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Aim: Implementation of Multistage Graphs.

Theory:

A multistage graph **G = (V, E)** is a directed graph where vertices are partitioned into **k** (where ***k*** **> 1**) number of disjoint subsets ***S* = {*s₁*,*s₂*,…,*sₖ*}** such that edge *(u, v)* is in E, then $u \in s_i$ and $v \in s_{1+1}$ for some subsets in the partition and $|s_1| = |s_k| = 1$.

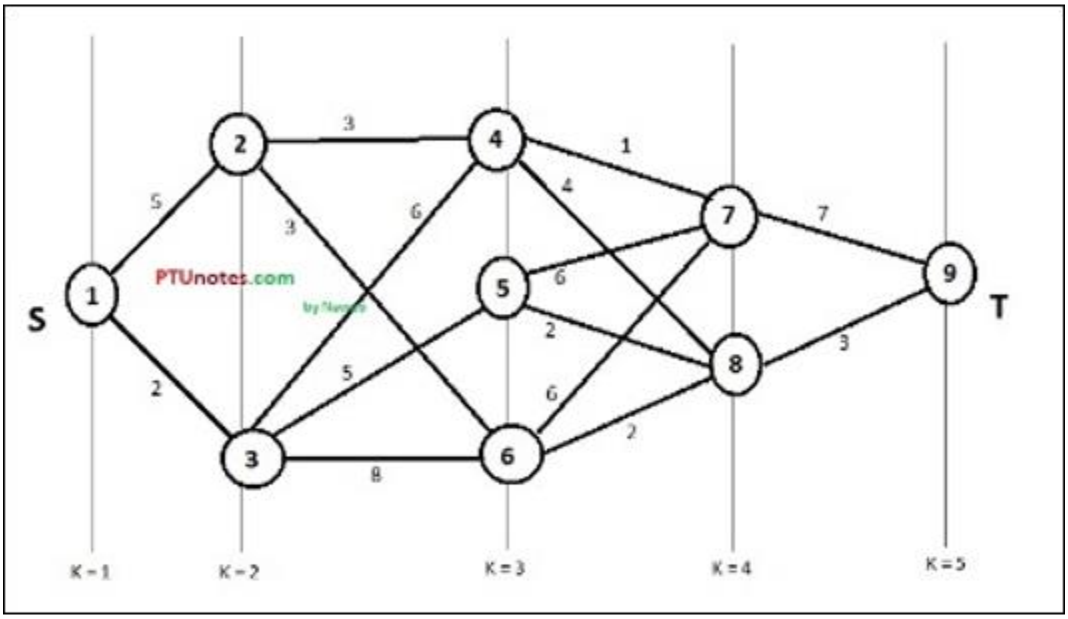
The vertex $s \in s_1$ is called the **source** and the vertex $t \in s_k$ is called **sink**.

G is usually assumed to be a weighted graph. In this graph, cost of an edge *(i, j)* is represented by *c(i, j)*. Hence, the cost of path from source ***s*** to sink ***t*** is the sum of costs of each edges in this path.

The multistage graph problem is finding the path with minimum cost from source ***s*** to sink ***t***.

Example

Consider the following example to understand the concept of multistage graph.



According to the formula, we have to calculate the cost **(i, j)** using the following steps

Step-1: Cost (K-2, j)

In this step, three nodes (node 4, 5. 6) are selected as **j**. Hence, we have three options to choose the minimum cost at this step.

Cost(3, 4) = min {c(4, 7) + Cost(7, 9),c(4, 8) + Cost(8, 9)} = 7

Cost(3, 5) = min {c(5, 7) + Cost(7, 9),c(5, 8) + Cost(8, 9)} = 5

Cost(3, 6) = min {c(6, 7) + Cost(7, 9),c(6, 8) + Cost(8, 9)} = 5

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Step-2: Cost (K-3, j)

Two nodes are selected as j because at stage k - 3 = 2 there are two nodes, 2 and 3. So, the value i = 2 and j = 2 and 3.

Cost(2, 2) = min {c(2, 4) + Cost(4, 8) + Cost(8, 9),c(2, 6) +

Cost(6, 8) + Cost(8, 9)} = 8

Cost(2, 3) = {c(3, 4) + Cost(4, 8) + Cost(8, 9), c(3, 5) + Cost(5, 8)+ Cost(8, 9), c(3, 6) + Cost(6, 8) + Cost(8, 9)} = 10

Step-3: Cost (K-4, j)

Cost (1, 1) = {c(1, 2) + Cost(2, 6) + Cost(6, 8) + Cost(8, 9), c(1, 3) + Cost(3, 5) + Cost(5, 8) + Cost(8, 9))} = 12

c(1, 3) + Cost(3, 6) + Cost(6, 8 + Cost(8, 9))} = 13

Hence, the path having the minimum cost is **1→ 3→ 5→ 8→ 9**.

