★ Discrete Probability Distribution

(i) Binomial Probability distribution → Mean ≠ Variance
(ii) Poisson Probability distribution → Mean = Variance
(iii) Negative binomial distribution
(iv) Hypergeometric probability distribution

Discrete random variable :: A variable whose value is determine by the outcomes of a random experiment is called random variable. A random variable is also called chances of outcome of the experiment.

If x denotes the number obtained then x is the random variable which can take any one of the values 1, 2, 3, 4, 5, 6 each with equal probability is 1/6 .

A random variable may be discrete or continuous. if the random variable takes an integer value such as 1, 2, 3, ... then it is called discrete random variable. For example :- no. of printing mistake, no. of telephone call, no. of pages etc.

If the variable takes all possible values within a certain interval, then the random variable is called continuous random variable.

For example :: The amount of rainfall, Height, Weight etc.

Let x be the discrete random variable which can assume the values $x_1, x_2, \ldots, x_n$ with each value of the variable x, we associate the number.

i.e. $p_i = P(x = x_i)$, $i = 1, 2, \ldots, n$ which is known as the probability of x_i and satisfied the following conditions.

$$p_i = P(x = x_i) \geq 0, \quad i = 1, 2, \ldots n$$

and $\sum_{i=1}^{n} p_i = 1$

The function $p_i = P(x = x_i)$ or $P(x)$ is called probability function or probability mass function (pmf) of discrete random variable x and the set of all possible order pair $[x, P(x)]$ is called discrete probability distribution.

$x = 3$, $P(x = 3) = 1/6$, probability of getting face 3

* Binomial distribution :-

Binomial distribution follows under the some conditions,

a The n trial (observation) are mutually independent.

b The outcomes of the trial results in dichotomous classification of the events, i.e. head and tails, success and failure etc.

c Probability associated with each of the n trial is equal.

d Probability of success is p, probability of failure q, then

$$p + q = 1$$

e The trials are independent i.e. outcome of one toss doesnot affect the outcome of any other tosses.

Binomial probability distribution :- Let x be the random variable which follow the binomial distribution, then the probability of x success which takes the value from 0, 1, 2 ...... n, it is called probability mass function of the binomial distribution with parameter n and p

i.e. $p(x) = p(x = x) = {}^nC_x p^x q^{n-x}$

where $x = 0, 1, 2, \ldots\ldots n$

Now we can define binomial distribution as, if x is a random variable then x is said to be binomial distribution

- If it assume only the possible integer i.e. $0, 1, \ldots\ldots, n$
- If probability mass function is given by

$$p(x = x) = p(x) = {}^nC_x p^x q^{n-x}$$

$$= \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

Mean of binomial distribution is np and variance npq.

Problem 1 Determine the binomial distribution for which the mean is 4 and variance 3.

=> Mean $(np) = 4$

Variance $(npq) = 3$

Now,

As we know,

$npq = 3$

or, $4 * q = 3$

$\therefore q = \frac{3}{4}$

Then,

$p + q = 1$

or, $p + \frac{3}{4} = 1$

$\therefore p = 1/4$

Again,

$np = 4$

or, $n * \frac{1}{4} = 4$

$\therefore n = 16$

Hence required binomial distribution is $X \sim B(16, 1/4)$.

Problem 2 For binomial distribution with $n = 4$, $P = 0.45$ find

a $P(X = 0)$

b $P(X \leq 2)$

c $P(X > 2)$

Soln:

$n = 4$

$P = 0.45$

$q = 0.55 \quad (1 - 0.45)$

a $p(x=0) = {}^nC_x\, p^x\, q^{n-x}$

$= {}^4C_0 * 0.45^0 * 0.55^{4-0}$

$= 1 * 1 * 0.55^4$

$= 0.092$

b $p(x \le 2) = p(2) + p(1) + p(0)$

Here,

$p(2) = 0.368$

$p(1) = 0.2993$

So,

$p(x \le 2) = 0.7593$

c $p(x > 2) = 1 - p(x \le 2)$

$= 0.2407$

Q The incidence of occupation disease in a road survey is such that the surveyer have 20 % of chance of suffering from it. What is the probability that out of six surveyer 4 or more will attack the disease?

