

```

+ using QuadGK √
+ begin
+     function I_of_p_1(p)
+         integrand(y) = sqrt((1 - y) / (p^2 + y))
+         val, _ = quadgk(integrand, 0.0, 1.0; rtol=1e-6, atol=1e-6)
+         return val
+     end

+     function solve_p0_1(a, b; tol=1e-6, maxiter=50)
+         fa = I_of_p_1(a) - 1
+         fb = I_of_p_1(b) - 1
+         if fa * fb > 0
+             error
+         end
+         for iter in 1:maxiter
+             m = (a + b) / 2
+             fm = I_of_p_1(m) - 1
+             @info "iter=$iter p=$m I(p)-1=$(fm)"
+             if abs(fm) < tol
+                 return m
+             end
+             if fa * fm < 0
+                 b = m
+                 fb = fm
+             else
+                 a = m
+                 fa = fm
+             end
+         end
+         return (a + b) / 2
+     end

# -----
# Run solver
# -----

p0 = solve_p0_1(-0.5, 0.0)
println("\nSolution p(0) = $p0")
end

```

① iter=1 p=-0.25 I(p)-1=0.1586812931495858

① iter=2 p=-0.375 I(p)-1=0.00746678908620968

① iter=3 p=-0.4375 I(p)-1=-0.057391792381185414

① iter=4 p=-0.40625 I(p)-1=-0.02577927908922506

① iter=5 p=-0.390625 I(p)-1=-0.009366090740190347

① iter=6 p=-0.3828125 I(p)-1=-0.0010028320346118225

① iter=7 p=-0.38790625 I(p)-1=0.003218592422551403

① iter=8 p=-0.380859375 I(p)-1=0.0011045450552582547

① iter=9 p=-0.3818359375 I(p)-1=5.002414580479453e-5

① iter=10 p=-0.38232421875 I(p)-1=-0.00047661185821501473

① iter=11 p=-0.382080078125 I(p)-1=-0.0002133458568137625

① iter=12 p=-0.3819580078125 I(p)-1=-8.167385842694497e-5

① iter=13 p=-0.38189697265625 I(p)-1=-1.5828107387982904e-5

① iter=14 p=-0.381866455078125 I(p)-1=1.7097206395977338e-5

① iter=15 p=-0.3818817138671875 I(p)-1=6.343463063718247e-7

② Solution p(0) = -0.3818817138671875

② 430 ms

```

+ begin
+     using Printf √
+
# Computes the interval value at midpoint
function I_of_p(p)
    integrand(y) = sqrt((1 - y) / (p^2 + y))
    val, err = quadgk(integrand, 0.0, 1.0; rtol=1e-6, atol=1e-6)
    return val, err
end

function solve_p0(a, b; tol=1e-10, maxiter=50)
    val_a, _ = I_of_p(a)
    fa = val_a - 1
    val_b, _ = I_of_p(b)
    fb = val_b - 1

    if fa * fb > 0
        error("Bisection interval does not bracket the root.")
    end

    for iter in 1:maxiter
        m = (a + b) / 2
        val, err = I_of_p(m)
        fm_interval_lo = val - err
        fm_interval_hi = val + err

        @printf("Iter %d: m = %.10f Integral[m] = [% .12f, %.12f]\n", iter, m,
                fm_interval_lo, fm_interval_hi)

        fm = val - 1
        if abs(fm) < tol
            println("Converged at iter $iter: root ≈ $m; Integral ∈
[$fm_interval_lo, $fm_interval_hi]")
            return a, b, (fm_interval_lo, fm_interval_hi)
        end

        if fa * fm < 0
            b = m
            fb = fm
        else
            a = m
            fa = fm
        end
    end

    println("Max iterations reached. Interval [$a, $b], last Integral ∈
[$fm_interval_lo, $fm_interval_hi]")
    return a, b, (fm_interval_lo, fm_interval_hi)
end

# Run
p_a, p_b, (lo, hi) = solve_p0(-0.5, 0.0)
println("\nRoot lies in [$p_a, $p_b] with last Integral ∈ [$lo, $hi]")
end

