School of Engineering and Applied Science (SEAS), Ahmedabad University

B.Tech(ICT) Semester V: Wireless Communication (CSE 311)

- **No** : BT S12

- **Group Members**
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- **Base Article Title** :

  1) S. Ghavami, "Anomaly Detection in Molecular Communications With Applications to Health Monitoring Networks" [1]

- **New Title** :

  2) "Health Monitoring Network with a modeled basic suboptimum two-tier structure and a multi-faceted analysis"

1 New Performance Analysis

We will attempt to further the design problem from the previous set up to determine the optimum threshold for the highest possible probability of detection and probability of false alarm. By devising the probability of error, which relies on the aforementioned probabilities, we can observe the pattern and attempt to find the least possible value.

Symbol	Description
P_D^{AC}	Probability of detection at AC
P_F^{AC}	Probability of false alarm at AC
P_e^{AC}	Probability of error at AC
P_{MD}^{AC}	Probability of misdetection at AC
P_D	Probability of detection at BC
P_F	Probability of false alarm at BC
P_e	Probability of error at BC
P_{MD}	Probability of misdetection at BC
m	At a particular AC (m^{th} AC)
M	No. of ACs
N	No. of observations at AC
y_{BC}	Received molecules by the BC
$y_{AC,m}$	Received molecules by the m^{th} AC
$\overline{\epsilon}_r$	Noise mean at BC
$\overline{\eta}$	Noise mean at ACs
p_r	Reception probability at ACs
q_r	Reception probability at BC
y_{BC}^{THR}	Decision threshold at BC

- ## DCs-AC model

As we know, biomarkers are transmitted by disease cells (DCs) and are received by Artificial Cells (ACs). The received number of biomarkers in the nth time slot at the mth AC can be modeled as

$$y_{AC,m}[n] = \text{Poiss}(pr.\lambda_{AC,m}[n] + pr.\overline{\eta}_m) \tag{1}$$

p_r: the probability of reception

$\overline{\eta}_m$: the mean of Poisson noise. Poiss(.) denotes the Poisson distribution.

$\lambda_{AC,m}[n]$: average number of bio-marker molecules receive at the m^{th} AC, nth time slot.

From our model, we have

$$p(y_{AC,m};\lambda_{AC,m}) = \prod_{n=1}^{N} \frac{exp\Big(-\lambda_{AC,m}[n] - p_r\overline{\eta}\Big)\Big(\lambda_{AC,m}[n] + p_r\overline{\eta}\Big)^{y_{AC,m}[n]}}{y_{AC,m}[n]!} \tag{2}$$

We consider $\lambda_{AC,m}^{H_0,min}$ and $\lambda_{AC,m}^{H_0,max}$ as a lower and upper bounds for the $\lambda_{AC,m}[.]$ under hypothesis test of H_0. Similarly, consider $\lambda_{AC,m}^{H_1^+,min}$ and $\lambda_{AC,m}^{H_1^-,max}$ as the lower and upper bounds for the $\lambda_{AC,m}[.]$ under hypothesis test of H_1^+ and H_1^- respectively, i.e. $\forall n$ we have,

$$
\begin{cases} H_0 : \lambda_{AC,m}^{H_0,min} \leq \lambda_{AC,m}[n] \leq \lambda_{AC,m}^{H_0,max} \\ H_1 : \begin{cases} H_1^- : \lambda_{AC,m}[n] \leq \lambda_{AC,m}^{H_1^-,max} \\ H_1^+ : \lambda_{AC,m}[n] \geq \lambda_{AC,m}^{H_1^+,min} \end{cases} \end{cases} \tag{3}
$$

The likelihood ratio of hypothesis test based on GLRT can be given as

$$
L_{AC,m} = \frac{p(y_{AC,m}; H_1)}{p(y_{AC,m}; H_0)} \overset{H_1}{>} \gamma \tag{4}
$$

Further it can put as

$$
L_{AC,m} = \frac{\prod_{n=1}^{N} exp\bigg(- \lambda_{AC,m}^{H_1}[n] - p_r \overline{\eta} \bigg) \bigg(\lambda_{AC,m}^{H_1}[n] + p_r \overline{\eta} \bigg)^{y_{AC,m}[n]}}{\prod_{n=1}^{N} exp\bigg(- \lambda_{AC,m}^{H_0}[n] - p_r \overline{\eta} \bigg) \bigg(\lambda_{AC,m}^{H_0}[n] + p_r \overline{\eta} \bigg)^{y_{AC,m}[n]}} \overset{H_1}{>} \gamma \tag{5}
$$

