

1a) Use the aggregate method in the aggregate method we consider the total cost across all the insertions and calculate the average cost per insertion.

When inserting the i^{th} element, if a resize operation is not needed the existing happens cost $O(1)$ as it involves copying the existing elements to the new table of size

$2K$ (K is number of resizes performed)

The cost $= O(n) \quad K = O(n \log n)$

cost per insertion $= O(\log n)$

Runtime per insertion $= O(\log n)$

Total time is $O(n) \log(n+1)$

b) Accounting method :- In the accounting method, we assign each insertion a higher "amortized" cost the store "credits" that pay for future resizing costs.

Pseudo code :-

for $i = 1$ to n

if table is full

new table = create new table

with size then copy elements

from old table to new table

table = new-table

insert element i into table

initial charge $= 0$

for $i = 1$ to n

charge $+= 2$

if table doubled, \ln size from

into $2m$

credits $+= m$

total charges $= 2 * n = O(n)$

total credits $= m + 2m = \frac{n}{2} * m = O(n)$

Amortized cost per insertion

$= \text{total} / n$

$= O(n/n)$

$= O(1)$

Runtime per insertion $O(1)$

total time $O(n)$