Complexity Analysis:

In a multi-stage graph algorithm for shortest path, we minimize cost for every *edge* exactly once. So the Time Complexity is $O(E)$O(E). However, in the worst case, we get a complete graph, which has edges $E=n*(n-1)/2$E=n*(n−1)/2, so worst time complexity then becomes $O(E)=O(n^2)$.

Source Code:

```
#include <stdio.h>
#include <limits.h>
#define n 8

int main()
{
    int stages = 4;
    printf("Number of stages=4 \n");
    // cost adjacency matrix
    int graph[n][n] =
        {{0, 1, 2, 5, 0, 0, 0, 0},
         {0, 0, 0, 0, 4, 11, 0, 0},
         {0, 0, 0, 0, 9, 5, 16, 0},
         {0, 0, 0, 0, 0, 0, 2, 0},
         {0, 0, 0, 0, 0, 0, 0, 18},
         {0, 0, 0, 0, 0, 0, 0, 13},
         {0, 0, 0, 0, 0, 0, 0, 2},
         {0, 0, 0, 0, 0, 0, 0, 0}};
```

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```
    int distance[n];
    int path[stages];
    int cost[n];
    cost[n - 1] = 0;
    for (int i = n - 2; i >= 0; i--)
    {
        int min = INT_MAX;
        for (int k = i + 1; k <= n - 1; k++)
        {
            if (graph[i][k] != 0 && graph[i][k] + cost[k] < min)
            {
                min = graph[i][k] + cost[k];
                distance[i] = k; //saving the vertex which gave minmum value
            }
        }
        cost[i] = min;
    }
    path[0] = 0; //first vertex will always be 1(added +1 in display)
    path[stages - 1] = n;
    for (int i = 1; i < stages - 1; i++)
    {
        path[i] = distance[path[i - 1]];
    }
    printf("Path is: ");
    for (int i = 0; i < stages; i++)
    {
        if (i == stages - 1)
        {
            printf("%d\n", path[i]); // last node
        }
        else
        {
            // adding +1 because we are starting from 0 and vertices starts
from 1
            printf("%d -> ", path[i] + 1);
        }
    }
    printf("Minimum Cost= %d", cost[0]);
    return 0;
}
```

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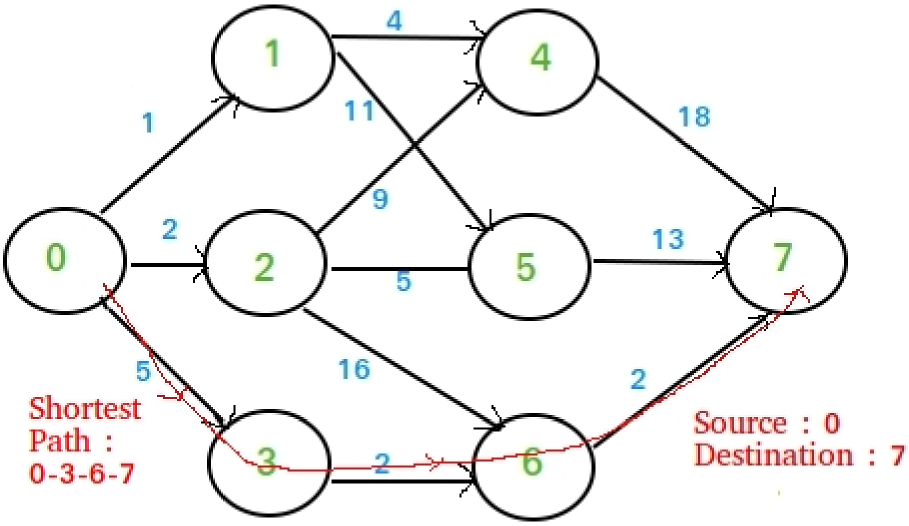
Output:




```
Number of stages=4
Path is: 1 -> 4 -> 7 -> 8
Minimum Cost= 9
PS C:\Users\zatak\OneDrive - Shri Vile Parle Kelavani Mandal\60003200163\DAA>
```

Conclusion: Thus, we have implemented multistage graph using dynamic programming and have also found optimal cost for the same. Also, the shortest path is being displayed correctly.