Now, the simplified decision rule deduced by bounding likelihood ratio is given as

$$
\begin{cases} H_0 : \gamma_l \leq \frac{1}{N} \sum_{n=1}^{N} y_{AC,m}[n] \leq \gamma_u \\ H_1 : \begin{cases} \frac{1}{N} \sum_{n=1}^{N} y_{AC,m}[n] > \gamma_u \quad \text{or} \\ \frac{1}{N} \sum_{n=1}^{N} y_{AC,m}[n] < \gamma_l \end{cases} \end{cases} \tag{6}
$$

where,

$$
\gamma_l := \lambda_{AC,m}^{H_0,min} + p_r \overline{\eta} - \gamma^{'} \tag{7}
$$

$$
\gamma_u := \lambda_{AC,m}^{H_0,max} + p_r \overline{\eta} + \gamma^{'} \tag{8}
$$

Therefore, the simplified test statistic can be defined as

$$
Y_{AC,m} := \frac{1}{N} \sum_{n=1}^{n=N} y_{AC,m}[n] \tag{9}
$$

Using it and their respective upper and lower bound, we can define the probability of false alarm and probability of detection as

- Probability of false alarm:

$$
P_F^{AC,m} = Pr\Big\{ \gamma_l \leq Y_{AC,m} \leq \gamma_u; H_0 \Big\} = Pr\Big\{ 0 \leq Y_{AC,m} \leq \gamma_u; H_0 \Big\} - Pr\Big\{ 0 \leq Y_{AC,m} < \gamma_l; H_0 \Big\} \tag{10}
$$

which can be simplified as,

$$P_F^{AC,m} = Pr\Bigg\{0 \le Poiss\Bigg(N\lambda_{AC,m}^{H_0,max} + Np_r\overline{\eta}\Bigg) \le N\gamma_u\Bigg\} - Pr\Bigg\{0 \le Poiss\Bigg(N\lambda_{AC,m}^{H_0,min} + Np_r\overline{\eta}\Bigg) < N\gamma_u\Bigg\}$$

$$P_F^{AC,m} = Poisscdf\Bigg(N\lambda_{AC,m}^{H_0,max} + Np_r\overline{\eta}, N\gamma_u\Bigg) - Poisscdf\Bigg(N\lambda_{AC,m}^{H_0,min} + Np_r\overline{\eta}, N\gamma_l\Bigg) \tag{11}$$

- Probability of detection:

$$P_D^{AC,m} = Pr\Big\{0 < Y_{AC,m} < \gamma_l; H_1^-\Big\}.Pr\Big\{H_1^-|H_1\Big\} + Pr\Big\{Y_{AC,m} > \gamma_u; H_1^+\Big\}.Pr\Big\{H_1^+|H_1\Big\} \tag{12}$$

Assuming a symmetric distribution, $Pr\Big\{H_1^-|H_1\Big\} = Pr\Big\{H_1^+|H_1\Big\} = 0.5$

$$P_D^{AC,m} = 0.5 \times Pr\Bigg\{Poiss\Bigg(N\lambda_{AC,m}^{H_1^+,min} + Np_r\overline{\eta}\Bigg) > N\gamma_u\Bigg\} + 0.5 \times Pr\Bigg\{0 \le Poiss\Bigg(N\lambda_{AC,m}^{H_1^-,max} + Np_r\overline{\eta}\Bigg) < N\gamma_l\Bigg\}$$

$$P_D^{AC,m} = \left(1 - \frac{Poisscdf\Bigg(N\lambda_{AC,m}^{H_1^+,min} + Np_r\overline{\eta}, N\gamma_u\Bigg)}{2}\right) + \left(\frac{Poisscdf\Bigg(N\lambda_{AC,m}^{H_1^-,max} + Np_r\overline{\eta}, N\gamma_l\Bigg)}{2}\right) \tag{13}$$

Here, there is room for error in two spaces. Either a signal is sent, and not received, which would be misdetection. Or a signal is not sent, yet is received, which would be false alarm. Hence, probability of error will be a summation of both, with β which is the prior probability as a controlling parameter.

$$\begin{aligned} P_e^{AC} &= P_{MD}^{AC}.\beta_{AC} + P_F^{AC}.(1 - \beta_{AC}) \\ P_e^{AC} &= (1 - P_D^{AC}).\beta_{AC} + P_F^{AC}.(1 - \beta_{AC}) \end{aligned} \tag{14}$$

Here, assuming a symmetric distribution, we take $\beta_{AC} = 0.5$.

