

AI1103 : Assignment 3

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Download all python codes from

<https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment3/codes>

and latex codes from

<https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment3/Assignment3.tex>

PROBLEM(GATE-26)

A fair dice is tossed two times. The probability that the second toss result in a value that is higher than the first toss is

- (A) $\frac{2}{36}$ (B) $\frac{2}{6}$ (C) $\frac{5}{12}$ (D) $\frac{1}{2}$

SOLUTION(GATE-26)

Given, a fair die, which is tossed twice. Let the random variable $X_i \in \{1, 2, 3, 4, 5, 6\}, i = 1, 2$, represent the outcome of the number on the die in the first, second toss. X_1, X_2 denote the result of the first, second toss respectively.

As the die is given to be fair, the probability mass function (PMF) is expressed as

$$p_{X_i}(n) = \Pr(X_i = n) = \begin{cases} \frac{1}{6}, & 1 \leq n \leq 6 \\ 0, & \text{otherwise} \end{cases} \quad (26.1)$$

The desired outcome $X_1 < X_2$, is of the form

$$X_1 \leq n-1, X_2 = n \quad (26.2)$$

The probability for this outcome is of the form

$$\begin{aligned} \Pr(X_1 \leq n-1, X_2 = n) \\ = \Pr(X_2 = n) \times \Pr(X_1 \leq n-1 | X_2 = n) \end{aligned} \quad (26.3)$$

As X_1, X_2 are independent, (26.3) simplifies to

$$\begin{aligned} \Pr(X_1 \leq n-1, X_2 = n) \\ = \Pr(X_2 = n) \times \Pr(X_1 \leq n-1) \end{aligned} \quad (26.4)$$

(26.4) can be rewritten as

$$\begin{aligned} \Pr(X_1 \leq n-1, X_2 = n) \\ = p_{X_2}(n) \times F_{X_1}(n-1) \end{aligned} \quad (26.5)$$

Using (26.1), the cumulative distribution function (CDF) is obtained to be

$$F_{X_i}(r) = \Pr(X_i \leq n) = \begin{cases} \frac{n}{6}, & 1 \leq n \leq 6 \\ 1, & n \geq 7 \\ 0, & \text{otherwise} \end{cases} \quad (26.6)$$

Hence the required probability is

$$\begin{aligned} \sum_{n=1}^{n=6} \Pr(X_1 \leq n-1, X_2 = n) \\ = \sum_{n=1}^{n=6} p_{X_2}(n) \times F_{X_1}(n-1) \end{aligned} \quad (26.7)$$

From (26.1) and (26.6),

$$\sum_{n=1}^{n=6} \Pr(X_1 \leq n-1, X_2 = n) = \sum_{n=1}^{n=6} \frac{1}{6} \times \frac{n-1}{6} \quad (26.8)$$

On solving (26.8), we get

$$\sum_{n=1}^{n=6} \Pr(X_1 \leq n-1, X_2 = n) = \frac{5}{12} \quad (26.9)$$

$$\therefore \Pr(X_1 < X_2) = \frac{5}{12} \text{ (option (C))} \quad (26.10)$$

P.T.O

TABLE 1: Theoretical probabilities for different possible cases

Condition	$X_1 < X_2$	$X_1 > X_2$	$X_1 = X_2$
Probability	$\frac{5}{12}$	$\frac{5}{12}$	$\frac{1}{12}$

Here is the plot describing the Theoretical vs Simulation results for the above mentioned cases

