① Void fun (int n){
int j = 1 , i = 0; ⟶ O(1)
while (i<n)
{
i = i+j;
j++;
}
}

i = 0 ⟶ i = 0+1 = 1 , j = 2
i = 1 ⟶ i = 1+2 = 3 , j = 3
i = 3 ⟶ i = 3+3 = 6 , j = 4
i = 6 ⟶ i = 6+4 = 10 , j = 5

i = k ⟶ i = k + x , j = n
Sum.

From the above pattern we can clearly see that it is just a sum of first n - numbers.

OR

j	1	2	3	4	------	n
i	1	3	6	10	------	k

$\frac{k(k+1)}{2} > n$

dominating term

$\frac{k^2+k}{2} > n$

$k^2 = n$

$k = \sqrt{n}$

T.C = O(k) or $O(\sqrt{n})$

T.C = $O(\sqrt{n})$

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Soln.2

```
int fib (int n) ⟶ T(n)
{
  if (n<=1) ⟶ O(1)
    return n;
  return fib(n-1) + fib(n-2); ⟶ T(n-1) + T(n-2)
}
```

$$T(n) = \begin{cases} 1 & , n <= 1 \\ T(n-1) + T(n-2) & , \text{otherwise.} \end{cases}$$

$$T(n) = T(n-1) + T(n-2) + c$$
$$= 2T(n-1) + c$$

[From the approximation $T(n-1) \sim T(n-2)$]

$$T(n) = 2(2T(n-2)+c) + c$$
$$T(n) = 4T(n-2) + 3c$$
$$= 8T(n-3) + 7c$$
$$\vdots$$
$$= 2^k T(n-k) + (2^k - 1)c$$

We know that $T(1) = 1$, so

$$T(n-k) = T(1) = 1$$

$$n - k = 1$$

$$\boxed{k = n-1}$$

We get

$$T(n) = 2^{n-1} T(1) + (2^{n-1} - 1)c$$

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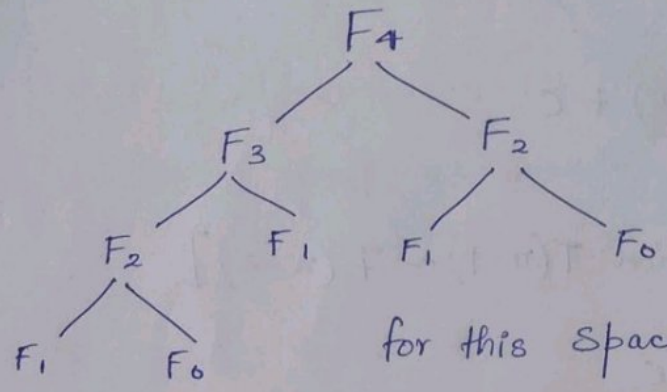
$T.C = O(2^n)$

Now, for space complexity we know that

space reqd. $\propto$ Max. depth of reccursive tree.

because that is the max. no. of elements that can be present in the implicit function call stack.

now, let's take an example F_4, so



for this Space comp. = $O(4)$

for n - elements, $S.C = O(n)$

Soln.3 (i) for $T.C = n \log n$.

```
A()
{ int i,j ;
  for (i=1 ; i<=n ; i++)        → O(n)
  { for(j=1; j<=n ; j=j*2)      → O(logn)
    { printf ("Hey");
    }
  }
}
```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Soln.4 $T(n) = T(n/4) + T(n/2) + Cn^2$

now, $T(n/4) \leq T(n/2)$

So we can write this equation as

$T(n) = T(n/2) + T(n/2) + cn^2$

$T(n) = 2T(n/2) + cn^2$

Comparing with Master equation we get

$a = 2, b = 2, k = 2$ & $p = 0$

now $\boxed{a < b^k}$ so, if $a < b^k$ & $p = 0$,

$2 < 2^2$

$T.C = \theta(n^k \log^p n)$

$= \theta(n^2 (\log n)^0)$

$\boxed{T.C = \theta(n^2)}$

Soln.5

```
int fun (int n){
    for (int i=1 ; i<=n ; i++)
    {for(int j=1 ; j< n ; j+=i)
    {// Some O(1) task
    }}}
```

j incrementation depends on i . ∴ We unroll all loops.

i = 1	i = 2	i = 3 ------	i = n
j = 1 to n	j = 1 to n	j = 1 to n	j = 1 to n
"n - times"	"n/2 times"	"n/3 times."	"1" times

So,

$$T.C = n + \frac{n}{2} + \frac{n}{3} + \cdots + \frac{n}{n}$$

$$= n\underbrace{\left(1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right)}_{\log n}$$

$$\boxed{T.C = O(n \log n)}$$

Soln.6

```
for (int i = 2 ; i <= n ; i = i^k)
{
  // Some O(1) stmt
}
```

$i = 2$	$i = 2^k$	$i = (2^k)^k = 2^{k^2}$
"2 to n times"	"2^k to n times"	"2^{k^2} to n times"

$$\cdots \quad i = 2^{k^{\log_k \log n}}$$

at that time, $2^{k^{\log_k (\log n)}} = 2^{\log n} = n$

Total $\boxed{T.C = O(\log \log n)}$

OR

for ~~k~~

$i = 2$	$i = 2^k$	$i = 2^{k^2}$ $\cdots$ $i = 2^{k^l}$

$$2^{k^l} = n$$

$$k^l = \log_2 n$$

$$\boxed{l = \log_k \log_2 n}$$

$$\boxed{T.C = O\left(\log_k \log_2 n\right)}$$

(ii) for , T.C = n^3

```
A()
{ int i,j,k;
for (i=0; i<=n; i++)      → O(n)
{for (j=0; j<=n; j++)     → O(n)
{ for (k=0; k<=n; k++)    → O(n)
  { printf("Hey");
  }
}
}
}
```

(iii) T.C = log (log n)