Analysis for an Agree-Disagree Model of a Political Party

by

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DECLARATION

I, C.M.A. Yashen Melaka , declare that the presented project report titled, "Analysis for an Agree-Disagree Model of a Political Party" is uniquely prepared by me based on the group project carried out under the supervision of Dr. L. W. Somathilake (B.Sc, PG.Dip, MPhil (SL), PhD (SL)) Department of Mathematics, Faculty of Science, University of Ruhuna, as a partial fulfillment of the requirements of the level III Industrial Mathematics course unit (IMT3b1β) of the Bachelor of Science (General) Degree in Department of Mathematics, Faculty of Science, University of Ruhuna, Sri Lanka. It has not been submitted to any other institution or study program by me for any other purpose.

Signature:.................................

Date:..................

SUPERVISOR'S RECOMMENDATION

I certify that this study was carried out by C.M.A. Yashen Melaka under my supervision.

Signature:. . . . .

Date:. . . . .

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ABSTRACT

Understanding the dynamics of public opinion is crucial for predicting electoral outcomes and shaping political strategies. This thesis presents a mathematical model that captures the interactions between three key populations regarding a political party: those who are Ignorant or indifferent (denoted as I), those who Agree with or support the party (denoted as A), and those who Disagree with or oppose the party (denoted as D). The model, grounded in a system of coupled ordinary differential equations (ODEs), describes the time evolution of these populations by incorporating factors such as transmission rates, interest loss, and interaction terms. Using the explicit Euler method for numerical integration, the model simulates the dynamics of these populations over time, revealing the conditions that drive shifts in public opinion. This approach provides insights into how various campaign strategies, societal influences, and external events impact the proportions of supporters, opponents, and undecided individuals. By modeling these dynamics, the study offers a quantitative framework to predict and analyze changes in public opinion, helping political strategists and researchers make informed decisions and refine their approaches. The model underscores the importance of mathematical tools in understanding the complexities of electoral politics and improving campaign effectiveness.

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CHAPTER 1
Introduction

Those who support, oppose, or remain Ignorant to a political party or ideology, plays a significant role in shaping electoral outcomes and political strategies. To effectively predict election results, develop campaign strategies, and assess the impact of political communication, it is crucial to understand the dynamics among these groups. This thesis presents a model that analytically captures the interactions between three key populations: those who are Ignorant or indifferent to the political party (denoted as I), those who Agree with or support the party (denoted as A), and those who Disagree with or oppose the party (denoted as D).

Public opinion's role in elections cannot be understated, as it offers insights into voter behavior, preferences, and the potential direction of electoral outcomes. This division into supportive, oppositional, and indifferent groups mirrors the segmentation of voter bases often seen in opinion polls, where the results help refine political strategies and messaging. By strategically engaging with these groups, political parties can influence perceptions, mobilize support, and potentially shift the balance of power in their favor. Understanding how these populations interact and respond to political messaging is, therefore, critical in navigating the complexities of modern electoral politics.

The model is grounded in a system of coupled ordinary differential equations (ODEs) that depict the time evolution of these distinct populations. These equations account for various factors influencing the transition of individuals between the states of being Ignorant, Agreeing, or Disagreeing with a political party. Such factors include transmission rates, interest loss factors, and interaction terms. By numerically integrating these equations using the explicit Euler method, the model simulates the dynamics of these populations over time, offering insights into the conditions that drive shifts in public opinion.

Understanding these dynamics is crucial for political strategists aiming to anticipate and influence changes in public opinion. The use of ODEs in this context mirrors the application of mathematical models in political science, where they are employed to predict voter behavior and refine campaign tactics. The explicit Euler method, known for its straightforward approach to numerical integration, allows for the continuous tracking of population changes, thereby enabling the model to reveal patterns and trends that might otherwise remain hidden. This approach underscores the importance of mathematical tools in dissecting the complexities of political movements and in crafting effective communication strategies.

The motivation for this study is to provide a quantitative framework that can be directly applied to real-world political scenarios. By simulating how various campaign strategies, societal influences, and external events impact the proportions of supporters, opponents, and undecided individuals within a population, the model serves as a valuable tool for political analysis. The aim is to uncover the underlying mechanisms that drive changes in public opinion, offering political strategists, analysts, and researchers the insights needed to make informed decisions.

CHAPTER 2
Literature Review

The study of public opinion dynamics is a critical area of research, particularly in the context of political science and social psychology. Understanding how public sentiment evolves over time can inform electoral strategies, policy development, and social movements. This literature review explores existing mathematical models that have been employed to capture the dynamics of public opinion, focusing on the specific approach presented in the provided abstract.

