CS517

Assignment Report

JPEG Compression

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Introduction

JPEG compression is a lossy type of compression algorithm which works by first dividing all the pixel values by a quantization matrix and then applying a special function called DCT(Discrete Cosine Transform) on this image. The DCT exploits the fact that natural images are typically characterized by a relatively small number of significant high-frequency components, which can be represented using fewer bits than the original pixel values.

Once compressed, we further compress this image in an array containing encoded values by encoding them in a zig-zag manner.

Some Terms:

DCT and IDCT: DCT(Discrete Cosine Transform) is a mathematical technique used to convert a signal, in this case an image, from the spatial domain to the frequency domain. In JPEG compression, DCT is used to compress the **high** frequency components of the image, as these components typically **contribute less** to the overall image quality as observed.

IDCT on the other hand is the inverse function for DCT, both have the same formula except for the scaling term.

DCT:

$$F_{(k)} = \sqrt{\frac{2}{N}} C_{(k)} \sum_{j=0}^{N-1} f_{(j)} * \cos\left[\frac{(2j+1)k\pi}{2N}\right] \qquad k = 0, \ldots, N-1$$

IDCT:

$$f_{(j)} = \sqrt{\frac{2}{N}} \sum_{k=0}^{N-1} C_{(k)} * F_{(k)} * \cos\left[\frac{(2j+1)k\pi}{2N}\right] \qquad j = 0, \ldots, N-1$$

where:

$$C_{(k)} = \begin{cases} \frac{1}{\sqrt{2}} & \text{for k=0} \\ 1 & \text{for k=1, \ldots ,N-1} \end{cases}$$

Quantization and Quantization Matrix: A quantization matrix is a matrix used in the quantization process of JPEG compression. It is applied to the output of the DCT (Discrete Cosine Transform) of the image. The purpose of the quantization matrix is to reduce the amount of information needed to represent the image by rounding off the values of the DCT coefficients.

PSNR: It is a measure of the quality of a reconstructed image, compared to the original image. It calculates the ratio between the maximum possible value of a signal and the noise that affects the fidelity of its representation. A **higher PSNR value indicates a better quality reconstruction**, while a **lower PSNR value indicates more noise** and loss of information in the reconstruction.

$$\boxed{\begin{aligned} PSNR &= 10 \cdot \log_{10}\left(\frac{MAX_I^2}{MSE}\right) \\ &= 20 \cdot \log_{10}\left(\frac{MAX_I}{\sqrt{MSE}}\right) \\ &= 20 \cdot \log_{10}(MAX_I) - 10 \cdot \log_{10}(MSE). \end{aligned}}$$

Compression Ratio: It is the ratio of the number of original bits used to send the information with the number of bits sent when the data was compressed.

<u>Quantization Matrices</u>:

The code has a function to resize standard 8x8 quantization matrix.

8x8:

```python
# QMatrix for 8x8 blocks

QMatrix = np.array([[16, 11, 10, 16, 24, 40, 51, 61],
                    [12, 12, 14, 19, 26, 58, 60, 55],
                    [14, 13, 16, 24, 40, 57, 69, 56],
                    [14, 17, 22, 29, 51, 87, 80, 62],
                    [18, 22, 37, 56, 68, 109, 103, 77],
                    [24, 35, 55, 64, 81, 104, 113, 92],
                    [49, 64, 78, 87, 103, 121, 120, 101],
                    [72, 92, 95, 98, 112, 100, 103, 99
                     ]])
```

16x16:

```
[[126 121 127 125 134 130 139 127 134 124 126 124 118 125 128 129]
 [107 117 108 132 118 118 111 141 112  90 102 125 146  98 120 134]
 [101 105  99 135 115 126 109 152 105  80 112 136 145  98 112 123]
 [100  89  94 134 113 144 109 145  95  74 111 148 147 100 113 108]
 [ 96 102  90 130 107 145 103 135  98  98 104 136 134 114 118 113]
 [131 136 142 137 136 128 140 138 135 117 114 129 127 130 132 135]
 [135 138 142 134 141 138 135 137 137 130 129 127 133 129 127 130]
 [137 131 141 138 135 125 131 132 131 126 123 127 128 128 126 129]
 [118  76 120 136 117  86 118 133  83  96  85  94 104  79  90  84]
 [102  55 110 128 112  60 108 130  76 110  92  90 103  78  86  88]
 [104  54 117 133 115  57 105 128  77  65  65  88 104  71  77  82]
 [ 95  57 117 123 115  57 102 135  71 114  94  87 107  87 115  99]
 [111 114 121 134 112  57 102 126 119 115 113 121 123 120 125 134]
 [100 101 110 118  99  58  84 115 108 107 110 122 116 117 117 124]
 [ 64  58  62  69  81 109  98  76  72  70  72  66  74  59  80  94]
 [ 90  83  84  90  87  91  91  94  93  93  87  84  86  74  82 103]]
```

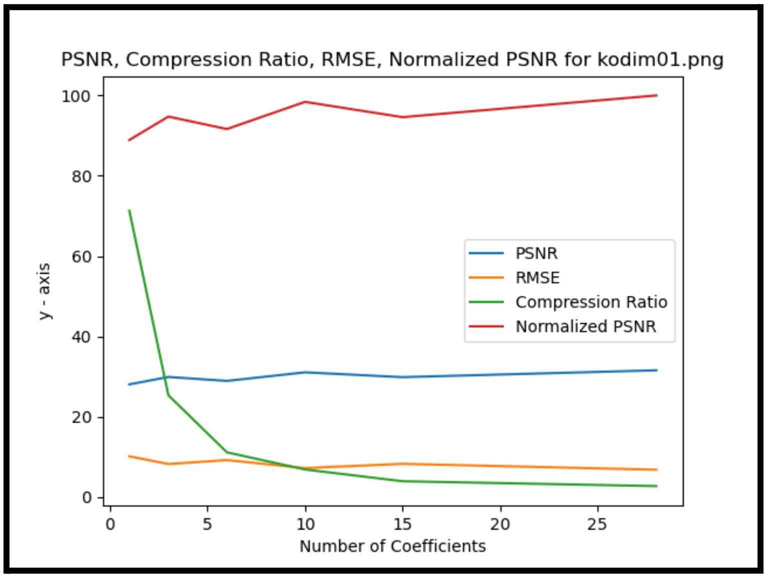
4x4:

```
[[114 127 112 121]
 [129 132 123 127]
 [103 105  87  91]
 [ 94  92  97 102]]
```

