Adversarial Training

IE643 - Final Lecture

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1. Adversarial Training

Adversarial Training

Explaining and Harnessing Adversarial Examples

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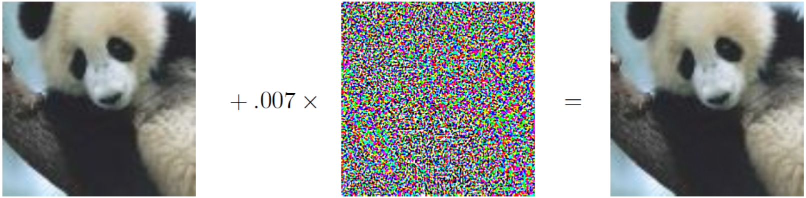
Adversarial Training



$\boldsymbol{x}$ "panda" 57.7% confidence

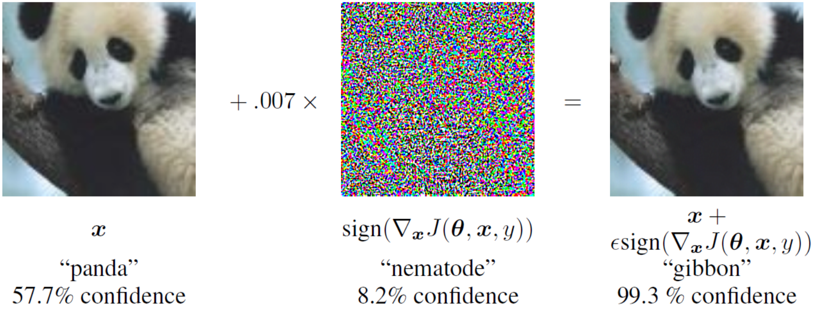
$\text{sign}(\nabla_{\boldsymbol{x}} J(\boldsymbol{\theta}, \boldsymbol{x}, y))$ "nematode" 8.2% confidence

$\boldsymbol{x} + \epsilon\text{sign}(\nabla_{\boldsymbol{x}} J(\boldsymbol{\theta}, \boldsymbol{x}, y))$ "gibbon" 99.3 % confidence

Note:

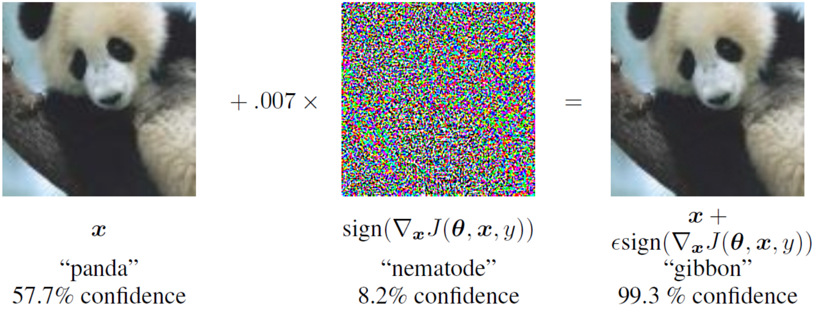
- $J(\boldsymbol{\theta}, \mathbf{x}, y)$ is the loss function.
- $\nabla_{\mathbf{x}} J(\boldsymbol{\theta}, \mathbf{x}, y)$ is the gradient of loss function with respect to the variations in input $\mathbf{x}$.

Adversarial Training



- Generating adversarial images: By using a perturbation $\mathbf{x} + \boldsymbol{\eta}$ where $\boldsymbol{\eta} = \epsilon\text{sign}(\nabla_{\mathbf{x}} J(\boldsymbol{\theta}, \mathbf{x}, y))$.

Adversarial Training



- Generating adversarial images: By using a perturbation $\mathbf{x} + \boldsymbol{\eta}$ where $\boldsymbol{\eta} = \epsilon \text{sign}(\nabla_{\mathbf{x}} J(\boldsymbol{\theta}, \mathbf{x}, y))$.
- This is called fast gradient sign method of generating adversarial examples.

Adversarial Training

- Adversarial training done using original images and adversarial images using an adversarial objective function:

$$\bar{J}(\boldsymbol{\theta}, \mathbf{x}, y) = \alpha J(\boldsymbol{\theta}, \mathbf{x}, y) + (1-\alpha) J(\boldsymbol{\theta}, \mathbf{x} + \epsilon \text{sign}(\nabla_{\mathbf{x}} J(\boldsymbol{\theta}, \mathbf{x}, y)), y).$$

- $\alpha = 0.5$ was used in experiments.

Adversarial Training

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- $\alpha = 0.5$ was used in experiments.
- Adversarial training helped in better regularization.
- Adversarial training did not reach zero error rate on adversarial images.

