AI1103 : Assignment 5

Yashas Tadikamalla - AI20BTECH11027

Download all python codes from

https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment5/codes

and latex codes from

https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment5/Assignment5.tex

GATE-2016-S2-CS-CS/IT-Problem(5)

Suppose that a shop has an equal number of LED bulbs of two different types. The probability of an LED bulb lasting more than 100 hours given that it is of Type 1 is 0.7, and given that it is of Type 2 is 0.4. The probability that an LED bulb chosen uniformly at random lasts more than 100 hours is

GATE-2016-S2-CS-CS/IT-Solution(5)

Let the random variable $X \in \{1, 2\}$ represent the type of the chosen bulb. $X = 1$ denotes a Type 1 bulb, while $X = 2$ denotes a Type 2 bulb. Given,

$$n(X = 1) = n(X = 2) \tag{5.1}$$

$$\Rightarrow p_X(1) = p_X(2) = \frac{1}{2} \tag{5.2}$$

Let the random variable $Y \in \{0, 1\}$ represent if a bulb lasts more than 100 hours. $Y = 1$ denotes that it lasts, while $Y = 0$ denotes that it doesn't. Given,

$$p_{Y|X}(1|1) = 0.7 \tag{5.3}$$

$$p_{Y|X}(1|2) = 0.4 \tag{5.4}$$

To find : $p_Y(1)$

$$p_Y(1) = p_{Y|X}(1|1)p_X(1) + p_{Y|X}(1|2)p_X(2) \tag{5.5}$$

$$p_Y(1) = (0.7)(0.5) + (0.4)(0.5) \tag{5.6}$$

$$\therefore p_Y(1) = 0.55 \tag{5.7}$$