ACKY CONSULTING

Project Scheduling Optimization Model

Group Members

Abhijit Agrawal 86948742

Christopher Lin 49213176

Kristy Yu 18105156

Yashika Kochhar 43596394

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Introduction

We live in the world of “time is money”, The travelling salesperson problem (TSP) is indirectly feeding into theory of saving time to make the maximum profit for completing multiple projects. It is one of the most complex problems that is solved by constant constraining. The travelling salesperson problem does not have a defined algorithm to find an optimal solution, thus it requires addition of constraints at each step for the problem to not go in a sub-loop. This project is a small iteration of many real-world problems with tens and thousands of variable and unimaginable constraints.
In this project, we are using the Gurobi model to find the different optimal routes along with the increasing hours for each result. The output gives us different routes combination to complete all the projects and the number of hours it will take to complete. As all the projects are aiming to complete as early as possible, profit incentives are given if the projects are started within the first 15 hours. We then use the profit from each project to calculate the total profit for completing all optimal routes. For this project's basis, ACKY is working on five projects and should complete them in forty hours. Additionally, ACKY can only do ten hours overtime if it is helping in maximizing the profits from working on additional projects, but they will have to incur the $50 cost per hour for each additional hour. Therefore, for profit calculation we must analyze the money each project is making individually, and will it be worth putting in the extra hours from the consultant?

Executive Summary

Business description

ACKY is a Vancouver based consulting organization, started by a Master of Business Analytics Graduate hoping to bring data driven solutions to local businesses around the city. ACKY plans to complete five projects per week by only working a maximum of fifty hours, ten hours over regular working hours (40) which will be paid in overtime. While consulting on project ACKY is also running an iteration of Travelling Salesperson Problem (TSP) which will be demonstrated in this project. We have built this model for ACKY to help determine the routes to take and which project to prioritize to maximize the profits.

Problem summary

The travelling salesperson problem (TSP) does not have an exact algorithm for the perfect optimal solution. It is a problem that is transferable through different industries, but the model must be constrained constantly with the use of "Lazy constraints" so that the optimal paths do not run-in sub-loops. We are trying to find the best possible route within the optimal hours and each project must be finished in a defined time. The consultant will be taking different hours to finish different projects as routes can be different/ there could be bus delays, i.e., not two times are same. He also wants to try and finish the project that will be the most profitable first. The problem has also been constrained to maintain that the consultant starts from project 1 (since its based in the same city as ACKY headquarters) and then progresses but always must come back to 1 to show completion. An example of optimal hours for each project with total hours and the profit from each project with total profit from one solution is provided below:

Optimal Path	Start time from project	Time taken from project to project	Profit from project	Optimal result
13	0	15	1200	40
34	15	4	1500	
45	19	6	500	
52	25	5	2700	
21	30	10	1700	
Total			7600	

Implementation

We have used the Gourbi optimizer to find solutions to the routes with the optimal value for hours. One key difference for this problem from the shortest path problem is that the optimizer always comes back to the project where the consultant started from. The first optimal solution to complete all projects is 40 hours after eliminating all sub-loops. We then moved on to the next best optimal solution by removing the previous result using constraints. Using all the optimal results we calculated the total profit from each route to create a pareto optimal trade off curve to maximize profits while minimizing time.

Values

As a consultation service provider, ACKY always optimize the time taken by their consultants to complete projects. They are often put in a situation by their clients to figure out how to maximize their profit by catering to important projects first. This model provides ACKY the opportunity to look at data and determine the best solution for themselves to make the maximum profit.
ACKY can also use this as a base model for as many projects as can be added. Part of this model can also be widely used in the logistics industry to solve transportation problems to find the optimal route. An example where this model can be applied is for load delivery by multinational companies who have multiple trucks travelling to different routes across the country.

Model and Formulation

Below is the required completion time for each project and their respective profit if completed before and after the first 15 hours:

Project	Completion time	Profit if started before 15 hours	Profit if started after 15 hours
1	8	$1,200	$1,000
2	4	$2,000	$1,700
3	3	$1,500	$1,500
4	1	$700	$500
5	4	$3,100	$2,700
Total	20	$8,500	$7,400

Below is the travel time (hours) between each project. Due to traffic restrictions, travel time from project A to project B might be different from the travel time from project B to project A.

