

AI1103 : Assignment 4

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Download all python codes from

<https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment4/codes>

and latex codes from

<https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment4/Assignment4.tex>

Here,

$$p = \frac{3}{4}, q = 1 - p = \frac{1}{4} \quad (32.8)$$

$$a = 1.8 \Rightarrow k \leq 0.8n \quad (32.9)$$

$$\therefore \lim_{n \rightarrow \infty} P(\bar{X}_n \leq 1.8) = \lim_{n \rightarrow \infty} \sum_{k=0}^{0.8n} \frac{{}^nC_k 3^k}{4^n} \quad (32.10)$$

On solving (32.10), we get

$$\lim_{n \rightarrow \infty} P(\bar{X}_n \leq 1.8) = 1 \quad (32.11)$$

GATE-2015-MA-PROBLEM(32)

Let X_1, X_2, \dots , be a sequence of independent and identically distributed random variables with $P(X_1 = 1) = \frac{1}{4}$ and $P(X_1 = 2) = \frac{3}{4}$. If $\bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i$, for $n = 1, 2, \dots$, then $\lim_{n \rightarrow \infty} P(\bar{X}_n \leq 1.8)$ is equal to

GATE-2015-MA-SOLUTION(32)

Given,

$$Pr(X_1 = 1) = \frac{1}{4}, Pr(X_2 = 2) = \frac{3}{4} \quad (32.1)$$

As X_1, X_2, \dots , are identically distributed random variables, $\forall i \in \{1, 2, \dots, n\}$

$$Pr(X_i = 1) = \frac{1}{4}, Pr(X_i = 2) = \frac{3}{4} \quad (32.2)$$

Also,

$$\therefore P(X_i = 1) + P(X_i = 2) = 1 \quad (32.3)$$

$$\therefore X_i \in \{1, 2\} \quad (32.4)$$

Let $X \in \{1, 2\}$ be the random variable denoting the value of $X_i, \forall i \in \{1, 2, \dots, n\}$. Let

$$n(X = 1) = n - k, n(X = 2) = k \quad (32.5)$$

$$\Rightarrow \bar{X}_n = \frac{1}{n} \sum_{i=1}^n X_i = \frac{n + k}{n} \quad (32.6)$$

To find : $\lim_{n \rightarrow \infty} Pr(\bar{X}_n \leq a)$

The cumulative distribution function (CDF) for a binomial distribution is given by

$$F_X(r) = Pr(X \leq r) = \sum_{k=0}^r {}^nC_k p^k q^{n-k} \quad (32.7)$$