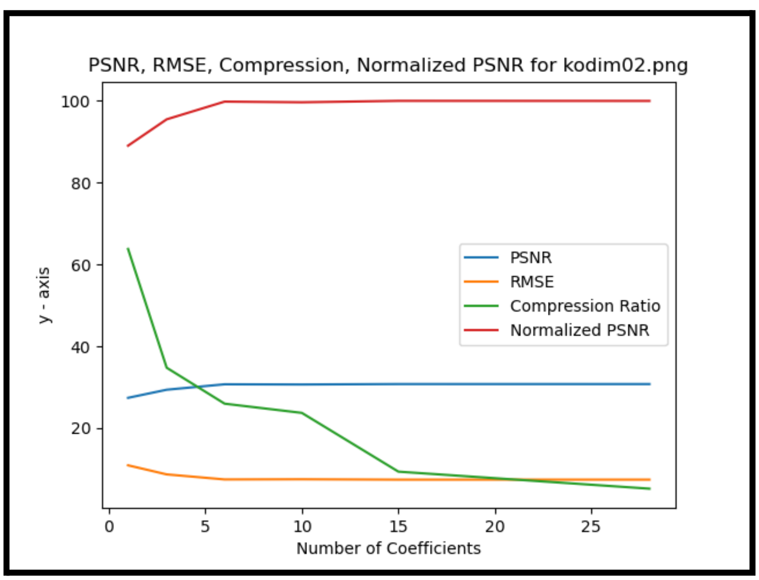
Graphs of PSNR, Compression Ratio, RMSE, Normalized PSNR with Number of Coefficients sent:

The calculations were done on several colored and grayscale images, here are some of their results:

