1 Method

So far we have done till a 7^{th}-degree polynomial. The way we have done it is - (1) Let's say we take n^{th} order polynomial for y

$$y = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$$

and use it in the differential equation

$$1 + (y')^2 + 2yy'' = 0$$

And then we equate the coefficients to 0, we get 2n - 1 equations. We take the first n - 1 equations of these 2n - 1 equations. We also have 2 other equations

$$\text{(i) } a_0 = 1 \qquad \text{(ii) } \sum_{i=0}^{n} a_i = 0$$

arising from the conditions

$$\text{(i) } y(0) = 1 \qquad \text{(ii) } y(1) = 0$$

So we have n + 1 equations and n + 1 variables. In the n - 1 equations we got from coefficient matching, a_i appears for the first time in the $i - 1^{\text{th}}$ equation. So we represent a_i as a function of $a_0, a_1, a_2, \ldots, a_{i-1}$. Finally, we represent each one of a_is $(i > 1)$ as a function of a_1: $a_2 = f_2(a_1)$, $a_3 = f_3(a_1)$, $\ldots$, $a_n = f_n(a_1)$.

Using $\sum_{i=0}^{n} a_i = 0$, we get

$$1 + a_1 + f_2(a_1) + \ldots + f_n(a_1) = 0$$

This is an n^{th} degree polynomial in a_1. We solve this numerically and substitute values to get all the coefficients.

2 Example (7th-degree polynomial)

For n = 7, we get these:
(1) The first 6 of the 13 equations we get from coefficient matching:

$$1 + 4a_0a_2 + (a_1)^2 = 0$$

$$12a_0a_3 + 8a_1a_2 = 0$$

$$\frac{1}{2}\left(48a_0a_4 + 36a_1a_3 + 16(a_2)^2\right) = 0$$

$$\frac{1}{6}\left(240a_0a_5 + 192a_1a_4 + 168a_2a_3\right) = 0$$

$$\frac{1}{24}\left(1440a_0a_6 + 1200a_1a_5 + 1056a_2a_4 + 504(a_3)^2\right) = 0$$

$$\frac{1}{120}\left(10080a_0a_7 + 8640a_1a_6 + 7680a_2a_5 + 7200a_3a_4\right) = 0$$

(2) a_is as functions of a_1:

$\mathrm{a}_2 = -\frac{1}{4}\left(1 + a_1^2\right)$

$\mathrm{a}_3 = \frac{1}{6}a_1\left(1 + a_1^2\right)$

$\mathrm{a}_4 = \frac{1}{48}\left(-(1 + a_1^2)^2 - 6a_1^2(1 + a_1^2)\right)$

$\mathrm{a}_5 = \frac{1}{240}\left(7(1 + a_1^2)^2a_1 - 4a_1\left(-(1 + a_1^2)^2 - 6a_1^2(1 + a_1^2)\right)\right)$

$\mathrm{a}_6 = \frac{1}{1440}\Big(\frac{11}{2}\left(1 + a_1^2\right)\Big(-\left(1 + a_1^2\right)^2 - 6a_1^2\left(1 + a_1^2\right)\Big) - 14\left(1 + a_1^2\right)^2a_1^2 - 5a_1\Big(7\left(1 + a_1^2\right)^2a_1 - 4a_1\Big(-\left(1 + a\right.$

Using $1 + a_1 + a_2 + \ldots + a_7 = 0$, we get

$$\frac{2089}{2880} + \frac{773}{630}a_1 - \frac{479}{960}a_1^2 + \frac{139}{280}a_1^3 - \frac{367}{960}a_1^4 + \frac{9}{20}a_1^5 - \frac{91}{576}a_1^6 + \frac{13}{72}a_1^7 = 0$$

which gives a_1 = -0.45043, further we get these:

$$a_2 = -0.30072, \quad a_3 = -0.09030, \quad a_4 = -0.06065, \quad a_5 = -0.04086, \quad a_6 = -0.03156, \quad a_7 = -0.02546$$

Using these coefficients, we get our 7th degree polynomial.

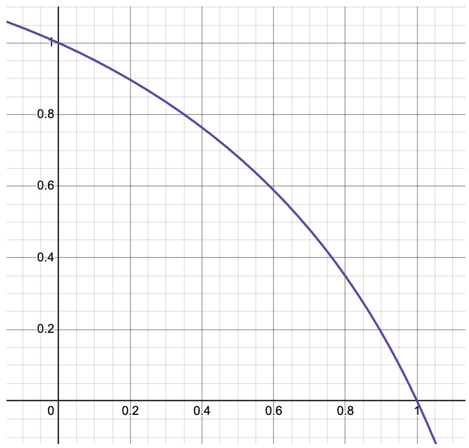

Figure 1: 7th degree polynomial plot