Assignment 3 Presentation

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AI20BTECH11027

Power of a point

The power of a point is a real number that reflects the relative distance of a given point from a given circle. Specifically, the power $\Pi(P)$ of a point P with respect to a circle C with center O and radius r is defined by

$$\Pi(P) = |PO|^2 - r^2 \tag{1}$$

- $\Pi(P) > 0 \Leftrightarrow P$ is outside the circle
- $\Pi(P) = 0 \Leftrightarrow P$ is on the circle
- $\Pi(P) < 0 \Leftrightarrow P$ is inside the circle

For a circle $S = 0$, (1) boils down to,

$$\Pi(P) = S_{11} \tag{2}$$

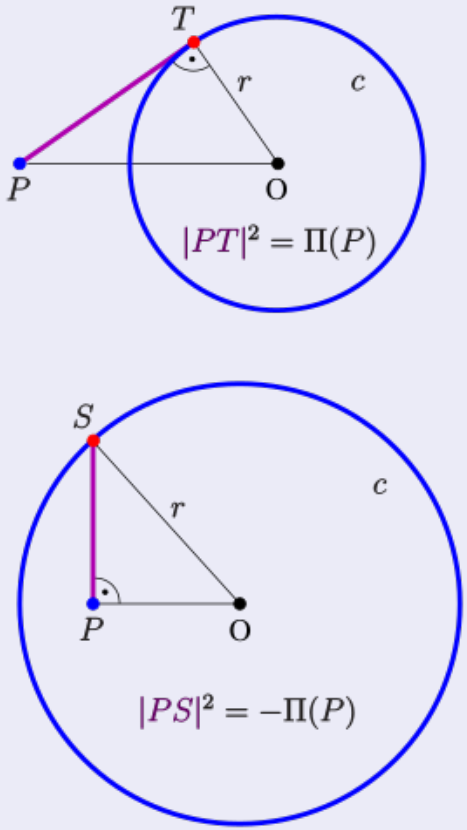

Figure: Geometric interpretation of power of a point with respect to a circle

Radical axis

The radical axis of two non-concentric circles is the set of points whose power with respect to the circles are equal. i.e, points for which,

$$\Pi_1(P) = \Pi_2(P) \tag{3}$$

For two non-concentric circles $S = 0, S' = 0$, the radical axis is given by

$$L = S - S' = 0 \tag{4}$$

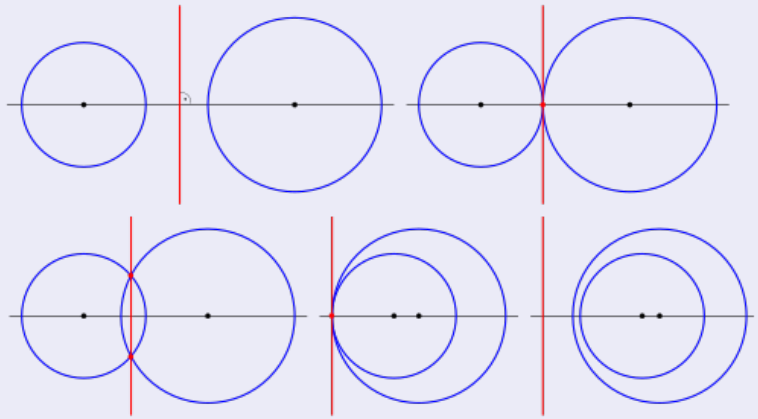

Figure: Variations of radical axis

Question

Ramsey/4.4 Systems of circles/Q4 (a)

Write down the equation of the radical axis of the following pair of circles:

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix}4 & -5\end{pmatrix}\mathbf{x} - 2 = 0$$

$$\mathbf{x}^T\mathbf{x} - \begin{pmatrix}5 & -6\end{pmatrix}\mathbf{x} = 0$$

Solution

Given, two circles with equations,

$$S = \mathbf{x}^T\mathbf{x} - \begin{pmatrix}4 & -5\end{pmatrix}\mathbf{x} - 2 = 0 \tag{5}$$

$$S' = \mathbf{x}^T\mathbf{x} - \begin{pmatrix}5 & -6\end{pmatrix}\mathbf{x} = 0 \tag{6}$$

To find: The radical axis of the pair of circles.
Using (4), the required equation is

$$\left(\mathbf{x}^T\mathbf{x} - \begin{pmatrix}4 & -5\end{pmatrix}\mathbf{x} - 2\right) - \left(\mathbf{x}^T\mathbf{x} - \begin{pmatrix}5 & -6\end{pmatrix}\mathbf{x} = 0\right) = 0 \tag{7}$$

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} - 2 = 0 \tag{8}$$

$\therefore L = \begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} - 2 = 0$ is the equation of the required radical axis.