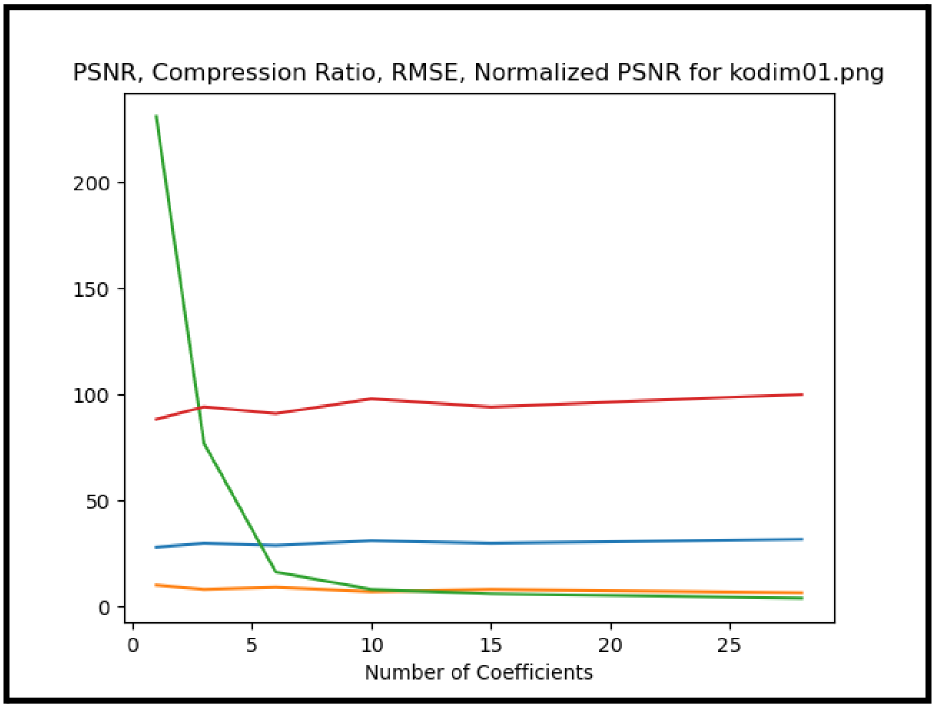
For **kodim01.png on 8x8 blocksize**



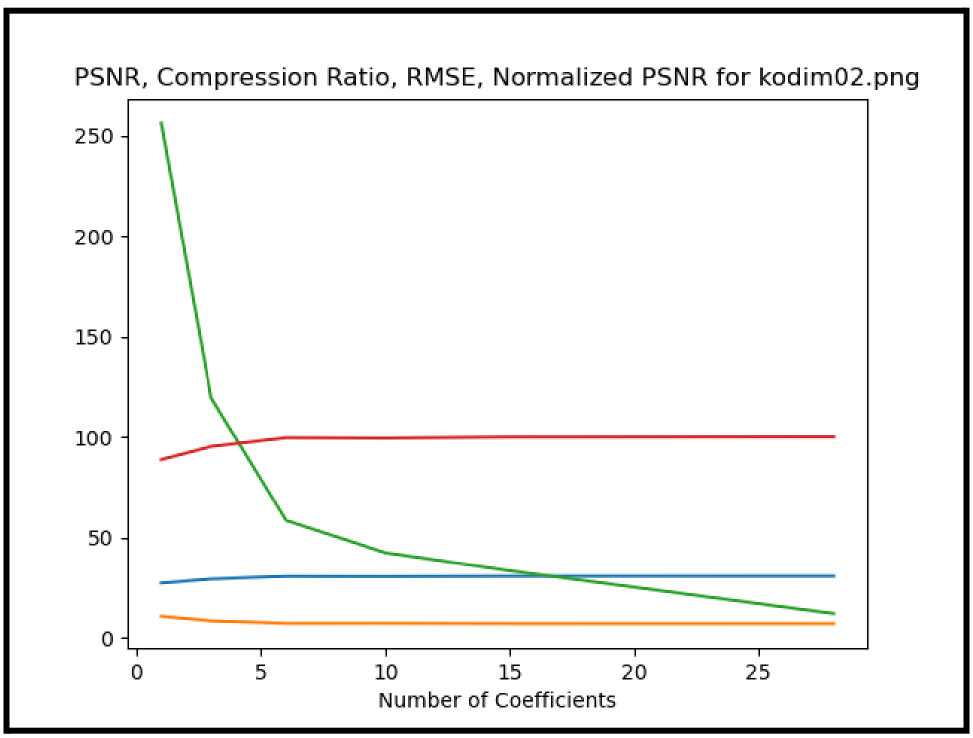
For **kodim02.png on 8x8 blocksize**



For **kodim01.png on 16x16 blocksize:**

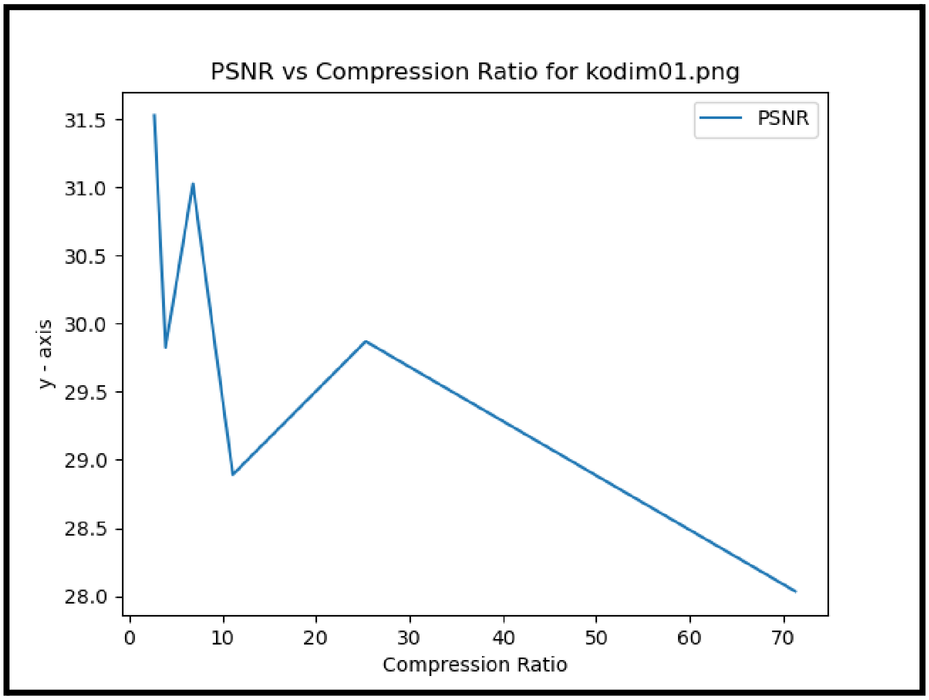


For **kodim02.png on 16x16 blocksize:**

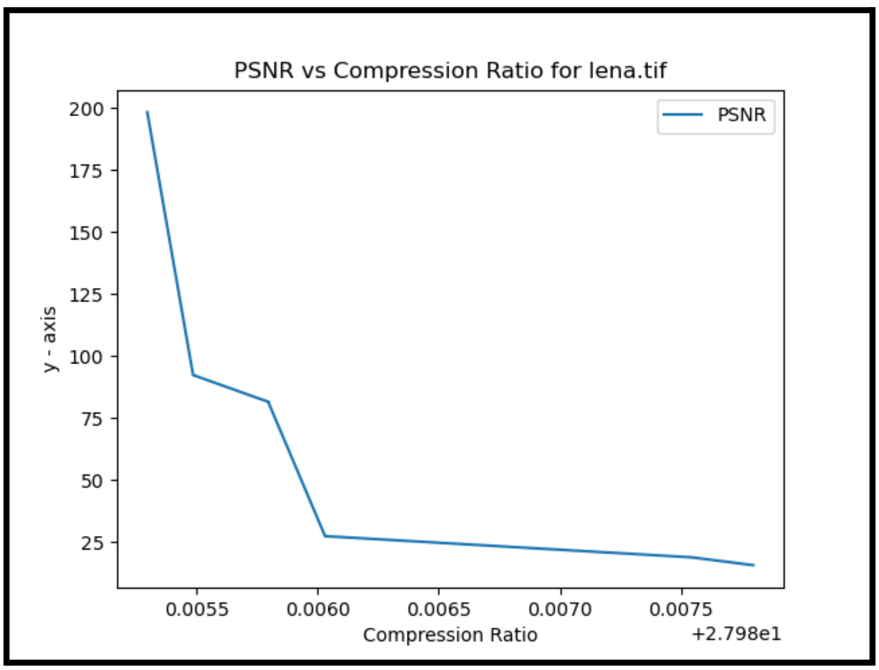


Graphs of PSNR vs Compression Ratio:

