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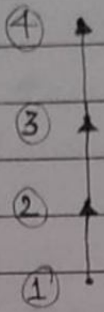
Assignment :- 02.

Q1] set $X = \{1, 2, 3, 4\}$.

$R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$.

Hasse diagram :



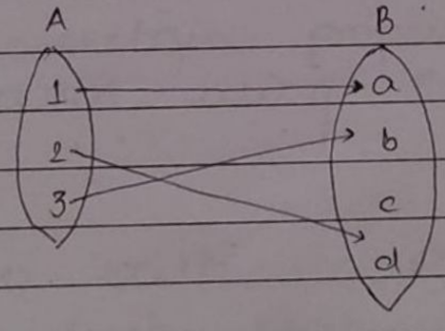
Since every element is related each other

∴ It is a chain.

∴ No antichain is present above.

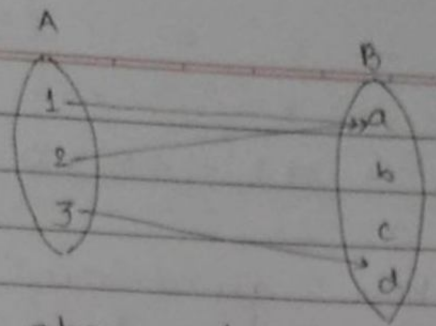
Q2] $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$

① $f = \{(1,a), (2,d), (3,b)\}$



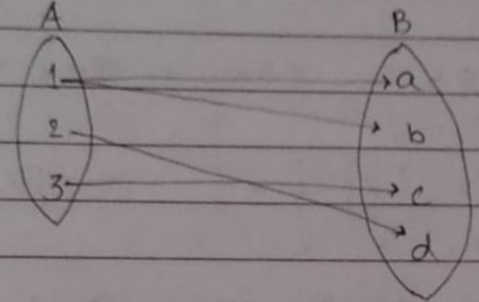
The above function is injective as every element in domain has not matched with that of codomain.

② $g = \{(1,a), (2,a), (3,d)\}$



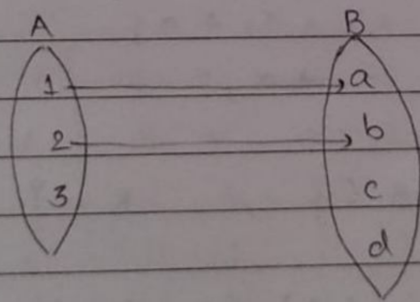
The above function is injective as every element in domain is occupied, where as not all the element in codomain are occupied.

(3) $h = \{(1,a), (1,b), (2,d), (3,c)\}$



The above relation is not a function, because an element in codomain will be not related to more than one element in domain.

(4) $j = \{(1,a), (2,b)\}$



The above relation is a function which is neither injective, surjective nor bijective.

Q3] $a_n = 11a_{n-1} - 39a_{n-2} + 45a_{n-3}$

for $a_0 = 5$, $a_1 = 11$, $a_2 = 25$.

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Sol^n: characteristic eq^n:

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0.$$

by solving the above quadratic we get

$$\lambda_1 = 5, \quad \lambda_2 = 3, \quad \lambda_3 = 3.$$

for a_0 : $5 = 5\alpha_1 + 3\alpha_2 + 3\alpha_3$

for a_1 : $11 = 5\alpha_1 + 3\alpha_2 + 3\alpha_3$

for a_2 : $25 = 25\alpha_1 + 9\alpha_2 + 9\alpha_3$

$$(5-\alpha_1) = \alpha_2 + \alpha_3 \quad —①$$

substituting the eq-① in a_1 :

$$\therefore \quad 11 = 5\alpha_1 + 3(5-\alpha_1)$$

$$11 = 5\alpha_1 + 15 - 3\alpha_1$$

$$-4 = 2\alpha_1$$

$$\alpha_1 = -2$$

$$\therefore \quad \{5-(-2)\} = \alpha_2 + \alpha_3$$

$$\therefore \quad 7 = \alpha_2 + \alpha_3$$

Q 4] $A = \{1, 2, 4, 5, 6\}$, $R = \{(x-y) \mid |x-y| = 2\}$

Sol^n: $R = \{(1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,6), (6,4)\}$

$$R_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

ROW {3} Col^m {3}

$$R_2 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Row $\{4\}$ Col^m $\{4\}$

