key matrix in the Hill cipher needs to have a multiplicative inverse.

Hill Cipher

- **Encryption :**

  Cipher Text =(Plain Text x Key) Mod 26

- **Decryption:**

  Plain Text =(Cipher Text x Key^{-1}) Mod 26

Encryption

- **Message:** ATTACK IS TONIGHT

Hill Cipher (Cont.)

- **Key** $= \begin{bmatrix} 3 & 10 & 20 \\ 20 & 9 & 17 \\ 9 & 4 & 17 \end{bmatrix}$

.....Encryption

- Message: ATTACK IS TONIGHT

  Assign : A-Z 0-25

$$\begin{bmatrix} A & T & T \\ A & C & K \\ I & S & T \\ O & N & I \\ G & H & T \end{bmatrix} = \begin{bmatrix} 0 & 19 & 19 \\ 0 & 2 & 10 \\ 8 & 18 & 19 \\ 14 & 13 & 8 \\ 6 & 7 & 19 \end{bmatrix}$$

Hill Cipher (Cont.)

.....Encryption

- Message: ATTACK IS TONIGHT
- **Cipher Text = (Plain Text x Key) Mod 26**

$$= \begin{bmatrix} 0 & 19 & 19 \\ 0 & 2 & 10 \\ 8 & 18 & 19 \\ 14 & 13 & 8 \\ 6 & 7 & 19 \end{bmatrix} \times \begin{bmatrix} 3 & 10 & 20 \\ 20 & 9 & 17 \\ 9 & 4 & 17 \end{bmatrix} \text{ Mod } 26$$

.....Encryption

Hill Cipher (Cont.)

- $= \begin{bmatrix} 0 & 19 & 19 \\ 0 & 2 & 10 \\ 8 & 18 & 19 \\ 14 & 13 & 8 \\ 6 & 7 & 19 \end{bmatrix} \times \begin{bmatrix} 3 & 10 & 20 \\ 20 & 9 & 17 \\ 9 & 4 & 17 \end{bmatrix} \text{ Mod } 26$

$C_{11} = 0*3 + 19*20 + 19*9 = 551 \text{ Mod } 26 = 05$

$C_{12} = 0*10 + 19*9 + 19*4 = 247 \text{ Mod } 26 = 13$

$C_{13} = 0*20 + 19*17 + 19*17 = 646 \text{ Mod } 26 = 22$

ATT → FNW *Similarly you can calculate other...*

Decryption

- **Plain Text = (Cipher Text x Key^{-1}) Mod 26**

Hill Cipher (Cont.)

- You need to Find : Key^{-1}
  - $Key^{-1} = [Det\ (Key)]^{-1} \times Adj\ (Key)$

Step 1 : Find Determinant of Key

Adj (Key)

Step 2 : Transpose Key Matrix

Step 3 : Find Minor

Step 4 : Find Co-Factor

…Decryption

- $Key = \begin{bmatrix} 3 & 10 & 20 \\ 20 & 9 & 17 \\ 9 & 4 & 17 \end{bmatrix}$

Hill Cipher (Cont.)

Step 1 : Find Determinant of Key

Step 2 : Transpose Key Matrix

Step 3 : Find Minor

Step 4 : Find Co-Factor

…Decryption

Step 1 : Find Determinant of Key

- $\text{Det (Key)} = \begin{bmatrix} 3 & 10 & 20 \\ 20 & 9 & 17 \\ 9 & 4 & 17 \end{bmatrix} \text{ Mod } 26 = 03$

$= 3*(9*17 - 17*4) - 10*(20*17 - 17*9) + 20*(20*4 - 9*9)$

$= (-1635) \text{ Mod } 26 = (-23) \text{ Mod } 26 = 03$

$[Det\ (Key)]^{-1} = 03^{-1} \text{ Mod } 26 = 09$

Hill Cipher (Cont.)

…Decryption

Step 2 : Transpose Key Matrix

$$\text{Key} = \begin{bmatrix} 3 & 10 & 20 \\ 20 & 9 & 17 \\ 9 & 4 & 17 \end{bmatrix}$$

$$\textbf{\textit{Trans(Key)}} = \begin{bmatrix} \mathbf{3} & \mathbf{20} & \mathbf{9} \\ \mathbf{10} & \mathbf{9} & \mathbf{4} \\ \mathbf{20} & \mathbf{17} & \mathbf{17} \end{bmatrix}$$

…Decryption

Hill Cipher (Cont.)

