

Master's Theorem:-

If $f(n) \in O(n^k)$ or $f(n) = c \cdot n^d$ where $d \geq 0$ is recurrence
 $T(n) = aT(n/b) + f(n)$ then.

$$T(n) \in \begin{cases} O(n^a) , & \text{if } (a < b^d) \\ O(n^d \log n) , & \text{if } (a = b^d) \\ O(n^a \log_b a) , & \text{if } (a > b^d) \end{cases}$$

1) $T(n) = 8T(n/2) + 1000n^2$

$a=8$, $b=2$, $f(n) = 1000n^2 = c \cdot n^d$, where $d=2$
 $b^d = 2^2 = 4$, Hence $a > b^d$

$T(n) \in O(n \log_b a)$ $\log_2 8 = 3$
 $\therefore T(n) \in O(n^3)$

2) $T(n) = 2T(n/2) + n^2$

$a=2$, $b=2$, $f(n) = n^2$, $d=2$

$b^d = 2^2 = 4$, $a < b^d$, $\therefore T(n) \in O(n^2)$

3) $T(n) = 2T(n/2) + 10n$

$a=2$, $b=2$, $f(n) = 10n$, $d=1$

$b^d = 2$, Hence $a = b^d$

$\therefore T(n) \in O(n^d \log n)$

$\therefore T(n) \in \Theta(n \log n)$