

Dijkstra's Algo

Apr-3 Dijkstra

classmate
Date
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Dijkstra's Algo

This used used to find all nodes shortest path from ~~the~~ src node. in ~~undirected~~ graph

Approach :- we need. 'distance' vector $dist$, unordered map 'Adj List', 'set' or 'min heap' to store nodes based on min distance to reach them (Cuz we need to fetch min dist node)

$\langle 1, 3 \rangle$
$\langle 5, 4 \rangle$
$\langle 1, 2 \rangle$
$\langle 0, 1 \rangle$

Set $\langle pair \langle int, int \rangle \rangle$

dist \swarrow node \uparrow

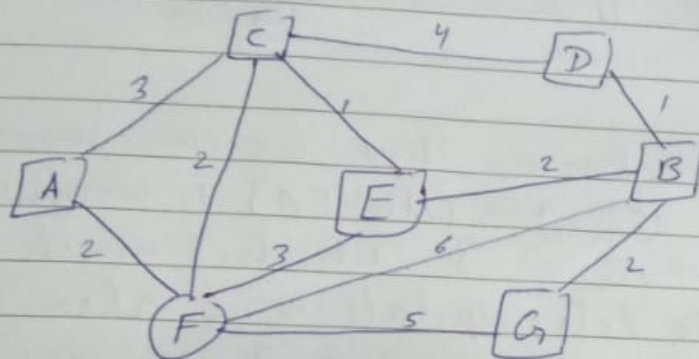
node : when we insert another node into set make sure to delete pair one

Dijkstra's Algo

works on 2. basis steps

- (i) relaxation.
- (ii) Choose new next.

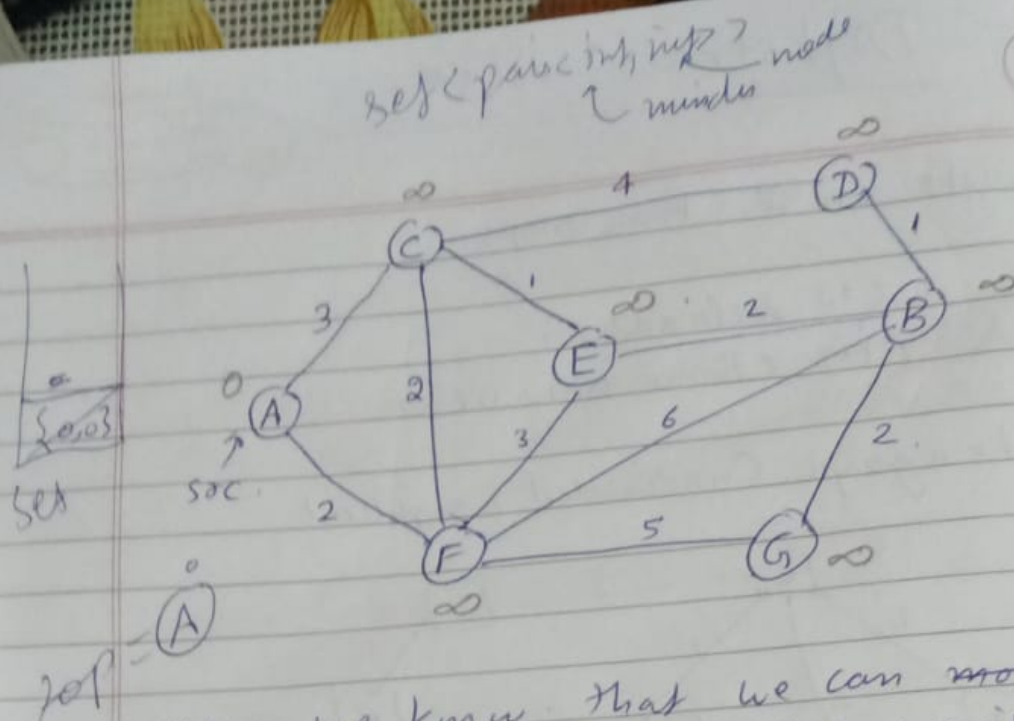
Let's take a graph. (undirected weighted).



Let's say we need to determine the shortest distance as well as shortest path from A to B, but for that we need to know the shortest distances to reach other nodes from A. & if we have it we can easily determine the final ans.