Step 3 : Find Minor

- Trans(Key) = $\begin{bmatrix} 3 & 20 & 9 \\ 10 & 9 & 4 \\ 20 & 17 & 17 \end{bmatrix}$ $\begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix}$ *To find Minor of*

$a11 = a22*a33 - a32*a23 = 85$

$a11 = 85$ $a12 = 90$ $a13 = (-10)$
$a21 = 187$ $a22 = (-129)$ $a23 = (-349)$
$a31 = (-1)$ $a32 = (-78)$ $a33 = (-173)$

$$\textbf{Minor} = \begin{bmatrix} 85 & 90 & -10 \\ 187 & -129 & -349 \\ -1 & -78 & -173 \end{bmatrix}$$

…Decryption

Hill Cipher (Cont.)

Step 4 : Find Co-Factor

$$\textbf{Minor} = \begin{bmatrix} 85 & 90 & -10 \\ 187 & -129 & -349 \\ -1 & -78 & -173 \end{bmatrix}$$

Put Sign According to (-1) $^{i+j}$

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix} \rightarrow \begin{bmatrix} 85 & -90 & -10 \\ -187 & -129 & 349 \\ -1 & 78 & -173 \end{bmatrix}$$

…Decryption

$$Key^{-1} = [Det\ (Key)]^{-1} \times Adj\ (Key)$$

Hill Cipher (Cont.)

$$Key^{-1} = [Det\ (Key)]^{-1} \times Adj\ (Key)$$

$$= 09 * \begin{bmatrix} 85 & -90 & -10 \\ -187 & -129 & 349 \\ -1 & 78 & -173 \end{bmatrix} Mod\ 26$$

$$= \begin{bmatrix} 765 & -810 & -90 \\ -1683 & -1161 & 3141 \\ -9 & 702 & -1557 \end{bmatrix} Mod\ 26 = \begin{bmatrix} 11 & 22 & 14 \\ 7 & 9 & 21 \\ 17 & 0 & 3 \end{bmatrix}$$

…Decryption

Hill Cipher (Cont.)

…Decryption

Finally Plain Text = (Cipher Text x Key^{-1}) Mod 26

$$= \begin{bmatrix} 5 & 13 & 22 \\ . & . & . \\ . & . & . \\ . & . & . \end{bmatrix} \times \begin{bmatrix} 11 & 22 & 14 \\ 7 & 9 & 21 \\ 17 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 520 & 227 & 409 \\ . & . & . \\ . & . & . \\ . & . & . \end{bmatrix} Mod\ 26 = \begin{bmatrix} 0 & 19 & 19 \\ . & . & . \\ . & . & . \\ . & . & . \end{bmatrix} = A\ T\ T$$

Similarly you can calculate other…

3.2.2 Continued

One-Time Pad

One of the goals of cryptography is perfect secrecy. A study by Shannon has shown that perfect secrecy can be achieved if each plaintext symbol is encrypted with a key randomly chosen from a key domain. This idea is used in a cipher called one-time pad, invented by Vernam.

3-3 TRANSPOSITION CIPHERS

A transposition cipher does not substitute one symbol for another, instead it changes the location of the symbols.

Note

A transposition cipher reorders symbols.

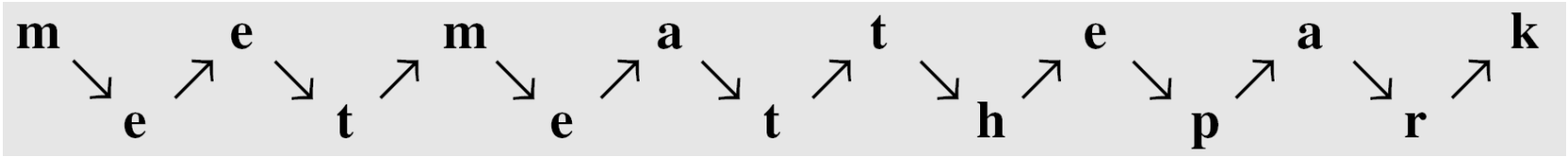
Topics discussed in this section:

3.3.1 Keyless Transposition Ciphers

Simple transposition ciphers, which were used in the past, are keyless.