Historical Overview

Early models of public opinion often relied on simple statistical methods or qualitative analysis. However, as the complexity of social phenomena became more apparent, researchers began to explore mathematical modeling as a means to quantify and predict public opinion trends. One of the pioneering works in this field was the work of Simon (1955), who introduced a model of opinion formation based on social influence and conformity [1].

Mathematical Modeling Frameworks

A variety of mathematical frameworks have been used to model public opinion dynamics. These include:

- **Agent-based models:** These models simulate the interactions between individual agents, each representing a person or a group. Agents can hold different opinions, and their interactions can lead to changes in these opinions.

- **Compartmental models:** These models divide the population into compartments based on their opinions or attitudes. The dynamics of the model are described by a system of differential equations that govern the flow of individuals between compartments.
- **Opinion dynamics models:** These models focus specifically on the evolution of opinions over time, often incorporating concepts such as social influence, conformity, and dissonance. [2]

The Proposed Model

The model presented in the abstract falls into the category of compartmental models. It divides the population into three compartments: Ignorant/Indifferent (I), Agree (A), and Disagree (D). The model incorporates factors such as transmission rates, interest loss, and interaction terms to describe the flow of individuals between these compartments.

Related Work

Several previous studies have used compartmental models to analyze public opinion dynamics. For example, DeGroot (1974) introduced a model where individuals update their opinions based on the average opinions of their neighbors [3]. More recent work has incorporated additional factors such as polarization, network structure, and the influence of external events [4, 5].

Contributions of the Proposed Model

The proposed model contributes to the existing literature in several ways:

- **Explicit inclusion of interest loss:** By incorporating a term for interest loss, the model captures the phenomenon where individuals may lose interest in a topic over time, leading to a decrease in their engagement.
- **Focus on political parties:** The model specifically addresses the dynamics of public opinion related to political parties, which is a central concern in political science.
- **Use of numerical integration:** The use of the explicit Euler method for numerical integration allows for the simulation of the model's dynamics over time, providing insights into the conditions that drive shifts in public opinion.

The proposed model offers a valuable contribution to the field of public opinion modeling. By providing a quantitative framework for analyzing the dynamics of public sentiment, it can help researchers and policymakers better understand the factors that influence electoral outcomes and shape political discourse. Future research could explore extensions of the model to incorporate additional factors, such as the role of media, the impact of misinformation, and the influence of social networks.

CHAPTER 3
Problem Statement

In a highly polarized political environment, understanding the dynamics of public opinion is a critical challenge for political parties, strategists, and policymakers. The population can generally be divided into three key groups:

- Ignorant (I): people who do not know about the poll or those who abstain from voting for personal reasons
- Agree (A): people in agreement with the idea being studied
- Disagree (D): people in disagreement with the idea being studied

The transitions between these groups are influenced by various factors, including media influence, social interactions, and external events.

However, there is a lack of comprehensive models that quantitatively describe how these populations evolve over time under the influence of these factors. Traditional models often overlook the complexity of interactions between these groups, failing to account for the nonlinear dynamics that can lead to sudden shifts in public opinion.

Therefore, the problem is to develop a mathematical model that accurately represents the interactions between the Ignorant, Agree, and Disagree populations. This model should be capable of simulating the time evolution of these groups, allowing for the exploration of how different parameters—such as transmission rates and interest loss factors—affect the overall dynamics. The ultimate goal is to provide a tool that can predict changes in public opinion and inform political strategies in a more nuanced and effective manner.

This thesis addresses this problem by formulating a system of coupled ordinary differential equations (ODEs) that encapsulate the interactions and transitions between the three populations. Through numerical integration and analysis of this model, the study seeks to uncover the key factors that drive shifts in public opinion and to provide a framework that can be adapted to various political contexts.

CHAPTER 4
Methodology

4.1 Model Formulation

The model is based on a system of coupled ordinary differential equations (ODEs) that describe the interaction between three distinct populations: Ignorant (I), Agree (A), and Disagree (D). The evolution of these populations over time is governed by parameters that capture the rates of transition and interaction between these states.

4.2 Parameter Definitions

The model uses six key parameters:

- α_1: Disagree to agree transmission rate
- α_2: Agree to disagree transmission rate
- β_1: Ignorant to agree transmission rate
- β_2: Ignorant to disagree transmission rate
- γ_1: Interest loss factor of agree individuals
- γ_2: Interest loss factor of disagree individuals

These parameters are provided by the user at the beginning of the simulation and are critical in determining the dynamics of the system.

