# Gate Assignment 4

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Download all python codes from

```
https://github.com/YashasTadikamalla/EE3900/blob
/main/GateAssignment4/codes
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and latex-tikz codes from

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https://github.com/YashasTadikamalla/EE3900/blob
/main/GateAssignment4/GateAssignment4.tex
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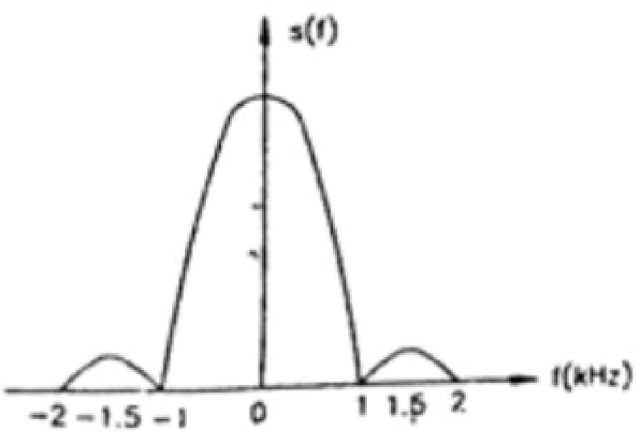
## 1 Problem (EC-1997 Q1.10)

A deterministic signal has the power spectrum given in the figure. The minimum sampling rate needed to completely represent this signal is




1) $1KHz$
2) $2KHz$
3) $3KHz$
4) None

## 2 Solution

**Definition 2.1** (Normalised sinc function)**.** *A normalised sinc function is defined as*

$$sinc(x) = \begin{cases} 1, & x = 0 \\ \dfrac{sin(\pi x)}{\pi x}, & x \neq 0 \end{cases} \tag{2.0.1}$$

**Definition 2.2** (Power spectrum)**.** *Power Spectral density, or simply, Power spectrum, denoted by $s(f)$ is defined as*

$$s(f) = |X(f)|^2 \tag{2.0.2}$$

**Theorem 2.1** (Sampling Theorem)**.** *If a signal contains no frequency components above W Hz, then the sampling rate at which the continuous time signal needs to be sampled uniformly, so as to completely recover the original signal is given by*

$$f_s \geq 2W \tag{2.0.3}$$

**Definition 2.3** (Nyquist rate)**.** *Minimum sampling rate is also called as Nyquist rate. It is given by*

$$f_s = 2W \tag{2.0.4}$$

Given, power spectrum of a deterministic signal.

$$s(f) = \begin{cases} sinc(f)^2, & |f| \leq 2KHz \\ 0, & else \end{cases} \tag{2.0.5}$$

From (2.2), Fourier transform of the given band limited signal is **truncated normalised sinc pulse**.

$$X(f) = \begin{cases} sinc(f), & |f| \leq 2KHz \\ 0, & else \end{cases} \tag{2.0.6}$$

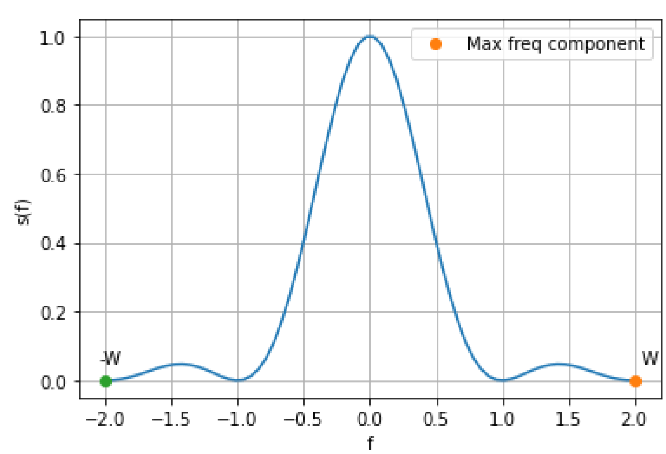



Fig. 4: Plot of s(f)

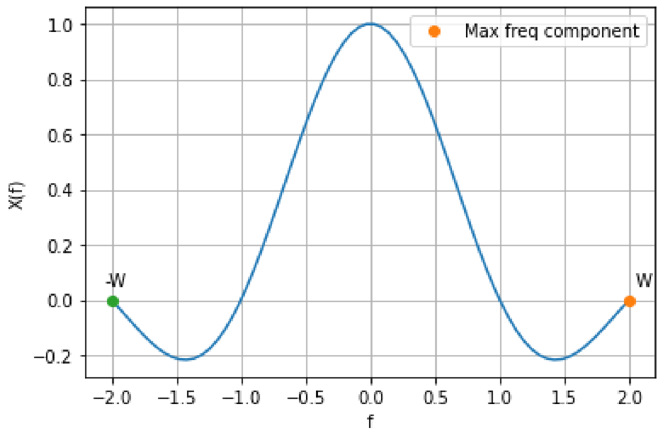



Fig. 4: Plot of X(f)

As no frequency component exceeds $2KHz$,

$$W = 2KHz \tag{2.0.7}$$

From (2.0.4),

$$f_s = 2W = 4KHz \tag{2.0.8}$$

Hence, option 4 is the correct answer.