Example 3.22

A good example of a keyless cipher using the first method is the rail fence cipher. The ciphertext is created reading the pattern row by row. For example, to send the message "Meet me at the park" to Bob, Alice writes



She then creates the ciphertext "MEMATEAKETETHPR".

3.3.1 Continued

Example 3.23

Alice and Bob can agree on the number of columns and use the second method. Alice writes the same plaintext, row by row, in a table of four columns.

m	e	e	t
m	e	a	t
t	h	e	p
a	r	k	

She then creates the ciphertext "MMTAEEHREAEKTTP".

3.3.1 Continued

Example 3.24

The cipher in Example 3.23 is actually a transposition cipher. The following shows the permutation of each character in the plaintext into the ciphertext based on the positions.

01	02	03	04	05	06	07	08	09	10	11	12	13	14	15
↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓	↓
01	05	09	13	02	06	10	13	03	07	11	15	04	08	12

The second character in the plaintext has moved to the fifth position in the ciphertext; the third character has moved to the ninth position; and so on. Although the characters are permuted, there is a pattern in the permutation: (01, 05, 09, 13), (02, 06, 10, 13), (03, 07, 11, 15), and (08, 12). In each section, the difference between the two adjacent numbers is 4.

3.3.2 Keyed Transposition Ciphers

The keyless ciphers permute the characters by using writing plaintext in one way and reading it in another way The permutation is done on the whole plaintext to create the whole ciphertext. Another method is to divide the plaintext into groups of predetermined size, called blocks, and then use a key to permute the characters in each block separately.

3.3.2 Continued

Example 3.25

Alice needs to send the message "Enemy attacks tonight" to Bob..

e n e m y a t t a c k s t o n i g h t z

The key used for encryption and decryption is a permutation key, which shows how the character are permuted.

