AI1103 : Assignment 4

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Download all python codes from

https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment4/codes

and latex codes from

https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment4/Assignment4.tex

GATE-2015-MA-Problem(32)

Let $X_1, X_2, \dots,$ be a sequence of independent and identically distributed random variables with $P(X_1 = 1) = \frac{1}{4}$ and $P(X_1 = 2) = \frac{3}{4}$. If $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$, for $n = 1, 2, \dots,$ then $\lim_{n\to\infty} P(\bar{X}_n \le 1.8)$ is equal to

GATE-2015-MA-Solution(32)

Given,

$$Pr(X_1 = 1) = \frac{1}{4}, Pr(X_2 = 2) = \frac{3}{4} \tag{32.1}$$

As $X_1, X_2, \dots,$ are identically distributed random variables, $\forall i \in \{1, 2, \dots, n\}$

$$Pr(X_i = 1) = \frac{1}{4}, Pr(X_i = 2) = \frac{3}{4} \tag{32.2}$$

Also,

$$\because P(X_i = 1) + P(X_i = 2) = 1 \tag{32.3}$$

$$\therefore X_i \in \{1, 2\} \tag{32.4}$$

Let $X \in \{1, 2\}$ be the random variable denoting the value of $X_i, \forall i \in \{1, 2, \dots, n\}$. Let

$$n(X = 1) = n - k, n(X = 2) = k \tag{32.5}$$

$$\Rightarrow \bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i = \frac{n+k}{n} \tag{32.6}$$

$$\therefore \bar{X}_n \le 1.8 \Rightarrow k \le 0.8n \tag{32.7}$$

The cumulative distribution function (CDF) for a binomial distribution is given by

$$F_X(r) = Pr(X \le r) = \sum_{k=0}^{r} {}^nC_k p^k q^{n-k} \tag{32.8}$$

$$\therefore P(\bar{X}_n \le 1.8) = \sum_{k=0}^{0.8n} {}^nC_k p^k q^{n-k} \tag{32.9}$$

where,

$$p = \frac{3}{4}, q = 1 - p = \frac{1}{4} \tag{32.10}$$

$$\Rightarrow \lim_{n\to\infty} P(\bar{X}_n \le 1.8) = \lim_{n\to\infty} \sum_{k=0}^{0.8n} \frac{{}^nC_k 3^k}{4^n} \tag{32.11}$$

On solving (32.11), we get

$$\lim_{n\to\infty} P(\bar{X}_n \le 1.8) = 1 \tag{32.12}$$