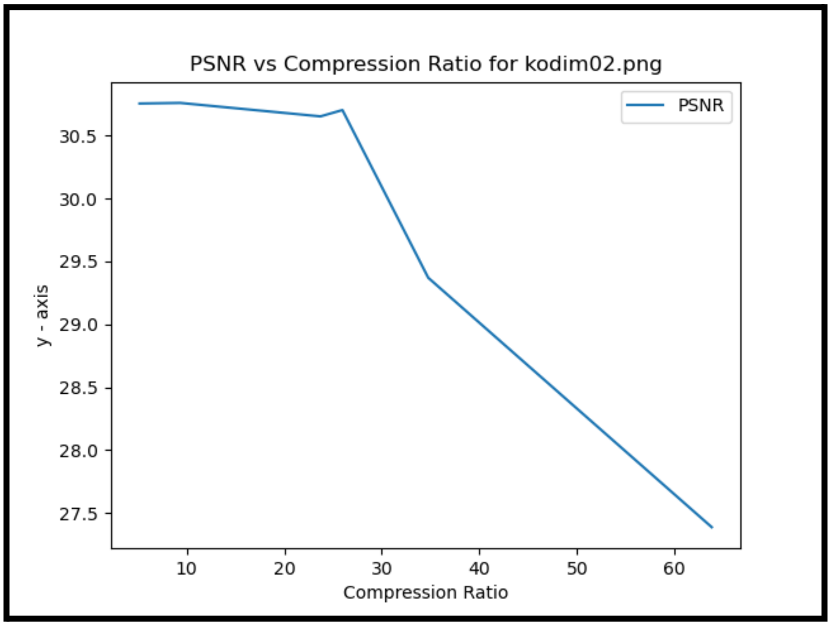
For **Kodim01.png on 8x8 blocksize:**



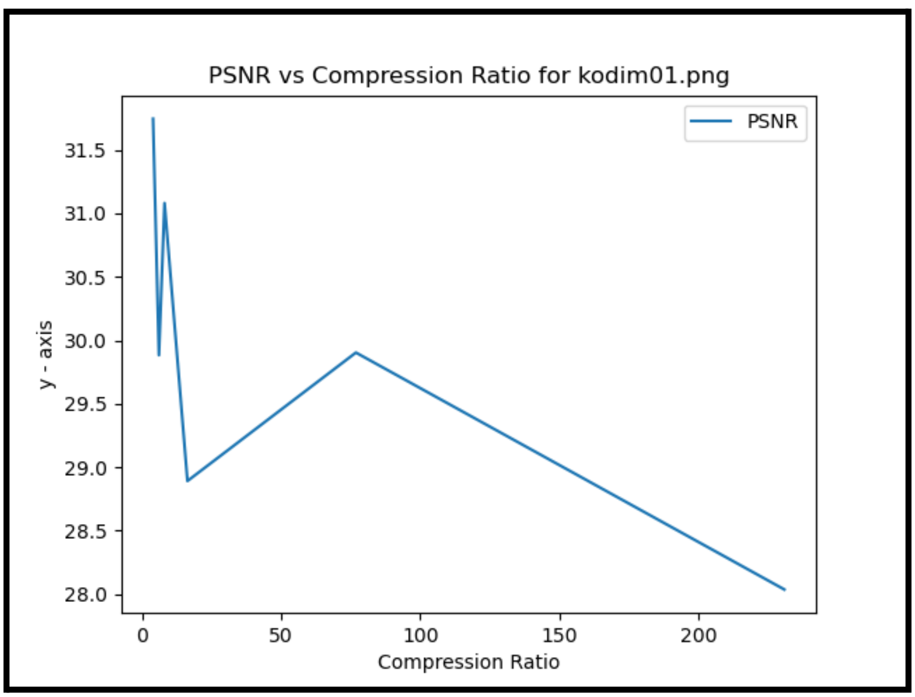
For **lena.tif on 8x8 blocksize:**



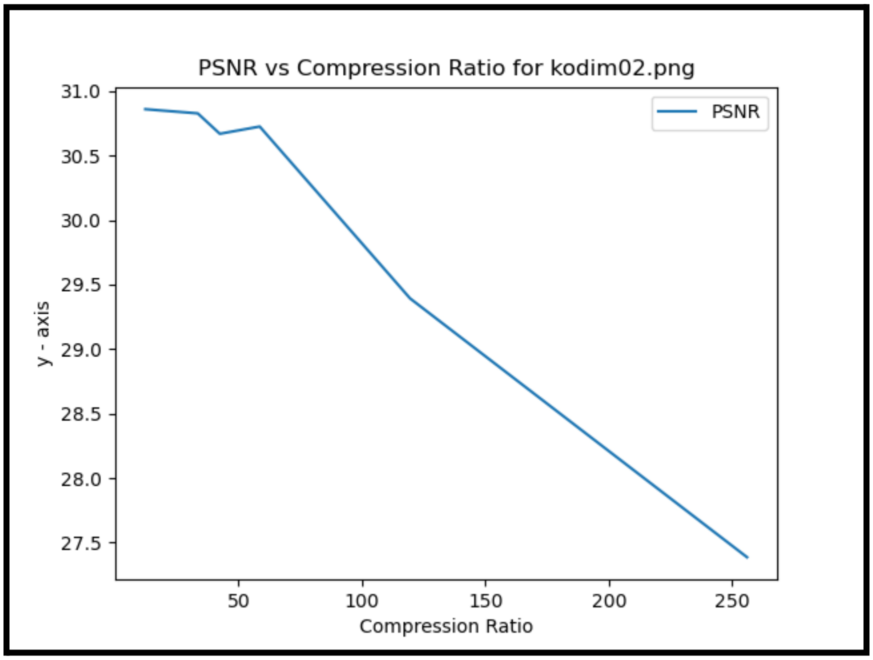
For **kodim02.png on 8x8 blocksize:**



For **kodim01.png on 16x16 blocksize:**

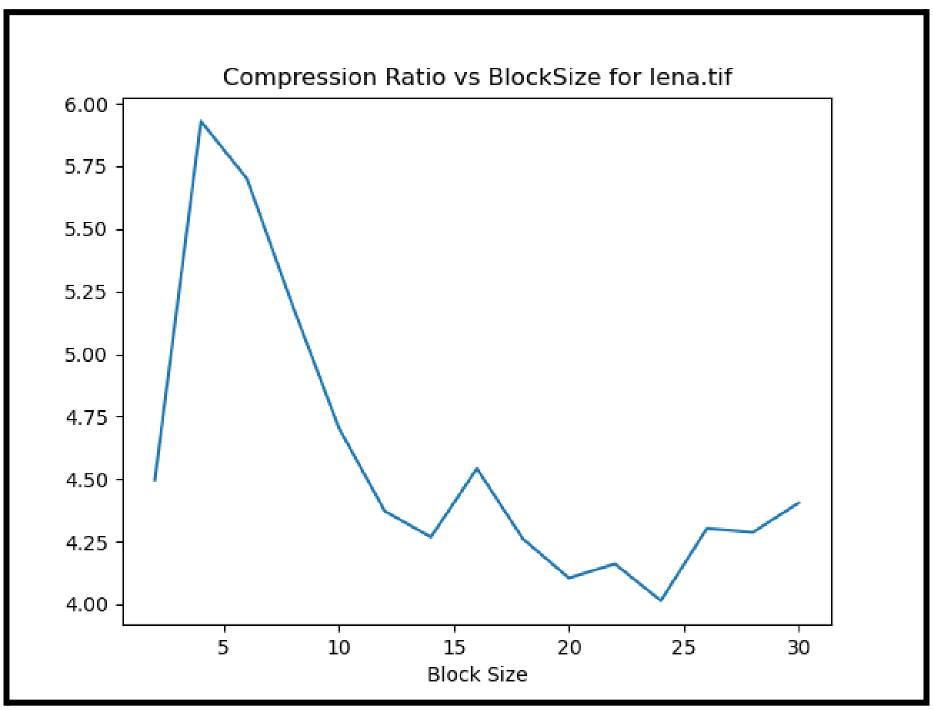


For **kodim02.png on 16x16 blocksize:**

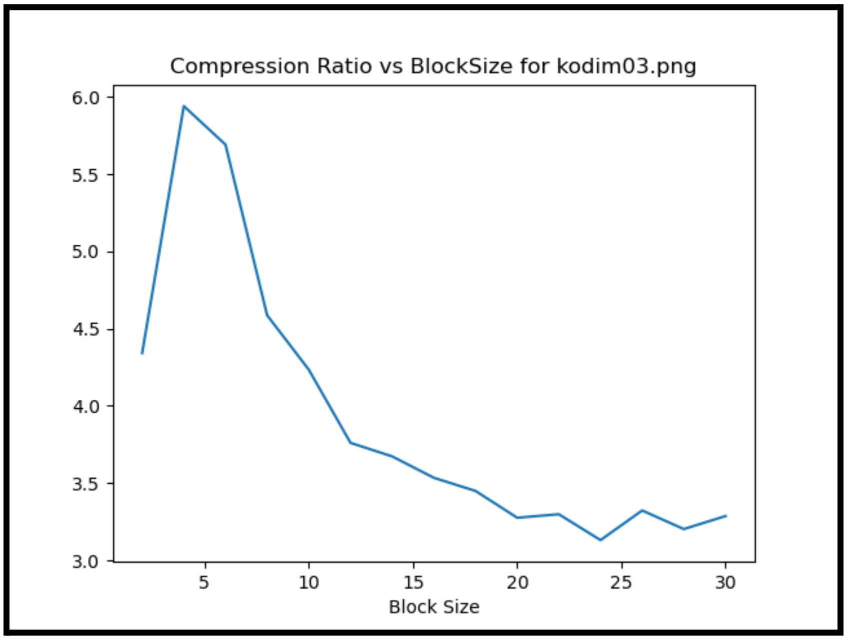


Graphs of Compression Ratio vs BlockSize:

For **lena.tif:**



For **kodim03.png:**



<u>Maximum Pixel Value after Compression:</u>

These are the maximum values for colored images after compression, i.e. after applying DCT and then Quantizing it on an 8x8 block.

```
Maximum value in image after DCT and quantization on image-kodim13.png on blocksize-8: 63
Maximum value in image after DCT and quantization on image-kodim19.png on blocksize-8: 50
Maximum value in image after DCT and quantization on image-kodim15.png on blocksize-8: 63
Maximum value in image after DCT and quantization on image-kodim08.png on blocksize-8: 62
Maximum value in image after DCT and quantization on image-kodim14.png on blocksize-8: 51
Maximum value in image after DCT and quantization on image-kodim16.png on blocksize-8: 60
Maximum value in image after DCT and quantization on image-kodim09.png on blocksize-8: 58
Maximum value in image after DCT and quantization on image-kodim03.png on blocksize-8: 52
Maximum value in image after DCT and quantization on image-kodim11.png on blocksize-8: 42
Maximum value in image after DCT and quantization on image-kodim17.png on blocksize-8: 51
Maximum value in image after DCT and quantization on image-kodim21.png on blocksize-8: 62
Maximum value in image after DCT and quantization on image-kodim10.png on blocksize-8: 63
Maximum value in image after DCT and quantization on image-kodim20.png on blocksize-8: 63
Maximum value in image after DCT and quantization on image-kodim06.png on blocksize-8: 62