Adversarial Training

- Adversarial training done using original images and adversarial images using an adversarial objective function:

$$\tilde{J}(\boldsymbol{\theta}, \mathbf{x}, y) = \alpha J(\boldsymbol{\theta}, \mathbf{x}, y) + (1 - \alpha) J(\boldsymbol{\theta}, \mathbf{x} + \epsilon \operatorname{sign}(\nabla_{\mathbf{x}} J(\boldsymbol{\theta}, \mathbf{x}, y)), y).$$

- $\alpha = 0.5$ was used in experiments.
- Adversarial training helped in better regularization.
- Adversarial training did not reach zero error rate on adversarial images.
- Larger models with more units in hidden layers, early stopping on adversarial validation set error, helped to decrease error rate from 89.4% on adversarial examples to 17.9%.

Adversarial Training

Other Experiments:

- Control experiments were performed by adding random uniform noise $\pm\epsilon$ to each pixel of an image to generate adversarial samples.
- This model achieved 86.2% error rate on the adversarial examples created by adding random uniform noise.
- But the model achieved 90.4% error rate on adversarial examples created by fast gradient sign method.

Adversarial Training

Other Experiments:

- Control experiments were performed by generating adversarial examples using small rotations or by addition of scaled gradient.
- Such experiments lead to smooth objective functions.
- However training on such adversarial examples was found to be ineffective.

Adversarial Training

Other Experiments:

- Experiments were performed by perturbing the activations of hidden layers (but not the output layer).
- When hidden layers have sigmoid activations, perturbing hidden layer activations helped to improve regularization.
- When hidden layer activations have ReLU type activation functions, the perturbations did not give much improvements.
- Exercise: Check why last layer was not considered for perturbations!

Adversarial Training

Observations:

- An adversarial example generated for one model is misclassified by other models too.

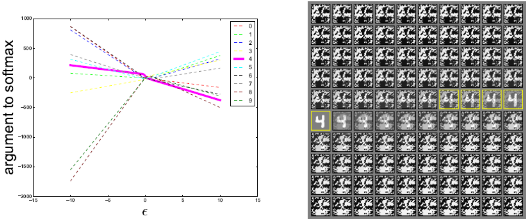

Figure 4: By tracing out different values of ϵ, we can see that adversarial examples occur reliably for almost any sufficiently large value of ϵ provided that we move in the correct direction. Correct classifications occur only on a thin manifold where $\boldsymbol{x}$ occurs in the data. Most of $\mathbb{R}^n$ consists of adversarial examples and *rubbish class examples* (see the appendix). This plot was made from a naively trained maxout network. Left) A plot showing the argument to the softmax layer for each of the 10 MNIST classes as we vary ϵ on a single input example. The correct class is 4. We see that the unnormalized log probabilities for each class are conspicuously piecewise linear with ϵ and that the wrong classifications are stable across a wide region of ϵ values. Moreover, the predictions become very extreme as we increase ϵ enough to move into the regime of rubbish inputs. Right) The inputs used to generate the curve (upper left = negative ϵ, lower right = positive ϵ, yellow boxes indicate correctly classified inputs).

Adversarial Training

Observations:

- When other models misclassify an adversarial example, the class labels given by different models are almost the same.
- One hypothesis to validate this observation: most neural networks try to approximate to a reference linear model on the training set.

Adversarial Training

Optimization problem:

- Adversarial training can be posed as the following training problem:

$$\min_{\theta} \sum_{i=1}^{N} \max_{\delta \in \Delta} \ell(f_\theta(x^i + \delta), y^i)$$

where $\Delta = \{\vartheta : \|\vartheta\|_\infty \leq \epsilon\}$ for some fixed $\epsilon > 0$.

Adversarial Training

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Note: FGSM can be seen to consider an inaccurate approximate solution to the inner problem as $\delta^* = \epsilon \text{sign}(\nabla_x \ell(f(x), y))$.

Adversarial Training

Towards Deep Learning Models Resistant to Adversarial Attacks

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Adversarial Training

PGD Method:

Algorithm 1 PGD adversarial training for T epochs, given some radius ϵ, adversarial step size α and N PGD steps and a dataset of size M for a network f_θ

for $t = 1 \dots T$ **do**
  for $i = 1 \dots M$ **do**
    // Perform PGD adversarial attack
    $\delta = 0$ *// or randomly initialized*
    for $j = 1 \dots N$ **do**
      $\delta = \delta + \alpha \cdot \mathrm{sign}(\nabla_\delta \ell(f_\theta(x_i + \delta), y_i))$
      $\delta = \max(\min(\delta, \epsilon), -\epsilon)$
    end for
    $\theta = \theta - \nabla_\theta \ell(f_\theta(x_i + \delta), y_i)$ *// Update model weights with some optimizer, e.g. SGD*
  end for
end for

Adversarial Training

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  where $\Delta = \{\vartheta : \|\vartheta\|_\infty \leq \epsilon\}$ for some fixed $\epsilon > 0$.

