Calculus Assignment:-

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Q#1:-

find Domain & Range:-

(i) $f(x) = \sqrt{4-\sqrt{x}}$

Sol

$4-\sqrt{x} \geq 0$

$-\sqrt{x} \geq \cancel{0}+4$

$\sqrt{x} \leq 4$

$x \leq 16$

Domain = $x \in [0, 16]$

for range:

put domain:-

$\sqrt{4-\sqrt{0}} \Rightarrow \sqrt{4} = 2.$

$\sqrt{4-\sqrt{16}} \Rightarrow \sqrt{0} = 0.$

Range = $[0, 12]$

Domain = $x \in [0, 16]$ Ans.

Range = $[0, 12]$

(ii) $f(x) = \frac{2-3x}{7-2x}$

Sol:

$f(x) = \frac{2-3x}{7-2x}$

$7-2x \neq 0$
$-2x \neq -7$
$x \neq \frac{7}{2}$

Domain = $x \in \mathbb{R} - \left\{\frac{7}{2}\right\}$

As $f(x)$ is linear both numerator & denominator the ~~co-efficient~~ of ratio of co-efficient will be eliminated.

Range = $x \in \mathbb{R} - \left\{\frac{3}{2}\right\}$ Ans

Domain = $x \in \mathbb{R} - \{7/2\}$
Range = $x \in \mathbb{R} - \{3/2\}$ Ans

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Q#2:- $f:(0,1) \to (0,2)$.

$\frac{4x}{3-x}$ Whether the function is one to one function.

Sol:

for one to one function

$f(x_1) = f(x_2)$.

$$\frac{4x_1}{3-x_1} = \frac{4x_2}{3-x_2}$$

$$= 4x_1(3-x_2) = 4x_2(3-x_1)$$

$$= \cancel{12}x_1 - \cancel{4x_1x_2} = \cancel{12}x_2 - \cancel{4x_1x_2}$$

$$= \boxed{x_1 = x_2}$$

Check Domain & range:-

$f:(0,1) \to (0,2)$.

$$f(0) \; \frac{4(0)}{3-0} = 0$$

$$f(1) \; \frac{4(1)}{3-1} = \frac{4}{2} = 2$$

$\Rightarrow$ Yes, it is one to one (Injective) function.

Q#3:-

Ans= Part (a). The function is discontinuous at $x=1$ & $x=2$.

part(b)= The function is discontinuous at $x=1$ & $x=2$
left hand & right hand limits are not Equal.

part(c):- The function is continuous on the intervals $(0,1)$, $(1,2)$ & $(2,3)$

part(d) The piecewise function is:-

$$f(x) = \begin{cases} -x+1 & \text{for } 0 \leq x < 1 \\ 2 & \text{for } 1 \leq x < 2 \\ -2x+4 & \text{for } 2 < x \leq 3 \end{cases}$$

Q#4: find value of a & b is the function f Continuous at the point $x=3$:

$$f(x)=\begin{cases} 4x^2+ax+b & \text{if } x<3 \\ a+b-2 & \text{if } x=3 \\ 2x^3-bx+a & \text{if } x>3 \end{cases}$$

$\lim_{x\to 3^-} 4x^2+ax+b$

$= 4(3)^2+a(3)+b$

$= 36+3a+b \rightarrow (i)$

$\lim_{x=3} = a+b-2 \rightarrow (ii)$

$\lim_{x\to 3^+} 2(3)^3-b(3)+a$

$= 54-3b+a \rightarrow (iii)$

Compare (i) & (ii)

$36+3a+b = a+b-2$

$3a-a+\cancel{b}-\cancel{b} = -2-36$

$2a = -38$

$\boxed{a=-19}$

Compare (ii) & (iii).

$a+b-2 = 54-3b+a$

$= \cancel{a}-\cancel{a}+b+3b = 54+2$

$= 4b = 56$ $\boxed{b=14}$

put $a = -19$, $b = 14$ & then check limit:-

$$f(x) = \begin{cases} 4x^2 - 19x + 14 & \text{if } x < 3 \\ -7 & \text{if } x = 3 \\ 2x^3 - 14x - 19 & \text{if } x > 3 \end{cases}$$

$$\lim_{x \to 3^-} 4(3)^2 - 19(3) + 14$$

$$\lim_{x \to 3^-} = -7$$

$$\lim_{x \to 3} = -7.$$

$$\lim_{x \to 3^+} 2(3)^3 - 14(3) - 19.$$

$$= -7.$$

$\Rightarrow a = -19$, $b = 14$ & function is Countinuous at $x = 3$.

Q#5:- Compute the limits of the following functions:

(a) $\lim_{x\to\infty} \frac{2x^4-4x^2+5}{3x^4-7x+2}$

Sol.

$$\lim_{x\to\infty} \frac{2x^4-4x^2+5}{3x^4-7x+2}$$

$$= \frac{\cancel{x^4}}{\cancel{x^4}}\left[\frac{2-\frac{4}{x^2}+\frac{5}{x^4}}{3-\frac{7}{x^3}+\frac{2}{x^4}}\right]$$

Applying $\lim x\to\infty$:-

$$= \left[\frac{2-\frac{4}{\infty^2}+\frac{5}{\infty^4}}{3-\frac{7}{\infty^3}+\frac{2}{\infty^4}}\right]$$

$$= \frac{2-0+0}{3-0+0}$$

$$= \frac{2}{3} \text{ Ans.}$$

(b) $M = \lim_{x \to 3^-} \dfrac{x^3 - 5x + 4}{x^3 - 8x - 3}$

Sol

$$\lim_{x \to 3^-} \frac{x^3 - 5x + 4}{x^3 - 8x - 3}$$

$$= \frac{(3)^3 - 5(3) + 4}{(3)^3 - 8(3) - 3}$$

$$= \frac{27 - 15 + 4}{27 - 24 - 3}$$

$$= \frac{16}{0}$$

$= \infty$ (limit doesn't exist)

(c) $M = \lim_{x \to 1} \dfrac{3x^3 - 7x^2 + 6x - 2}{x - 1}$

Sol:

$$\lim_{x \to 1} \frac{3x^3 - 7x^2 + 6x - 2}{x - 1}$$

$$\frac{\cancel{(x-1)}(3x^2 - 4x + 2)}{\cancel{(x-1)}}$$

$$\lim_{x \to 1} (3x^2 - 4x + 2).$$

$= 3(1)^2 - 4(1) + 2.$

$= 3 - 4 + 2.$

$= 1$ Ans.