AI1103 : Assignment 2

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Download all python codes from

https://github.com/YashasTadikamalla/AI1103/tree/main/Assignment2/codes

and latex codes from

https://github.com/YashasTadikamalla/AI1103/blob/main/Assignment2/Assignment2.tex

Problem(5.25)

A bag contains 2 white and 1 red balls. One ball is drawn at random and then put back in the box after noting its colour. The process is repeated again. If X denotes the number of red balls recorded in the two draws, describe X.

Solution(5.25)

Given, a bag containing 2 white and 1 red balls. Let the random variable $X_i \in \{0,1\}, i = 1,2$, represent the outcome of the colour of the ball drawn in the first, second attempts. $X_i = 0, X_i = 1$ denote a white ball, red ball being drawn respectively, in the i^{th} attempt.

We know, for an event E with a sample space S, the probability for it to occur is given by

$$Pr(E) = \frac{n(E)}{n(S)} \tag{5.25.1}$$

where $n(E), n(S)$ denote the number of favourable outcomes(i.e, event E), total number of outcomes respectively.

As the ball drawn in the first attempt is replaced in the bag, for both the attempts, the number of balls of a specified colour, and their probability mass function's (pmf's) remain the same. i.e,

$$n(X_i = 0) = 2 \tag{5.25.2}$$

$$n(X_i = 1) = 1 \tag{5.25.3}$$

$$\therefore n(X_i = 0) + n(X_i = 1) = 3 \tag{5.25.4}$$

and

$$\Pr(X_i = i) = \begin{cases} \dfrac{n(X_i = 0)}{n(X_i = 0) + n(X_i = 1)} = \frac{2}{3}, & i = 0 \\ \dfrac{n(X_i = 1)}{n(X_i = 0) + n(X_i = 1)} = \frac{1}{3}, & i = 1 \\ 0, & otherwise \end{cases} \tag{5.25.5}$$

Define

$$X = X_1 + X_2 \tag{5.25.6}$$

so that $X \in \{0,1,2\}$ represents a random variable denoting the number of red balls drawn in both the attempts. Then, X has a binomial distribution with

$$Pr(X = k) = \binom{n}{k} p^k q^{n-k} \tag{5.25.7}$$

where,

$$n = 2 \tag{5.25.8}$$

p = probability of success = probability of drawing a red ball = $Pr(X_i = 1)$

$$p = \frac{1}{3} \tag{5.25.9}$$

q = probability of failure = $1 - p$

$$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3} \tag{5.25.10}$$

Hence, on substituting, we get

$$Pr(X = 0) = \binom{2}{0}\left(\frac{1}{3}\right)^0\left(\frac{2}{3}\right)^{2-0}$$

$$= 1\left(\frac{1}{1}\right)\left(\frac{4}{9}\right) = \frac{4}{9} \tag{5.25.11}$$

$$Pr(X = 1) = \binom{2}{1}\left(\frac{1}{3}\right)^1\left(\frac{2}{3}\right)^{2-1}$$

$$= 2\binom{1}{3}\binom{2}{3} = \frac{4}{9} \quad (5.25.12)$$

$$Pr(X = 2) = \binom{2}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^{2-2}$$

$$= 1\left(\frac{1}{9}\right)\left(\frac{1}{1}\right) = \frac{1}{9} \quad (5.25.13)$$

Therefore, the pmf for X (theoretical) is

$$\Pr(X = i) = \begin{cases} \frac{4}{9}, & i = 0 \\ \frac{4}{9}, & i = 1 \\ \frac{1}{9}, & i = 2 \\ 0, & otherwise \end{cases} \quad (5.25.14)$$

Condition	$X = 0$	$X = 1$	$X = 2$
Probability	$\binom{2}{0}p^0q^2$	$\binom{2}{1}p^1q^1$	$\binom{2}{2}p^2q^0$

So, if we conduct this experiment 9000 times, theoretically, we get

$$n(X = 0) = Pr(X = 0)(9000) = 4000 \quad (5.25.15)$$
$$n(X = 1) = Pr(X = 1)(9000) = 4000 \quad (5.25.16)$$
$$n(X = 2) = Pr(X = 2)(9000) = 1000 \quad (5.25.17)$$

Here are the plots describing X, after the experiment is conducted 9000 times.

Theoretical vs Simulation