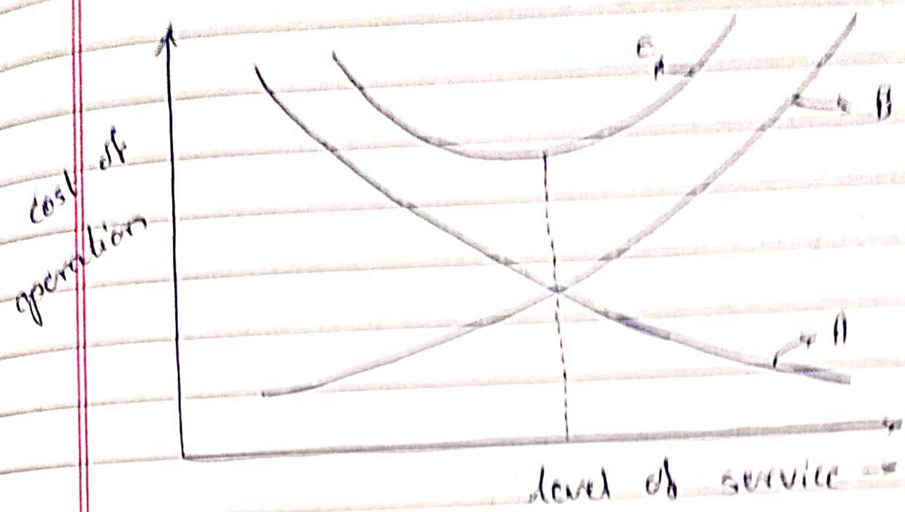
Unit VI

Queueing Theory



A → cost of waiting
B → cost of operation
C → total expected cost

The arrival time distribution:
A) Poisson distribution
B) Exponential distribution
C) Erlang distribution

Queueing Process:

1) Static: FCFS / LCFS

2) Dynamic:
- SIRO (service in random order)
- Priority service
- Preemptive priority → emergency
- Non preemptive

Service process:
1) arrangements
2) distribution
3) service behaviour
4) management policies

Types of queues:
i) Single queue
ii) Single queue multiple facility
iii) Single queue multiple facility in parallel
iv) multiple queue
v) multiple queue in service.

Problems based on:
i) Time related → waiting time
ii) No. of customers → in queue or waiting in queue
iii) Time customers & services → probability
iv) Cost related.

Notations:
i) n = no. of customers in the system (waiting & in service)
ii) P_n = probability of n customers in system.
iii) λ = average (expected) customer's arrival time or rate or average no. of arrivals per unit of time in the queueing system
iv) μ = average (expected) service rate or avg no. of customers served per unit time at the place of service.
v) $\rho = \frac{\lambda}{\mu}$ (ρ is traffic intensity, λ is avg. service completion time, μ is avg. inter arrival time)

vi) Probability of no. of customers in the system

$$P_0 = 1 - \frac{\lambda}{\mu}$$

vii) S = no. of service channels

viii) N = maximum no. of customers allowed in a system

ix) L_s = Avg (expected) no. of customers in the system (waiting & inservice)

x) L_q = Avg (expected) no. of customers in the queue.

xi) L = Avg. (expected) length of non-empty queue)

xii) W_s = Avg. waiting time in the system.

xiii) W_q = Avg. waiting time in the queue.

xiv) P_w = Probability that an arriving customer has to wait (system being busy)

$$P_w = 1 - P_0 = \frac{\lambda}{\mu}$$

xv) For steady state condition:

$$\rho = \frac{\lambda}{\mu} < 1$$

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Measure of Performance:

1) $L_q = \sum_{n=0} n P_n$ & $L_q = \sum_{n=1} (n-s) P_n$

2) Expected no. of customer in the system.

$$L_s = L_q + \frac{\lambda}{\mu} \quad \text{or} \quad L_s = L_q + \rho$$

3) Expected waiting time:

$$W_s = W_q + \frac{1}{\mu}$$

W_q = avg. waiting time.

$1/\mu$ = expected service time

4) Probability of B in the system longer than time

$$P(T>t) = e^{-(\mu-\lambda)\lambda t}$$

$$P(T \leq t) = 1 - P(T>t)$$

T = time spent in the system

t = specified time period

e = 2.178

Probability of exact n customers in the system

$$P_n = \left(1 - \frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu}\right)^n$$

Probability that the no. of customers in the system, n exceeds a given number r is given by

$$P(n > r) = \left(\frac{\lambda}{\mu}\right)^{r+1}$$

Expected no. of customers in the system is equal to the average no. of arrival per unit of time multiplied by the time spend by customer in the system.

$$L_s = \lambda W_s \quad \text{or} \quad W_s = W_q + \frac{1}{\mu}$$

$$L_q = L_s - \frac{\lambda}{\mu} = \lambda W_q \quad \text{or} \quad W_q = W_s - \frac{1}{\mu} = \frac{1}{\lambda} L_q$$

$$L_s = \frac{\rho}{1-\rho}$$

Probability P_n of n customers in the queueing system at any time can be

$$L_s = \sum_{n=0}^{\infty} n P_n \Rightarrow W_s = \frac{L_s}{\lambda} \Rightarrow W_q = W_s - \frac{1}{\mu}$$

Probability of waiting time $\geq n$

$$= \int_n^{\infty} \left(\frac{\lambda}{\mu}\right)(\mu - \lambda) \cdot e^{-(\mu-\lambda)t} \, dt.$$