- ## ACs-BC model

We know, The ACs identify anomaly as proposed here,

$$
\begin{cases}
H_0 : x^{H_0,min} \le x_{D_0} \le x^{H_0,max} \\
H_1 : \begin{cases} H_1^- : x_{D_0} \le x^{H_1^-,max} \\ H_1^+ : x_{D_0} \ge x^{H_1^+,min} \end{cases}
\end{cases}
\tag{15}
$$

Moreover, the BC uses the following hypothesis test to detect anomaly,

$$
\begin{cases}
W_0 : \sum_{m=1}^{M} \lambda_{BC,m} < k\overline{\lambda}_{BC} \\
W_1 : \sum_{m=1}^{M} \lambda_{BC,m} \ge k\overline{\lambda}_{BC}
\end{cases}
\tag{16}
$$

where W_0 and W_1 are events corresponding to H_0 and H_1 in the BC. k is the minimum number of ACs, which detect anomaly in the DC-ACs link. We define $\overline{\lambda}_{BC}$ as the average number of received molecules at the BC using (9). It can be given for $\forall m \in 1, ..., M$,

If the ACs-BC link is perfect the presence of anomaly is alarmed when at least k numbers of ACs alarm the hypothesis H1 by transmitting x_{AC},m(t) molecules. The decision rule for the BC with Poisson observation, where y_{BC}^{THR} is the decision threshold at the BC, is given as

$$
\begin{cases}
H_0 : y_{BC} < y_{BC}^{THR} \\
H_1 : y_{BC} \ge y_{BC}^{THR}
\end{cases}
\tag{17}
$$

Then, we drive a lower bound for probability of detection and an upper bound of false alarm for the perfect ACs-BC link.
By considering a symmetric topology, the probabilities of detection and false alarm of the noise free ACs-BCs will be:

$$
Q_D := Pr\left\{ \sum_{m=1}^{M} \lambda_{BC,m} \ge k\overline{\lambda}_{BC}; H_1 \right\} = \sum_{m=k}^{M} p'_m
\tag{18}
$$

$$
Q_F := Pr\left\{ \sum_{m=1}^{M} \lambda_{BC,m} \ge k\overline{\lambda}_{BC}; H_0 \right\} = \sum_{m=k}^{M} p''_m
\tag{19}
$$

where k is the minimum number of ACs which should detect the anomaly in order to enable the BC.

Now, P_D can be obtained as,

$$P_D = Pr\Big\{y_{BC} \geq y_{BC}^{THR}; H_1\Big\}$$

From Total Probability theorem,

$$= Pr\Big\{y_{BC} \geq y_{BC}^{THR}|W_0; H_1\Big\}Pr\Big\{W_0; H_1\Big\} + Pr\Big\{y_{BC} \geq y_{BC}^{THR}|W_1; H_1\Big\}Pr\Big\{W_1; H_1\Big\}$$

From the definitions of Q_D, W_0 and W_1,

$$= Pr\left\{y_{BC} \geq y_{BC}^{THR}|\sum_{m=1}^{M}\lambda_{BC,m} < k\overline{\lambda}_{BC}; H_1\right\}(1-Q_D) + Pr\left\{y_{BC} \geq y_{BC}^{THR}|\sum_{m=1}^{M}\lambda_{BC,m} \geq k\overline{\lambda}_{BC}; H_1\right\}Q_D$$

Using Bayes Theorem,

$$= \frac{\sum_{i=0}^{k-1}\left(Pr\left\{y_{BC} \geq y_{BC}^{THR}|\sum_{m=1}^{M}\lambda_{BC,m} = i\overline{\lambda}_{BC}; H_1\right\} \times Pr\left\{\sum_{m=1}^{M}\lambda_{BC,m} = i\overline{\lambda}_{BC}; H_1\right\}\right)}{Pr\left\{\sum_{m=1}^{M}\lambda_{BC,m} < k\overline{\lambda}_{BC}; H_1\right\}} \times (1-Q_D)$$

$$+ \frac{\sum_{i=k}^{M}\left(Pr\left\{y_{BC} \geq y_{BC}^{THR}|\sum_{m=1}^{M}\lambda_{BC,m}[n] = i\overline{\lambda}_{BC}; H_1\right\} \times Pr\left\{\sum_{m=1}^{M}\lambda_{BC,m} = i\overline{\lambda}_{BC}; H_1\right\}\right)}{Pr\left\{\sum_{m=1}^{M}\lambda_{BC,m} \geq k\overline{\lambda}_{BC}; H_1\right\}} \times Q_D$$