- Landscape of local maxima of the inner maximization problem was investigated to check the behavior.
- Done by restarting PGD from different points in the ℓ_∞ balls around the data points.
- Existence of multiple maxima; in each case, the loss plateaus quickly.
- Exercise: How to check for distinct maxima?

Adversarial Training

Loss values for different restarts:

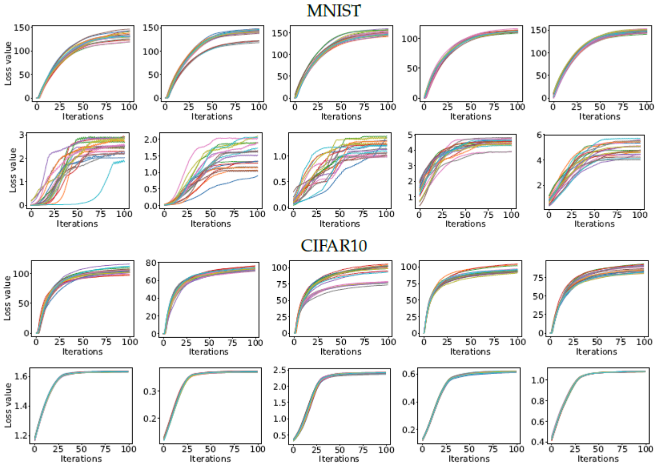


Figure : Loss function value over PGD iterations for 20 random restarts on random examples. The 1st and 3rd rows correspond to standard networks, while the 2nd and 4th to adversarially trained ones.

Adversarial Training

Loss value concentration over multiple restarts:

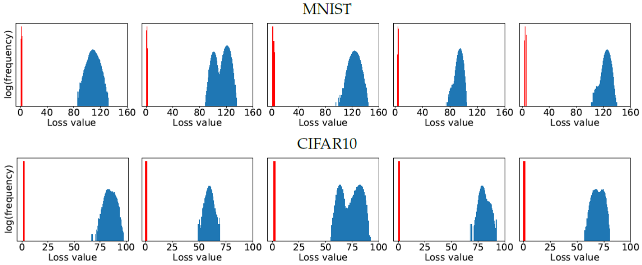
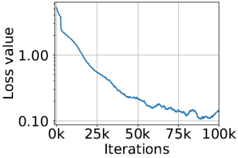


Figure : Values of the local maxima given by the cross-entropy loss for five examples from the MNIST and CIFAR10 evaluation datasets. For each example, we start projected gradient descent (PGD) from 10^5 uniformly random points in the ℓ_∞-ball around the example and iterate PGD until the loss plateaus. The blue histogram corresponds to the loss on a standard network, while the red histogram corresponds to the adversarially trained counterpart. The loss is significantly smaller for the adversarially trained networks, and the final loss values are very concentrated without any outliers.

The average loss of final iterate over 10^5 restarts follows a concentrated distribution without extreme outliers.

Adversarial Training

Loss behavior on adversarial examples:



(a) MNIST (b) CIFAR10

Figure : Cross-entropy loss on adversarial examples during training. The plots show how the adversarial loss on training examples evolves during training the MNIST and CIFAR10 networks against a PGD adversary. The sharp drops in the CIFAR10 plot correspond to decreases in training step size. These plots illustrate that we can consistently reduce the value of the inner problem of the saddle point formulation (2.1), thus producing an increasingly robust classifier.

MNIST:

- Training was done on CNN with two conv layers with 32 and 64 filters respectively, followed by 2×2 max-pooling and fully connected layer of size 1024.
- $\epsilon = 0.3$
- 40 iterations of PGD with step size 0.01.

Adversarial Training

Loss behavior on adversarial examples:

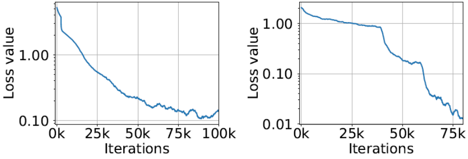


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Figure : Cross-entropy loss on adversarial examples during training. The plots show how the adversarial loss on training examples evolves during training the MNIST and CIFAR10 networks against a PGD adversary. The sharp drops in the CIFAR10 plot correspond to decreases in training step size. These plots illustrate that we can consistently reduce the value of the inner problem of the saddle point formulation (2.1), thus producing an increasingly robust classifier.

CIFAR:

- Training was done on Resnet.
- $\epsilon = 8$
- 40 iterations of PGD with step size 0.02.

Adversarial Training

Types of adversarial attakcs:

- Black box attacks: when the adversary has no knowledge of the architecture used for training.
- White box attacks: when the adversary has complete knowledge of the architecture used for training.

Adversarial Training

Types of adversarial examples:

- Non-targeted: add perturbation to an image so that the network predicts a different class label other than the original class label.
- Targeted: add perturbation to an image so that the network predicts a particular class label of the adversary's choice different from the original class label.