

Homework 4

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Due date: Tuesday, December 4

1. **Survival of root canal filled teeth** Deep caries or restorations in teeth could lead to pulpal involvement, necessitating root canal therapy or extraction. In a retrospective dental study, the primary interest is to assess the impact of pulpal involvement on tooth survival. In this data analysis, a Cox model is fitted using the survival time of the teeth as the response variable. The covariates included in the model are

$$\text{MOLAR} = \begin{cases} 1 & \text{molar tooth} \\ 0 & \text{otherwise,} \end{cases} \quad \text{ROOT} = \begin{cases} 1 & \text{root canal treatment applied} \\ 0 & \text{otherwise,} \end{cases}$$

and three mutually exclusive categories of proximal contacts

$$\text{PC1} = \begin{cases} 1 & \text{nonbridge abutment with one proximal contacts} \\ 0 & \text{otherwise,} \end{cases}$$

$$\text{PC2} = \begin{cases} 1 & \text{nonbridge abutment with two proximal contacts} \\ 0 & \text{otherwise,} \end{cases}$$

$$\text{PCABUT} = \begin{cases} 1 & \text{bridge abutment} \\ 0 & \text{otherwise,} \end{cases}$$

and the number of pockets larger than 5 mm (POCKET). Use the attached the `coxph` output to answer the following questions:

- a. (2 points) Suppose the log-partial likelihood for the model is -581.4417, what is the log-partial likelihood for the reduced model with no covariates?

$$G = 2\{l_p(\hat{\beta}) - l_p(0)\}$$

$$97.66 = 2(-581.4417 - l_p(0))$$

$$l_p(0) = -630.2717$$

- b. (2 points) What is the hazard ratio that compares teeth with bridge adutment with those without?

$$HR(\text{PCABUT} = 1, \text{PCABUT} = 0) = \exp(\beta_{\text{PCABUT}}) = 0.5941$$

- c. (2 points) What is the 95% confidence interval for the hazard ratio in #2?

$$\beta_{\text{PCABUT}} = -0.5207, se(\beta_{\text{PCABUT}}) = 0.5114$$

$$\beta_{\text{PCABUT}} \in [-0.5207 - 1.96 * 0.5114, -0.5207 + 1.96 * 0.5114] = [-1.523044, 0.481644]$$

$$\exp(\beta_{\text{PCABUT}}) \in [\exp(-1.523044), \exp(0.481644)] = [0.218047, 1.618733]$$

- d. (2 points) What is the hazard ratio that compares teeth with nonbridge abutment and one proximal contacts with those with nonbridge abutment and two proximal contacts?

$$HR(\text{PC1}, \text{PC2}) = \exp(\beta_{\text{PC1}} - \beta_{\text{PC2}}) = \exp(-0.7587 - (-1.5423)) = 2.18934$$

- e. (2 points) What is the hazard ratio that compares molar teeth with non-molar teeth **among those underwent root canal treatment**?

$$\begin{aligned} HR(\text{MOLAR} = 1, \text{MOLAR} = 0 | \text{ROOT} = 1) &= \exp(\beta_{\text{MOLAR}} + \beta_{\text{MOLAR:ROOT}}) \\ &= \exp(-0.8440 + 0.6645) = 0.8357 \end{aligned}$$