# AI1103 : Assignment 8

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Download all python codes from

https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment8/codes

and latex codes from

https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment8/Assignment8.tex

## CSIR-UGC NET-Dec 2016-Problem(104)

$A$ and $B$ play a game of tossing a fair coin. $A$ starts the game by tossing the coin once and $B$ then tosses the coin twice, followed by $A$ tossing the coin once and $B$ tossing the coin twice and this continues until a head turns up. Whoever gets the first head wins the game. Then,

1) $P(B \text{ Wins}) > P(A \text{ Wins})$
2) $P(B \text{ Wins}) = 2P(A \text{ Wins})$
3) $P(A \text{ Wins}) > P(B \text{ Wins})$
4) $P(A \text{ Wins}) = 1 - P(B \text{ Wins})$

## CSIR-UGC NET-Dec 2016-Solution(104)

Given, a fair coin is tossed till heads turns up.

$$p = \frac{1}{2}, q = \frac{1}{2} \tag{104.1}$$

Let's define a Markov chain $\{X_n, n = 0, 1, 2, \dots\}$, where $X_n \in S = \{1, 2, 3, 4, 5\}$, such that

TABLE 1: States and their notations

| Notation | State |
|---|---|
| $S = 1$ | $A$'s turn |
| $S = 2$ | $B$'s first turn |
| $S = 3$ | $B$'s second turn |
| $S = 4$ | $A$ wins |
| $S = 4$ | $B$ wins |

The state transition matrix for the Markov chain is

$$P = \begin{bmatrix} 0 & 0.5 & 0 & 0.5 & 0 \\ 0 & 0 & 0.5 & 0 & 0.5 \\ 0.5 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \tag{104.2}$$

$1, 2, 3$ are transient states, while $4, 5$ are absorbent states. Now let's define $a_i, b_i$ as the absorption probabilities in states $4, 5$ respectively, if we start from state $i$. Then,

$$a_i = \sum_k a_k p_{ik}, \forall i \in S \tag{104.3}$$

$$b_i = \sum_k b_k p_{ik}, \forall i \in S \tag{104.4}$$

Also, it is evident that

$$a_4 = 1, a_5 = 0 \tag{104.5}$$

$$b_4 = 0, b_5 = 1 \tag{104.6}$$

From (104.2), (104.3), (104.5)

$$a_1 = \frac{1}{2}a_2 + \frac{1}{2} \tag{104.7}$$

$$a_2 = \frac{1}{2}a_3 \tag{104.8}$$

$$a_3 = \frac{1}{2}a_1 \tag{104.9}$$

Solving the above recurrence relations, we get

$$Pr(A - Wins) = a_1 = \frac{4}{7} \tag{104.10}$$

Similarly, from (104.2), (104.4), (104.6)

$$b_1 = \frac{1}{2}b_2 \tag{104.11}$$

$$b_2 = \frac{1}{2}b_3 + \frac{1}{2} \tag{104.12}$$

$$b_3 = \frac{1}{2}b_2 + \frac{1}{2} \tag{104.13}$$

Solving the above recurrence relations, we get

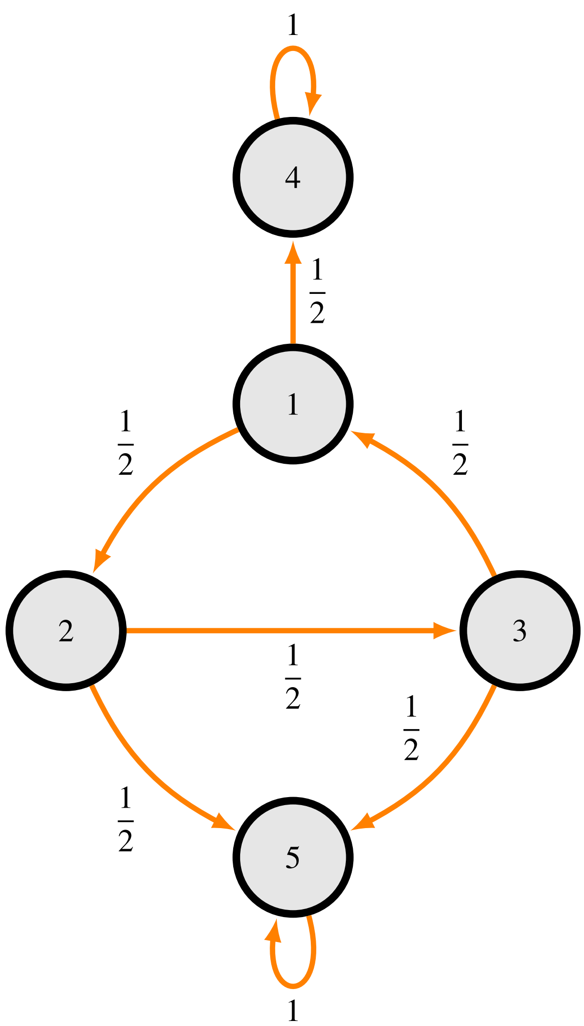
$$Pr(B - Wins) = b_1 = \frac{3}{7} \tag{104.14}$$

From (104.10), (104.14)

$$Pr(A - Wins) = 1 - Pr(B - Wins) \quad (104.15)$$
$$Pr(A - Wins) > Pr(B - Wins) \quad (104.16)$$

Therefore, options 3), 4) are correct.

**Markov chain diagram**




Theoretical vs Simulation plot