Gate Assignment 4

Yashas Tadikamalla - AI20BTECH11027

Download all python codes from

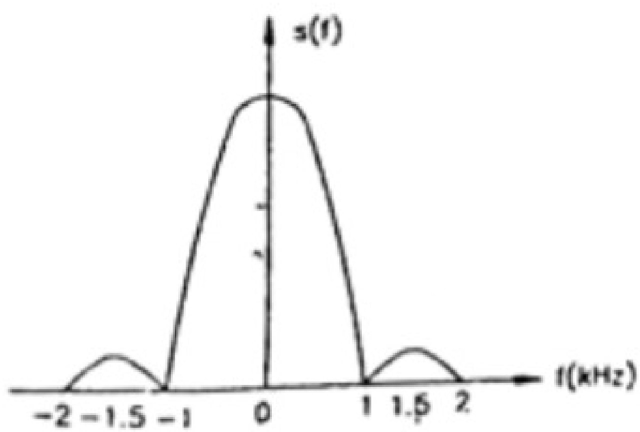
https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment4/codes

and latex-tikz codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment4/GateAssignment4.tex

1 Problem (EC-1997 Q1.10)

A deterministic signal has the power spectrum given in the figure. The minimum sampling rate needed to completely represent this signal is



1) $1KHz$
2) $2KHz$
3) $3KHz$
4) None

2 Solution

Definition 2.1 (Normalised sinc function)**.** *A normalised sinc function is defined as*

$$sinc(x) = \begin{cases} 1, & x = 0 \\ \frac{sin(\pi x)}{\pi x}, & x \neq 0 \end{cases} \tag{2.0.1}$$

Definition 2.2 (Power spectrum)**.** *Power Spectral density, or simply, Power spectrum, denoted by $s(f)$ is defined as*

$$s(f) = |X(f)|^2 \tag{2.0.2}$$

Theorem 2.1 (Sampling Theorem)**.** *If a signal contains no frequency components above W Hz, then the sampling rate at which the continuous time signal needs to be sampled uniformly, so as to completely recover the original signal is given by*

$$f_s \geq 2W \tag{2.0.3}$$

Definition 2.3 (Nyquist rate)**.** *Minimum sampling rate is also called as Nyquist rate. It is given by*

$$f_s = 2W \tag{2.0.4}$$

Given, power spectrum of a deterministic signal. From (2.2), Fourier transform of the given band limited signal is **truncated normalised sinc pulse**. As no frequency component exceeds $2KHz$,

$$W = 2KHz \tag{2.0.5}$$

From (2.0.4),

$$f_s = 2W = 4KHz \tag{2.0.6}$$

Hence, option 4 is the correct answer.
To verify the validity of (2.1), let's see what happens if we sample at a rate lower than Nyquist rate.
Let our original continuous time signal be $x(t)$. Consider impulse train $x_i(t)$ given by

$$x_i(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \tag{2.0.7}$$

where T_s is the sampling period. ($f_s = \frac{1}{T_s}$ is the sampling frequency). The sampled signal would be

$$x_s(t) = x(t)x_i(t) \tag{2.0.8}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) \tag{2.0.9}$$

Also, from (2.0.8)

$$X_s(f) = X(f) * X_i(f) \tag{2.0.10}$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(f - kf_s) \tag{2.0.11}$$

$X_s(f)$ consists periodically repeated copies of $X(f)$, shifted by integer multiples of f_s. For our example, $X(f)$ is the **truncated normalised sinc pulse**. Let W be the the maximum frequency component.

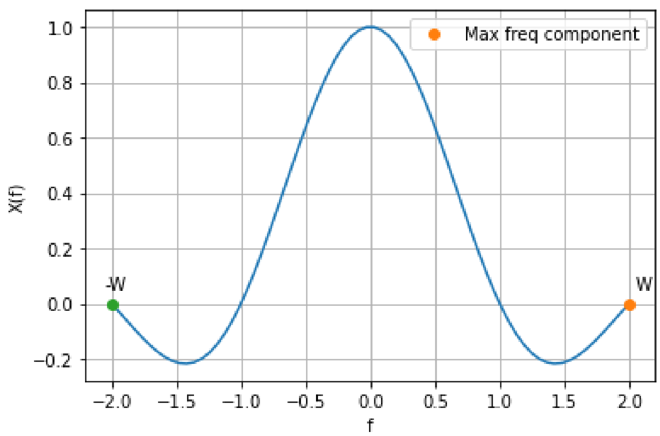


Fig. 4: Plot of X(f)