Maximum value in image after DCT and quantization on image-kodim18.png on blocksize-8: 57
Maximum value in image after DCT and quantization on image-kodim23.png on blocksize-8: 63
Maximum value in image after DCT and quantization on image-kodim01.png on blocksize-8: 49
Maximum value in image after DCT and quantization on image-kodim05.png on blocksize-8: 60
Maximum value in image after DCT and quantization on image-kodim04.png on blocksize-8: 52
Maximum value in image after DCT and quantization on image-kodim07.png on blocksize-8: 45
Maximum value in image after DCT and quantization on image-kodim02.png on blocksize-8: 55
Maximum value in image after DCT and quantization on image-kodim24.png on blocksize-8: 63
Maximum value in image after DCT and quantization on image-kodim22.png on blocksize-8: 56
Maximum value in image after DCT and quantization on image-kodim12.png on blocksize-8: 62
```

Here the **maximum value** obtained for all these images is **63**.

Similarly, observing the maximum values for grayscale images.

```
Maximum value in image after DCT and quantization on image-Fig81a.tif on blocksize-8: 37
Maximum value in image after DCT and quantization on image-lena.tif on blocksize-8: 56
Maximum value in image after DCT and quantization on image-fingerprint.tif on blocksize-8: 47
Maximum value in image after DCT and quantization on image-checkerboard1024.tif on blocksize-8: 63
Maximum value in image after DCT and quantization on image-matches-random.tif on blocksize-8: 63
Maximum value in image after DCT and quantization on image-zoneplate.tif on blocksize-8: 64
Maximum value in image after DCT and quantization on image-Fig81c.tif on blocksize-8: 0
Maximum value in image after DCT and quantization on image-book-cover.tif on blocksize-8: 62
Maximum value in image after DCT and quantization on image-matches-aligned.tif on blocksize-8: 63
Maximum value in image after DCT and quantization on image-Fig81b.tif on blocksize-8: 35
```

