⇒ Calculus & Analytical Geometry Assignment:- Date:

Name: Yash-Raj Roll-no:- 24K-0737

Q#1: Compute the following Limits:-

(a) $\lim_{x\to 0}\left(\frac{1}{x\sin x} - \frac{1}{x\tan x}\right)$

Sol:

$$\lim_{x\to 0}\left(\frac{1}{x\sin x} - \frac{1}{x\tan x}\right)$$

$$\lim_{x\to 0}\left(\frac{1}{x\sin x} - \frac{1}{x\tan x}\right)$$

$$\lim_{x\to 0}\frac{\tan x - \sin x}{x\sin x\tan x}$$

$$\frac{\lim_{x\to 0} \tan x - \sin x}{\lim_{x\to 0} x\sin x\tan x}.$$

$f'(x) = \sin x\tan x + x(\cos x\tan x + \sin x\sec^2 x)$

$$\lim_{x\to 0}\frac{\sec^2 x - \cos x}{\sin x\tan x + x(\cos x\tan x + \sin x\sec^2 x)}.$$

$$\frac{0}{0} = 0 \text{ Ans}$$

Roll. No:- 24K-0737

(b) $\lim_{x\to\infty}\left(x-\sqrt{x^2+3x}\right)$

sol:

$$\lim_{x\to\infty}\left(x-\sqrt{x^2+3x}\right).$$

$$\lim_{x\to\infty} x - \lim_{x\to\infty}\sqrt{x^2+3x}$$

$$1\cancel{\infty} - \frac{1}{2\sqrt{x^2+3x}} \times 2x+3x.$$

$$\lim_{x\to\infty}\frac{2\sqrt{x^2+3x} - 2x+3\cancel{x}}{2\sqrt{x^2+3x}}.$$

$$= \frac{-3}{2} \quad \text{Ans.}$$

(c) $\lim_{x\to\infty}\left(x-\ln(x^2+3)\right).$

sol:

$$\lim_{x\to\infty}\left(x-\ln(x^2+3)\right).$$

$$1 - \frac{1}{x^2+3} \times 2x.$$

$$= \frac{x^2+3-2x}{x^2+3}.$$

$$= \infty \quad \text{Ans.}$$

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Q#2:- first derivative test for local extrema of $f(x) = 5x^{3/2} - x^{5/2}$

Sol:

$$f(x) = 5x^{3/2} - x^{5/2}$$

$$f'(x) = \frac{15}{2}x^{1/2} - \frac{5}{2}x^{3/2}$$

$$f'(x) = 0$$

$$= \frac{15}{2}x^{1/2} - \frac{5}{2}x^{3/2} = 0.$$

$$= \frac{15}{\cancel{2}}x^{1/2} = \frac{5}{\cancel{2}}x^{3/2}$$

$$= \cancel{15}x^{1/2} = \cancel{5}x^{3/2}$$

$$= 3x^{1/2} = x^{3/2}$$

$$= 3 = \frac{x^{3/2}}{x^{1/2}}$$

$$= 3 = x$$

$$\boxed{x = 3}$$

→ $f'(x)$ is positive for $x > 3$ since $x^{1/2}$ dominates.
→ $f'(x)$ is negative for $x > 3$ since $x^{3/2}$ dominates.

Therefore $x = 3$ is local maximum.
at $x = 0$ is a critical point where $f'(x) = 0$ which is also local minimum.

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Q#3:- Determine intervals of concavity & inflexion points for $f(x) = x^3 - 6x^2 + 9x + 30$.

Sol:

$$f(x) = x^3 - 6x^2 + 9x + 30$$
$$f'(x) = 3x^2 - 12x + 9.$$

$$f''(x) = 6x - 12,$$

$\boxed{f''(x) = 0} \rightarrow$ To find inflexion points

$$6x - 12 = 0$$
$$x = \frac{\cancel{12}\,2}{\cancel{6}\,1}$$
$$\boxed{x = 2}$$

• at $x = 2$, there is inflexion point.

(b) find Concavity:-

$f''(x) < 0$ (Concave down) for $x < 2$.
$f''(x) > 0$ (Concave up) for $x > 2$.

$\boxed{(-\infty, 2) \cup (2, \infty)}$ Ans

$(-\infty, 2) \rightarrow$ Concave down

$(2, \infty) \rightarrow$ Concave up.

FAST

Name: Yash Raj Roll-no: 24K-0737 Date:

Q#4:

Sol:

Two planes flying at the same height

(i) Airplane A is flying east at 250 miles/hr.

(ii) Airplane B is flying north at 300 miles/hr.

$A(x) = 250t + 30$ at $t = 0$.

$B(x) = 300t + 40$ at $t = 0$.

$\Rightarrow$ find distance b/w the airplanes.

$$d^2 = x^2 + y^2$$

$$d^2 = (250t + 30)^2 + (300t + 40)^2$$

Taking derivative on B.Sides:

$$\frac{d}{dt}(d^2) = \frac{d}{dt}(x^2 + y^2)$$

$$2d\frac{dd}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

$$d = \sqrt{x^2 + y^2} = \sqrt{30^2 + 40^2} = \sqrt{900 + 1600} = \sqrt{2500} = 50 \text{ miles}$$

$$50\frac{dd}{dt} = (30)(250) + (40)(300)$$

$$= \frac{19500}{50} = \boxed{390 \text{ miles/hr}} \text{ Ans}$$

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Q#5.-

Compute Left & Right Riemann Sums:-

$f(x) = \sqrt{9-(x-3)^2}$ on the interval $[0,6]$ using 4 subintervals.

$\Delta x = \frac{6-0}{4} = 1.5$

$x_0 = 0$ $x_2 = 3$ $x_4 = 6$

$x_1 = 1.5$ $x_3 = 4.5$

$f(x) = \sqrt{9-(x-3)^2}$

$f(x_0) = \sqrt{9-(0-3)^2} = \sqrt{9-9} = 0$

$f(x_1) = \sqrt{9-(1.5-3)^2} = \sqrt{9-2.25} = \sqrt{6.75} = 2.598$

$f(x_2) = \sqrt{9-(3-3)^2} = \sqrt{9-0} = \sqrt{9} = 3$

$f(x_3) = \sqrt{9-(4.5-3)^2} = \sqrt{9-(1.5)^2} = \sqrt{9-2.25} = 2.598$

$f(x_4) = \sqrt{9-(6-3)^2} = \sqrt{9-9} = 0$

$L_4 = \Delta x\left(f(x_0) + f(x_1) + f(x_2) + f(x_3)\right)$

$L_4 = 1.5\,(0+2.598+3+2.598)$

$= 1.5 \times 8.196$

$\boxed{L_4 = 12.294}$

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Right Riemann Sum:-

$$R_4 = \Delta x\left(f(x_1)+f(x_2)+f(x_3)+f(x_4)\right)$$

$$f(x_4) = f(6) = \sqrt{9-(6-3)^2} = \sqrt{9-9} = 0$$

$$R_4 = 1.5(2.598 + 3 + 2.598 + 0)$$
$$= 1.5 \times 8.196$$

$$\boxed{R_4 = 12.294}$$

$$\boxed{L_4 = 12.294} = \boxed{R_4 = 12.294}$$