AI1103 : Assignment 3

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Download all python codes from

https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment3/codes

and latex codes from

https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment3/Assignment3.tex

PROBLEM(GATE-26)

A fair dice is tossed two times. The probability that the second toss result in a value that is higher than the first toss is

(A)$\frac{2}{36}$ (B)$\frac{2}{6}$ (C)$\frac{5}{12}$ (D)$\frac{1}{2}$

SOLUTION(GATE-26)

Given, a fair die, which is tossed twice. Let the random variable $X_i \in \{1,2,3,4,5,6\}, i = 1,2,$ represent the outcome of the number on the die in the first, second toss respectively.

The probability mass function (PMF) for a fair die is expressed as

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{6}, & 1 \le n \le 6 \\ 0, & otherwise \end{cases} \tag{26.1}$$

Using (26.1), the cumulative distribution function (CDF) is obtained to be

$$F_{X_i}(r) = \Pr(X_i \le n) = \begin{cases} \frac{n}{6}, & 1 \le n \le 6 \\ 1, & n \ge 7 \\ 0, & otherwise \end{cases} \tag{26.2}$$

As the desired outcome $X_1 < X_2$, is of the form

$$X_1 \le n-1, X_2 = n \tag{26.3}$$

the required probability $Pr(X_1 < X_2)$ is given by

$$\sum_{n=1}^{n=6} p_{X_2}(n) \times Pr(X_1 \le n-1/X_2 = n) \tag{26.4}$$

Since X_1, X_2 are independent,

$$Pr(X_1 \le n-1/X_2 = n) = Pr(X_1 \le n-1) \tag{26.5}$$

Therefore, (26.4) simplifies to

$$Pr(X_1 < X_2) = \sum_{n=1}^{n=6} p_{X_2}(n) \times F_{X_1}(n-1) \tag{26.6}$$

Substituting (26.1) and (26.2), we get

$$Pr(X_1 < X_2) = \sum_{n=1}^{n=6} \frac{1}{6} \times \frac{n-1}{6} \tag{26.7}$$

On solving, we get

$$Pr(X_1 < X_2) = \frac{5}{12}(\text{option (C)}) \tag{26.8}$$

TABLE 1: Cases and their theoretical probabilities

Case	$X_1 < X_2$	$X_1 > X_2$	$X_1 = X_2$
Probability	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

Theoretical vs Simulation results for the above mentioned cases