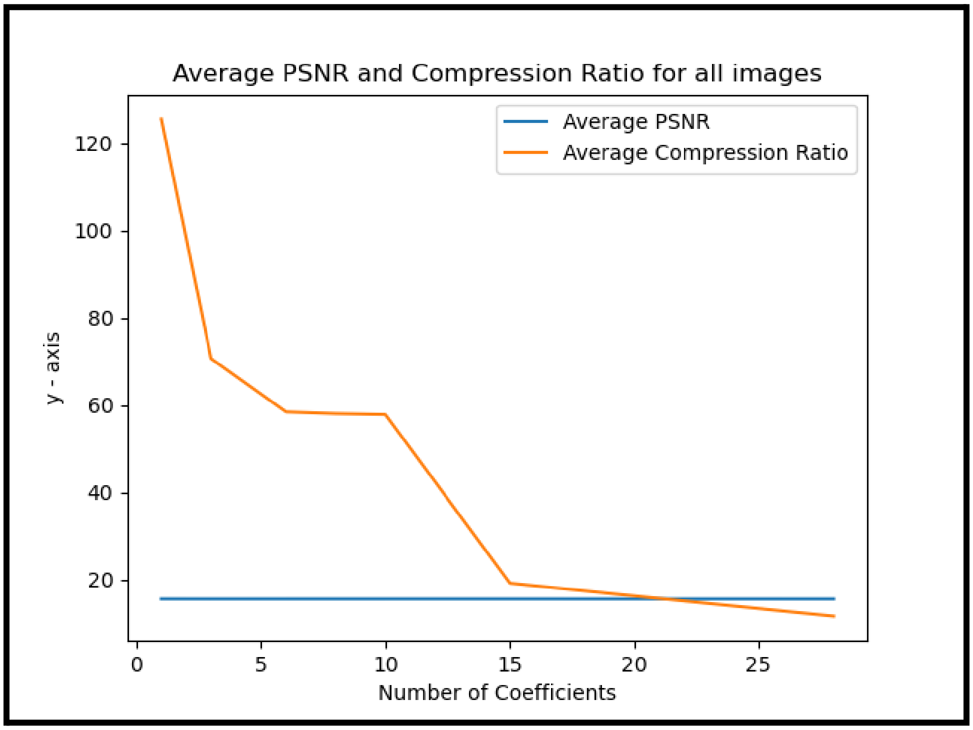
In this too, the **maximum value** obtained is **63**.

Since **63** can be represented using **8 bits**. So, **8 bits** are sufficient to send all the pixel values of the image because all the pixel values will always be less than or equal to 63.

Average Compression Ratio, Average (+Normalized) PSNR with Number of coefficients sent:

Common Plot for both with varying Number of Coefficients:

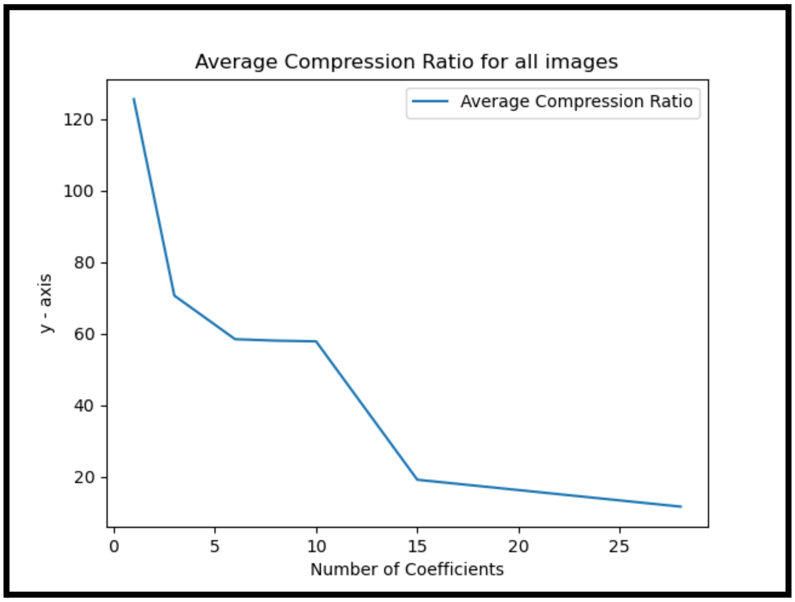
(Normalized PSNR is multiplied by 100 for better view)



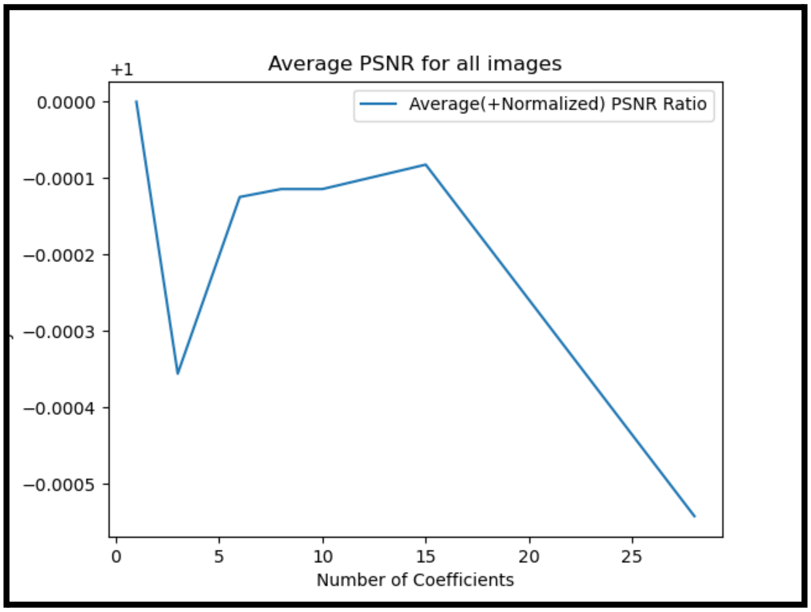
It is not visible because of high values of Average Compression Ratio, but the value of Average PSNR slightly increases.

Here are the graphs separately:

Average Compression:

