Gate Assignment 1

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Download all python codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment1/codes

and latex-tikz codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/GateAssignment1/GateAssignment1.tex

1 Problem (EC-2013 Q8)

The impulse response of a system is $h(t) = tu(t)$. For an input $u(t-1)$, the output is

1) $\frac{t^2}{2}u(t)$
2) $\frac{t(t-1)}{2}u(t-1)$
3) $\frac{(t-1)^2}{2}u(t-1)$
4) $\frac{t^2-1}{2}u(t-1)$

2 Solution

Given,

$$h(t) = tu(t) \tag{2.0.1}$$

$$x(t) = u(t-1) \tag{2.0.2}$$

i.e,

$$h(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases} \tag{2.0.3}$$

$$x(t) = \begin{cases} 1, & t \geq 1 \\ 0, & t < 1 \end{cases} \tag{2.0.4}$$

To find: $y(t)$. We know,

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(k)x(t-k)dk \tag{2.0.5}$$

$$= \int_{-\infty}^{0} h(k)x(t-k)dk + \int_{0}^{\infty} h(k)x(t-k)dk \tag{2.0.6}$$

Substituting (2.0.3) in (2.0.6),

$$y(t) = \int_{0}^{\infty} k\,[x(t-k)]\,dk \tag{2.0.7}$$

From (2.0.4), we can observe that, $\forall t < 1$,

$$y(t) = 0 \tag{2.0.8}$$

Also, for $t \geq 1$,

$$y(t) = \int_{0}^{\infty} k\,[x(t-k)]\,dk = \left[\frac{k^2}{2}\right]_{0}^{t-1} \tag{2.0.9}$$

$$\therefore y(t) = \begin{cases} \frac{(t-1)^2}{2}, & t \geq 1 \\ 0, & t < 1 \end{cases} \tag{2.0.10}$$

$$\therefore y(t) = \frac{(t-1)^2}{2}u(t-1) \tag{2.0.11}$$

Option 3 is the correct answer.

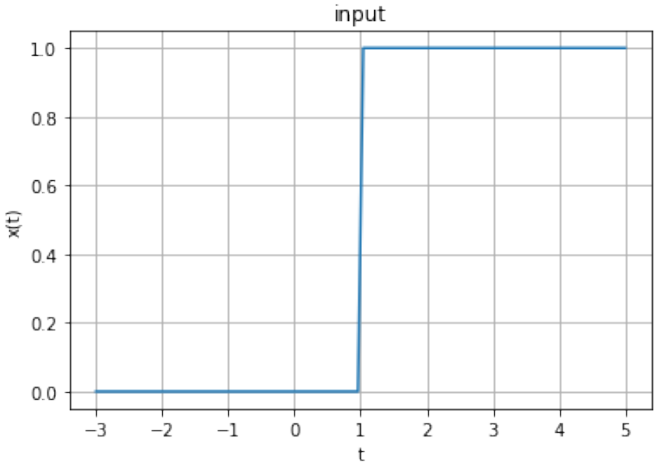

Fig. 4: Plot of $x(t)$

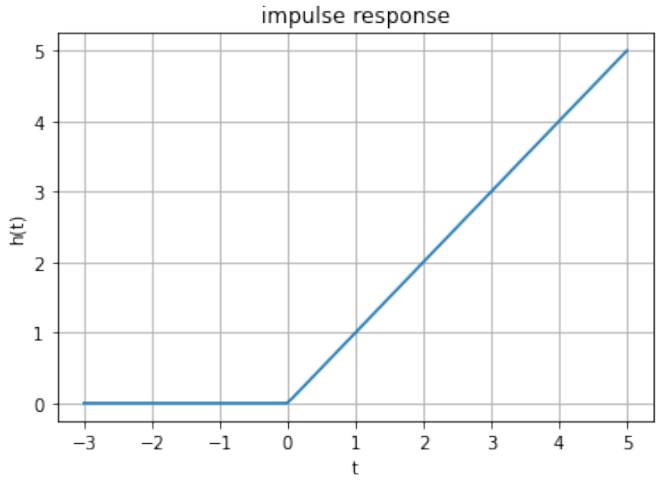

Fig. 4: Plot of $h(t)$

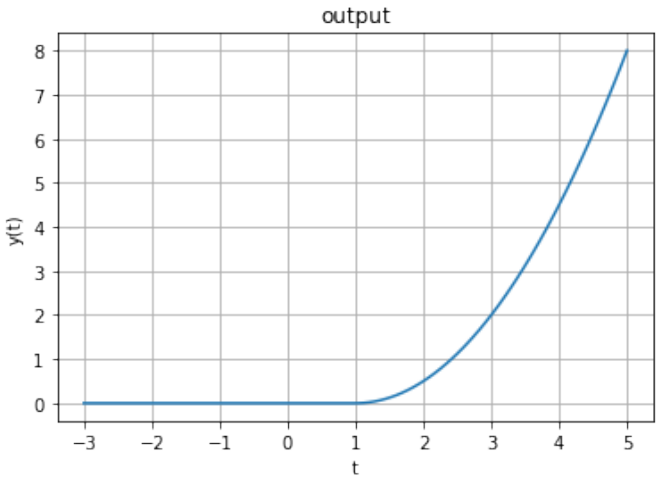

Fig. 4: Plot of $y(t)$