

# Gate Assignment 4

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Download all python codes from

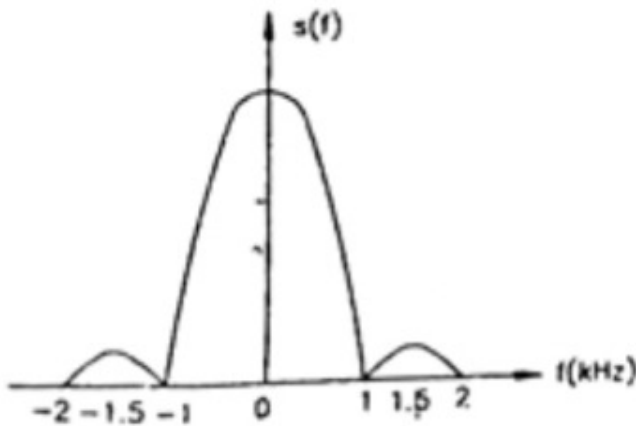
<https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment4/codes>

and latex-tikz codes from

<https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment4/GateAssignment4.tex>

## 1 PROBLEM (EC-1997 Q1.10)

A deterministic signal has the power spectrum given in the figure. The minimum sampling rate needed to completely represent this signal is



- 1) 1KHz
- 2) 2KHz
- 3) 3KHz
- 4) None

## 2 SOLUTION

**Definition 2.1** (Normalised sinc function). A *normalised sinc function* is defined as

$$\text{sinc}(x) = \begin{cases} 1, & x = 0 \\ \frac{\sin(\pi x)}{\pi x}, & x \neq 0 \end{cases} \quad (2.0.1)$$

**Definition 2.2** (Power spectrum). *Power Spectral density, or simply, Power spectrum, denoted by  $s(f)$  is defined as*

$$s(f) = |X(f)|^2 \quad (2.0.2)$$

**Theorem 2.1** (Sampling Theorem). *If a signal contains no frequency components above  $W$  Hz, then the sampling rate at which the continuous time signal needs to be sampled uniformly, so as to completely recover the original signal is given by*

$$f_s \geq 2W \quad (2.0.3)$$

**Definition 2.3** (Nyquist rate). *Minimum sampling rate is also called as Nyquist rate. It is given by*

$$f_s = 2W \quad (2.0.4)$$

Given, power spectrum of a deterministic signal. From (2.2), Fourier transform of the given band limited signal is **truncated normalised sinc pulse**. As no frequency component exceeds 2KHz,

$$W = 2KHz \quad (2.0.5)$$

From (2.0.4),

$$f_s = 2W = 4KHz \quad (2.0.6)$$

Hence, option 4 is the correct answer.

To verify the validity of (2.1), let's see what happens if we sample at a rate lower than Nyquist rate.

Let our original continuous time signal be  $x(t)$ . Consider impulse train  $x_i(t)$  given by

$$x_i(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \quad (2.0.7)$$

where  $T_s$  is the sampling period. ( $f_s = \frac{1}{T_s}$  is the sampling frequency). The sampled signal would be

$$x_s(t) = x(t)x_i(t) \quad (2.0.8)$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) \quad (2.0.9)$$

Also, from (2.0.8)

$$X_s(f) = X(f) * X_i(f) \quad (2.0.10)$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(f - kf_s) \quad (2.0.11)$$

$X_s(f)$  consists periodically repeated copies of  $X(f)$ , shifted by integer multiples of  $f_s$ . For our example,  $X(f)$  is the **truncated normalised sinc pulse**. Let  $W$  be the maximum frequency component.