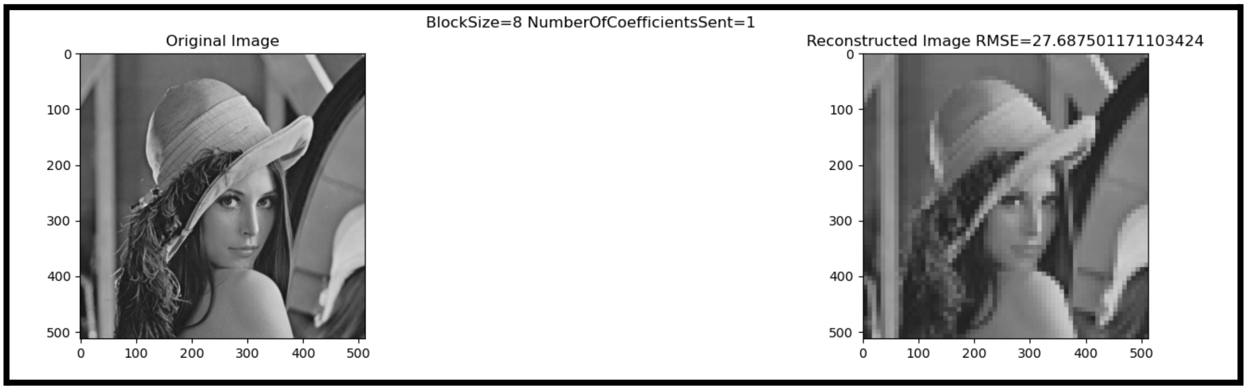


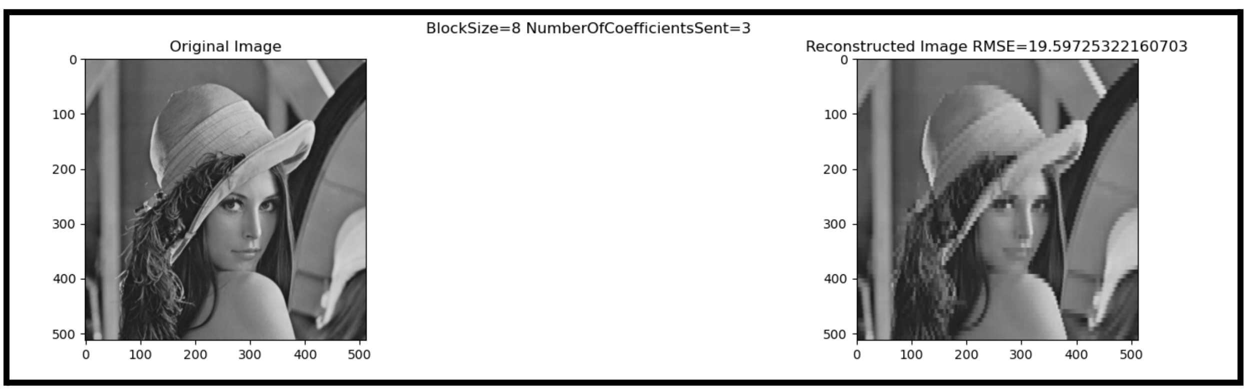
Average(+Normalized) PSNR:

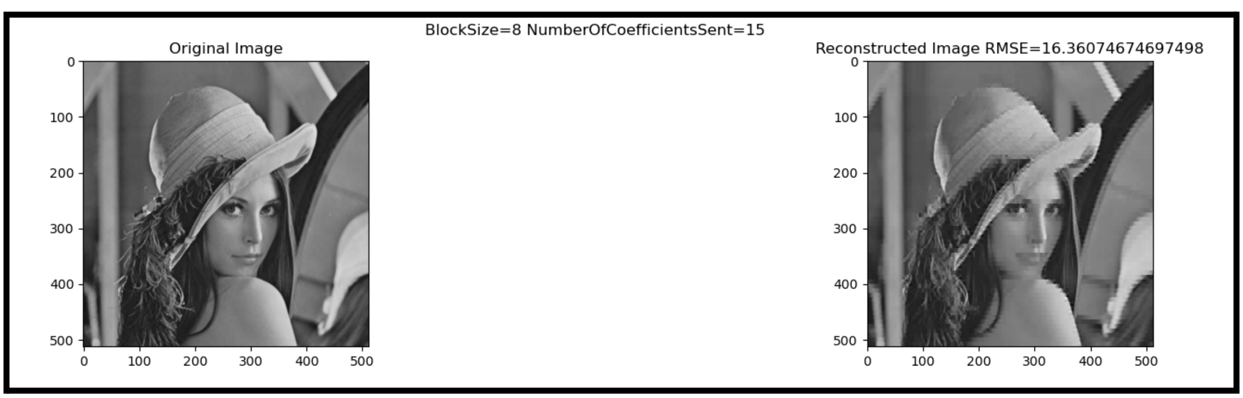


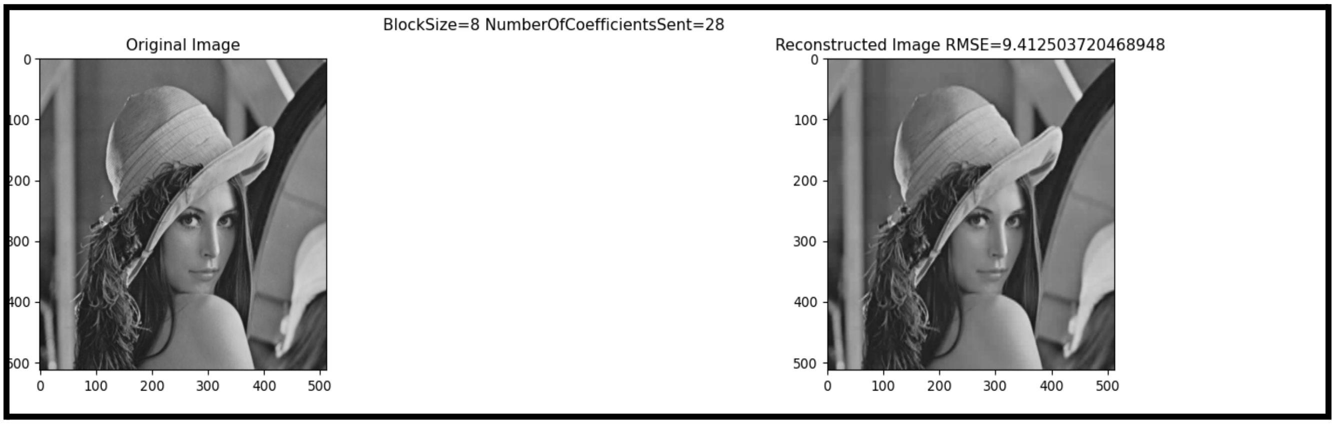
Results:

Here are some results obtained after JPEG compression on grayscale and color images:

