Assignment : Parameter estimation.

1. Let us consider a random sample $(X_1, X_2, X_3 \ldots X_n)$, where

$\mu = Q_1$ (mean) and $n^2 = Q_2$ (variance)

So likelihood f^n $L(Q_1, Q_2) = \prod_{i=1}^{n} \dfrac{1}{\sqrt{2\pi Q_2}} \, e^{-\dfrac{(x_i - Q_1)^2}{2Q_2}}$.

to max. (logarithm)

$$\ln L(Q_1, Q_2) = \sum_{i=1}^{n} \left[-\dfrac{1}{2} \ln(2\pi Q_2) - \dfrac{(x_i - Q_1)^2}{2Q_2} \right].$$

① $\dfrac{d \ln L(Q_1, Q_2)}{dQ_1} = \sum_{i=1}^{n} \left[\dfrac{(x_i - Q_1)^2}{Q_2} \right] = 0.$

$\Rightarrow \quad x_i - nQ_1 = 0$

$$\boxed{Q_1 = \dfrac{\sum_{i=1}^{n} x_i}{n}} \quad (\text{mean}).$$

⑪ $\dfrac{d \ln L(Q_1, Q_2)}{dQ_2} = \sum_{i=1}^{n} \left[-\dfrac{1}{2Q_2} + \dfrac{(x_i - Q_1)^2}{2Q_2^2} \right] = 0.$

$\dfrac{n}{2Q_2} = \dfrac{1}{2Q_2^2} \sum_{i=1}^{n} (x_i - Q_1)^2.$

$$\boxed{Q_2 = \dfrac{1}{n} \sum_{i=1}^{n} (x_i - Q_1)^2}$$

(variance).

$$\boxed{\text{Parameter estimation}}$$

2. Binomial distribution $\quad B(n,\theta)$.

$$p = \theta \qquad q = 1-\theta$$

Probability mass function.

$$f(x; n, \theta) = {}^{n}C_{x}\, \theta^{x} (1-\theta)^{n-x}.$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i}\, \theta^{x_i} (1-\theta)^{n-x_i}.$$

taking log.

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left[\ln {}^{n}C_{x_i} + x_i \ln\theta + (n-x_i)\ln(1-\theta) \right]$$

diff. w.r.t θ.

$$\frac{d(\ln(L(\theta)))}{d\theta} = \sum_{i=1}^{m} \left[\frac{x_i}{\theta} - \frac{(n-x_i)}{1-\theta} \right] = 0.$$

To find θ. $\displaystyle\sum_{i=1}^{n} \left[\frac{(1-\theta)x_i - \theta(n-x_i)}{\theta(1-\theta)} \right] = 0$

$$\sum_{i=1}^{n} \left((1-\theta)x_i - \theta(n-x_i) \right) = 0.$$

$$\theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i \cdot m.$$

$$\boxed{\theta = \dfrac{\displaystyle\sum_{i=1}^{n} x_i}{n \cdot m}}$$