soln: Given,

$n = 6$

Probability of suffering from an occupational disease

$(p) = 20\,\% = 0.20$

" " not suffering " " " "

$(q) = 80\,\% = 0.80$

$p(x \ge 4) = ?$

From Binomial distribution,

$p(x) = {}^nC_x\, p^x\, q^{n-x}$

Now,

We know,

$p(x \ge 4) = p(x=4) + p(x=5) + p(x=6)$

$$= {}^{6}C_{4} * (0.20)^{4} * (0.20)^{6-4}$$

$$+ {}^{6}C_{5} * (0.20)^{5} * (0.20)^{6-5}$$

$$+ {}^{6}C_{6} * (0.20)^{6} * (0.20)^{6-6}$$

$=$

* **Poisson distribution** :: The poisson distribution explain discrete random variable for which the probability of occurance of an event is small and the total number of possible cases is very large. For example :: death, no. of plane accidents etc.

(i) all events are mutually exclusive events, events are independent.

(ii) the probability of occurance of an single events within a specified time period is generally proportional to the length of the time period or time interval.

(iii) Very small portion of the time period under consideration the probability of occurance of two or more events in negligible.

* Poisson distribution as a limiting case of binomial distribution ::

Under the following condition,

(i) n, the number of trials be indefinitly large, $n \to \infty$

(ii) P, the probability occurance of P is very small, $P \to 0$

(iii) mean, np = finite = λ (lando)

$\therefore P = \lambda/n$

Above conditions, the binomial probability function tends to probability function of the poisson distribution.

$$P(x) = P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

where x be the occurance of the event and $\lambda = np$.

$e = 2.71828$

$x! = x(x-1)(x-2) \cdots 3 \cdot 2 \cdot 1$

then in poisson distribution if x be the random variable then x is said to be poisson distribution if it assume only the positive integer and probability mass function is given by

$$P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

Symbollicaly,

$$X \sim P(\lambda)$$

mean = Variance

i.e. λ

Problem: Given $\lambda = 4.2$ for a poisson distribution find

ⓐ $P(X = 8)$

ⓑ $P(X \leq 2)$

ⓒ $P(X > 2)$

soln: Given,

$$\lambda = 4.2$$

We know,

$$P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$$

ⓐ $P(X = 8)$

When $x = 8$, then

$$P(X = 8) = \frac{e^{-4.2} * (4.2)^{8}}{8!}$$

$$= 0.036$$

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soln

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(b) $p(x \le 2) = p(2) + p(1) + p(0)$

$$= \frac{e^{-4.2} * (4.2)^2}{2!} + \frac{e^{-4.2} * (4.2)^1}{1!} + \frac{e^{-4.2} * (4.2)^0}{0!}$$

$= 0.21$

(c) $p(x > 2) = 1 - p(x \le 2)$

$= 1 - 0.21$

$= 0.79$

Problem 2 If 4 % of bulb manufactured by a company are defective find the probability that in a sample of 125 bulbs
(a) Non - defective
(b) 3 bulbs will be defective

soln :- Here,

$n = 125$
$p = 4\%$
$= 0.04$

$\left.\begin{matrix} \\ \\ \end{matrix}\right\}$ $n \to$ large, $p \to$ small

$\therefore$ Poisson distribution

$\lambda = np$
$= 125 * 0.04$
$= 5$

(a) Non - defective : $x = 0$

$$p(x = 0) = \frac{e^{-5} * 5^0}{0!}$$

$= 6.73 * 10^{-3}$

(b) $p(x = 3) = \frac{e^{-5} * 5^3}{3!} = 0.14$

Problem 3 The average number of network error experienced in a day on a local area network (LAN) is distributed with an average 2.4. What is the probability that in any given day @ zero network error will occur

ⓑ exactly one network error will occur

ⓒ at least one " " " "

soln:- Here,

$\lambda = 2.4$

Poisson distribution, Pmf is

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$= \frac{e^{-2.4}\lambda^{x}}{x!} = \frac{e^{-2.4} * 2.4^{x}}{x!}$$