$R = \{(1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,1), (6,4)$
$(4,4), (3,3)\}$.

$$R_3 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 0 & 1 & 0 & 1 \\ 5 & 0 & 0 & 1 & 0 & 0 & 0 \\ 6 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Row $\{1,3,5\}$ Col^m $\{1,3,5\}$

$R = \{(1,1), (1,3), (1,5)$
$(3,1), (3,3), (3,5)$
$(5,1), (5,3), (5,5)\}$

$$R_4 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 3 & 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 0 & 1 & 0 & 1 \\ 5 & 1 & 0 & 1 & 0 & 1 & 0 \\ 6 & 0 & 0 & 0 & 1 & 0 & 0 \end{array}$$

Row $\{2,4,6\}$ Col^m $\{2,4,6\}$

$R = \{(2,2), (2,4), (2,6)$
$(4,2), (4,4), (4,6)$
$(6,2), (6,4), (6,6)\}$

$$R_5 = \begin{array}{c|cccccc} & 1 & 2 & 3 & 4 & 5 & 6 \\ \hline 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 2 & 0 & 1 & 0 & 1 & 0 & 1 \\ 3 & 1 & 0 & 1 & 0 & 1 & 0 \\ 4 & 0 & 1 & 0 & 1 & 0 & 1 \\ 5 & 1 & 0 & 1 & 0 & 1 & 0 \\ 6 & 0 & 1 & 0 & 1 & 0 & 1 \end{array}$$

Row $\{1,3,5\}$ Col^m $\{1,3,5\}$

$R = \{(1 1), (1 3), (1,5),$
$(3,1) (3 3) (3 5)$
$(5 1) (5 3) (5 5)\}$

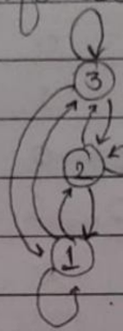
$$R_6 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$\therefore$ R = { (1,1), (1,3), (1,5), (2,2), (2,4), (2,6),
(3,3), (3,5), (4,2), (4,4), (4,6), (5,5),
(5,3), (5,1), (6,6), (6,2), (6,4) }.

Q5] ① Equivalence Relation:-

A relation on set A is called an equivalence relation if it is reflexive, symmetric and transitive.

consider the following diagram:



R=2

R = { (1,1), (2,2), (3,3),
(1,2), (2,1), (2,3),
(3,2), (1,3), (3,1) }

It contains all the elements which have properties of reflexive, transitive and symmetric.

② Partial ordering relation:-

A relation R on a set S is called partial ordering or partial order if it is reflexive, antisymmetric and transitive. A set S together with partial ordering R is called partial ordered set.

Consider a set A = { 3, 2, 4, 6 } with relation
R = { (2,4), (3,6), (3,3), (2,2), (4,6), (4,4),
(2,6), (6,6) }.

It contains properties of reflexive [(3,3) (4,4) (6,6)]
transitive [(3,3), (3,6)] and antisymmetric

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Q6] set = { 1, 2, 3, 4, 5, 6 }

R = { (i,j) | |i-j| = 2 }

Sol^n:

∴ R = { (1,3) (3,1) (2,4) (4,2) (3,5) (5,3) (4,6) (6,4) }

∴ The above element are in form of (a,b) and (b,a) where a ∈ R and b ∈ R and a ≠ b.

∴ The relation is symmetric T^n R = ?

