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Part 1 :- Shift Distribution Problem

Assumption :-

1) ~~I~~ In shift Distribution constrian we have assumed sufficiant availability of manpower to fill the optimal result

2) As we are only concerned with minimizing total shift couont we will not consider any OT

3) As we are only concerned with shift distribution we will not consider bundling shift to make schedules

Parameters :-

T : Total number of Time ~~Pediod~~ Period in Planning horizon
$4 \times 24 \times 14 = 1{,}344$

r_t = minimum number of ATCO's requried at time Period t
$\forall\ t = 1 \ldots T$

S_i = length of shift type i in number of Period
$S_1 = 32$ (8 hrs Shift)
$S_2 = 40$ (10 hrs shift)

Decision Variable :-

~~x_{it} : {0,1}, 1 if shift type i starts at time Period t, 0 other wise~~

x_{it} = number of shifts of type i starting at time Priod t (Positive integer)

Objective function :-

minimize the total number of shifts

$$\min \sum_{i=1}^{2} \sum_{t=0}^{T} x_{it}$$

Subject to :-

1) Coverage requirment :

$$\sum_{i=1}^{2} \sum_{\tau=\max(1,\,t-S_i+1)}^{t} x_{i\tau} \geq r_t \quad \forall\ t = 1 \ldots T$$

2) No shift should start if it is not going to finsh in time horizon

$$x_{1t} = 0 \quad \forall\ t = T-S_1+1 \ldots T \quad (\text{last } 31 \text{ time Priod})$$
$$x_{2t} = 0 \quad \forall\ t = T-S_2+1 \ldots T \quad (\text{Last } 39 \text{ time Priod})$$

3) Non negativity constrain

$$x_{it} \geq 0 \quad \forall\ i = 1, 2 \quad \forall\ t = 1, 2 \ldots T$$

Part 2 :- Shift Scheduling Problem

Asumptions :-

1) Each ATCO can only be assighed to one type of schedule
2) Overtime is allowed as continuation of shift only
3) a mandatory gap of 8 hrs is requird be for starting new Shift after a shift or OT

Parameters

T = Total number of Period in Planning horizon :- 1,344

γ_t = minimum staffing required at time Period t

S_i = length of shift type i in number of Period

$S_1 = 32$ (8 hrs shift)
$S_2 = 40$ (10 hrs shift)

$C_1 = 80 \times C$ - Cost of 10 x 8 hrs schedule
$C_2 = 80 \times C$ - cost of 8 x 10hrs schedule

C is hourly rate

$C_o = 1.5 \times C$ hourly rate for OT

i = 1 to N number of standard schedule
k = 1 to m number of comprssed schedule
J_1 = 1 to 10 number of shift in standard schedule
J_2 = 1 to 8 for comprssed schedule

Decision Variables :-

$S_i = \{0, 1\}$ 1 if ith standard schedule is Populated
0 other wise

$C_k = \{0, 1\}$ 1 if k^{th} standard schedule is Popolated
0 other wise

$x_{iJ_1t} = \{0, 1\}$ 1 if J_1^{th} shift of i^{th} standard schedule is starting at t
0 otherwise

$y_{kJ_2t} = \{0, 1\}$ 1 if J_2^{th} shift of k^{th} compressed schedule is starting at t
0 otherwise

$Os_{iJ_1t} = \{0, 1\}$ 1 if ot of J_1^{th} shift of standard schedul i in in Process during time Period t
0 other wise

$Oc_{kJ_2t} = \{0, 1\}$ 1 if OT of J_2^{th} shift of ~~sto~~ compressed schedule k is in process during time period t
0 otherwise

Objective function :- Minimize total cost of the schedule

$$\min \sum C_1 S_i + \sum C_2 C_k + \sum_{i=1}^{N} \sum_{J_1=1}^{10} \sum_{t=1}^{T} C_o\, Os_{iJ_1t}$$
$$+ \sum_{k=1}^{M} \sum_{J_2=1}^{8} \sum_{t=1}^{T} C_o\, Oc_{kJ_2t}$$

Subject to :-

1) Coverage requirment

$$\sum_{i=1}^{N}\sum_{J_1=1}^{10}\sum_{T=(\max(1,t-31))}^{t} x_{iJ_1t} + \sum_{i=1}^{N}\sum_{J_1=1}^{10} Os_{iJ_1t}$$

$$+ \sum_{k=1}^{N}\sum_{J_2=1}^{8}\sum_{T=(\max(1,t-39))}^{t} y_{kJ_2t} + \sum_{k=1}^{M}\sum_{J_2=1}^{8} Oc_{kJ_2t} \geq \gamma_t \quad \forall t=1..T$$

2) OT contiuation constrain

$$Os_{iJ_1t} \leq Os_{iJ_1(t-1)} + x_{iJ_1(t-32)} \quad \forall i=1...N,\ J_1=1...10,\ t=33...T$$

$$Oc_{kJ_2t} \leq Oc_{kJ_2(t-1)} + y_{kJ_2(t-40)} \quad \forall k=1..M,\ \forall J_2=1...8,\ \forall t=41...T$$

$$\left.\begin{aligned} Os_{iJ_1t} &= 0 \quad \forall t=1...32 \\ Oc_{kJ_2t} &= 0 \quad \forall t=1...40 \end{aligned}\right\} \text{ot can not start befor 1st shifts end}$$

3) 8 hrs gap constrains :-

$$M(1-x_{ij_1t}) \geq \sum_{j_1=1}^{10} \sum_{\tau=\max(1,t-63)}^{t-1} x_{ij_1\tau} + \sum_{j_1=1}^{10} \sum_{\tau=\max(1,t-32)}^{t-1} Os_{ij_1\tau}$$

$\forall i = 1 \ldots N$

$\forall j_1 = 1 \ldots 10$

$\forall t = 1 \ldots T$

$$M(1-y_{kj_2t}) \geq \sum_{j_2=1}^{8} \sum_{\tau=\max(1,t-71)}^{t-1} y_{kj_2\tau} + \sum_{j_2=1}^{p} \sum_{\tau=\max(1,t-32)}^{t-1} Oc_{kj_2\tau}$$

$\forall k = 1 \ldots m$

$\forall j_2 = 1 \ldots 8$

$\forall t = 1 \ldots T$

here M is sufficiently large number

4) Sift validation and schedule fulfilment constrain

$$\sum_{j_1=1}^{10} \sum_{t=1}^{T} x_{ij_1t} = 10S_i \quad \forall i$$

$$\sum_{j_2=1}^{8} \sum_{t=1}^{T} y_{kj_2t} = 8C_k \quad \forall k$$

$$\sum_{j_1=1}^{10} x_{ij_1t} \leq 1 \quad \forall i = 1 \ldots N \quad \forall t = 1 \ldots T$$

$$\sum_{j_2=1}^{p} y_{kj_2t} \leq 1 \quad \forall k = 1 \ldots N \quad \forall t = 1 \ldots T$$

Only 1 shift form schedule will start in 1 period

5) all shifts should finish in Planning hoizon

$x_{ij_2t} = 0$ for $\forall t = T - 30$ to T (last 31 Period)

$y_{kj_2t} = 0$ for $\forall t = T - 38$ to T (last 39 Period)