4.3 Calculation of Basic Reproductive Numbers

Two important ratios, R_{D0} and R_{A0}, are calculated to characterize the initial state of the system:

4.3.1 Calculation of R_{D0}

The initial ratio R_{D0} is given by:

$$R_{D0} = \frac{\alpha_2\beta_1 - \alpha_2\gamma_1 + \beta_2\gamma_1}{\alpha_1\beta_1 - \alpha_1\gamma_1 + \beta_1\gamma_2} \tag{4.1}$$

This ratio reflects the balance between different interaction and transition terms involving the populations and their parameters.

4.3.2 Calculation of R_{A0}

Similarly, the initial ratio R_{A0} is computed as:

$$R_{A0} = \frac{\alpha_1\beta_2 - \alpha_1\gamma_2 + \beta_1\gamma_2}{\alpha_2\beta_2 - \alpha_2\gamma_2 + \beta_2\gamma_1} \tag{4.2}$$

Like R_{D0}, this ratio represents the balance of interactions but is specific to the Agree population.

4.4 Numerical Integration Using the Euler Method

The explicit Euler method is used to numerically integrate the system of ODEs. This method updates the state of each population at discrete time intervals, allowing us to simulate the dynamics over time.

4.4.1 Initial Conditions

The initial conditions for the populations are set as follows:

$$\begin{aligned} N &= 100 \quad \text{(Total population)} \\ I(0) &= 10 \\ A(0) &= 45 \\ D(0) &= 45 \end{aligned}$$

4.4.2 Time Parameters

The simulation runs over a total time $T = 13$ units, with a time step $\Delta t = 0.1$ units, resulting in 130 time steps.

4.4.3 Differential Equations

The differential equations governing the evolution of $I(t)$, $A(t)$, and $D(t)$ are:

$$I' = -\beta_1 \frac{A \cdot I}{N} - \beta_2 \frac{D \cdot I}{N} + \gamma_1 \cdot A + \gamma_2 \cdot D \tag{4.3}$$

$$A' = \beta_1 \frac{A \cdot I}{N} + \alpha_1 \frac{A \cdot D}{N} - \alpha_2 \frac{A \cdot D}{N} - \gamma_1 \cdot A \tag{4.4}$$

$$D' = \beta_2 \frac{D \cdot I}{N} + \alpha_2 \frac{A \cdot D}{N} - \alpha_1 \frac{A \cdot D}{N} - \gamma_2 \cdot D \tag{4.5}$$

4.4.4 Population Update

For each time step, the populations are updated using:

$$I(t + \Delta t) = I(t) + I'(t) \cdot \Delta t$$

$$A(t + \Delta t) = A(t) + A'(t) \cdot \Delta t$$

$$D(t + \Delta t) = D(t) + D'(t) \cdot \Delta t$$

where $I'(t)$, $A'(t)$, and $D'(t)$ are the derivatives computed from the equations above.

4.5 Data Storage and Analysis

The results of the simulation, including the time evolution of $I(t)$, $A(t)$, and $D(t)$, are recorded in an Excel spreadsheet for further analysis. This allows for a detailed examination of how the populations interact over time and the impact of different parameter values on the system's dynamics.

CHAPTER 5
Results

5.1 (1)

Basic Reproductive number (1)

R_{D0} - 1.9900595629395006

R_{A0} - 2.0730284404625503

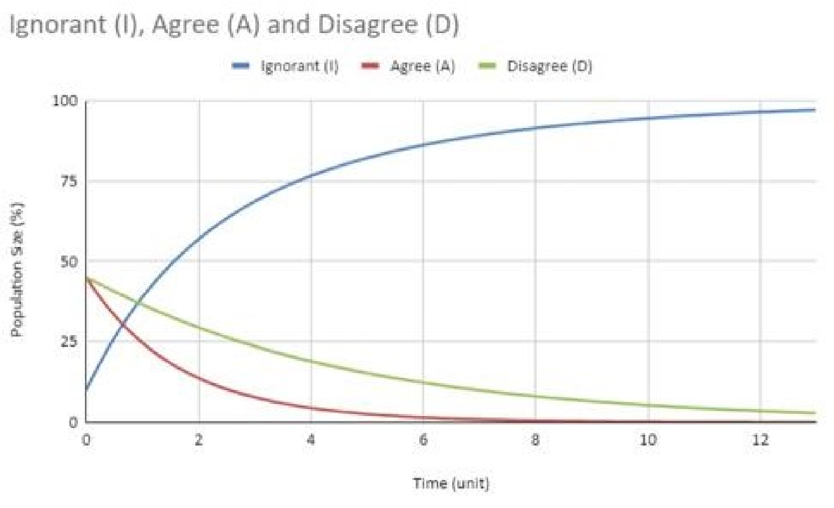

Figure 5.1: Normal condition

Here we keep the baseline parameter values for our initial conditions. The resulting graph reveals a fascinating shift. The Ignorant population steadily increase over time, while both the Agree and Disagree populations gradually decline. This pattern suggests a compelling dynamic at play. Individuals are more frequently shifting from Agree and Disagree positions back into ignorance, rather than the other way around.