Encryption ↓

3	1	4	5	2
1	2	3	4	5

↑ Decryption

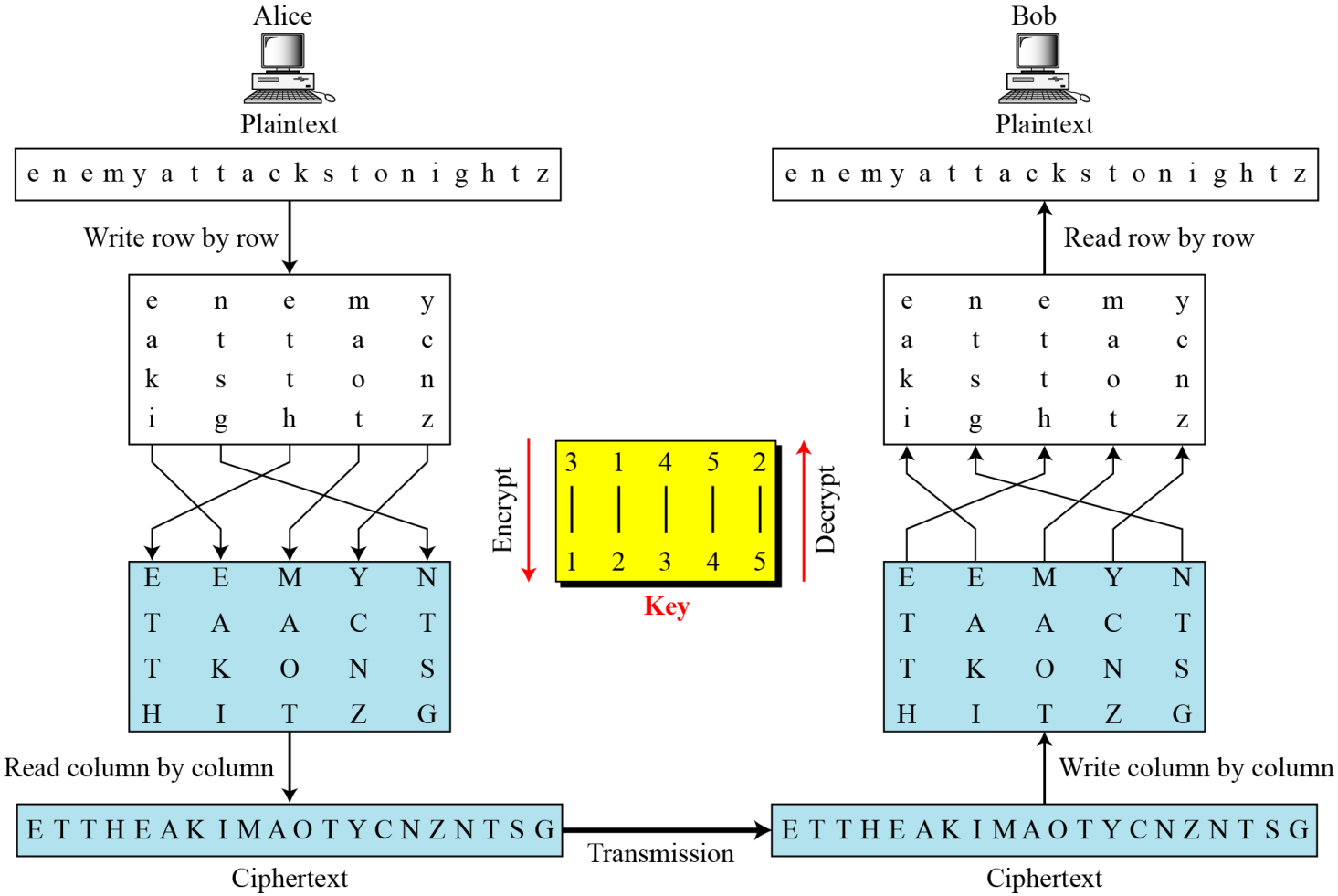
The permutation yields

E E M Y N T A A C T T K O N S H I T Z G

3.3.3 Combining Two Approaches

Example 3.26

Figure 3.21

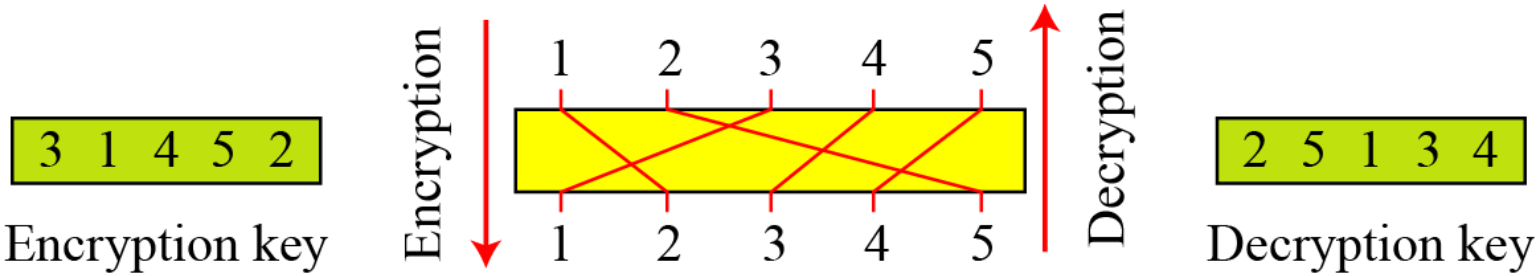


3.3.3 Continued

Keys

In Example 3.27, a single key was used in two directions for the column exchange: downward for encryption, upward for decryption. It is customary to create two keys.

