Assignment 4

Q5 Report

Log-likelihood equation using cross entropy

$$J(\vec{w}) = \sum_i y_i log(p_1(\vec{w}, \vec{x}_i)) + (1 - y_i) log(p_0(\vec{w}, \vec{x}_i))$$

where,

$$p_1(\vec{w}, \vec{x}_i) = \sigma(\vec{w}.\vec{x}_i)$$
$$p_0(\vec{w}, \vec{x}_i) = 1 - \sigma(\vec{w}.\vec{x}_i)$$

Maximising Log-likelihood is same as minimising -J(w)

$$\frac{d}{d\vec{w}}(-J(\vec{w})) = \sum_i (\sigma(\vec{w}.\vec{x}_i) - y_i)\vec{x}_i$$

Using Gradient-descent to find optimal parameters

$$\vec{w}^{(i+1)} = \vec{w}^{(i)} - \gamma \frac{d}{d\vec{w}}(-J(\vec{w}))$$

By using tolerance of 0.001 and cross entropy,
Weights after one iteration: [-1.01899756, 1.53210518, 0.51181202]
Final weights: [-18.07774125, 26.47820683,   6.58909946]

a

$$\vec{w} = [-1, 1.5, 0.5]$$
$$P(\hat{y} = 1|x_1, x_2) = \frac{1}{1 + e^{-(-1+1.5x_1+0.5x_2)}}$$
$$P(\hat{y} = 0|x_1, x_2) = 1 - \frac{1}{1 + e^{-(-1+1.5x_1+0.5x_2)}}$$
$$J(\vec{w}) = \sum_i y_i log(P(\hat{y}_i = 1|\vec{x}_i)) + (1 - y_i)log(P(\hat{y}_i = 0|\vec{x}_i))$$

where

$$\vec{x} = [1, x_1, x_2]$$

b

$$\vec{w} = [-1.01899756, 1.53210518, 0.51181202]$$
$$P(\hat{y} = 1|x_1, x_2) = \frac{1}{1 + e^{-(-1.01899756+1.53210518x_1+0.51181202x_2)}}$$
$$P(\hat{y} = 0|x_1, x_2) = 1 - \frac{1}{1 + e^{-(-1.01899756+1.53210518x_1+0.51181202x_2)}}$$
$$J(\vec{w}) = \sum_i y_i log(P(\hat{y}_i = 1|\vec{x}_i)) + (1 - y_i)log(P(\hat{y}_i = 0|\vec{x}_i))$$

where

$$\vec{x} = [1, x_1, x_2]$$

c

Prediction: [1, 1, 0, 1, 1, 1]
Actual: [0, 0, 0, 1, 1, 1]
Accuracy: (TP+TN)/(TP+TN+FP+FN)=4/6=0.67
Precision: TP/(TP+FP)=3/5=0.6
Recall: TP/(TP+FN)=3/3=1