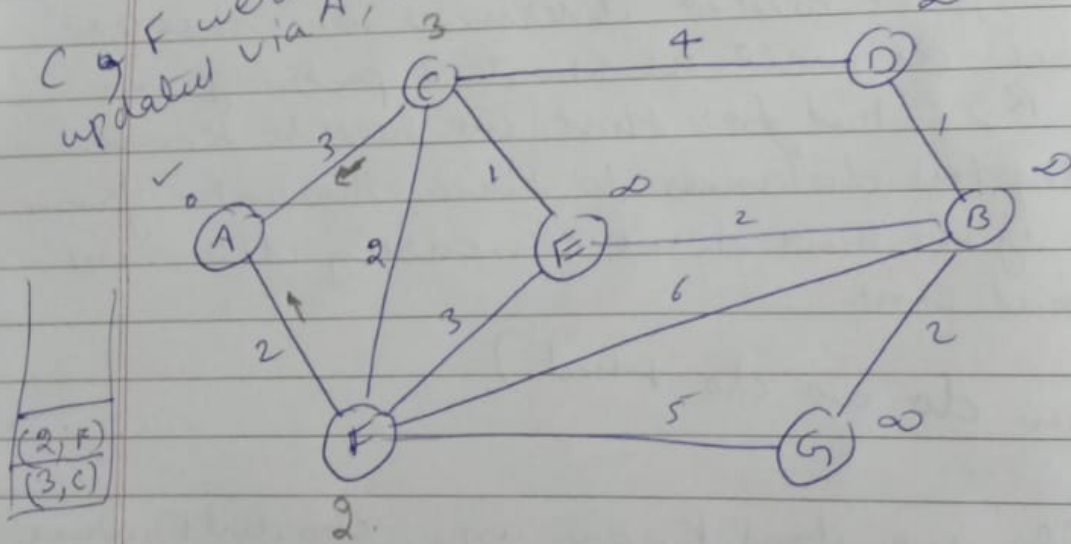
But how do we do that?

Initially we don't know any shortest distance for any node except start, so let's take all the other nodes distance as ' ∞ ' & src's distance as '0'



now we know that we can move & reach C via $(\text{dist}[A] + \text{weight from A to C})$ i.e. $0 + 3 \Rightarrow 3$ which is much less than ∞ , so let's update it; also we can reach F via $0 + 2 = 2 < \infty$.

C & F were updated via A, let's print is below



now let's pick the next node (with min dist)
which is node F ($\text{dis}(F) < \text{dis}(C)$).

~~Since C is~~

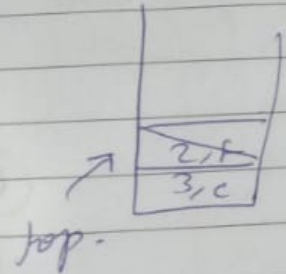
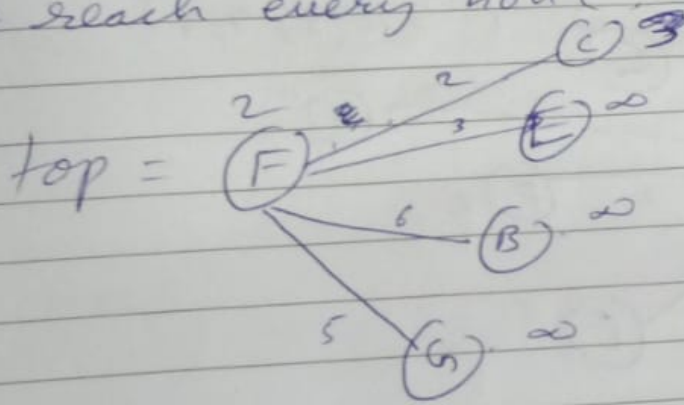
Imp note

We need to remove any prev entries for a given 'node' & update new entries for it because the min distance for that node is not the same.

note: Here we picked F rather than C. because ~~the~~ currently acc to our known edges we can reach next step with least dist of 2.