Figure 3.22 *Encryption/decryption keys in transpositional ciphers*



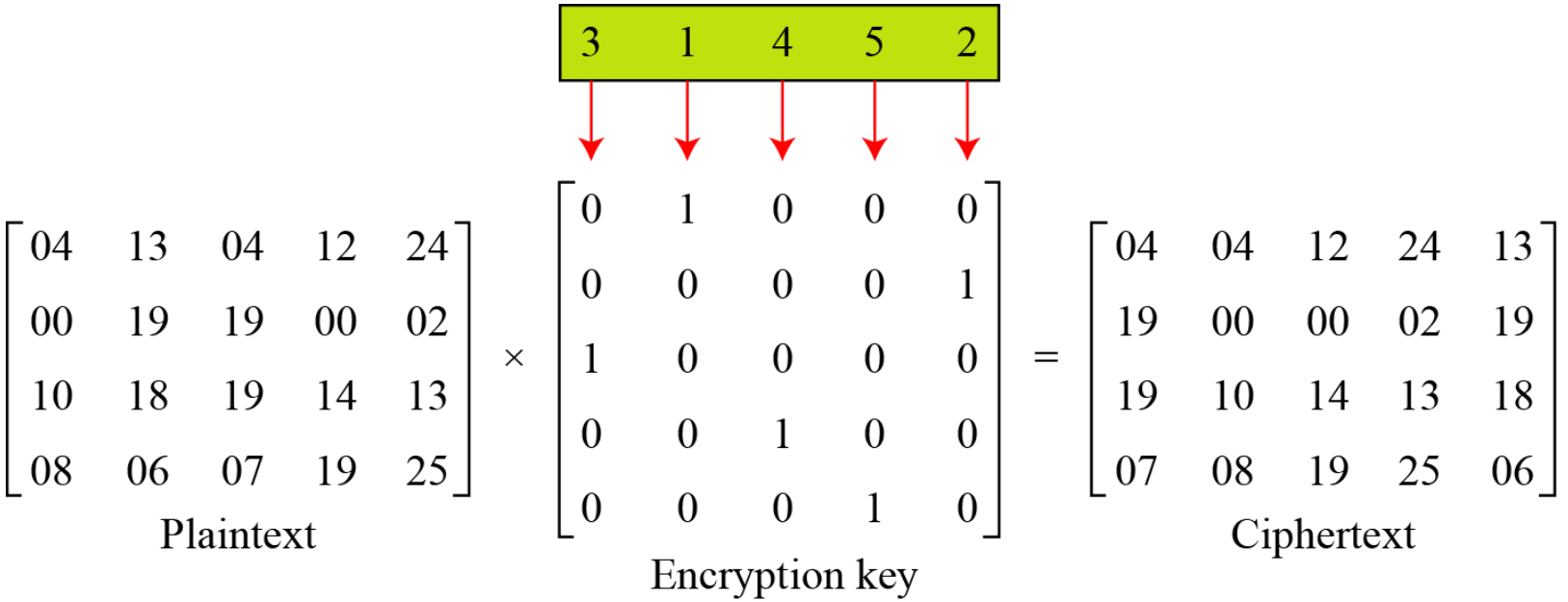
3.3.3 Continued

Using Matrices

We can use matrices to show the encryption/decryption process for a transposition cipher.

Example 3.27

Figure 3.24 ***Representation of the key as a matrix in the transposition cipher***