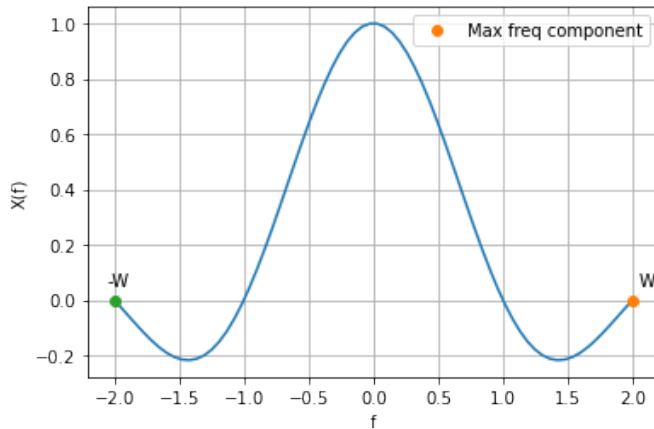


Fig. 4: Plot of  $X(f)$

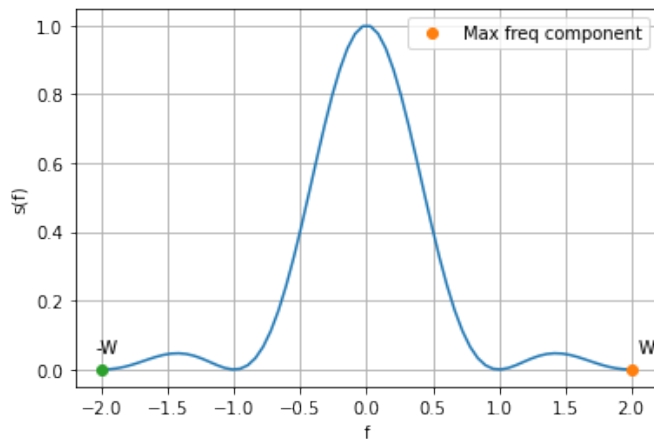


Fig. 4: Plot of  $s(f)$

- Case 1:  $W > 2f_s$ :  
The copies don't overlap. Hence,  $x(t)$  can be recovered from  $x_s(t)$  using a low pass filter.
- Case 2:  $W = 2f_s$ :  
The copies just touch each other, but don't overlap. So,  $x(t)$  can be recovered from  $x_s(t)$  using a low pass filter.
- Case 3:  $W < 2f_s$ :  
As the copies overlap, they get added. So, we cannot reconstruct the original signal  $x(t)$ . This gives rise to situation called aliasing.

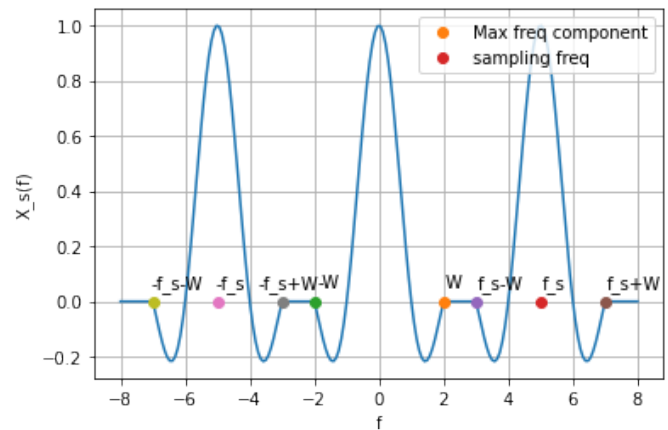


Fig. 4:  $W > 2f_s$

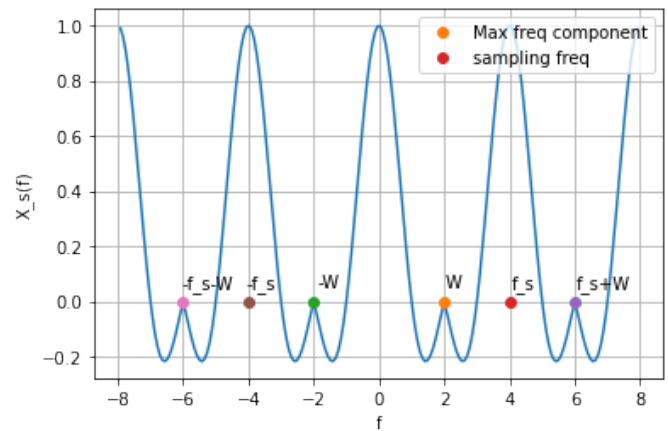


Fig. 4:  $W = 2f_s$

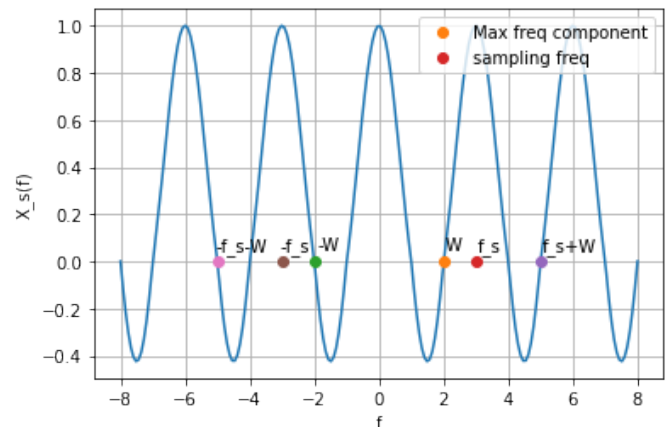


Fig. 4:  $W < 2f_s$

Hence, we need to sample at a rate higher than Nyquist rate, in order to be able to recover the original signal.