```
using Symbolics
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n = 3

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```

$[x, y, \mathtt{y1}, \mathtt{y2}, a]$

```
@variables x y y1 y2 a[0:n]
```

y_taylor =

$$a_0 + a_1 x + x^2 a_2 + x^3 a_3$$

```
y_taylor  = series(a, x)
```

y1_taylor =

$$a_1 + 2a_2 x + 3x^2 a_3$$

```
y1_taylor = expand_derivatives(Differential(x)(y_taylor))
```

y2_taylor =

$$2a_2 + 6a_3 x$$

```
y2_taylor = expand_derivatives(Differential(x)(y1_taylor))
```

diff_eq1 =

$$1 - 2\left(1 - y\right)\mathtt{y2} + \mathtt{y1}^2 = 0$$

```
diff_eq1 = 1 + y1^2 - 2*y2*(1 - y) ~ 0
```

diff_eq2 =

$$1 + \mathtt{y1}^2 - 2\left(1 - a_0 - a_1 x - x^2 a_2 - x^3 a_3\right)\mathtt{y2} = 0$$

```
diff_eq2 = substitute(diff_eq1, y => y_taylor)
```

diff_eq3 =

$$1 - 2\left(1 - a_0 - a_1 x - x^2 a_2 - x^3 a_3\right)\mathtt{y2} + \left(a_1 + 2a_2 x + 3x^2 a_3\right)^2 = 0$$

```
diff_eq3 = substitute(diff_eq2, y1 => y1_taylor)
```

diff_eq_taylor =

$$1+\left(a_1+2a_2x+3x^2a_3\right)^2-2\left(2a_2+6a_3x\right)\left(1-a_0-a_1x-x^2a_2-x^3a_3\right)=0$$

```
diff_eq_taylor = substitute(diff_eq3, y2 => y2_taylor)
```

diff_eq_taylor1 =

$$1-4\left(1-a_0\right)a_2+\left(a_1\right)^2+\left(-12\left(1-a_0\right)a_3+8a_1a_2\right)x+\frac{1}{2}x^2\left(36a_1a_3+16\left(a_2\right)^2\right)+28x^3a_2$$

```
diff_eq_taylor1 = taylor(diff_eq_taylor, x, 0:2n - 2)
```

coeff_eq =

$$\left[1-4\left(1-a_0\right)a_2+\left(a_1\right)^2=0,\ -12\left(1-a_0\right)a_3+8a_1a_2=0,\ \frac{1}{2}\left(36a_1a_3+16\left(a_2\right)^2\right)=0,\ 28a_2a_3=0,\ 21(a\right.$$

```
coeff_eq = taylor_coeff(diff_eq_taylor1, x, 0:2n - 2)
```

coeff_eq1 = $\left[1-4\left(1-a_0\right)a_2+\left(a_1\right)^2=0\right]$

```
coeff_eq1 = coeff_eq[1:(n-2)]
```

deriv_at1_expr =

$$a_1+2a_2+3a_3$$

```
deriv_at1_expr = substitute(y1_taylor, x => 1)   # plug x = 1 into the derivative
```

deriv_at1_eq =

$$a_1+2a_2+3a_3=-0.3818$$

```
deriv_at1_eq    = deriv_at1_expr ~ -0.3818               # make it an equation
```

$$\left[1-4\left(1-a_0\right)a_2+\left(a_1\right)^2=0,\ a_1+2a_2+3a_3=-0.3818\right]$$

```
push!(coeff_eq1, deriv_at1_eq)
```

solutions = Dict()

```
solutions = Dict{Any, Any}()
```

```
# solutions[a[0]] = 0
```

```
for k in 2:n
    eq = coeff_eq[k-1]                         # take equation
    sol = solve_for(eq, a[k])                   # solve for a_k
    sol_sub = substitute(sol, solutions)        # substitute previous results
    sol_simplified = simplify(sol_sub)          # clean up
    solutions[a[k]] = sol_simplified            # save final version
end
```

Dict(a_2 a_3)

$$\Rightarrow \frac{1+(a_1)^2}{4\,(1-a_0)}, \quad \Rightarrow \frac{a_1\left(1+(a_1)^2\right)}{6(1-a_0)^2}$$

```
1 solutions
```

a1_poly =

$$a_0 + a_1 + a_2 + a_3$$

```
1 a1_poly = sum(a[i] for i in 0:n)
```

a1_poly1 =

$$a_0 + a_1 + \frac{a_1\left(1+(a_1)^2\right)}{6(1-a_0)^2} + \frac{1+(a_1)^2}{4\,(1-a_0)}$$

```
1 a1_poly1 = substitute(a1_poly, solutions)
```

poly_in_a1 =

$$\frac{3 + 9a_0 + 14a_1 - 24(a_0)^2 - 24a_0a_1 + 3(a_1)^2 + 12(a_0)^3 + 12(a_0)^2a_1 - 3(a_1)^2a_0 + 2(a_1)^3}{(3-3a_0)\,(4-4a_0)}$$

```
1 poly_in_a1 = simplify(expand(a1_poly1))
```