Project	1	2	3	4	5
1	-	5	7	6	2
2	6	-	11	8	1
3	12	9	-	1	14
4	10	7	3	-	5
5	5	1	13	2	-

Objective

Minimize the total time to complete all the projects

Variables

The input variable is set as the format:

$$x_{ij}$$
$$where\ i, j\ \in \{1, 2, 3, 4, 5\}, x\ is\ a\ binary\ variable$$

The i, j represents the number of the project, and the x represents whether the route from i to j should be used. As the route should always be from different nodes, the variables $x_{11}, x_{22}, x_{33}, x_{44}, x_{55}$ have been removed, leading to 20 variables in total.

Constraints

a) Net outflow should be 1 for each node
b) Net inflow should be 1 for each node
c) Lazy Constraints
d) Non-negativity

Objective formula

$$\begin{aligned} Minimize \quad & 13\,x_{12} + 15\,x_{13} + 14\,x_{14} + 10\,x_{15} \\ & + 10\,x_{21} + 15\,x_{23} + 12\,x_{24} + 5\,x_{25} \\ & + 15\,x_{31} + 12\,x_{32} + 4\,x_{34} + 17\,x_{35} \\ & + 11\,x_{41} + 8\,x_{42} + 4\,x_{43} + 6\,x_{45} \\ & + 9\,x_{51} + 5\,x_{52} + 17\,x_{53} + 6\,x_{54} \end{aligned}$$

The coefficient represents the **time of completing project *i* and traveling to project *j***. For example, the coefficient of $x12$ is the sum of the project 1 completion time (8 hours) and travel time from project 1 to project 2 (5 hours).

Algebraic formulation

a) Net outflow should be 1 for each node.

$$x_{12} + x_{13} + x_{14} + x_{15} = 1$$
$$x_{21} + x_{23} + x_{24} + x_{25} = 1$$
$$x_{31} + x_{32} + x_{34} + x_{35} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{45} = 1$$
$$x_{51} + x_{52} + x_{53} + x_{54} = 1$$

b) Net inflow should be 1 for each node

$$x_{21} + x_{31} + x_{41} + x_{51} = 1$$
$$x_{12} + x_{32} + x_{42} + x_{52} = 1$$
$$x_{13} + x_{23} + x_{43} + x_{53} = 1$$
$$x_{14} + x_{24} + x_{34} + x_{54} = 1$$
$$x_{15} + x_{25} + x_{35} + x_{45} = 1$$

c) Lazy constraints

After running the above model, we find that the optimal solution is including two subtours, pair {3,4} and pair {2,5}. To eliminate that, we add two lazy constraints below:

$$x_{34} + x_{43} \leq 1$$
$$x_{25} + x_{52} \leq 1$$

d) Non-negativity

Model Results

Based on the result calculated by Gurobi using the sheet *"OS1"*, the optimal solution for the shortest path is:

Project 1 → Project 3 → Project 4 → Project 5 → Project 2 → Project 1

And the minimum time for completing the projects and traveling is 40 working hours. In this optimal solution, only project 1 can be started within the first 15 hours and get the higher profit of $1200. The total profit for finishing all the projects is $7600. Detail as below:

Project Order	Profit
1	$1,200
3	$1,500
4	$500
5	$2,700
2	$1,700
Total	$7,600

Further Modeling

To reach a better profit, the company is interested in knowing the 2nd, 3rd, 4th … optimal solutions and how the company profit varies in response to different solutions. We change the model by adding constraints to prevent the pervious optimal solutions.

For example, to get the 2nd optimal solution, we add the constraints below:
(sheet *"OS2"*)

$$x_{13} + x_{34} + x_{45} + x_{52} + x_{21} \leq 4$$

The 2nd optimal solution is below, changing the order of project 2 and 5, with total time of 41 working hours. And the total profit remains $7,600.

Project 1 → Project 3 → Project 4 → <u>Project 2 → Project 5</u> → Project 1

We keep continuously add new constraints to find the next optimal solution and their respective profits. (Detail of all solutions and profits are attached in the appendix.)