Now,

ⓐ $x = 0$

$$P(X = 0) = \frac{e^{-2.4} * 2.4^{0}}{0!}$$

$$= 0.09$$

ⓑ $x = 1$

$$P(X = 1) = \frac{e^{-2.4} * (2.4)^{1}}{1!}$$

$$= 0.218$$

ⓒ $P(X \geq 1) = 1 - P(0)$

$= 1 - 0.09$

$= 0.91$

* Fitting a Poisson distribution :- If a series of trial is repeated 'N' times and satisfied the condition of poisson distribution, then expected or theoretical frequency is given by

$$f(x) = N * P(x = x) = N * \frac{e^{-\lambda} \lambda^{x}}{x!}$$

Problem 4 Fit the poisson distribution to the following data

x	0	1	2	3	4	
f	120	60	12	4	4	$\Sigma f = N = 200$

Soln :-

$$f(x) = N \cdot \frac{e^{-\lambda} \lambda^{x}}{x!}$$

$$\bar{x} = \frac{\Sigma f x}{N} = 0.5 = \lambda$$

x	$f(x) = N * \frac{e^{-\lambda} \lambda^{x}}{x!}$
0	$f(0) = 200 * e^{-0.5} * (0.5)^0 / 0! = 121.3 \approx 121$
1	$f(1) = 200 * e^{-0.5} * (0.5)^1 / 1! = 60.65 \approx 61$
2	$f(2) = 200 * e^{-0.5} * (0.5)^2 / 2! = 15.16 \approx 15$
3	$f(3) = 200 * e^{-0.5} * (0.5)^3 / 3! = 2.52 \approx 3$
4	$f(4) = 200 * e^{-0.5} * (0.5)^4 / 4! = 0.31 \approx 0$
	Total = 200

Hence, total expected freq $\Sigma f(x) = 200$

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* Hypergeometric distribution :-

$$p(X = x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

An experiment in which the events are stachastically ↗ additionally dependent although random where the probability of success doesn't remain the same from trial to trial is called hypergeometric experiment.

In this experiment draw a random sample of size n from a population size N without replacement, then find the probability selecting x item having a certain characteristic from total number of item with the same character in a population i.e. x = number of items having the certain characteristic i.e. number of success in random sample of size n is called hypergeometric random variable and its probability distribution is called hypergeometric distribution. Problem

A random variable x is said to have hypergeometric distribution with parameter N, M and n. then

$$p(X = x) = p(x, N, M, n) = \begin{cases} \dfrac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}} & x = 0, 1 \ldots \ldots (M, N) \text{ minimum} \\ 0 & \text{otherwise} \end{cases}$$

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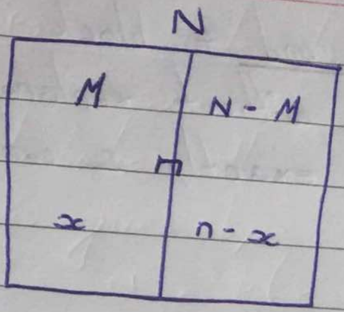

stachastically
characteristic
Without replacement } Removed => Binomial distribution

M is character of N = 44 students
↳ like pass - fail etc

Problem 1 From a lot of 10 items containing 3 defectives, a sample of 4 items is drawn a random without replacement, find the probability that the sample contains
(i) exactly 2 defective item
(ii) at least 1 defective item
=) Here,

$N = 10$

$n = 4$

$x = 2$

$M = 3$

Now,

(i) $P(x = 2) = P(2, 10, 4, 3)$

$$= \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

${}^{M}C_{x}$ ${}^{N-M}C_{n-x}$ ${}^{N}C_{n}$

(ii) $P(x \geq 1) = P(1) + P(2) + P(3) =$

Problem 2 A box contain 5 white and 7 black ball. If 5 balls are drawn from the box without replacement, what is the probability of drawing exactly 2 white and 3 black balls?