To verify the validity of (2.1), let's see what happens if we sample at a rate lower than Nyquist rate.
Let our original continuous time signal be $x(t)$.

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{2\pi f t j} df \tag{2.0.9}$$

Consider impulse train $x_i(t)$ given by

$$x_i(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \tag{2.0.10}$$

where $T_s$ is the sampling period. ($f_s = \frac{1}{T_s}$ is the sampling frequency). The sampled signal would be

$$x_s(t) = x(t)x_i(t) \tag{2.0.11}$$

$$= x(t) \sum_{n=-\infty}^{\infty} \delta(t - nT_s) \tag{2.0.12}$$

$$= \sum_{n=-\infty}^{\infty} x(nT_s)\delta(t - nT_s) \tag{2.0.13}$$

Also, from (2.0.11)

$$X_s(f) = X(f) * X_i(f) \tag{2.0.14}$$

$$= \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X(f - kf_s) \tag{2.0.15}$$

$X_s(f)$ consists periodically repeated copies of $X(f)$, shifted by integer multiples of $f_s$. Let $W$ be the the maximum frequency component.

- Case 1: $W > 2f_s$:
  The copies don't overlap. Hence, $x(t)$ can be recovered from $x_s(t)$ using an ideal low pass filter.
- Case 2: $W = 2f_s$:
  The copies just touch each other, but don't overlap. So, $x(t)$ can be recovered from $x_s(t)$ using an ideal low pass filter.
- Case 3: $W < 2f_s$:
  As the copies overlap, they get added. So, we cannot reconstruct the original signal $x(t)$. This gives rise to situation called aliasing.

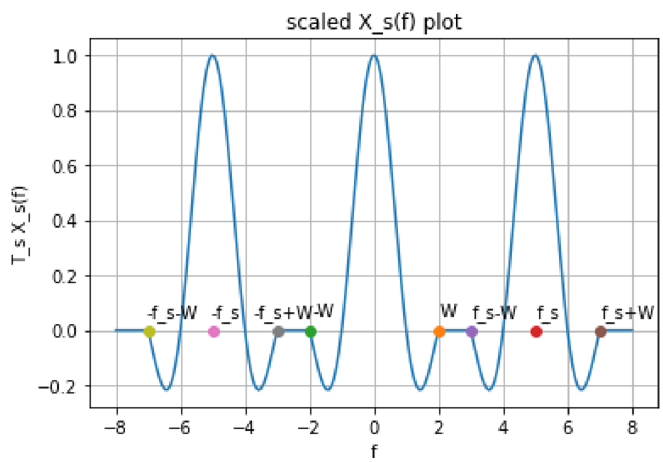



Fig. 4: $W > 2f_s$

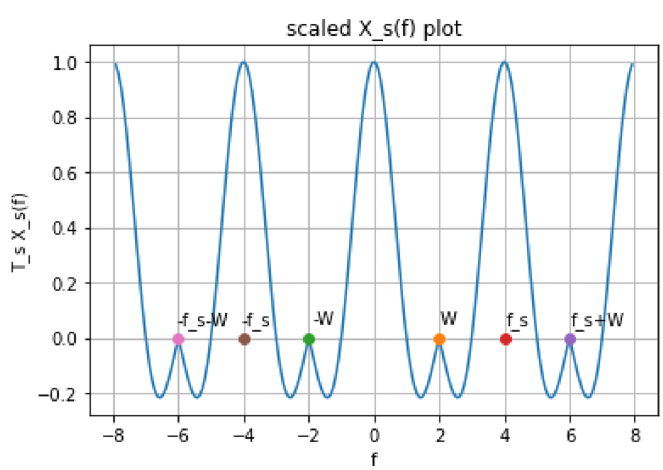



Fig. 4: $W = 2f_s$

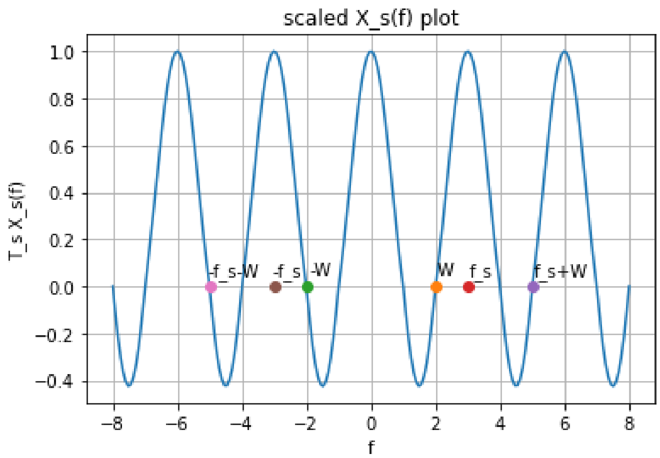



Fig. 4: $W < 2f_s$

Hence, we need to sample at a rate greater than Nyquist rate to be able to recover the original signal.

$$x(t) = \int_{-2}^{2} sinc(f)e^{2\pi f t j} df \tag{2.0.16}$$

We cannot find a closed form expression for $x(t)$, but we can calculate it numerically using python.

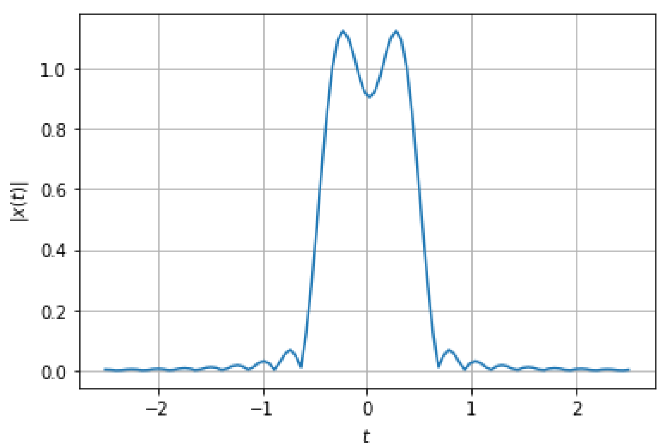



Fig. 4: Plot of $|x(t)|$

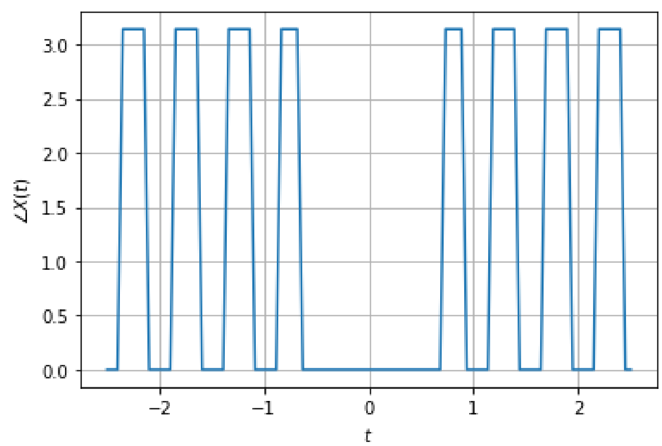



Fig. 4: Plot of $\angle x(t)$