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# AI1103: Assignment 2

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Download all python codes from

https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment2/codes

and latex codes from

https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment2/Assignment2.tex

# PROBLEM(5.25)

A bag contains 2 white and 1 red balls. One ball is drawn at random and then put back in the box after noting its colour. The process is repeated again. If X denotes the number of red balls recorded in the two draws, describe X.

### Solution(5.25)

Given, a bag containing 2 white and 1 red balls. Let the random variable  $X_i \in \{0, 1\}, i = 1, 2$ , represent the outcome of the colour of the ball drawn in the first, second attempts.  $X_i = 0, X_i = 1$  denote a white ball, red ball being drawn respectively, in the  $i^{th}$  attempt.

As the ball drawn in the first attempt is replaced in the bag, for both the attempts, the number of balls of a specified colour, and their probability mass function's (pmf's) remain the same. i.e,

$$n(X_i = 0) = 2 (5.25.1)$$

$$n(X_i = 1) = 1 (5.25.2)$$

$$\therefore n(X_i = 0) + n(X_i = 1) = 3$$
 (5.25.3)

and

$$\Pr(X_i = j) = \begin{cases} \frac{2}{3}, & j = 0\\ \frac{1}{3}, & j = 1\\ 0, & otherwise \end{cases}$$
 (5.25.4)

Define

$$X = X_1 + X_2 \tag{5.25.5}$$

so that  $X \in \{0, 1, 2\}$  represents a random variable denoting the number of red balls drawn in both the attempts. Then, X has a binomial distribution with

$$Pr(X = k) = {}^{n}C_{k}p^{k}q^{n-k}$$
 (5.25.6)

where,

$$n = 2$$
 (5.25.7)

p = probability of success = probability of drawinga red ball =  $Pr(X_i = 1)$ 

$$p = \frac{1}{3} \tag{5.25.8}$$

q = probability of failure = 1 - p

$$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$$
 (5.25.9)

Hence, on substituting and simplifying, we get

$$Pr(X = 0) = \frac{4}{9}, Pr(X = 1) = \frac{4}{9}, Pr(X = 2) = \frac{1}{9}$$
(5.25.10)

Therefore, the pmf for X (theoretical) is

$$\Pr(X = i) = \begin{cases} \frac{4}{9}, & i = 0\\ \frac{4}{9}, & i = 1\\ \frac{1}{9}, & i = 2\\ 0, & otherwise \end{cases}$$
 (5.25.11)

TABLE 0: Probability distribution of X

Condition	X = 0	X = 1	X = 2
Probability	$^2C_0p^0q^2$	${}^2C_1p^1q^1$	$^2C_2p^2q^0$

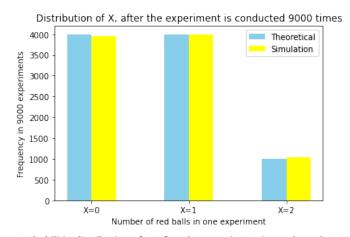
So, if we conduct this experiment 9000 times, theoretically, we get

TABLE 0: Frequency distribution of X

Condition	X = 0	X = 1	X = 2
Frequency	4000	4000	1000

Here are the plots describing X, after the experiment is conducted 9000 times.

# Theoretical vs Simulation



Probability distribution of X, after the experiment is conducted 9000 times