Pareto Optimal Solution

Objective

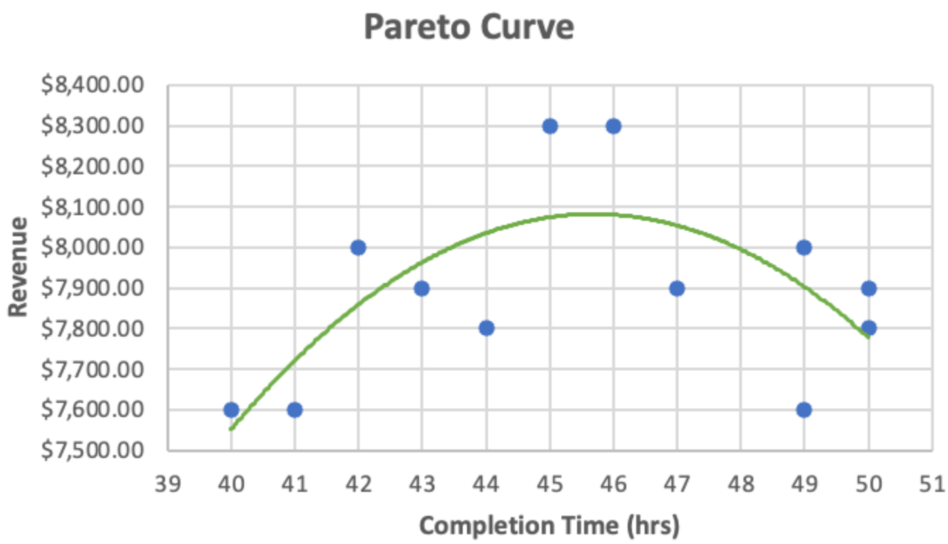
Minimize time for completion while maximizing profit

Method

We forced the Gurobi model to come up with new optimal solutions by invalidating the old solution with the help of added constraints. This was done for Gurobi to provide us with the next best optimal solution for the set of projects. This new solution could be a tie with the total time for the route being the same, or it could be a situation where the time of completion given in the result could be longer than the previous one. Although it is counterintuitive to try to get an answer that is not optimal, in the real world there are a lot of situations where you would prefer that. Some routes might be unrealistic, some projects might have different deadlines, and as in our case, some projects have a higher payout depending on the completion time.

For each new optimal solution, we calculated the profit ACKY would make based on when each project was started. Due to the time sensitivity of some of the projects assigned to the company, some clients were willing to pay ACKY more money if they started their project within 15 work hours which meant higher profit margins. Unfortunately, an employee at ACKY is only paid for 40 hours of work, post which overtime kicks in (max overtime allowed is 10 hours). ACKY would not like to pay this extra overtime unless it meant that they were able to make more money overall. So, we developed a pareto optimal trade-off curve to evaluate the profit ACKY could make through different optimal routes and to check if ACKY should just follow the route that takes the shortest time, or if they should ask their consultants to take a route that kicks in overtime but caters to some clients within the 15 hours to maximize profit.

Pareto Curve	
Completion Time (hours)	Revenue
40	$7,600
41	$7,600
42	$8,000
43	$7,900
44	$7,800
45	**$8,300**
46	$8,300
47	$7,900
49	$8,000
49	$7,600
50	$7,900
50	$7,800



We found out that the maximum profit is achieved with 45 hours of work for the combination of projects available to ACKY. The line of best fit represents the same with the curve increasing up to 45 and then falling back down. The shortest time to complete all projects is 40 hours and avoids any overtime but it is also the point providing us with the least profit possible. Although the longest time (50 hours) provides us with better profit value, it does not maximize our profits while also incurring the most amount of overtime. Between these points, depending on the route we must take, the profit may be higher or lower, making it reasonable to pay the overtime.

Recommendations and Conclusions

Depending on the above analysis and the pareto curve, we would have the following suggestions for ACKY:

ACKY should ask their consultant to finish project 1 first since that is in Vancouver, where the company is based in. In order to maximize the additional profits offered by projects 2 and 5, the consultant should travel to finish project 5 after 1, followed by project 2 which will enable ACKY to get the extra profit from those clients having started the projects within the first 15 work hours. Finally, the consultant should finish projects 3 and 4 before returning to Vancouver and this entire route would take a total of 45 hours and earn ACKY a profit of $8,300.

