AI1103 : Assignment 2

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Download all python codes from

https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment2/codes

and latex codes from

https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment2/Assignment2.tex

Problem(5.25)

A bag contains 2 white and 1 red balls. One ball is drawn at random and then put back in the box after noting its colour. The process is repeated again. If X denotes the number of red balls recorded in the two draws, describe X.

Solution(5.25)

Given, a bag containing 2 white and 1 red balls. Let the random variable $X_i \in \{0,1\}, i = 1,2$, represent the outcome of the colour of the ball drawn in the first, second attempts. $X_i = 0, X_i = 1$ denote a white ball, red ball being drawn respectively, in the i^{th} attempt.

As the ball drawn in the first attempt is replaced in the bag, for both the attempts, the number of balls of a specified colour, and their probability mass function's (pmf's) remain the same. i.e,

$$n(X_i = 0) = 2 \quad (5.25.1)$$

$$n(X_i = 1) = 1 \quad (5.25.2)$$

$$\therefore n(X_i = 0) + n(X_i = 1) = 3 \quad (5.25.3)$$

and

$$\Pr(X_i = j) = \begin{cases} \frac{2}{3}, & j = 0 \\ \frac{1}{3}, & j = 1 \\ 0, & otherwise \end{cases} \quad (5.25.4)$$

Define

$$X = X_1 + X_2 \quad (5.25.5)$$

so that $X \in \{0,1,2\}$ represents a random variable denoting the number of red balls drawn in both the attempts. Then, X has a binomial distribution with

$$Pr(X = k) = {}^nC_k p^k q^{n-k} \quad (5.25.6)$$

where,

$$n = 2 \quad (5.25.7)$$

p = probability of success = probability of drawing a red ball = $Pr(X_i = 1)$

$$p = \frac{1}{3} \quad (5.25.8)$$

q = probability of failure = $1 - p$

$$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3} \quad (5.25.9)$$

Hence, on substituting and simplifying, we get

$$Pr(X = 0) = \frac{4}{9}, Pr(X = 1) = \frac{4}{9}, Pr(X = 2) = \frac{1}{9} \quad (5.25.10)$$

Therefore, the pmf for X (theoretical) is

$$\Pr(X = i) = \begin{cases} \frac{4}{9}, & i = 0 \\ \frac{4}{9}, & i = 1 \\ \frac{1}{9}, & i = 2 \\ 0, & otherwise \end{cases} \quad (5.25.11)$$

TABLE 1: Probability distribution of X

Condition	$X = 0$	$X = 1$	$X = 2$
Probability	${}^2C_0 p^0 q^2$	${}^2C_1 p^1 q^1$	${}^2C_2 p^2 q^0$

So, if we conduct this experiment 9000 times, theoretically, we get

TABLE 2: Frequency distribution of X

Condition	$X = 0$	$X = 1$	$X = 2$
Frequency	4000	4000	1000

Here are the plots describing X, after the experiment is conducted 9000 times.

Theoretical vs Simulation