Name :- Yash sharma.
Section :- CS t SPL-2.
Roll no :- 32.

Assignment - 1

Quy what do you understand by Asymptotic Notations. Define Asymptotic Notation with Examples.

Soln. Asymptotic Notation are the mathematical Notation used to describe the running time of an Algorithm when the input tends towards a particular value. Or a limiting value.

There are mainly three asymptotic Notation.

- Big - O notation :- It represents the upper bound of the running time of an algo. Thus it gives the worst Case Complexity of an Algorithm.

- Omega notation :- It represents the lower bound of running time of any algorithm. Thus it provides the best Case Complexity of an algorithm.

- ## Theta Notation (θ - Notation)

Theta Notation enclose the function from above and below. Since it repersents the upper bound and the lower bound of the running time of an algo. It is used for analyzing the average - case complexity of an algo.

Ques. what should be time Complexity of

Sol^n.
```
for (i=1 to n).
{

    i = i * 2;

}
```

$i = 1, 2, 4, 8 \text{ ----- } n$
$= 2^0, 2^1, 2^2, 2^3 \text{ ----- } \neq 2^k$

looks like g.p

$a = 1$ (first term)

$r = \dfrac{t_2}{t_1} = \dfrac{4}{2} = 2$ ~~b/w terms~~

$t_k = a \cdot r^{k-1}.$

$n = 1 * 2^{k-1}$

$n = \dfrac{2^k}{2}$

$\left[\because a^{b-c} = \dfrac{a^b}{a^c} \right]$

$$2n = 2^k$$

take log both side

$$\log_2(2n) = k \log_2 2$$

$$k = \log_2(2n)$$

$$k = \log_2(2) + \log_2(n)$$

$$k = \log_2(n) + 1$$

$$k = \log_2(n)$$

$$T.C = O(\log_2(n)) \quad \underline{Ans}$$

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Sol⁹

$$T.(n) = \{ 3T(n-1) \text{ if } n>0 \text{ otherwise } 1 \}$$

$$T(n) = 3T(n-1) \quad n>0$$
$$T(n) = 1$$

by using Substitution method.

$$T.(n) = 3T(n-1) \quad — ①$$

put $n = n-1$ in eqⁿ ②.

$$T.(n-1) = 3T(n-1-1).$$

$$T(n-1) = 3T.(n-2) \quad — ③$$

put ③ in ①

$$T(n) = 3(3T(n-2)).$$

$$T(n) = 9T(n-2) \quad -④$$

put $n = n-2$ in eq

$$T(n-2) = 3T(n-2-1)$$

$$= 3T(n-3). \quad -⑤$$

put ⑤ in ④

$$T(n) = 9(3T(n-3)$$

$$= 27T(n-3).$$

$$---$$

$$----$$

$$---$$

$$3^3 T(n-3)$$

$$3^k T(n-k). \quad - 6$$

put $n-k = 1$

$k = n-1$ ~~$= k$~~ put in eqn ⑥

$$3^k T(n-k+1) = 3^{n-1} T(1).$$

$$= O(3^n) \quad (\because k = n)$$

Ans

Ques 4. $T(n) = 2T(n-1) -1$ $n > 0$.
$T(1) = 1$.

Sol^n using forward Subs.

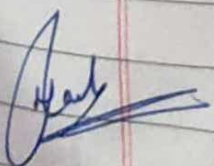$T(0) = 1$.

$T(1) = 2T(0) -1 = 1$

$T(2) = 2T(2-1) -1 = 1$

$T(3) = 2T(3-1) -1 = 1$

$T(4) = 2T(3) -1 = 1$

So $T(n) = 1$ for any value of n.

i.e it remains constant

$T \cdot C = O(1)$ Ans

Que 5 what should be time complexity of

```
int i=1, S=1.

while (S<=n)
{
    i++
    s = s+i
}
```

Sol^n let.

$$1+3+6+10 - - - K = n$$

$$\frac{K(K+1)(K+2)}{6} = n$$

$$O(K^3) = n.$$

$$= O(\sqrt[3]{n}) \quad \underline{Ans}$$

Que 6 Time Complexity of

```
void fn (int n)
{
    int i, Count = 0;

    for (int i=1; i*i <=n; i++)
    {
        Count ++;      O(1)
    }
```

Solⁿ

a.

$$i^2 <= n.$$
$$i <= \sqrt{n}$$

$$i = 1, 2, 3, 4, --- \sqrt{n}.$$

$$\sum_{i=1}^{n} 1 + 2 + 3 + 4 + ---- + \sqrt{n}$$

$$T(n) = \frac{\sqrt{n} \times (\sqrt{n} + 1)}{2}$$

$$T(n) = \frac{n \times \sqrt{n}}{2.}$$

$$T(n) = O(n) \quad \underline{Ans}$$

Quy 7   Time complexity of :-

```
void fn(int n)
{
    int i, j, k, count = 0;
    for (i = n/2 ; i <= n ; ++i)
    {
        for (i = 1 ; j <= n ; j = j*2)
        {
            for (k = 1 ; k <= n ; k = k*2)
                count ++;
        }
    }
}
```

Sol[n].

i	j	K.
1.	$\log n$	$\log n * \log n$.
2	$\log n$	$\log n * \log n$
n	$\log n$	$\log n * \log n$.

$$\Rightarrow \quad O(n * \log n * \log n.)$$

$$= \quad O(n \log^2 n) \text{ Ans}.$$

Que.8 Time Complexity of

function (int n)

{

   int (n==1)

   return; $O(1)$

   for (i=1 to n) i=1,2,3----n $\Rightarrow$ O(n)
   {

      for (j=1 to n) j=1,2,3---n = O(n)
      {

         print ("*")

      }

      function (n-3) $\rightarrow$ T(n/3)

   }

$$T(n) = T(n/3) + n^2$$

$$a = 1 \quad , b = 3 \quad f(n) = n^2$$

$$C = \log_3 1 = 0$$

$$n^0 = 1 \quad > f(n) = n^2$$

$$T(n) = \Theta(n^2) \quad \text{Ans}$$

Qy Time Complexity of.

void function (int n)
{
    for (i=1 to n)
    {
        for (j=1; j<=n; j=j+i)
        {
            print ("*"); O(1)
        }
    }
}

$$\sum_{i=1}^{n}$$

$$i=1 \Rightarrow j = 1, 2, 3, 4 - - - - - n = n$$

$$i=2 \Rightarrow j= 1, 3, 5 - - - - - n = n/2$$

$$i=3 \Rightarrow j= 1, 4, 7, - - - n = n/3$$

S.

$$T.C = \sum_{i=1}^{n} \sum_{j=1 (j=j+1)}^{n} 1.$$

for $i=n \rightarrow j=1$ _ _ _ _ 1

$$= \sum_{j=n}^{1} n + \frac{n}{2} + \frac{n}{3} + \frac{n}{4} + \cdots + 1$$

$$= \sum_{j=n}^{1} n \left[1 + \frac{1}{2} + \frac{1}{3} + \cdots \frac{1}{n} \right]$$

$$= \sum_{j=n} n (\log n)$$

$$T(n) = O(n \log n)$$

Q10) for function n^k and c^n, what is asymptotic relationship between two function? _ find out the value of C and no for which relation holds.

Sol:- as given n^k & c^n

relation b/w n^k & c^n is

$$n^k = O(m)$$
or $n^k \le am$

$\forall n \ge n_0$ & for constant $a > 0$

for $\underset{c\ =\ 2}{n_0 = 1}$

$1^k \le a2^1$

$n_0 = 1$ & $c = 2$