Gate Assignment 3

Yashas Tadikamalla - AI20BTECH11027

Download all python codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment3/codes

and latex-tikz codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment3/GateAssignment3.tex

1 Problem (EC-2005 Q25)

A linear system is equivalently represented by two sets of state equations:

$$\dot{X} = AX + BU \text{ and } \dot{W} = CW + DU$$

Eigenvalues of the representations are also computed as $[\lambda]$ and $[\mu]$. Which of the following is true?

1) $[\lambda] = [\mu]$ and $X = W$
2) $[\lambda] = [\mu]$ and $X \neq W$
3) $[\lambda] \neq [\mu]$ and $X = W$
4) $[\lambda] \neq [\mu]$ and $X \neq W$

2 Solution

Theorem 2.1. *Consider the n-dimensional continuous time LTI system*

$$\dot{X} = AX + BU \text{ and } Y = CX + DU \tag{2.0.1}$$

Let T be an $n \times n$ real non-singular matrix and let $\bar{X} = TX$. Then the state equation

$$\dot{\bar{X}} = \bar{A}\bar{X} + \bar{B}U, Y = \bar{C}\bar{X} + \bar{D}U \tag{2.0.2}$$

where $\bar{A} = TAT^{-1}, \bar{B} = TB, \bar{C} = CT^{-1}, \bar{D} = D$ is said to be equivalent to (2.0.1).

Proof. Given, $\dot{X} = AX + BU$ and $Y = CX + DU$, T is a non-singular matrix such that $\bar{X} = TX$. The same system can be defined using $\bar{X}$ as the state,

$$\dot{\bar{X}} = T\dot{X} = TAX + TBU \tag{2.0.3}$$
$$= TAT^{-1}\bar{X} + TBU \tag{2.0.4}$$
$$Y = CX + DU = CT^{-1}\bar{X} + DU \tag{2.0.5}$$

□

Theorem 2.2. *Equivalent state space representations have the same set of eigen values*

Proof.

$$\bar{\triangle}(\lambda) = det(\lambda I - \bar{A}) \tag{2.0.6}$$
$$= det(\lambda TT^{-1} - TAT^{-1}) \tag{2.0.7}$$
$$= det(T(\lambda I - A)T^{-1}) \tag{2.0.8}$$
$$= det(\lambda I - A) = \triangle(\lambda) \tag{2.0.9}$$

□

Given,

$$\dot{X} = AX + BU \tag{2.0.10}$$
$$\dot{W} = CW + DU \tag{2.0.11}$$

represent the same system. Hence, using (2.1) and (2.2), we can conclude that

$$[\lambda] = [\mu] \text{ and } X \neq W$$

Hence, option 2 is the correct answer.