

# AI1103 : Assignment 8

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Download all python codes from

<https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment8/codes>

and latex codes from

<https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment8/Assignment8.tex>

## CSIR-UGC NET-DEC 2016-PROBLEM(104)

$A$  and  $B$  play a game of tossing a fair coin.  $A$  starts the game by tossing the coin once and  $B$  then tosses the coin twice, followed by  $A$  tossing the coin once and  $B$  tossing the coin twice and this continues until a head turns up. Whoever gets the first head wins the game. Then,

- 1)  $P(B \text{ Wins}) > P(A \text{ Wins})$
- 2)  $P(B \text{ Wins}) = 2P(A \text{ Wins})$
- 3)  $P(A \text{ Wins}) > P(B \text{ Wins})$
- 4)  $P(A \text{ Wins}) = 1 - P(B \text{ Wins})$

## CSIR-UGC NET-DEC 2016-SOLUTION(104)

Given, a fair coin is tossed till heads turns up.

$$p = \frac{1}{2}, q = \frac{1}{2} \quad (104.1)$$

Given, for  $m \in \mathbb{W}$

TABLE 1: Toss and its player

Toss	Player
$(3m+1)^{th}$	$A$
$(3m+2)^{th}$	$B$
$(3m+3)^{th}$	$B$

Define  $X \sim \text{Geometric}(p)$ . Then, the probability that someone wins at the  $k^{th}$  trial is

$$p_X(k) = pq^{k-1} \quad (104.2)$$

Let  $Y \in \{1, 2\}$  denote the winning player.  $Y = 1$  denotes  $A$  wins, while  $Y = 2$  denotes  $B$  wins. From (1), (104.2),

$$p_Y(1) = \sum_{m=0}^{\infty} p_X(3m+1) \quad (104.3)$$

$$= \sum_{m=0}^{\infty} pq^{3m} = \frac{p}{1-q^3} \quad (104.4)$$

$$p_Y(2) = \sum_{m=0}^{\infty} (p_X(3m+2) + p_X(3m+3)) \quad (104.5)$$

$$= \sum_{m=0}^{\infty} (pq^{3m+1} + pq^{3m+2}) \quad (104.6)$$

$$= \frac{pq(1+q)}{1-q^3} \quad (104.7)$$

So,

$$p_Y(1) + p_Y(2) = \frac{p(1+q+q^2)}{1-q^3} = \frac{p}{1-q} \quad (104.8)$$

Substituting (104.1) in (104.8), we get

$$p_Y(1) + p_Y(2) = 1 \quad (104.9)$$

$$\Rightarrow p_Y(1) = 1 - p_Y(2) \quad (104.10)$$

Solving (104.4), (104.7) with (104.1), we get

$$p_Y(1) = 0.5714, p_Y(2) = 0.4285 \quad (104.11)$$

$$\Rightarrow p_Y(1) > p_Y(2) \quad (104.12)$$

Therefore, options 3), 4) are correct.  
P.T.O

Theoretical vs Simulation plot

