Gate Assignment 1

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Download all python codes from

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https://github.com/YashasTadikamalla/EE3900/blob
    /main/GateAssignment1/codes
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and latex-tikz codes from

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https://github.com/YashasTadikamalla/EE3900/blob
    /main/GateAssignment1/GateAssignment1.tex
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1 Problem (EC-2013 Q8)

The impulse response of a system is $h(t) = tu(t)$. For an input $u(t-1)$, the output is

1) $\frac{t^2}{2}u(t)$
2) $\frac{t(t-1)}{2}u(t-1)$
3) $\frac{(t-1)^2}{2}u(t-1)$
4) $\frac{t^2-1}{2}u(t-1)$

2 Solution

Given,

$$h(t) = tu(t) \tag{2.0.1}$$

$$x(t) = u(t-1) \tag{2.0.2}$$

i.e,

$$h(t) = \begin{cases} t, & t \in \mathbb{Z}^+ \cup \{0\} \\ 0, & t \in \mathbb{Z}^- \end{cases} \tag{2.0.3}$$

$$x(t) = \begin{cases} 1, & t \in \mathbb{Z}^+ \\ 0, & t \in \mathbb{Z}^- \cup \{0\} \end{cases} \tag{2.0.4}$$

To find: $y(t)$. We know,

$$y(t) = h(t) * x(t) = \sum_{k=-\infty}^{\infty} h(k)x(t-k) \tag{2.0.5}$$

$$= \sum_{k=-\infty}^{-1} h(k)x(t-k) + \sum_{k=0}^{\infty} h(k)x(t-k) \tag{2.0.6}$$

Substituting (2.0.3) in (2.0.6),

$$y(t) = \sum_{k=0}^{\infty} k\,[x(t-k)] \tag{2.0.7}$$

From (2.0.4), we can observe that, $\forall t \leq 0$,

$$y(t) = 0 \tag{2.0.8}$$

Also, for $t \geq 1$,

$$y(1) = \sum_{k=0}^{\infty} k\,[x(1-k)] = \sum_{k=0} k \tag{2.0.9}$$

$$y(2) = \sum_{k=0}^{\infty} k\,[x(2-k)] = \sum_{k=0}^{1} k \tag{2.0.10}$$

$$y(3) = \sum_{k=0}^{\infty} k\,[x(3-k)] = \sum_{k=0}^{2} k \tag{2.0.11}$$

$$\vdots \tag{2.0.12}$$

$$y(t) = \sum_{k=0}^{\infty} k\,[x(t-k)] = \sum_{k=0}^{t-1} k \tag{2.0.13}$$

$$\therefore y(t) = \begin{cases} \frac{t(t-1)}{2}, & t \in \mathbb{Z}^+ \\ 0, & t \in \mathbb{Z}^- \cup \{0\} \end{cases} \tag{2.0.14}$$

$$\therefore y(t) = \frac{t(t-1)}{2}u(t-1) \tag{2.0.15}$$

Option 2 is the correct answer.

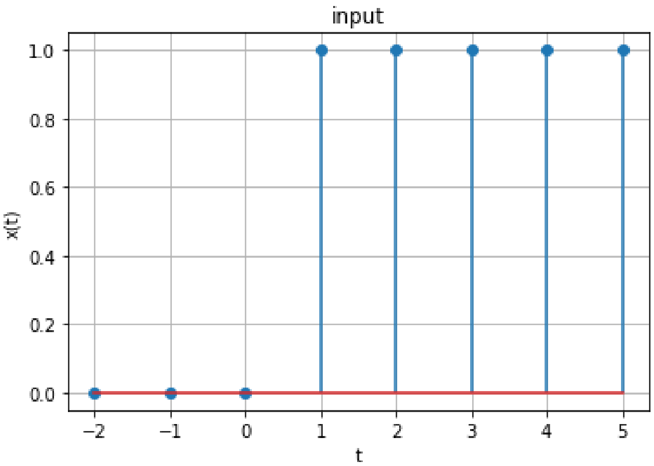

Fig. 4: Plot of $x(t)$

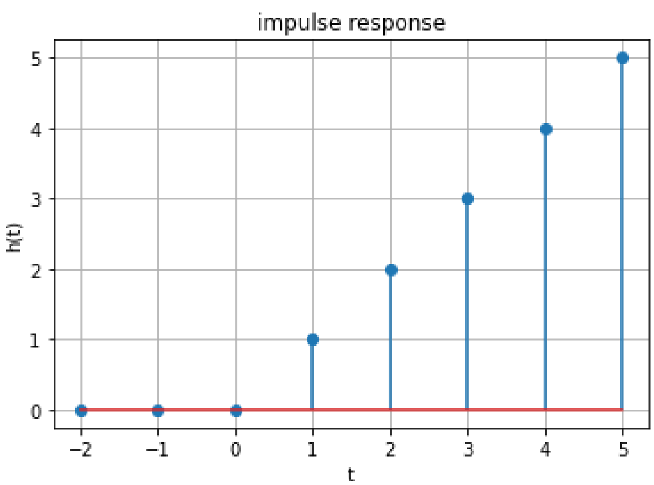

Fig. 4: Plot of $h(t)$

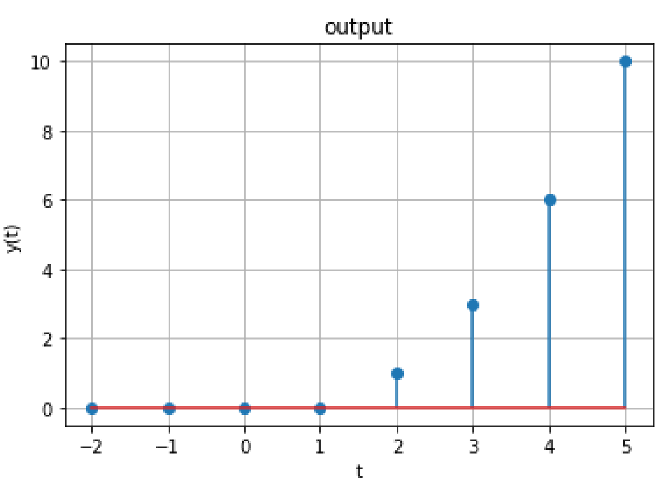

Fig. 4: Plot of $y(t)$