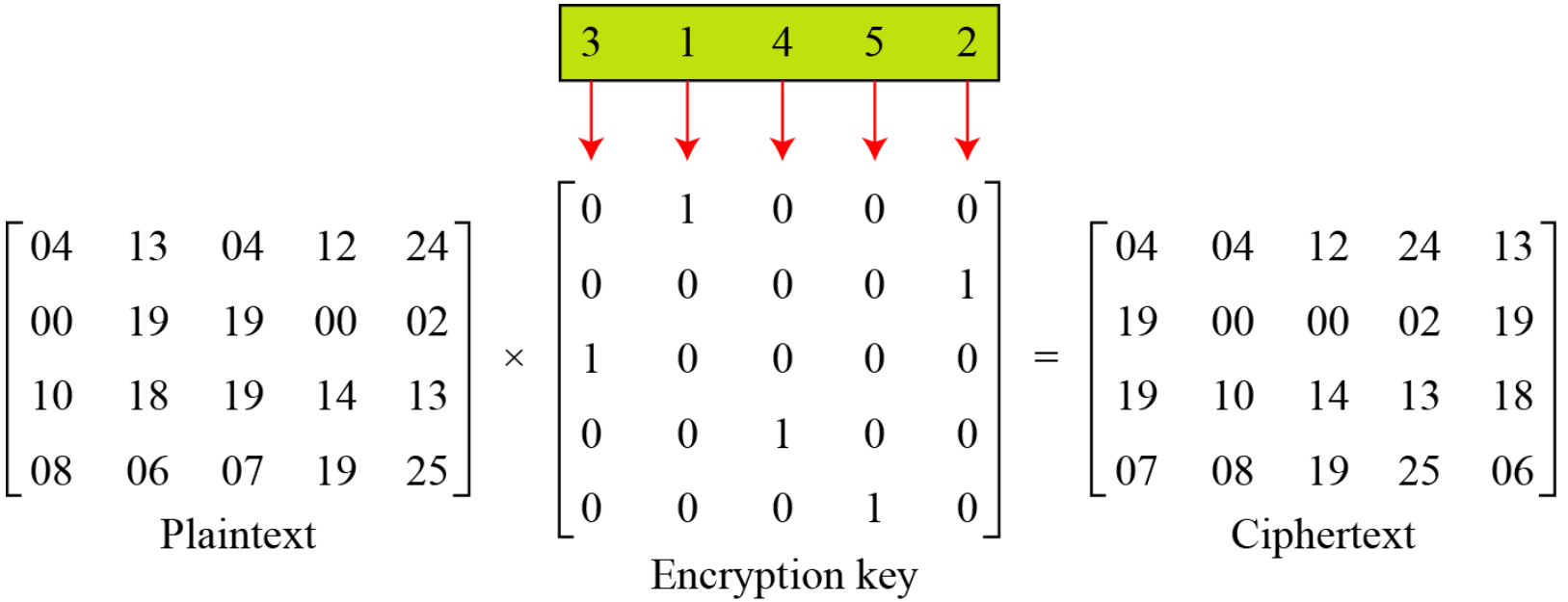


3.3.3 Continued

Example 3.27

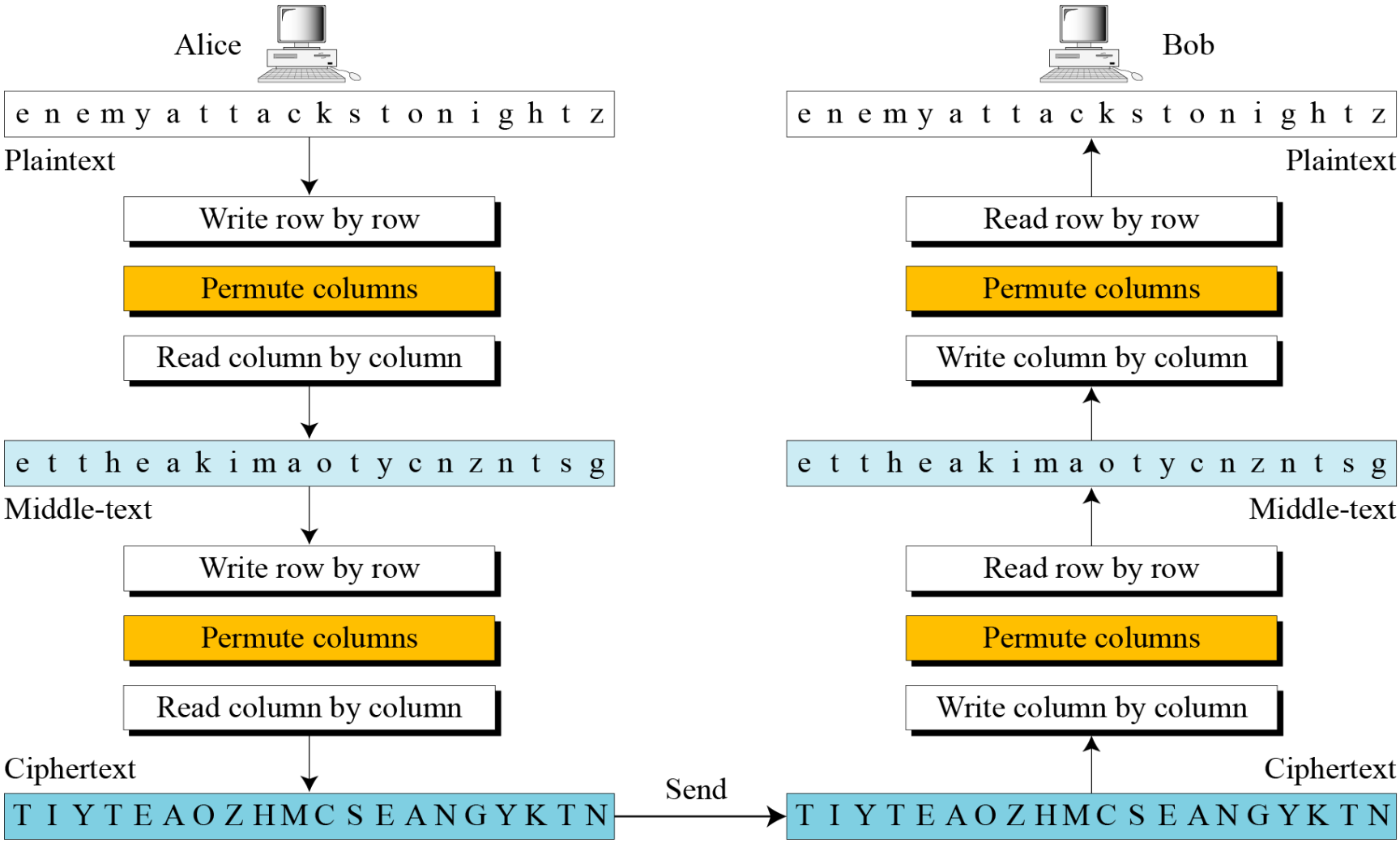
Figure 3.24 shows the encryption process. Multiplying the 4 × 5 plaintext matrix by the 5 × 5 encryption key gives the 4 × 5 ciphertext matrix.

