Assignment 4

Yashas Tadikamalla - AI20BTECH11027

Download all python codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/Assignment4/codes

and latex-tikz codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/Assignment4/Assignment4.tex

1 Problem (Linear Forms Q2.33)

If the lines

$$\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$$
$$\frac{x-3}{3k}=\frac{y-1}{1}=\frac{z-6}{-5}$$

are coplanar, find the value of k.

2 Solution

Given, two coplanar lines,

$$\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2} \tag{2.0.1}$$
$$\frac{x-3}{3k}=\frac{y-1}{1}=\frac{z-6}{-5} \tag{2.0.2}$$

For them,

$$\mathbf{m_1}=\begin{pmatrix}-3\\2k\\2\end{pmatrix}, \mathbf{m_2}=\begin{pmatrix}3k\\1\\-5\end{pmatrix} \tag{2.0.3}$$
$$\mathbf{A_1}=\begin{pmatrix}1\\2\\3\end{pmatrix}, \mathbf{A_2}=\begin{pmatrix}3\\1\\6\end{pmatrix} \tag{2.0.4}$$

where $\mathbf{m_1}, \mathbf{m_2}$ are the respective direction vectors and $\mathbf{A_1}, \mathbf{A_2}$ are the respective points through which the lines pass. To find: k.

If two lines are coplanar, they must be either parallel or intersecting. If the given lines are parallel, then

$$\mathbf{m_1}=\alpha\mathbf{m_2} \tag{2.0.5}$$
$$\implies \begin{pmatrix}-3\\2k\\2\end{pmatrix}=\begin{pmatrix}3k\alpha\\\alpha\\-5\alpha\end{pmatrix} \tag{2.0.6}$$

From (2.0.6),

$$\alpha=2k \tag{2.0.7}$$
$$3k\alpha=-3 \implies \alpha=\frac{-1}{k} \tag{2.0.8}$$

Clearly, such an α does not exist. Hence, the lines must be intersecting. In that case,

$$(\mathbf{A_1}-\mathbf{A_2})^T(\mathbf{m_1}\times\mathbf{m_2})=0 \tag{2.0.9}$$

$\mathbf{m_1}\times\mathbf{m_2}$ is obtained by row reducing the matrix,

$$\begin{pmatrix}-3 & 2k & 2\\3k & 1 & -5\end{pmatrix} \xleftrightarrow{R_2\leftarrow\frac{R_2+kR_1}{1+2k^2}} \begin{pmatrix}-3 & 2k & 2\\0 & 1 & \frac{2k-5}{1+2k^2}\end{pmatrix} \tag{2.0.10}$$
$$\xleftrightarrow{R_1\leftarrow\frac{R_1-2kR_2}{-3}} \begin{pmatrix}1 & 0 & \frac{10k+2}{-3(1+2k^2)}\\0 & 1 & \frac{2k-5}{1+2k^2}\end{pmatrix} \tag{2.0.11}$$

Substituting in (2.0.9),

$$\begin{pmatrix}-2\\1\\-3\end{pmatrix}^T\begin{pmatrix}\frac{10k+2}{3(1+2k^2)}\\\frac{5-2k}{1+2k^2}\\1\end{pmatrix}=0 \tag{2.0.12}$$
$$\implies -\frac{20k+4}{3+6k^2}+\frac{5-2k}{1+2k^2}=3 \tag{2.0.13}$$
$$\implies \frac{11-26k}{3+6k^2}=3 \tag{2.0.14}$$
$$\implies 9k^2+13k-1=0 \tag{2.0.15}$$

Solving this, we get

$$k=\frac{-13\pm\sqrt{205}}{18} \tag{2.0.16}$$

For the above values of k, the lines are intersecting, and hence, coplanar.

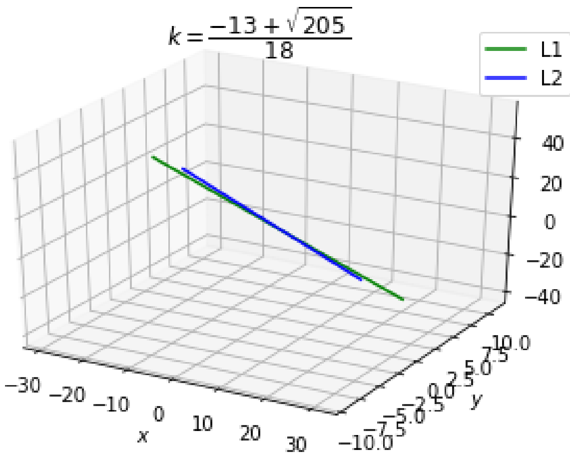

Fig. 0: Intersecting lines - Case1

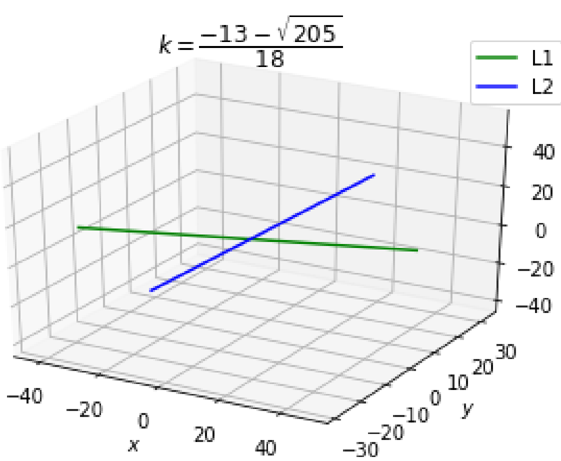

Fig. 0: Intersecting lines - Case2