AI1103 : Assignment 8

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Download all python codes from

```
https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment8/codes
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and latex codes from

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https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment8/Assignment8.tex
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CSIR-UGC NET-Dec 2016-Problem(104)

A and B play a game of tossing a fair coin. A starts the game by tossing the coin once and B then tosses the coin twice, followed by A tossing the coin once and B tossing the coin twice and this continues until a head turns up. Whoever gets the first head wins the game. Then,

1) $P(B \text{ Wins}) > P(A \text{ Wins})$
2) $P(B \text{ Wins}) = 2P(A \text{ Wins})$
3) $P(A \text{ Wins}) > P(B \text{ Wins})$
4) $P(A \text{ Wins}) = 1 - P(B \text{ Wins})$

CSIR-UGC NET-Dec 2016-Solution(104)

Given, a fair coin is tossed till heads turns up.

$$p = \frac{1}{2}, q = \frac{1}{2} \tag{104.1}$$

Given, for $m \in \mathbb{W}$

TABLE 1: Toss and its player

Toss	Player
$(3m+1)^{th}$	A
$(3m+2)^{th}$	B
$(3m+3)^{th}$	B

Define $X \sim Geometric(p)$. Then, the probability that someone wins at the kth trial is

$$p_X(k) = pq^{k-1} \tag{104.2}$$

Let $Y \in \{1, 2\}$ denote the winning player. $Y = 1$ denotes A wins, while $Y = 2$ denotes B wins. From (1), (104.2),

$$p_Y(1) = \sum_{m=0}^{\infty} p_X(3m+1) \tag{104.3}$$

$$= \sum_{m=0}^{\infty} pq^{3m} = \frac{p}{1-q^3} \tag{104.4}$$

$$p_Y(2) = \sum_{m=0}^{\infty} (p_X(3m+2) + p_X(3m+3)) \tag{104.5}$$

$$= \sum_{m=0}^{\infty} (pq^{3m+1} + pq^{3m+2}) \tag{104.6}$$

$$= \frac{pq(1+q)}{1-q^3} \tag{104.7}$$

So,

$$p_Y(1) + p_Y(2) = \frac{p(1+q+q^2)}{1-q^3} = \frac{p}{1-q} \tag{104.8}$$

Substituting (104.1) in (104.8), we get

$$p_Y(1) + p_Y(2) = 1 \tag{104.9}$$

$$\Rightarrow p_Y(1) = 1 - p_Y(2) \tag{104.10}$$

Solving (104.4),(104.7) with (104.1), we get

$$p_Y(1) = 0.5714, p_Y(2) = 0.4285 \tag{104.11}$$

$$\Rightarrow p_Y(1) > p_Y(2) \tag{104.12}$$

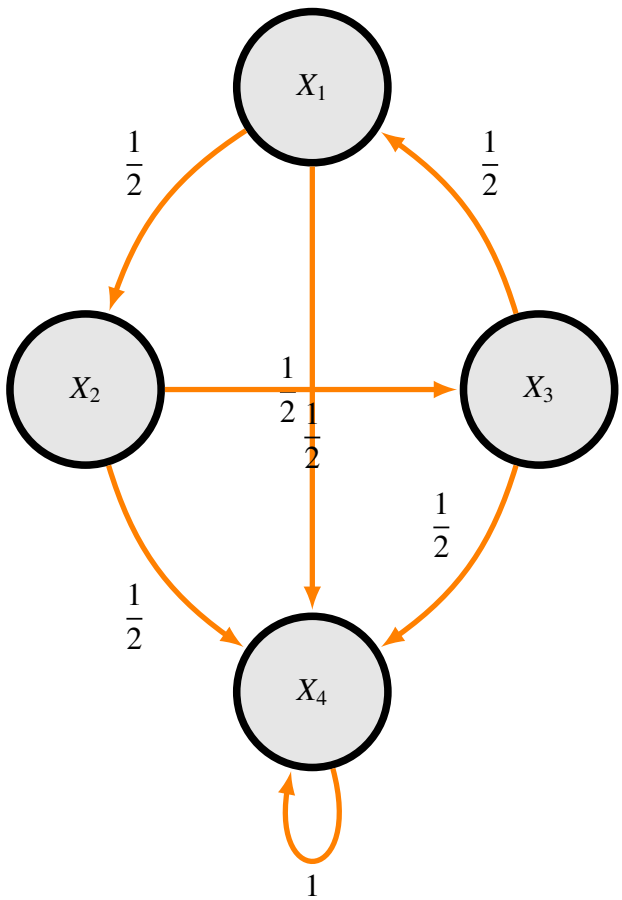
Therefore, options 3), 4) are correct.

TABLE 2: States and their notations

Notation	State
X_1	A's turn
X_2	B's first turn
X_3	B's second turn
X_4	A player wins

The state transition matrix for the Markov process is

$$P = \begin{array}{c} \\ X_1 \\ X_2 \\ X_3 \\ X_4 \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & X_3 & X_4 \end{array} \\ \begin{bmatrix} 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

The three states X_1, X_2, X_3 are transient, while X_4 is absorbent.

Markov chain diagram