Figure 3.24 *Representation of the key as a matrix in the transposition cipher*



Double Transposition Ciphers

Figure 3.25 *Double transposition cipher*



3-4 STREAM AND BLOCK CIPHERS

The literature divides the symmetric ciphers into two broad categories: stream ciphers and block ciphers. Although the definitions are normally applied to modern ciphers, this categorization also applies to traditional ciphers.

Topics discussed in this section:

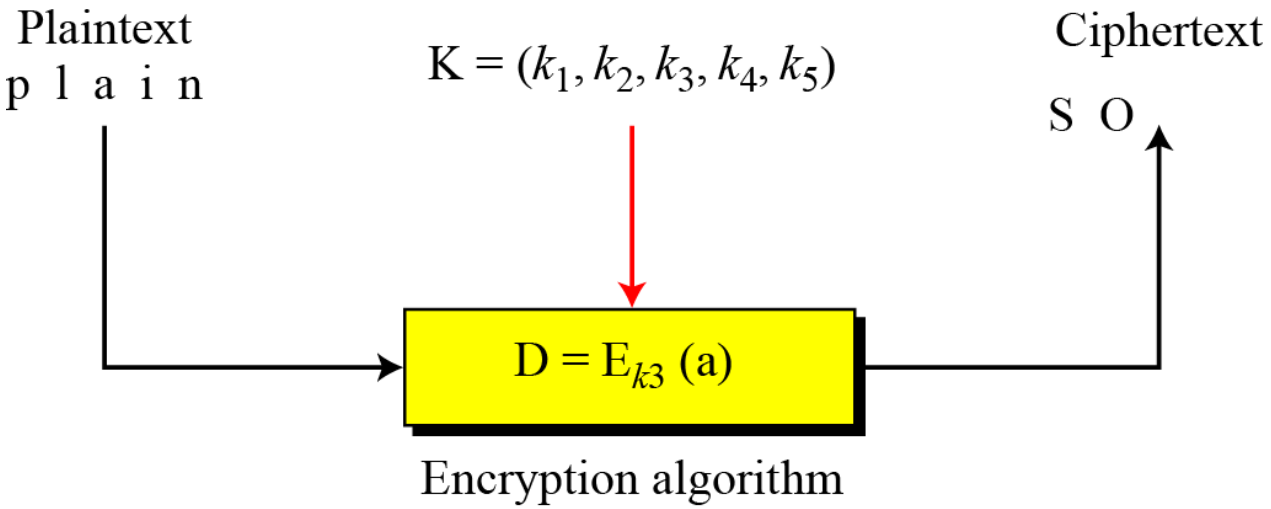
3.4.1 Stream Ciphers

Call the plaintext stream P, the ciphertext stream C, and the key stream K.

$$P = P_1P_2P_3, \ldots \qquad C = C_1C_2C_3, \ldots \qquad K = (k_1, k_2, k_3, \ldots)$$

$$C_1 = E_{k1}(P_1) \qquad C_2 = E_{k2}(P_2) \qquad C_3 = E_{k3}(P_3) \ldots$$

Figure 3.26 ***Stream cipher***



Example 3.30

Additive ciphers can be categorized as stream ciphers in which the key stream is the repeated value of the key. In other words, the key stream is considered as a predetermined stream of keys or K = (k, k, …, k). In this cipher, however, each character in the ciphertext depends only on the corresponding character in the plaintext, because the key stream is generated independently.

Example 3.31

The monoalphabetic substitution ciphers discussed in this chapter are also stream ciphers. However, each value of the key stream in this case is the mapping of the current plaintext character to the corresponding ciphertext character in the mapping table.