This does NOT mean that our shortest path is $A \rightarrow F$
 , no -

we are only finding the min distance required
 to reach every node



$$2 + 2 < 3 \quad \times$$

$$2 + 3 < E(\infty) \quad \checkmark \quad \text{update } E.$$

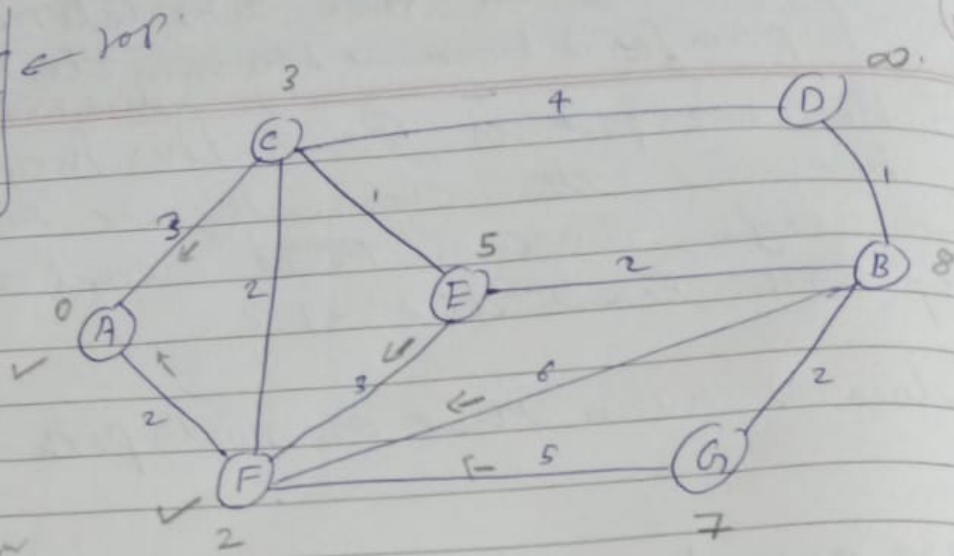
$$2 + 6 < B(\infty) \quad \checkmark \quad \text{" } B.$$

$$2 + 5 < G(\infty) \quad \checkmark \quad \text{" } G.$$

3, C
5, E
7, G
8, B
2, F

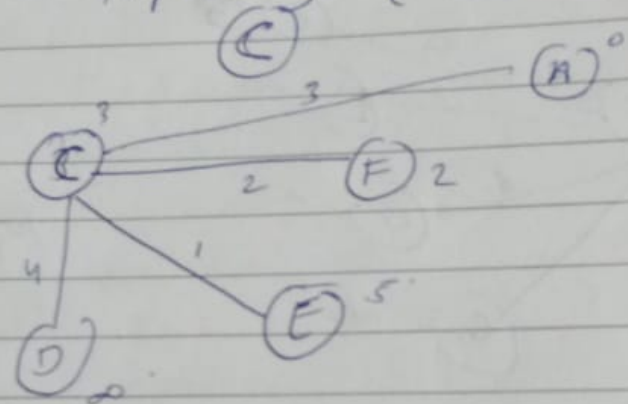
S, C
5, E
7, G
8, B

Set



mark down
from B, E, G
to F
now
C, B, F, G are
updates
with
F

top = 3 (3 < 5, 7, 8)



