

# AI1103 : Assignment 2

Yashas Tadikamalla - AI20BTECH11027

Download all python codes from

<https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment2/codes>

and latex codes from

<https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment2/Assignment2.tex>

## PROBLEM(5.25)

A bag contains 2 white and 1 red balls. One ball is drawn at random and then put back in the box after noting its colour. The process is repeated again. If  $X$  denotes the number of red balls recorded in the two draws, describe  $X$ .

## SOLUTION(5.25)

Given, a bag containing 2 white and 1 red balls. Let the random variable  $X_i \in \{0, 1\}, i = 1, 2$ , represent the outcome of the colour of the ball drawn in the first, second attempts.  $X_i = 0, X_i = 1$  denote a white ball, red ball being drawn respectively, in the  $i^{th}$  attempt.

Define

$$X = X_1 + X_2 \quad (5.25.1)$$

so that  $X \in \{0, 1, 2\}$  represents a random variable denoting the number of red balls drawn in both the attempts. Then,  $X$  has a binomial distribution with

$$Pr(X = k) = {}^nC_k p^k q^{n-k} \quad (5.25.2)$$

where,

$$n = 2 \quad (5.25.3)$$

$p$  = probability of success = probability of drawing a red ball =  $Pr(X_i = 1)$

$$p = \frac{1}{3} \quad (5.25.4)$$

$q$  = probability of failure =  $1 - p$

$$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3} \quad (5.25.5)$$

Hence, on substituting and simplifying, we get

$$Pr(X = 0) = \frac{4}{9} \quad (5.25.6)$$

$$Pr(X = 1) = \frac{4}{9} \quad (5.25.7)$$

$$Pr(X = 2) = \frac{1}{9} \quad (5.25.8)$$

Using (5.25.2), we get the following probability distribution.

TABLE 1: Probability distribution of  $X$

Condition	$X = 0$	$X = 1$	$X = 2$
Probability	${}^2C_0 p^0 q^2$	${}^2C_1 p^1 q^1$	${}^2C_2 p^2 q^0$

So, if we conduct this experiment 9000 times, theoretically, we get the following frequency distribution.

TABLE 2: Frequency distribution of  $X$

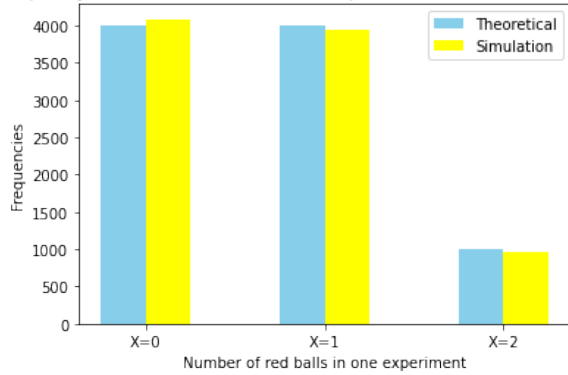
Condition	$X = 0$	$X = 1$	$X = 2$
Frequency	4000	4000	1000

P.T.O

Here are the plots describing  $X$ , after the experiment is conducted 9000 times.

### Theoretical vs Simulation

Frequency distribution of  $X$ , after the experiment is conducted 9000 times



Probability distribution of  $X$ , after the experiment is conducted 9000 times