Solution Contd.

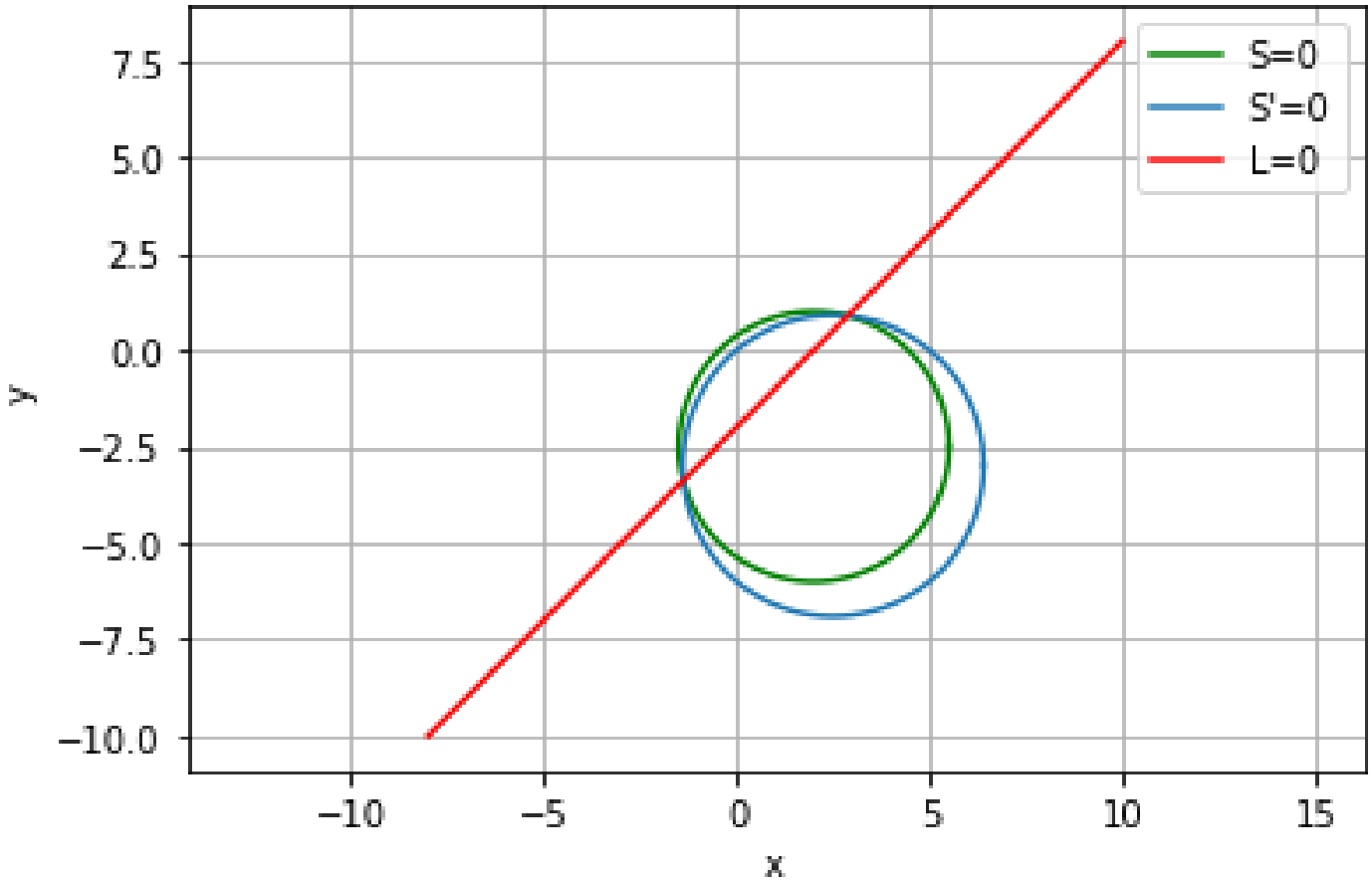

Figure: Pair of Circles and their radical axis