=) Calculus Assignment 03.
Name:- Yash Raj
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Q#1:-

$y = x$ and $y = 2x^2 - 1$

Sol:-

Using disk method:-

$$V = \pi \int_a^b [f(x)^2 - g(x)^2]\, dx.$$

Bond of integration:-

$x = 2x^2 - 1$, $2x^2 - x - 1 = 0$.

$(2x+1)(x-1) = 0$.

=) Points of intersection are $x = -1/2$ and $x = 1$

so $a = -1/2$, $b = 1$

=) $f(x) = x$ $\quad x \in [-1/2, 1]$

=) $g(x) = 2x^2 - 1$

$$V = \pi \int_{-1/2}^{1} [x^2 - (2x^2 - 1)^2]\, dx$$

$(2x^2 - 1)^2 = 4x^4 - 4x^2 + 1$

$x^2 - (4x^4 - 4x^2 + 1)$

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$$V = \pi \int_{-1/2}^{1} (-4x^4 + 5x^2 - 1)\,dx$$

$$= \pi \left[\frac{-4}{5}x^5 + \frac{5}{3}x^3 - x \right]_{-1/2}^{1}$$

At $x = 1$

$$\pi\left(\frac{-4}{5}(1)^5 + \frac{5}{3}(1)^3 - 1\right) - \pi\left(\frac{-4}{5}(-1/2)^5 + \frac{5}{3}\left(\frac{-1}{2}\right)^3 + \frac{1}{2}\right)$$

$$= \pi\left(\frac{-2}{15}\right) - \pi\left(\frac{19}{60}\right)$$

$$= \pi\left(\frac{-2}{15} - \frac{19}{60}\right)$$

$$\boxed{= \pi\left(\frac{-8 - 19}{60}\right)}$$ Ans.

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Q#2:- $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{\sqrt{k^2+1}}$

Soln

$$\frac{\frac{1}{2\sqrt{k}}}{\frac{2k}{2\sqrt{k^2+1}}} \Rightarrow \frac{1}{2\sqrt{k}} \div \frac{2k}{2\sqrt{k^2+1}}$$

$$\Rightarrow \frac{1}{2\sqrt{k}} \cdot \frac{2\sqrt{k^2+1}}{2k}$$

$$\Rightarrow \frac{\sqrt{k^2+1}}{2k\sqrt{k}} \Rightarrow \frac{\sqrt{k^2+1/k}}{2k\sqrt{k}}$$

$$= \frac{\sqrt{1+\frac{1}{k^2}}}{2\sqrt{k}}$$

Applying limit:-

$$\frac{\sqrt{(1+\frac{1}{\infty^2})}}{2\sqrt{\infty}} = \frac{\sqrt{1+0}}{2\sqrt{\infty}} = \frac{1}{\infty} = 0.$$

$\Rightarrow$ Divergence test fails, the test is inconclusive as $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{\sqrt{k^2+1}} = 0$. The divergence test states that $\lim_{k \to \infty} a_k \neq 0$, then series diverges

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Q#3:

$$\sum_{k=0}^{\infty} -ke^{-k}$$

Sol:

$$\sum_{k=0}^{\infty} -ke^{-k}$$

for integral test $f(x)$ should be positive, continuous & decreasing for $x \geq 0$.

$a_k = -ke^{-k}$ (not positive).

Applying integral test as $a_k = ke^{-k}$

$f(x) = xe^{-k}$.

Now $f(x)$ is positive $x \geq 0$, $f(x)$ is continous, decreasing as $f'(x) = e^{-x} - x \cdot e^{-x}$

$= e^{-x}(1-x)$.

$$\int_0^{\infty} xe^{-x} dx.$$

$u = x$ $v = e^{-x}$.

$\frac{du}{dx} = 1$ $\int v dx = e^{-x}$.

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$$\int u \cdot v\,dx = u\int v\,dx - \int\left[\frac{d}{du}(u) \cdot \int v\,dx\right] dx$$

$$\int_0^\infty xe^{-x} = x\int e^{-x} - \int(1)e^{-x}dx$$

$$= -xe^{-x} - e^{-x}\,dx$$

$$\lim_{a\to\infty} = \left[-x^{-ex} - e^{-x}\right]$$

$$\left[(0-0) - (0-1)\right]$$

$$= 1$$

$$\int_0^\infty xe^{-x}dx = 1$$

=) The integral converges to 1.

=) By integral test, the series converges absolutely

$$\sum_{k=0}^{\infty} -ke^{-k} = \sum_{k=0}^{\infty} ke^{-k}.$$

So the original series $\sum_{k=0}^{\infty} -ke^{-k}$ also converges.

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