This can be seen in the pareto optimal trade off curve as well as we always try to choose a point that is up and left when trying to maximize the profit (y-axis) while minimizing the completion time (x-axis). This approach does result in ACKY spending extra money to pay their consultant 5 hours of overtime but if ACKY tries to avoid paying that overtime they only make $7,600 in profit when trying to finish the route in 40 hours.

With an overtime cost of $50/hr, ACKY would incur an additional cost of $250 for the overtime by selecting a route that is not optimal, but they would receive an additional profit of $700 from their clients making their net profit $450. This shows us the importance of considering the trade-off between paying overtime and maximizing profits, and ACKY can also build on this model in the future by updating the model with any new projects that they might receive to always maximize their net profit while also minimizing their route.

Appendix

1st: Total time is 40 working hours. And the total profit is $7600.

Project 1 → Project 3 → Project 4 → Project 5 → Project 2 → Project 1

Project	Profit
1	$1,200
3	$1,500
4	$500
5	$2,700
2	$1,700
Total	$7,600

2nd: Total time is 41 working hours. And the total profit is $7600.

Project 1 → Project 3 → Project 4 → Project 2 → Project 5 → Project 1

Project	Profit
1	$1,200
3	$1,500
4	$500
2	$1,700
5	$2,700
Total	$7,600

3rd: Total time is 42 working hours. And the total profit is $8000.

Project 1 → Project 5 → Project 4 → Project 3 → Project 2 → Project 1

Project	Profit
1	$1,200
5	$3,100
4	$500
3	$1,500
2	$1,700
Total	$8,000

4th: Total time is 43 working hours. And the total profit is $7900.

Project 1 → Project 2 → Project 5 → Project 4 → Project 3 → Project 1

Project	Profit
1	$1,200
2	$2,000
5	$2,700
4	$500
3	$1,500
Total	$7,900

5th: Total time is 44 working hours. And the total profit is $7800.

Project 1 → Project 4 → Project 3 → Project 2 → Project 5 → Project 1

Project	Profit
1	$1,200
4	$700
3	$1,500
2	$1,700
5	$2,700
Total	$7,800

6th: Total time is 45 working hours. And the total profit is $8300.

Project 1 → Project 5 → Project 2 → Project 3 → Project 4 → Project 1

Project	Profit
1	$1,200
5	$3,100
2	$2,000
3	$1,500
4	$500
Total	$8,300

7th: Total time is 46 working hours. And the total profit is $8300.

Project 1 → Project 5 → Project 2 → Project 4 → Project 3 → Project 1

Project	Profit
1	$1,200
5	$3,100
2	$2,000
4	$500
3	$1,500
Total	$8,300

8th: Total time is 47 working hours. And the total profit is $7900.

Project 1 → Project 2 → Project 3 → Project 4 → Project 5 → Project 1

Project	Profit
1	$1,200
2	$2,000
3	$1,500
4	$500
5	$2,700
Total	$7,900

9th: Total time is 49 working hours. And the total profit is $8000.

Project 1 → Project 5 → Project 3 → Project 4 → Project 2 → Project 1

Project	Profit
1	$1,200
5	$3,100
3	$1,500
4	$500
2	$1,700
Total	$8,000

10th: Total time is 49 working hours. And the total profit is $7600.

Project 1 → Project 3 → Project 2 → Project 5 → Project 4 → Project 1

Project	Profit
1	$1,200
3	$1,500
2	$1,700
5	$2,700
4	$500
Total	$7,600

11th: Total time is 50 working hours. And the total profit is $7900.

Project 1 → Project 2 → Project 5 → Project 3 → Project 4 → Project 1

Project	Profit
1	$1,200
2	$2,000
5	$2,700
3	$1,500
4	$500
Total	$7,900

12th: Total time is 50 working hours. And the total profit is $7800.

Project 1 → Project 4 → Project 3 → Project 5 → Project 2 → Project 1

Project	Profit
1	$1,200
4	$700
3	$1,500
5	$2,700
2	$1,700
Total	$7,800