Assignment 4

Yashas Tadikamalla - AI20BTECH11027

Download all python codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/Assignment4/codes

and latex-tikz codes from

https://github.com/YashasTadikamalla/EE3900/blob/main/Assignment4/Assignment4.tex

1 Problem (Linear Forms Q2.33)

If the lines

$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}$$
$$\frac{x-3}{3k} = \frac{y-1}{1} = \frac{z-6}{-5}$$

are coplanar, find the value of k.

2 Solution

Given, two coplanar lines,

$$\frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2} \tag{2.0.1}$$
$$\frac{x-3}{3k} = \frac{y-1}{1} = \frac{z-6}{-5} \tag{2.0.2}$$

For them,

$$\mathbf{m_1} = \begin{pmatrix} -3 \\ 2k \\ 2 \end{pmatrix}, \mathbf{m_2} = \begin{pmatrix} 3k \\ 1 \\ -5 \end{pmatrix} \tag{2.0.3}$$
$$\mathbf{A_1} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \mathbf{A_2} = \begin{pmatrix} 3 \\ 1 \\ 6 \end{pmatrix} \tag{2.0.4}$$

where $\mathbf{m_1}, \mathbf{m_2}$ are the respective direction vectors and $\mathbf{A_1}, \mathbf{A_2}$ are the respective points through which the lines pass. To find: k.

If two lines are coplanar, they must be either parallel or intersecting. If the lines are parallel, the matrix

$$\begin{pmatrix} -3 & 2k & 2 \\ 3k & 1 & -5 \end{pmatrix} \tag{2.0.5}$$

should have $rank = 1$.

$$\implies \begin{vmatrix} -3 & 2k \\ 3k & 1 \end{vmatrix} = 0 \tag{2.0.6}$$
$$\implies 6k^2 + 3 = 0 \tag{2.0.7}$$

This is not possible. Hence, the lines must be intersecting. In that case,

$$\left(\mathbf{A_1} - \mathbf{A_2}\right)^T \left(\mathbf{m_1} \times \mathbf{m_2}\right) = 0 \tag{2.0.8}$$

$\mathbf{m_1} \times \mathbf{m_2}$ is obtained by row reducing the matrix,

$$\begin{pmatrix} -3 & 2k & 2 \\ 3k & 1 & -5 \end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2 + kR_1}{1+2k^2}} \begin{pmatrix} -3 & 2k & 2 \\ 0 & 1 & \frac{2k-5}{1+2k^2} \end{pmatrix} \tag{2.0.9}$$
$$\xleftrightarrow{R_1 \leftarrow \frac{R_1 - 2kR_2}{-3}} \begin{pmatrix} 1 & 0 & \frac{10k+2}{-3(1+2k^2)} \\ 0 & 1 & \frac{2k-5}{1+2k^2} \end{pmatrix} \tag{2.0.10}$$

Substituting in (2.0.8),

$$\begin{pmatrix} -2 \\ 1 \\ -3 \end{pmatrix}^T \begin{pmatrix} \frac{10k+2}{3(1+2k^2)} \\ \frac{5-2k}{1+2k^2} \\ 1 \end{pmatrix} = 0 \tag{2.0.11}$$
$$\implies -\frac{20k+4}{3+6k^2} + \frac{5-2k}{1+2k^2} = 3 \tag{2.0.12}$$
$$\implies \frac{11-26k}{3+6k^2} = 3 \tag{2.0.13}$$
$$\implies 9k^2 + 13k - 1 = 0 \tag{2.0.14}$$

Solving this, we get

$$k = \frac{-13 \pm \sqrt{205}}{18} \tag{2.0.15}$$

For the above values of k, the lines are intersecting, and hence, coplanar.

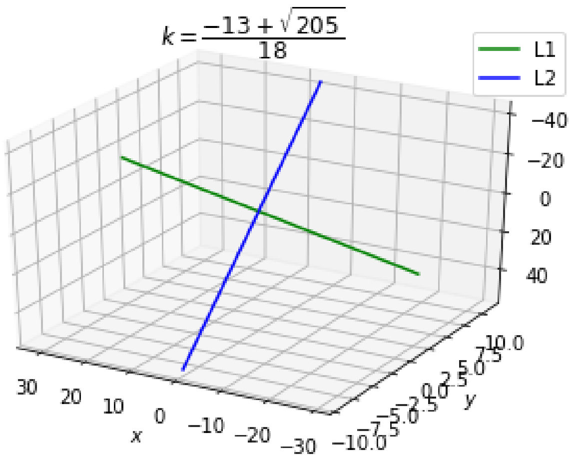

Fig. 0: Intersecting lines - Case1

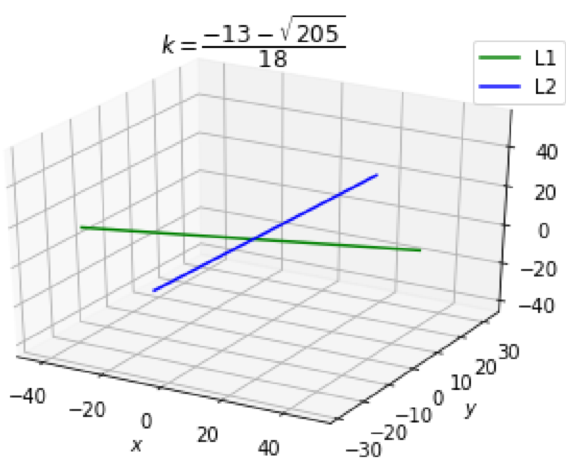

Fig. 0: Intersecting lines - Case2