5.2 (2)

Basic Reproductive number (2)

R_{D0} - -1.9920715552102632

R_{A0} - 0.0657218612173364

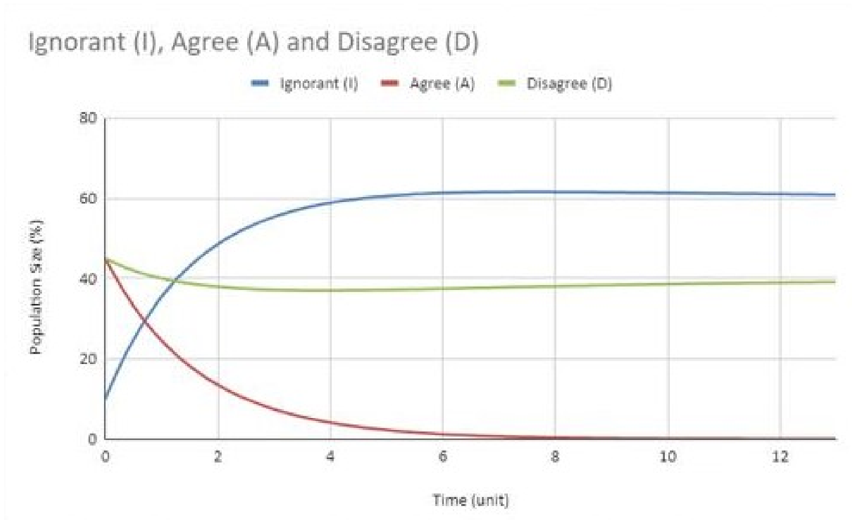

Figure 5.2: Increment in rate of ignorant to disagree

In this scenario, we amplified the parameter controlling the rate at which Ignorant individuals are swayed to disagree. The graph reveals a noticeable drop in the Ignorant population as more people are convinced to join the Disagree population. However, the Disagree population sees only a slight decrease, as some members eventually shift into the Agree category. Over time, the numbers for both the Ignorant and Disagree groups stabilize, suggesting that the system has reached a dynamic equilibrium. At this point, the transitions between these groups are balanced, leading to steady-state conditions where no further significant changes in population sizes are expected.

5.3 (3)

Basic Reproductive number (3)

R_{D0} - 3.9721707603737126

R_{A0} - 0.40282191947205415

Figure 5.3: Increment in rate of agree to disagree.

The "Ignorant" group grows over time, while the "Agree" group grows initially but eventually stabilizes. The "Disagree" group declines significantly. This could represent a scenario where information or persuasion is spreading through a population, leading to a shift in opinions.

CHAPTER 6
Discussion and Conclusion

The mathematical model developed in this study offers a robust framework for analyzing the dynamics of public opinion in a highly polarized political environment. By categorizing the population into Ignorant, Agree, and Disagree groups, the model provides a simplified but accurate representation of the complex interactions that occur within political campaigns.

One of the key strengths of the model is that its ability to incorporate non-linear dynamics, which are crucial for capturing the sudden shifts and oscillations in public opinion that are often observed in real-world scenarios. Traditional linear models fail to account for these complexities. This leads to oversimplified predictions that may not align with actual trends. We addresses this limitation with coupled ordinary differential equations (ODEs) in this project, allowing for a more accurate simulation of the transitions between the three populations.

The results of the numerical integration suggest that media influence, social interactions, and external events are significant drivers of opinion shifts. The model's sensitivity to these factors highlights the importance of understanding the underlying mechanisms that govern public opinion. For instance, a high transmission rate from the Ignorant to the Agree or Disagree populations can lead to rapid polarization, while a high interest loss factor can cause disillusionment and apathy, resulting in an increase in the Ignorant population.

Moreover, the model demonstrates the potential for feedback loops, where the growth of the Agree or Disagree population further influences the media and social interactions, creating a reinforcing cycle that can either stabilize or destabilize public opinion. This insight is particularly valuable for political strategists and policymakers, as it emphasizes the need for careful management of public communication and engagement strategies to avoid unintended consequences.

