AI1103 : Assignment 8

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CSIR-UGC NET-Dec 2016-Problem(104)

A and *B* play a game of tossing a fair coin. *A* starts the game by tossing the coin once and *B* then tosses the coin twice, followed by *A* tossing the coin once and *B* tossing the coin twice and this continues until a head turns up. Whoever gets the first head wins the game. Then,

1) $P(B$ Wins$) > P(A$ Wins$)$
2) $P(B$ Wins$) = 2P(A$ Wins$)$
3) $P(A$ Wins$) > P(B$ Wins$)$
4) $P(A$ Wins$) = 1 - P(B$ Wins$)$

CSIR-UGC NET-Dec 2016-Solution(104)

Given, a fair coin is tossed till heads turns up.

$$p = \frac{1}{2}, q = \frac{1}{2} \qquad (104.1)$$

Given, for $m \in \mathbb{W}$

TABLE 1: Toss and its player

Toss	Player
$(3m + 1)^{th}$	A
$(3m + 2)^{th}$	B
$(3m + 3)^{th}$	B

Define $X \sim Geometric(p)$. Then, the probability that someone wins at the kth trial is

$$p_X(k) = pq^{k-1} \qquad (104.2)$$

Let $Y \in \{1, 2\}$ denote the winning player. $Y = 1$ denotes *A* wins, while $Y = 2$ denotes *B* wins. From (1), (104.2),

$$p_Y(1) = \sum_{m=0}^{\infty} p_X(3m + 1) \qquad (104.3)$$

$$= \sum_{m=0}^{\infty} pq^{3m} = \frac{p}{1 - q^3} \qquad (104.4)$$

$$p_Y(2) = \sum_{m=0}^{\infty} (p_X(3m + 2) + p_X(3m + 3)) \qquad (104.5)$$

$$= \sum_{m=0}^{\infty} (pq^{3m+1} + pq^{3m+2}) \qquad (104.6)$$

$$= \frac{pq(1 + q)}{1 - q^3} \qquad (104.7)$$

So,

$$p_Y(1) + p_Y(2) = \frac{p(1 + q + q^2)}{1 - q^3} = \frac{p}{1 - q} \qquad (104.8)$$

Substituting (104.1) in (104.8), we get

$$p_Y(1) + p_Y(2) = 1 \qquad (104.9)$$
$$\Rightarrow p_Y(1) = 1 - p_Y(2) \qquad (104.10)$$

Solving (104.4),(104.7) with (104.1), we get

$$p_Y(1) = 0.5714, p_Y(2) = 0.4285 \qquad (104.11)$$
$$\Rightarrow p_Y(1) > p_Y(2) \qquad (104.12)$$

Therefore, options $3), 4)$ are correct.

TABLE 2: States and their notations

Notation	State
X_1	A's turn
X_2	B's first turn
X_3	B's second turn
X_4	A player wins

The state transition matrix for the Markov process is

$$P = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{cccc} X_1 & X_2 & X_3 & X_4 \\ \left[\begin{array}{cccc} 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{array} \right] \end{array}$$

The three states X_1, X_2, X_3 are transient, while X_4 is absorbent.

Markov chain diagram