```

Iter 1: m = -0.2500000000 Integral[m] ∈ [1.158680663755, 1.158681922544] ②

Iter 2: m = -0.3750000000 Integral[m] ∈ [1.007466235589, 1.0074657342584]

Iter 3: m = -0.4375000000 Integral[m] ∈ [0.942607776390, 0.942608638847]

Iter 4: m = -0.4062500000 Integral[m] ∈ [0.974220256956, 0.974221184865]

Iter 5: m = -0.3906250000 Integral[m] ∈ [0.990633412092, 0.990634046428]

Iter 6: m = -0.3828125000 Integral[m] ∈ [0.998996646399, 0.998997689535]

Iter 7: m = -0.3789062500 Integral[m] ∈ [1.003218055968, 1.003219128877]

Iter 8: m = -0.3808593750 Integral[m] ∈ [1.001104016294, 1.001105073816]

Iter 9: m = -0.3818359375 Integral[m] ∈ [1.000049499041, 1.000050549251]

Iter 10: m = -0.3823242188 Integral[m] ∈ [0.999522864820, 0.999523911464]

Iter 11: m = -0.3820800781 Integral[m] ∈ [0.999786129933, 0.999787178353]

Iter 12: m = -0.3819580078 Integral[m] ∈ [0.999917801485, 0.999918850798]

Iter 13: m = -0.381896972675 Integral[m] ∈ [0.99993647012, 0.99998469773]

Iter 14: m = -0.3818664551 Integral[m] ∈ [1.000016572214, 1.000017622199]

Iter 15: m = -0.3818817139 Integral[m] ∈ [1.000000109410, 1.000001159283]

Iter 16: m = -0.3818893433 Integral[m] ∈ [0.999991878160, 0.999992927977]

Iter 17: m = -0.3818855286 Integral[m] ∈ [0.999995993772, 0.999997043617]

Iter 18: m = -0.3818836212 Integral[m] ∈ [0.999998051588, 0.999999101447]

Iter 19: m = -0.3818826675 Integral[m] ∈ [0.999999080499, 1.000000130364]

Iter 20: m = -0.3818821907 Integral[m] ∈ [0.999999594964, 1.00000044823]

Iter 21: m = -0.3818824291 Integral[m] ∈ [0.999999337726, 1.000000387593]

Iter 22: m = -0.3818823099 Integral[m] ∈ [0.999999463240, 1.000000616208]

Iter 23: m = -0.3818822503 Integral[m] ∈ [0.999999530647, 1.000000680516]

Iter 24: m = -0.3818822801 Integral[m] ∈ [0.999999498493, 1.000000648362]

Iter 25: m = -0.3818822950 Integral[m] ∈ [0.999999482417, 1.000000632285]

Iter 26: m = -0.3818823025 Integral[m] ∈ [0.999999474378, 1.000000624247]

Iter 27: m = -0.3818822987 Integral[m] ∈ [0.999999478398, 1.000000628266]

Iter 28: m = -0.3818823006 Integral[m] ∈ [0.999999476388, 1.000000626266]

Iter 29: m = -0.3818823015 Integral[m] ∈ [0.999999475883, 1.000000625281]

Iter 30: m = -0.3818823020 Integral[m] ∈ [0.999999474481, 1.000000624749]

Iter 31: m = -0.3818823018 Integral[m] ∈ [0.999999475132, 1.000000625000]

Converged at iter 31: root ≈ -0.3818823017645627; Integral ∈ [0.9999947513191818, 1.0000006250002422]

Root lies in [-0.38188230190739337, -0.3818823015317321] with last Integral ∈ [0.999994751319188, 1.0000006250002422]

```

+ begin
+     f(x) = sin(x + exp(x))
+     val, err = quadgk(f, 0.0f, 8.0)
+     @printf("Integral : %.10f\n", val)
+     @printf("Error bound: %.10e\n", err)
+     @printf("Interval containing the integral: [%10f, %10f]\n", val - err, val +
+             err)
+ end

```

② Integral : 0.3474001727  
Error bound: 5.0976381923e-09  
Interval containing the integral: [0.3474001676, 0.3474001778]