AI1103 : Assignment 3

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Download all python codes from

https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment3/codes

and latex codes from

https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment3/Assignment3.tex

Problem(GATE-26)

A fair dice is tossed two times. The probability that the second toss result in a value that is higher than the first toss is

(A)$\frac{2}{36}$ (B)$\frac{2}{6}$ (C)$\frac{5}{12}$ (D)$\frac{1}{2}$

Solution(GATE-26)

Given, a fair die, which is tossed twice. Let the random variable $X_i \in \{1,2,3,4,5,6\}, i = 1,2$, represent the outcome of the number on the die in the first, second toss respectively.

The probability mass function (PMF) for a fair die is expressed as

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{6}, & 1 \le n \le 6 \\ 0, & otherwise \end{cases} \tag{26.1}$$

Using (26.1), the cumulative distribution function (CDF) is obtained to be

$$F_{X_i}(r) = \Pr(X_i \le r) = \begin{cases} \frac{r}{6}, & 1 \le r \le 6 \\ 1, & r \ge 7 \\ 0, & otherwise \end{cases} \tag{26.2}$$

$$X_1 < X_2 \Rightarrow X_2 = k, X_1 \le k-1 \tag{26.3}$$

$\because X_1, X_2$ are independent,

$$Pr(X_1 < X_2) = \sum_{X_2} F_{X_1}(X_2 - 1) \tag{26.4}$$

After unconditioning (26.4), we get

$$Pr(X_1 < X_2) = \sum_{k=1}^{6} p_{X_2}(k) F_{X_1}(k-1) \tag{26.5}$$

Substituting (26.1) and (26.2), we get

$$Pr(X_1 < X_2) = \sum_{k=1}^{6} \frac{1}{6}\left(\frac{k-1}{6}\right) \tag{26.6}$$

On solving, we get

$$Pr(X_1 < X_2) = \frac{5}{12} \text{(option (C))} \tag{26.7}$$

TABLE 1: Cases and their theoretical probabilities

Case	$X_1 < X_2$	$X_1 > X_2$	$X_1 = X_2$
Probability	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{6}$

Theoretical vs Simulation results for the above mentioned cases