5, E
7, G
8, B

old set

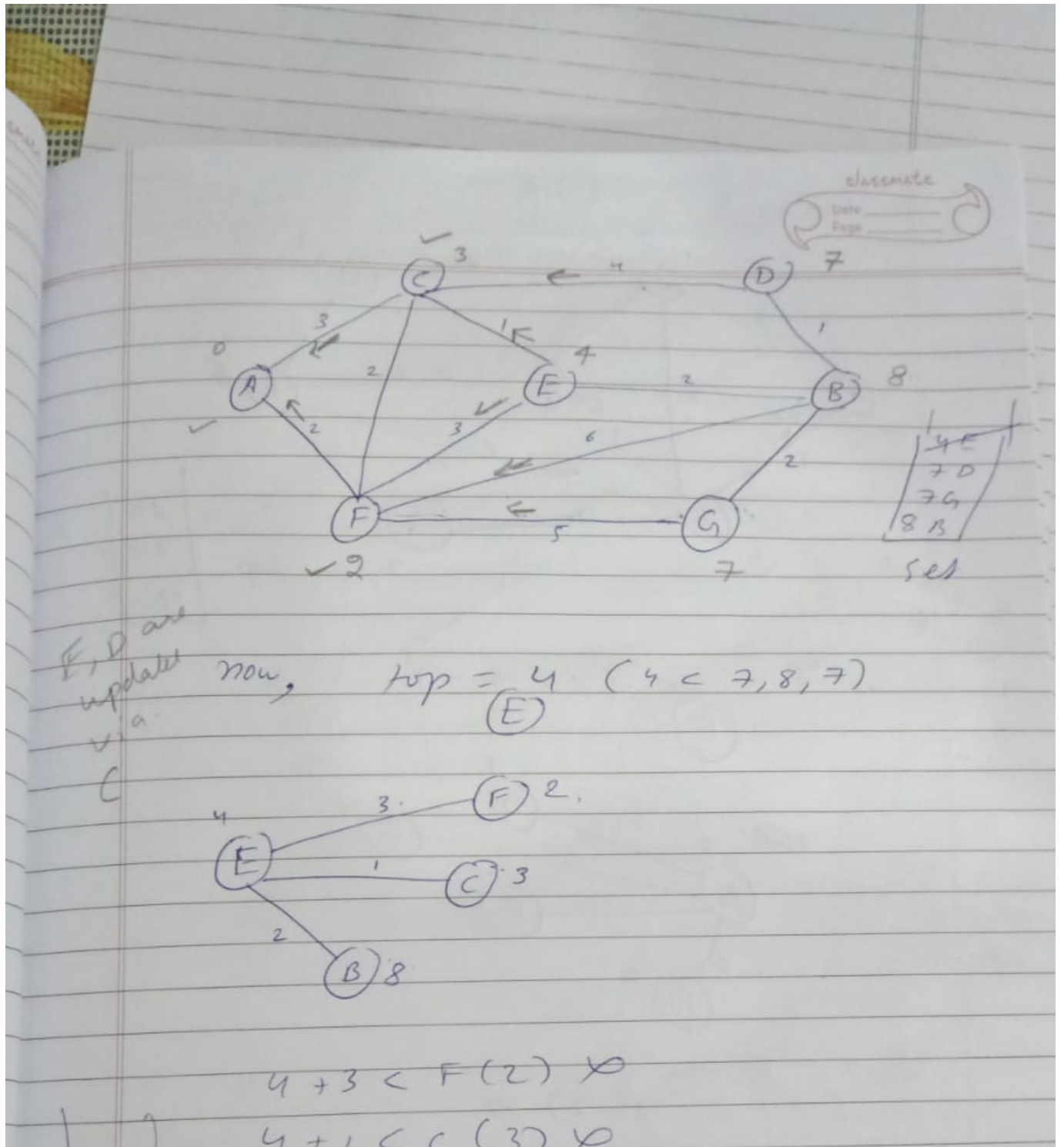
$3 + 3 < 0 (A) \times$
 $3 + 2 < F (2) \times$
 $3 + 1 < E (5) \checkmark$ update
 $3 + 4 < D (\infty) \checkmark$ " D

to be removed

4 E ← new value for E (5, E).

4 E
7 D
7 G
8 B

new set.



7D
7G
8B

deleted

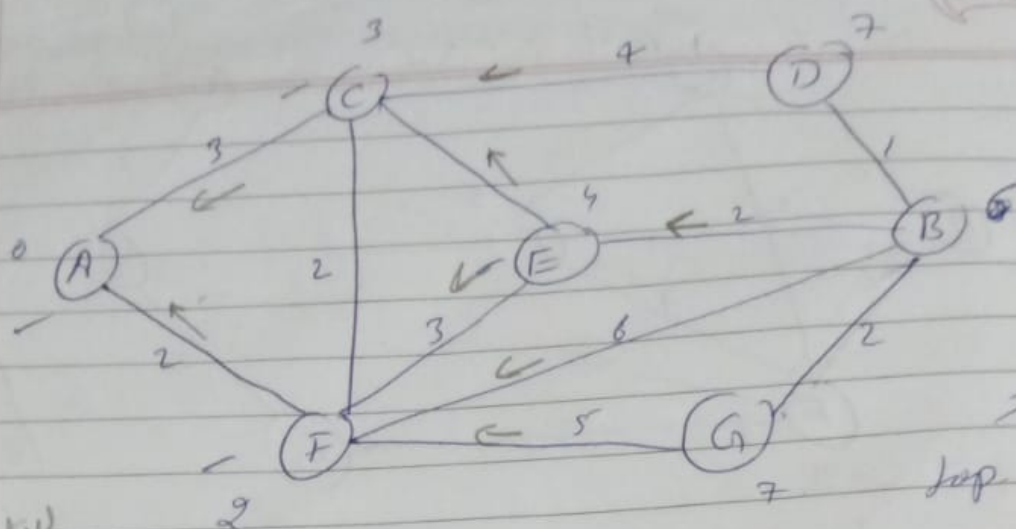
$4 + 2 < B(8)$ ✓ update B.