=) $x = 2$

$N = 5 + 7 = 12$

$n = 5$

$M = 5$

Now,

$$P = \frac{\binom{5}{2}\binom{7}{3}}{\binom{12}{5}}$$

$= 0.44$ #

Problem 3 Five cards are drawn from an ordinary deck of 52 cards without replacement. What is the probability that in the 5 cards drawn exactly 3 are spade?

=) $N = 52$

$n = 5$

$x = 3$

$M = 13$

Now,

$$P = \frac{\binom{13}{3}\binom{39}{2}}{\binom{52}{5}}$$

$= 0.08$ #

Problem 4 If a committee of 5 is to be selected from group of 6 men and 9 women, find the probability that the committee consist 3 men and $\boxed{2 \text{ women}}$ ↳ No need to give automatically

=) Here,

$N = 6 + 9 = 15$ total $M = 6$ men

$n = 5$ committee $x = 3$ men

$$p(x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$$

$$= \frac{\binom{6}{3}\binom{15-6}{5-3}}{\binom{15}{5}}$$

$=$

simple prob $\frac{{}^6C_3 \times {}^9C_2}{{}^{15}C_5}$

binomial
No of success = variable

No of trial - variable

* Negative binomial distribution :-

An experiment which the trials are repeated until a fixed number of success occur is called negative binomial experiment. The probability that at least n trials will be required to get a specified number of k success, so it means the probability that there are x failure preceding the k success in $x + k$ trials.

The random variable x in which represents the number of trials needed to produce k success is called a negative binomial random variable. This random variable is the inverse of the binomial random variable in binomial distribution, the number of trial n is kept fixed and the number of success x is random variable. but in negative binomial distribution the number of trial is random variable and the number of success is fixed.

A random variable x is said to be follow the negative binomial distribution with parameter k and p, if its probability mass function is,

$$P(X=x) = p(x) = \begin{cases} \binom{x+k-1}{k-1} p^k q^x & , x = 0, 1, 2, \ldots k \\ 0 & \text{otherwise} \end{cases}$$

$p+q=1$
$p=1-q$

$n = k+x$

Mean $= \dfrac{kq}{p}$

Variance $= \dfrac{kq}{p^2}$

∴ Mean < Variance

n trial
k success
x failure

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Problem 1 The probability of hitting the target at any trial is 0.2. If the shooter aims at a target find the probability that the fifth fire is second hit.

soln: Let x denote the number of misfire (failures) preceding k^{th} hit (success) in $(n+k)$ trial.

$$P = 0.2, \quad k = 2, \quad x + k = 5 \Rightarrow x = 3$$

The probability of x misfire preceding second hit is given by

$$P(x) = \binom{n + x - 1}{k - 1} p^k q^x$$

$$P(3) = \binom{5 - 1}{2 - 1} 0.2^2 * 0.8^3$$

$$= 0.0819$$

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k = success

x = failure

Total trial (n) = x + k

p(k) → fixed

p(x)

Problem 2 A machine is known to produce 5 % defective items. A quality control engineer is examining the items at random.

(i) What is the probability that at least 4 items are to be examined inorder to get 2 defective.

(ii) Find the mean and variance of the number of items to be the examined in order to get 2 defectives.

⇒

$$P(X \geq 4) = \sum_{x=4}^{\infty} P(x)$$

x = fail no. of

$$= 1 - \sum_{x=2}^{3} P(x)$$

$$\boxed{\sum_{x=2}^{3} P(x) + \sum_{x=4}^{\infty} P(x) = 1}$$

$n = x+k = x$ let x.

r +.

0.9923

$$P(2) = \binom{n-1}{k-1} p^k q^x = \binom{4-1}{2-1} 0.05^2 \; 0.95^4$$

$n = 4$

$x = 4$

let us consider x denotes the number of items needed to examine to get 2 defective.

$$P(x+r \geq 4) = 1 - P(x+r \leq 3)$$

$$= 1 - P(x \leq 1)$$

$$= 1 - \{P(x=0) + P(x=1)\}$$

$r = 2$

$x + r \leq 3$

$x + 2 \leq 3$

$x \leq 3-2$

$x \leq 1$