Introduction to AI and ML

Matrix Project

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Question:Q71 of JEE Main 2013(Code P)

The circle passing through (1,-2) and touching the axis of x at (3,0) also passes through the point:

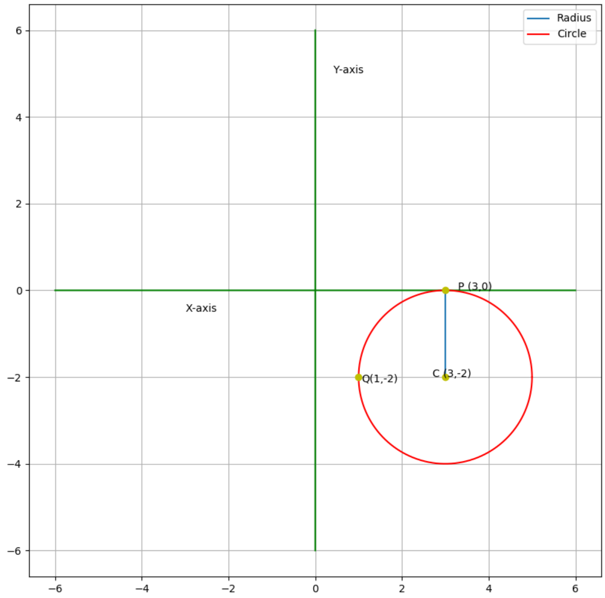
a)(-5,2) b)(2,-5) c)(5,-2) d)(-2,5)

Matrix form of the question

The circle passing through $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ and touching the axis of x at $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ also passes through the point:

1) $\begin{bmatrix} -5 \\ 2 \end{bmatrix}$ 2) $\begin{bmatrix} 2 \\ -5 \end{bmatrix}$ 3) $\begin{bmatrix} 5 \\ -2 \end{bmatrix}$ 4) $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$

Figure 1



Solution

As x-axis is tangent to circle at $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$,

Also we know, if PQ $\perp$ AB, $(P - Q)^T(A - B) = 0$,thus

$$\begin{bmatrix} k & 0 \end{bmatrix} (\mathbf{C} - \begin{bmatrix} 3 \\ 0 \end{bmatrix}) = 0$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{C} = 3 \tag{1}$$

As standard form of circle in matrix form is:

$$||\mathbf{x} - \mathbf{C}||^2 = r^2$$

$$\mathbf{x}^{\mathbf{T}}\mathbf{x} - 2\mathbf{C}^{\mathbf{T}}\mathbf{x} = r^2 - \mathbf{C}^{\mathbf{T}}\mathbf{C}$$

Given, $\begin{bmatrix} 3 & 0 \end{bmatrix}$ and $\begin{bmatrix} 1 & -2 \end{bmatrix}$ lie of the circle,

$$\begin{bmatrix} 3 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \end{bmatrix} - 2\mathbf{C}^{\mathbf{T}} \begin{bmatrix} 3 \\ 0 \end{bmatrix} = r^2 - \mathbf{C}^{\mathbf{T}}\mathbf{C} \tag{2}$$

$$\begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix} - 2\mathbf{C}^{\mathbf{T}} \begin{bmatrix} 1 \\ -2 \end{bmatrix} = r^2 - \mathbf{C}^{\mathbf{T}}\mathbf{C} \tag{3}$$

From above two equations:

$$9 - 2\mathbf{C}^{\mathbf{T}} \begin{bmatrix} 3 \\ 0 \end{bmatrix} = 5 - 2\mathbf{C}^{\mathbf{T}} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\mathbf{C}^{\mathbf{T}} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 \quad or \quad \begin{bmatrix} 1 & 1 \end{bmatrix} \mathbf{C} = 1 \tag{4}$$

From equations (1) and (4):

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \mathbf{C} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Using elementary transformations on augmented matrix:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 3 \end{bmatrix} \leftrightarrow \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 2 \end{bmatrix} \leftrightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & -1 & 2 \end{bmatrix} \leftrightarrow \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix}$$

Then

$$\mathbf{C} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

Substituting the value of **C** in equation 2:

$$9 - 2\begin{bmatrix}3 & -2\end{bmatrix}\begin{bmatrix}3\\0\end{bmatrix} = r^2 - \begin{bmatrix}3 & -2\end{bmatrix}\begin{bmatrix}3\\-2\end{bmatrix}$$

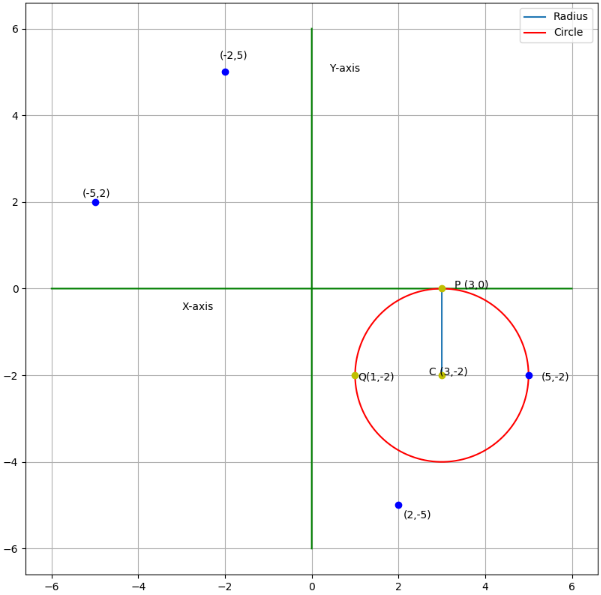
$$r = 2$$

So,the equation of circle becomes:

$$||\mathbf{x} - \begin{bmatrix}3\\-2\end{bmatrix}||^2 = 4$$

Clearly,the only point satisfying the circle equation is: $\begin{bmatrix}5\\-2\end{bmatrix}$

Figure 2


Radius
Circle
(-2,5)
Y-axis
(-5,2)
P (3,0)
X-axis
Q(1,-2)
C (3,-2)
(5,-2)
(2,-5)
6
4
2
0
−2
−4
−6
−6
−4
−2
0
2
4
6