

Code not executed in Safe preview

```
1 using Symbolics
```

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```
1 n = 20
```

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```
1 @variables x y y1 y2 a[0:n]
```

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```
1 y_taylor = series(a, x)
```

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```
1 y1_taylor = expand_derivatives(Differential(x)(y_taylor))
```

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```
1 y2_taylor = expand_derivatives(Differential(x)(y1_taylor))
```

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```
1 diff_eq1 = 1 + y1^2 - 2*y2*(1 - y) ~ 0
```

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```
1 diff_eq2 = substitute(diff_eq1, y => y_taylor)
```

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```
1 diff_eq3 = substitute(diff_eq2, y1 => y1_taylor)
```

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```
1 diff_eq_taylor = substitute(diff_eq3, y2 => y2_taylor)
```

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```
1 diff_eq_taylor1 = taylor(diff_eq_taylor, x, 0:n - 2)
```

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```
1 coeff_eq = taylor_coeff(diff_eq_taylor1, x, 0:n - 2)
```

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```
1 solutions = Dict{Any, Any}()
```

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```
1 solutions[a[0]] = 0 # enforce a0 = 1
```

```
1 # for k in 2:n
2 #   eq = coeff_eq[k-1]
3 #   sol = solve_for(eq, a[k])
4 #   sol_sub = substitute(sol, solutions)
5 #   sol_simplified = simplify(sol_sub)
6 #   solutions[a[k]] = sol_simplified
7 # end
```

take equation
solve for a_k
substitute previous results
clean up
save final version

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```
1 for k in 2:n
2   eq = coeff_eq[k-1]                      # equation for coefficient
3   sol = solve_for(eq, a[k])                # expression for a[k]
4
5   # only substitute numeric values (like a0), not previous a[j]
6   rhs_expr = substitute(sol, Dict(a[0] => solutions[a[0]]))
7
8   # keep as symbolic recurrence (in terms of a1, a2, ...)
9   rhs_simplified = simplify(rhs_expr)
10
11  # save recurrence form
12  solutions[a[k]] = rhs_simplified
13 end
```

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```
1 solutions
```

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```
1 a1_poly = sum((-1)^i * a[i] for i in 0:n)
```

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```
1 a1_poly1 = substitute(a1_poly, solutions)
```

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```
1 poly_in_a1 = simplify(expand(a1_poly1))
```