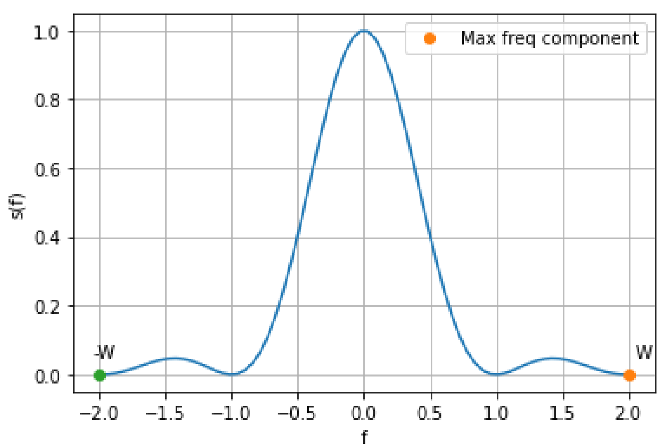


Fig. 4: Plot of s(f)

- Case 1: $W > 2f_s$:
  The copies don't overlap. Hence, $x(t)$ can be recovered from $x_s(t)$ using a low pass filter.
- Case 2: $W = 2f_s$:
  The copies just touch each other, but don't overlap. So, $x(t)$ can be recovered from $x_s(t)$ using a low pass filter.
- Case 3: $W < 2f_s$:
  As the copies overlap, they get added. So, we cannot reconstruct the original signal $x(t)$. This gives rise to situation called aliasing.

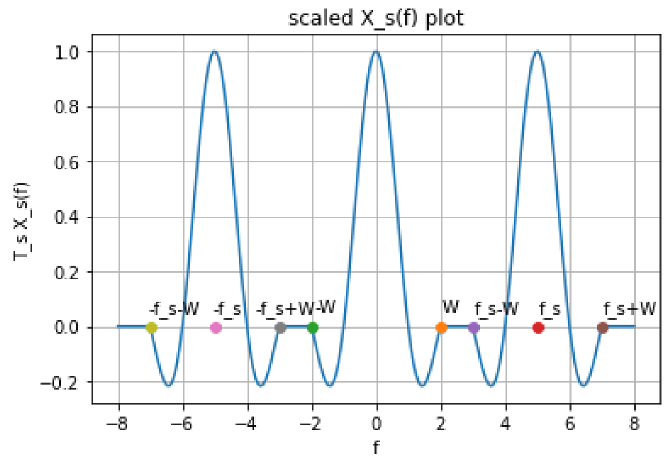


Fig. 4: $W > 2f_s$

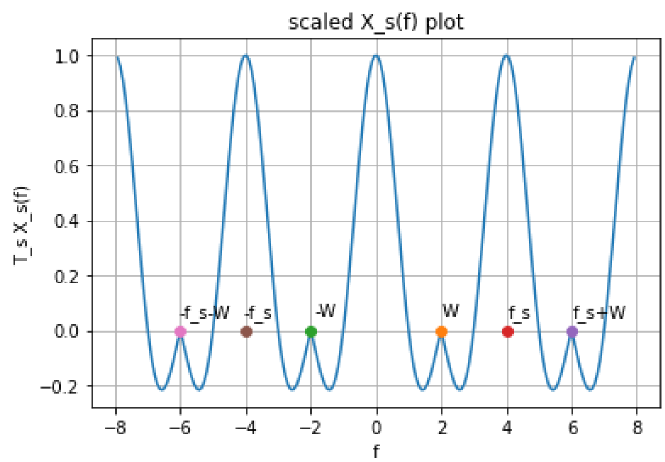


Fig. 4: $W = 2f_s$

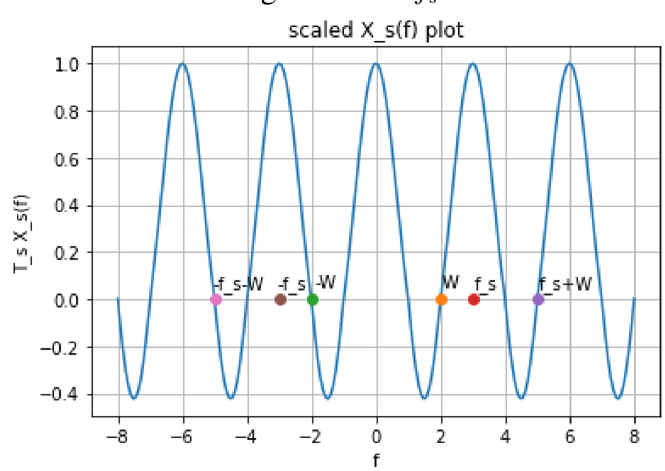


Fig. 4: $W < 2f_s$

Hence, we need to sample at a rate greater than Nyquist rate to be able to recover the original signal.