old set

6B
7D
7G

new entry for B. (6, B deleted)

new set.



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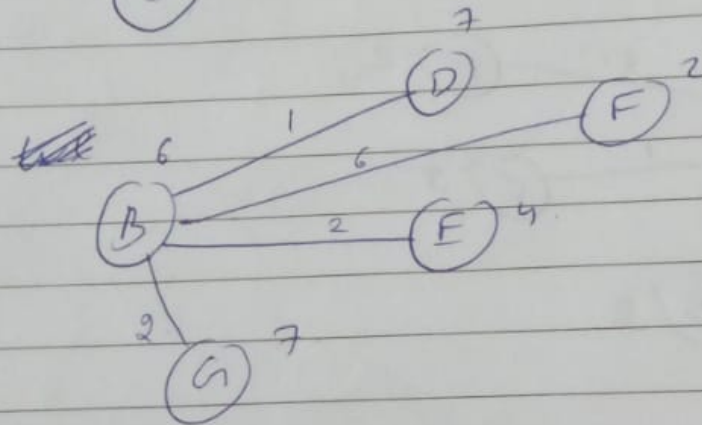
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B updated
in E.

lap

6B
7D
7G

top = 6 (6 < 7, 7)

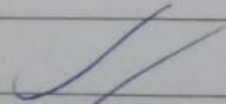
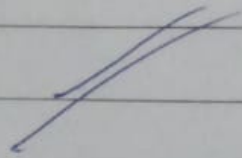


$$6 + 1 < D(7) \quad \times$$

$$6 + 6 < F(2) \quad \times$$

$$6 + 2 < E(4) \quad \times$$

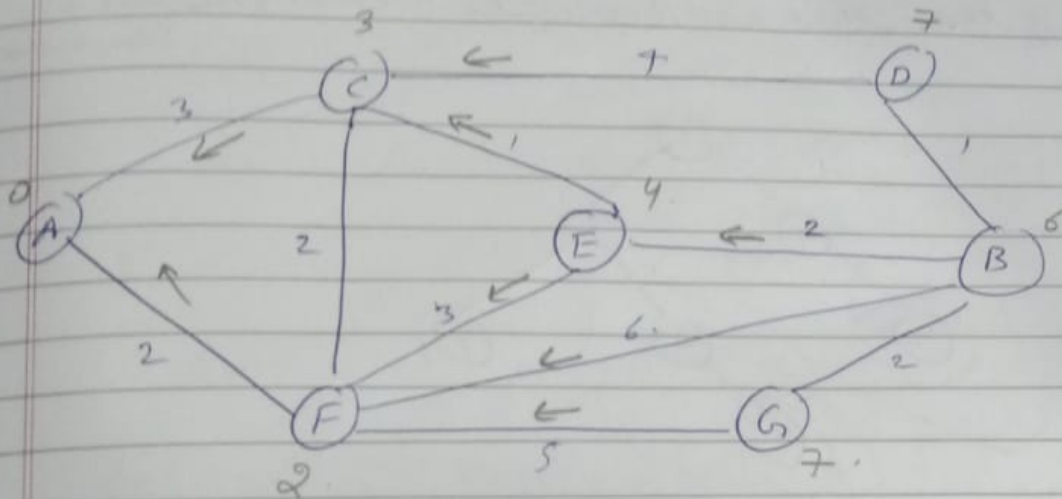
$$6 + 2 < G(7) \quad \checkmark$$



T: O(E log V)

C: O(E + V)

finally we have graph as.



