Homework 4

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Due date: Tuesday, December 3

1. **Survival of root canal filled teeth** Deep caries or restorations in teeth could lead to pulpal involvement, necessitating root canal therapy or extraction. In a retrospective dental study, the primary interest is to assess the impact of pulpal involvement on tooth survival. In this data analysis, a Cox model is fitted using the survival time of the teeth as the response variable. The covariates included in the model are

$$\text{MOLAR} = \begin{cases} 1 & \text{molar tooth} \\ 0 & \text{otherwise,} \end{cases} \quad \text{ROOT} = \begin{cases} 1 & \text{root canal treatment applied} \\ 0 & \text{otherwise,} \end{cases}$$

and three mutually exclusive categories of proximal contacts

$$\text{PC1} = \begin{cases} 1 & \text{nonbridge abutment with one proximal contacts} \\ 0 & \text{otherwise,} \end{cases}$$

$$\text{PC2} = \begin{cases} 1 & \text{nonbridge abutment with two proximal contacts} \\ 0 & \text{otherwise,} \end{cases}$$

$$\text{PCABUT} = \begin{cases} 1 & \text{bridge abutment} \\ 0 & \text{otherwise,} \end{cases}$$

and the number of pockets larger than 5 mm (POCKET). Use the attached the `coxph` output to answer the following questions:

a. (2 points) Suppose the log-partial likelihood for the model is -581.4417, what is the log-partial likelihood for the reduced model with no covariates?
b. (2 points) What is the hazard ratio that compares teeth with bridge adutment with those without?
c. (2 points) What is the 95% confidence interval for the hazard ratio in #2?
d. (2 points) What is the hazard ratio that compares teeth with nonbridge abutment and one proximal contacts with those with nonbridge abutment and two proximal contacts?
e. (2 points) What is the hazard ratio that compares molar teeth with non-molar teeth **among those underwent root canal treatment**?

```
> fm <- Surv(TIME, DELTA) ~ (MOLAR + ROOT)^2 + PC1 + PC2 + PCABUT + POCKET
> coxph(fm, data = dendata)
Call:
coxph(formula = fm, data = dendata)

              coef exp(coef) se(coef)     z       p
MOLAR      -0.8440    0.4300   0.5135 -1.64 0.10022
ROOT        1.5449    4.6876   0.3221  4.80 1.6e-06
PC1        -0.7587    0.4683   0.4202 -1.81 0.07100
PC2        -1.5423    0.2139   0.4242 -3.64 0.00028
PCABUT     -0.5207    0.5941   0.5114 -1.02 0.30860
POCKET      0.1463    1.1576   0.0814  1.80 0.07215
MOLAR:ROOT  0.6645    1.9435   0.5440  1.22 0.22192

Likelihood ratio test=97.66  on 7 df, p=<2e-16
n= 404, number of events= 109
```

Figure 1: `coxph` output

Answer

a.

$$\begin{aligned} G &= 2 \cdot \{l_p(\hat{\beta}) - l_p(0)\} \\ 97.66 &= 2 \cdot \{-581.4417 - l_p(0)\} \\ l_p(0) &= -630.2717 \end{aligned}$$

b. The hazard ratio that compares teeth with bridge adutment with those without is 0.5941.

c.

$$exp[\hat{\beta} \pm 1.96 \cdot \widehat{SE}(\hat{\beta})] = exp[-0.5207 \pm 1.96 \cdot 0.5114] = [0.218, 1.619]$$

95% confidence interval for the hazard ratio in #2 is $[0.218, 1.619]$.

d. The hazard ratio that compares teeth with nonbridge abutment and one proximal contacts with those with nonbridge abutment and two proximal contacts is

$$e^{(-0.7587+1.5423)} = e^{0.7826} = 2.1893$$

e. The hazard ratio that compares molar teeth with non-molar teeth among those underwent root canal treatment is

$$e^{(-0.8440+0.6645)} = e^{-0.1795} = 0.8357$$