Q7] $f(n) = n+1$

$g(n) = 2n$

$h(n) = \begin{cases} 0 & \text{otherwise} \\ 1 & \text{if } n \text{ is odd.} \end{cases}$

Sol

$$
\begin{aligned}
f \circ f &= f(f(n)) \\
&= f(n+1) \\
&= (n+1)+1 \\
&= n+2
\end{aligned}
$$

$$
\begin{aligned}
f \circ g &= f(g(n)) \\
&= g(n)+1 \\
&= 2n+1.
\end{aligned}
$$

$$
\begin{aligned}
g \circ h &= g(h(n)) \\
&= 2(h(n)) \\
&= \begin{cases} 0 & \text{otherwise} \\ 2 & \text{if } n \text{ is odd} \end{cases}
\end{aligned}
$$

$$
\begin{aligned}
f \circ g \circ h &= f((g \circ h)) \\
&= \begin{cases} 2(2)+1 = 5 & \text{otherwise} \\ 0+1 = 1 & \text{if } n \text{ is odd} \end{cases}
\end{aligned}
$$

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$h(g(n)) = h(2n)$

$\Rightarrow$ since $2n$ is always even.

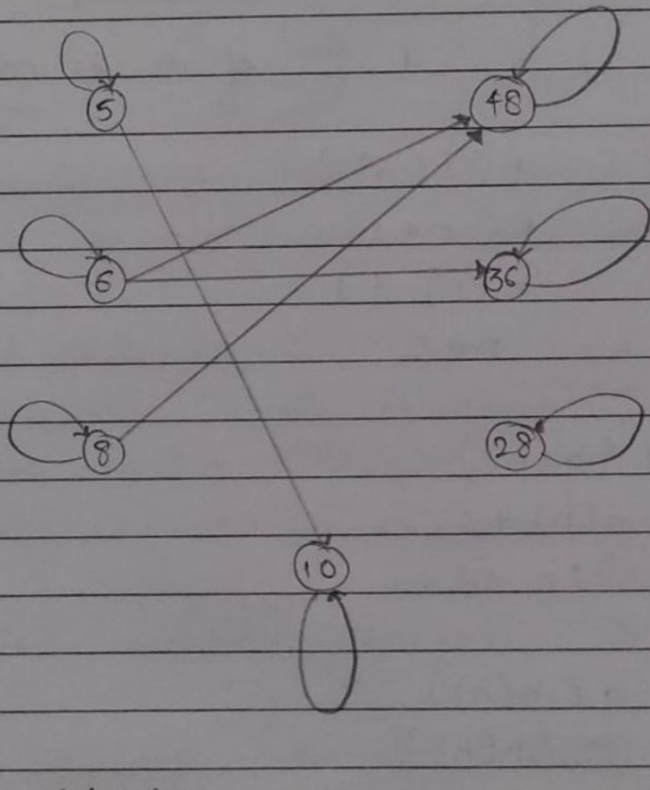
$\therefore h(g(n)) = 0$ always.

Q 8] set $A = \{5, 6, 8, 10, 28, 36, 48\}$

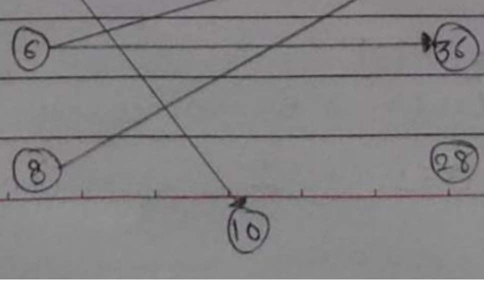
$R = \{(a,b) \mid a \text{ is divisor of } b\}$

$\therefore R = \{(5,10)\ (6,36)\ (8,48)\ (5,5)\ (6,6)\ (8,8)$
$(10,10)\ (28,28)\ (36,36)\ (48,48)\ (6,48)\}$

DIGRAPH :-



HASSE DIAGRAM



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