\therefore (A) to (B) min dist = 6 (shortest distance)

now track way back from (B) to (A) via parents

$B \rightarrow E \rightarrow C \rightarrow A$ (Shortest path)



Code :-


```
#include <bits/stdc++.h>
```

```
/*[✓★[App-1.1] more faster then prv] App-1
```

```
Title = Dijkstra's Algo Using Set<pair(minDis to reach node ,node) >
```

explanation :- what we do here is we use a set(minDis to reach node,node) and using this every time we pop the front

of this set, we will get the pair with least 'minDis'.

(we can also use minHeap instead of this), so every time we will follow a 2 step process after fetching the front

step 1 -> Relaxation :- here if the minDis[front] + distance between neigh and front is lesser then the minDis[neigh] then we will update the minDis[neigh]

and also insert this new pair for node
-> 'neigh' into the set, make sure to delete the prv entry of the 'neigh' from the set if exists any

step 2 -> find the next node to do the same process (for this we will choose the node with the min Distance, which is not yet explored i.e present in the set)

note: .find() .insert() .erase() takes $O(\log N)$ times to perform because we are using 'ordered_set' not an unordered_set

☑T : $O(E \cdot \log V)$ - traversing each edge takes E and inserting the verticies into set will take $\log V$

☑S : $O(E+V)$

```
*/
```

```
// fun.2
```

```
void createAdjList(vector<vector<int>> &vec, unordered_map<int,list<pair<int,int>>> &AdjList){
```

```
    for(int i=0; i < vec.size(); i++){
        int node1 = vec[i][0];
        int node2 = vec[i][1];
        int weight = vec[i][2];
```

```
        // undirected graph
```

```
        AdjList[node1].push_back({node2, weight});
```

```
        AdjList[node2].push_back({node1, weight});
```

```
    }
```

```
}
```

```
// fun.3
```

```
void dijkstraMinDis(unordered_map<int,list<pair<int,int>>> &AdjList,
```

```

set<pair<int,int>> &set, vector<int> &minDist, int src){

    // initially we can reach the src itself in 0 distance, and insert its pair{0,src}
    into the set also
    minDist[src] = 0;
    set.insert({0,src});

    // run a loop while set is non empty
    while(!set.empty()){

        // fetch the top node of set (the one with the minimum distance)
        pair<int,int> frontPair = *(set.begin());
        set.erase(set.begin()); // pop the front

        int frontMinDis = frontPair.first;
        int frontNode = frontPair.second;

        // explore all the neighbours of this frontnode
        for(auto neighPair:AdjList[frontNode]){

            int neigh = neighPair.first;
            int weight = neighPair.second; // this weight is the 'distance from
            frontNode to neigh'

            // now update the min distance to reach the 'neigh' node if and only if
            the new path distance is smaller then the prv.
            if(minDist[frontNode] + weight < minDist[neigh]){

                // now after a node's distance to reach is updated, we need to update
                that 'neigh''s min distance in the set also, so for that (find if there already exist
                a node 'neigh' in the set, if yes then delete it) and then insert the new pair into
                the set

                auto prvEntry = set.find({minDist[neigh] ,neigh});
                if(prvEntry != set.end()){ // another entry for 'neigh' exists
                    set.erase(prvEntry);
                }

                // updation of minDist of 'neigh'
                minDist[neigh] = minDist[frontNode] + weight;

                // insertion of new pair in set
                set.insert({minDist[neigh],neigh});
            }
        }
    }
}

// main function
vector<int> dijkstra(vector<vector<int>> &vec, int vertices, int edges, int source) {

```

```

    // we need a 'AdjList', a 'minDist' (to return at the end), and a
    set<pair<int,int>> to store the {minDis to reach node, node}
    unordered_map<int,list<pair<int,int>>> AdjList;
    vector<int> minDist(vertices,INT_MAX); // initially all nodes can be reached only
in inf distance
    set<pair<int,int>> set; // order this on basis of the pair.first i.e min dist

    // create AdjList by fun.2
    createAdjList(vec, AdjList);

    // now to apply dijkstra call fun.3
    dijkstraMinDis(AdjList, set, minDist, source);

    return minDist;

}

```

----- END -----