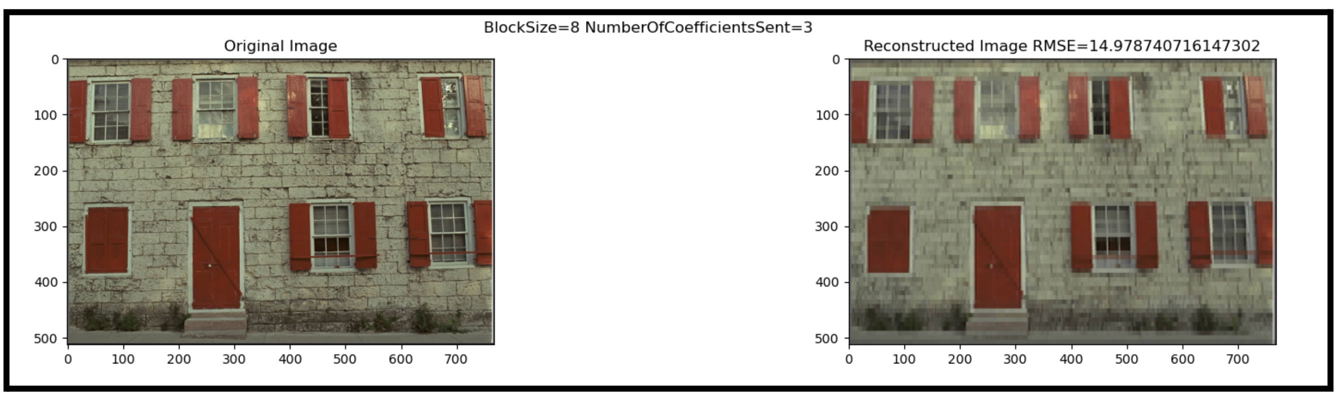


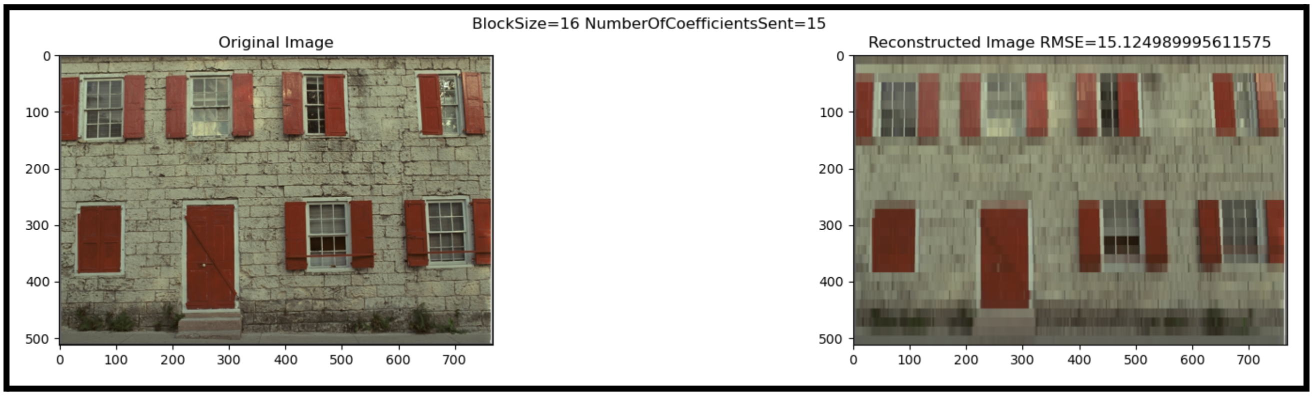


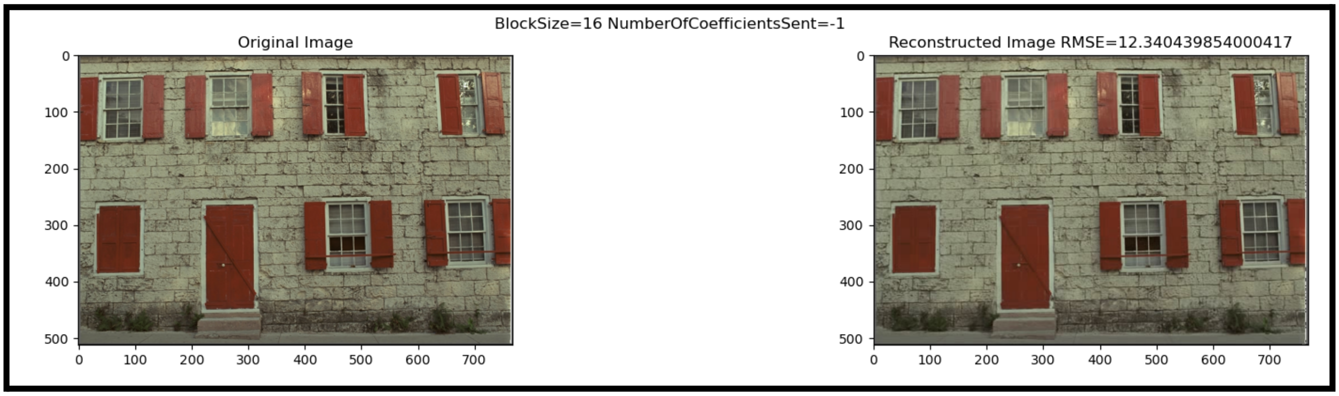



BlockSize=8 NumberOfCoefficientsSent=28
Original Image
Reconstructed Image RMSE=9.412503720468948
0 100 200 300 400 500


BlockSize=8 NumberOfCoefficientsSent=1
Original Image
Reconstructed Image RMSE=15.483443000221666
0 100 200 300 400 500
0 100 200 300 400 500 600 700


BlockSize=8 NumberOfCoefficientsSent=3
Original Image
Reconstructed Image RMSE=14.978740716147302
0 100 200 300 400 500
0 100 200 300 400 500 600 700


BlockSize=16 NumberOfCoefficientsSent=15
Original Image
Reconstructed Image RMSE=15.124989995611575


BlockSize=16 NumberOfCoefficientsSent=-1
Original Image
Reconstructed Image RMSE=12.340439854000417

Observations:

1. PSNR and Compression Ratio have an inverse relationship with each other when varying the number of coefficients.
2. We can reduce the amount of bits sent for each pixel by observing the maximum pixel values in our compressed image.
3. Converting to YCbCr increases the amount of zeros obtained in the image after compression.
4. When the number of coefficients sent are changed, the compression ratio decreases, RMSE decreases slightly, PSNR increases slightly.
5. When varying the blocksize, the compression ratio achieves a maximum value and then decreases.

Conclusion:

Overall it was a great learning experience, where I learnt about JPEG compression, encoding in ZigZag manner, decoding these ZigZag encoded values. I also learnt about PSNR and its application. Moreover, I also learnt about how different factors like, number of Coefficients sent, BlockSize affect the Compression Ratio, PSNR and Error in an image. The analysis done gave me insights on how sending different numbers of coefficients gives different compression ratios and how it changes.