AI1103 : Assignment 3

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Download all python codes from

https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment3/codes

and latex codes from

https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment3/Assignment3.tex

Problem(GATE-26)

A fair dice is tossed two times. The probability that the second toss result in a value that is higher than the first toss is

(A)$\frac{2}{36}$ (B)$\frac{2}{6}$ (C)$\frac{5}{12}$ (D)$\frac{1}{2}$

Solution(GATE-26)

Given, a fair die, which is tossed twice. Let the random variable $X_i \in \{1,2,3,4,5,6\}, i = 1,2$, represent the outcome of the number on the die in the first, second toss. X_1, X_2 denote the result of the first, second toss respectively.

As the die is given to be fair, the probability mass function (pmf) is expressed as

$$\Pr(X_i = n) = \begin{cases} \frac{1}{6}, & 1 \le n \le 6 \\ 0, & otherwise \end{cases} \tag{26.1}$$

The desired outcome $X_1 < X_2$, is of the form

$$X_1 = n, X_2 > n \tag{26.2}$$

The probability for this outcome is of the form

$$Pr(X_1 = n, X_2 > n) = Pr(X_1 = n) \times Pr(X_2 > n / X_1 = n) \tag{26.3}$$

As X_1, X_2 are independent, (26.3) simplifies to

$$Pr(X_1 = n, X_2 > n) = Pr(X_1 = n) \times Pr(X_2 > n) \tag{26.4}$$

Using (26.1), we can conclude that

$$\Pr(X_i > n) = \begin{cases} \frac{6-n}{6}, & 1 \le n \le 6 \\ 1, & n \le 0 \\ 0, & otherwise \end{cases} \tag{26.5}$$

Hence the required probability is

$$\sum_{n=1}^{n=6} Pr(X_1 = n, X_2 > n) = \sum_{n=1}^{n=6} Pr(X_1 = n) \times Pr(X_2 > n) \tag{26.6}$$

From (26.1) and (26.5),

$$\sum_{n=1}^{n=6} Pr(X_1 = n, X_2 > n) = \sum_{n=1}^{n=6} \frac{1}{6} \times \frac{6-n}{6} \tag{26.7}$$

On solving (26.7), we get

$$\sum_{n=1}^{n=6} Pr(X_1 = n, X_2 > n) = \frac{5}{12} \tag{26.8}$$

$$\therefore Pr(X_1 < X_2) = \frac{5}{12}(\text{option (C)}) \tag{26.9}$$

TABLE 1: Theoretical probabilities for different possible cases

Condition	$X_1 < X_2$	$X_1 > X_2$	$X_1 = X_2$
Probability	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

Here is the plot describing the Theoretical vs Simulation results for the above mentioned cases