However, the model also has its limitations. The simplification of the population into only three groups may overlook the nuances of political behavior, such as the presence of moderate or undecided individuals who do not fit neatly into the Agree or Disagree categories. Additionally, the model assumes homogeneity within each group, which may not accurately reflect the diversity of opinions and behaviors within a real population. Future research could address these limitations by incorporating more detailed population subgroups and exploring the effects of heterogeneous interactions.

In conclusion, the mathematical model developed in this thesis provides a valuable tool for predicting and analyzing the evolution of public opinion in a polarized political environment. By capturing the non-linear dynamics of group interactions, the model offers a more nuanced understanding of the factors that drive opinion shifts, which can inform more effective political strategies. Nevertheless, the model's simplifications suggest that further refinement and extension are necessary to fully capture the complexity of public opinion dynamics.

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APPENDIX A
Java Codes

A.1 Analysis For an Agree-disagree Model of a Political Party

```java

import java.io.FileOutputStream;
import java.io.IOException;
import org.apache.poi.ss.usermodel.*;
import org.apache.poi.xssf.usermodel.XSSFWorkbook;
import java.util.Scanner;


public class EulerBasic3 {

    public static void main(String[] args) {

        //Taking values for the parameters of the differential
    equations from the keyboard
        Scanner scanner = new Scanner(System.in);

        System.out.println("Enter alpha_1:");
        double alpha1 = scanner.nextDouble();

        System.out.println("Enter alpha_2:");
        double alpha2 = scanner.nextDouble();
```

```java

        System.out.println("Enter beta_1:");
        double beta1 = scanner.nextDouble();

        System.out.println("Enter beta_2:");
        double beta2 = scanner.nextDouble();

        System.out.println("Enter gamma_1:");
        double gamma1 = scanner.nextDouble();

        System.out.println("Enter gamma_2:");
        double gamma2 = scanner.nextDouble();

        // Close the scanner after reading all inputs
        scanner.close();

        // Calculating R_D0
        double numeratorRD0 = alpha2 * beta1 - alpha2 * gamma1
 + beta2 * gamma1;
        double denominatorRD0 = alpha1 * beta1 - alpha1 *
gamma1 + beta1 * gamma2;
        double RD0 = numeratorRD0 / denominatorRD0;


```

```
        // Calculating R_A0
        double numeratorRA0 = alpha1 * beta2 - alpha1 * gamma2 + beta1 * gamma2;
        double denominatorRA0 = alpha2 * beta2 - alpha2 * gamma2 + beta2 * gamma1;
        double RA0 = numeratorRA0 / denominatorRA0;

        System.out.println("R_D0 = " + RD0);
        System.out.println("R_A0 = " + RA0);

        double N = 100.0;  // Total population

        // Initial values of I, A and D
        double I = 10.0;
        double A = 45.0;
        double D = 45.0;

        // Time parameters for euler method
        double dt = 0.1;  // Time step
        double T = 13;  // Total time

        // Number of iterations
        int steps = (int)(T / dt);

```

```
        // Creating a new Workbook
        Workbook workbook = new XSSFWorkbook();
        // Creating a new sheet
        Sheet sheet = workbook.createSheet("Euler Method
    Results");

        // Creating header row
        Row header = sheet.createRow(0);
        header.createCell(0).setCellValue("Time (t)");
        header.createCell(1).setCellValue("Ignorant (I)");
        header.createCell(2).setCellValue("Agree (A)");
        header.createCell(3).setCellValue("Disagree (D)");

        // Euler method loop
        for (int i = 0; i <= steps; i++) {
            double time = i * dt;

            // Creating a new row in the sheet
            Row row = sheet.createRow(i + 1);
            row.createCell(0).setCellValue(time);
            row.createCell(1).setCellValue(I);
            row.createCell(2).setCellValue(A);
            row.createCell(3).setCellValue(D);

```

```java
            double I_prime = -beta1 * (A * I) / N - beta2 * (D * I) / N + gamma1 * A + gamma2 * D;
            double A_prime = beta1 * (A * I) / N + alpha1 * (A * D) / N - alpha2 * (A * D) / N - gamma1 * A;
            double D_prime = beta2 * (D * I) / N + alpha2 * (A * D) / N - alpha1 * (A * D) / N - gamma2 * D;

            // Update values
            I += I_prime * dt;
            A += A_prime * dt;
            D += D_prime * dt;
        }

        // Writing the output to an external excel file
        try (FileOutputStream fileOut = new FileOutputStream("EulerResults.xlsx")) {
            workbook.write(fileOut);
        } catch (IOException e) {
            e.printStackTrace();
        } finally {
            try {
                workbook.close();
            } catch (IOException e) {
                e.printStackTrace();
```

```
            }
        }

        System.out.println("Results can be seen on EulerResults.xlsx");
    }
}
```