Example 3.32

Vigenere ciphers are also stream ciphers according to the definition. In this case, the key stream is a repetition of m values, where *m* is the size of the keyword. In other words,

$$K = (k_1, k_2, \ldots k_m, k_1, k_2, \ldots k_m, \ldots)$$

Example 3.33

We can establish a criterion to divide stream ciphers based on their key streams. We can say that a stream cipher is a monoalphabetic cipher if the value of k_i does not depend on the position of the plaintext character in the plaintext stream; otherwise, the cipher is polyalphabetic.

Example 3.33 (Continued)

- Additive ciphers are definitely monoalphabetic because k_i in the key stream is fixed; it does not depend on the position of the character in the plaintext.

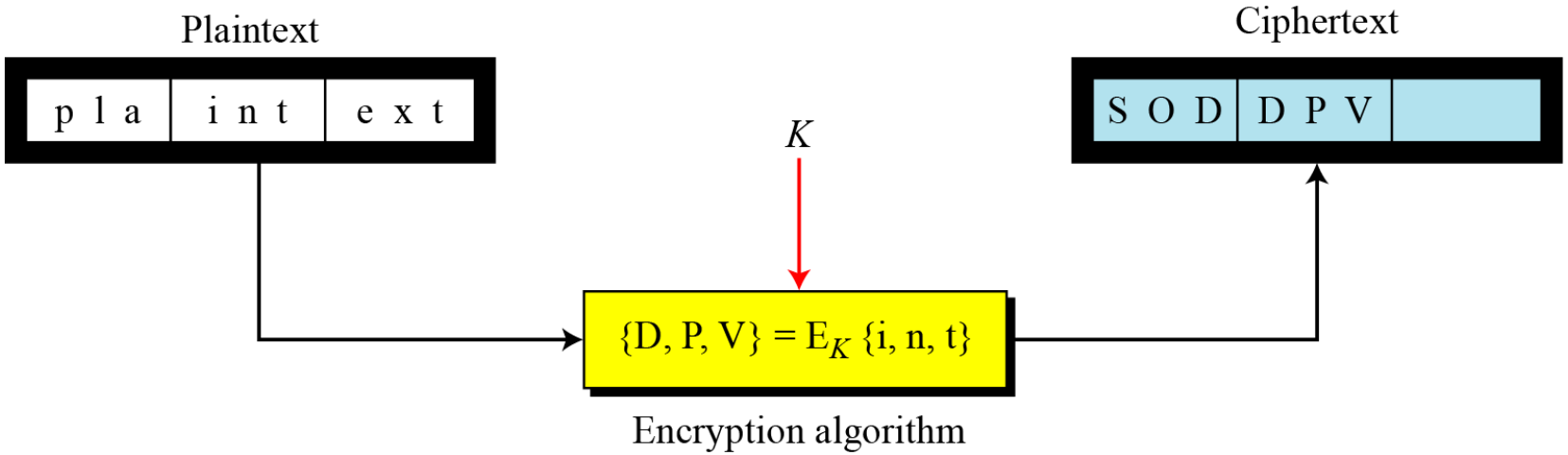
- Monoalphabetic substitution ciphers are monoalphabetic because k_i does not depend on the position of the corresponding character in the plaintext stream; it depends only on the value of the plaintext character.

- Vigenere ciphers are polyalphabetic ciphers because k_i definitely depends on the position of the plaintext character. However, the dependency is cyclic. The key is the same for two characters *m* positions apart.

3.4.2 Stream Ciphers

In a block cipher, a group of plaintext symbols of size *m* (*m* > 1) are encrypted together creating a group of ciphertext of the same size. A single key is used to encrypt the whole block even if the key is made of multiple values. Figure 3.27 shows the concept of a block cipher.

Figure 3.27 ***Block cipher***



Example 3.34

Playfair ciphers are block ciphers. The size of the block is $m = 2$. Two characters are encrypted together.

Example 3.35

Hill ciphers are block ciphers. A block of plaintext, of size 2 or more is encrypted together using a single key (a matrix). In these ciphers, the value of each character in the ciphertext depends on all the values of the characters in the plaintext. Although the key is made of $m \times m$ values, it is considered as a single key.

Example 3.36

From the definition of the block cipher, it is clear that every block cipher is a polyalphabetic cipher because each character in a ciphertext block depends on all characters in the plaintext block.

3.4.3 Combination

In practice, blocks of plaintext are encrypted individually, but they use a stream of keys to encrypt the whole message block by block. In other words, the cipher is a block cipher when looking at the individual blocks, but it is a stream cipher when looking at the whole message considering each block as a single unit.