Following the Poisson Distribution of y_{BC},

$$= \frac{1}{\sum_{i=0}^{k-1}p_i'}\sum_{i=0}^{k-1}p_i'\left(1 - \frac{\Gamma(\lceil y_{BC}^{THR}\rceil, i\overline{\lambda}_{BC}) + q_r\overline{\epsilon}}{\lceil y_{BC}^{THR} - 1\rceil!}\right) \times (1-Q_D) + \frac{1}{\sum_{i=k}^{M}p_i'}\sum_{i=k}^{M}p_i'\left(1 - \frac{\Gamma(\lceil y_{BC}^{THR}\rceil, i\overline{\lambda}_{BC}) + q_r\overline{\epsilon}}{\lceil y_{BC}^{THR} - 1\rceil!}\right)Q_D \tag{20}$$

$$\therefore\ P_D = \sum_{i=0}^{M}p_i'\left(1 - \frac{\Gamma(\lceil y_{BC}^{THR}\rceil, i\overline{\lambda}_{BC}) + q_r\overline{\epsilon}}{\lceil y_{BC}^{THR} - 1\rceil!}\right) \tag{21}$$

In a similar way, the probability of false alarm is computed as,

$$P_F = Pr\Big\{y_{BC} \geq y_{BC}^{THR}; H_0\Big\}$$

$$= Pr\Big\{y_{BC} \geq y_{BC}^{THR}|W_0; H_0\Big\}Pr\Big\{W_0; H_0\Big\} + Pr\Big\{y_{BC} \geq y_{BC}^{THR}|W_1; H_0\Big\}Pr\Big\{W_1; H_0\Big\}$$

$$= Pr\left\{y_{BC} \geq y_{BC}^{THR}| \sum_{m=1}^{M} \lambda_{BC,m} < k\bar{\lambda}_{BC}; H_0\right\}(1 - Q_F) + Pr\left\{y_{BC} \geq y_{BC}^{THR}| \sum_{m=1}^{M} \lambda_{BC,m} \geq k\bar{\lambda}_{BC}; H_0\right\}Q_F$$

$$= \frac{\sum_{i=0}^{k-1}\left(Pr\left\{y_{BC} \geq y_{BC}^{THR}| \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC}; H_0\right\} \times Pr\left\{\sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC}; H_0\right\}\right)}{Pr\left\{\sum_{m=1}^{M} \lambda_{BC,m} < k\bar{\lambda}_{BC}; H_0\right\}}(1 - Q_F)$$

$$+ \frac{\sum_{i=k}^{M}\left(Pr\left\{y_{BC} \geq y_{BC}^{THR}| \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC}; H_0\right\} \times Pr\left\{\sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC}; H_0\right\}\right)}{Pr\left\{\sum_{m=1}^{M} \lambda_{BC,m} \geq k\bar{\lambda}_{BC}; H_0\right\}}Q_F$$

$$= \frac{1}{\sum_{i=0}^{k-1} p_i''} \sum_{i=0}^{k-1} p_i'' \left(1 - \frac{\Gamma(\lceil y_{BC}^{THR}\rceil, i\bar{\lambda}_{BC}) + q_r\bar{\epsilon}}{\lceil y_{BC}^{THR} - 1\rceil!}\right) \times (1 - Q_F) + \frac{1}{\sum_{i=k}^{M} p_i''} \sum_{i=k}^{M} p_i'' \left(1 - \frac{\Gamma(\lceil y_{BC}^{THR}\rceil, i\bar{\lambda}_{BC}) + q_r\bar{\epsilon}}{\lceil y_{BC}^{THR} - 1\rceil!}\right)Q_F$$

$$= \sum_{i=0}^{M} p_i'' \left(1 - \frac{\Gamma(\lceil y_{BC}^{THR}\rceil, i\bar{\lambda}_{BC}) + q_r\bar{\epsilon}}{\lceil y_{BC}^{THR} - 1\rceil!}\right) \tag{22}$$

Now, to compute the Probability of Error(P_e), as a function of β will be the prior probability of AC received at BC.

$$
\begin{aligned}
P_e &= P_{MD} \times \beta + P_F \times (1-\beta) \\
P_e &= (1-P_D) \times \beta + P_F \times (1-\beta)
\end{aligned}
\tag{23}
$$

Here, assuming a symmetric distribution, we take $\beta = 0.5$.

2 New Numerical Results

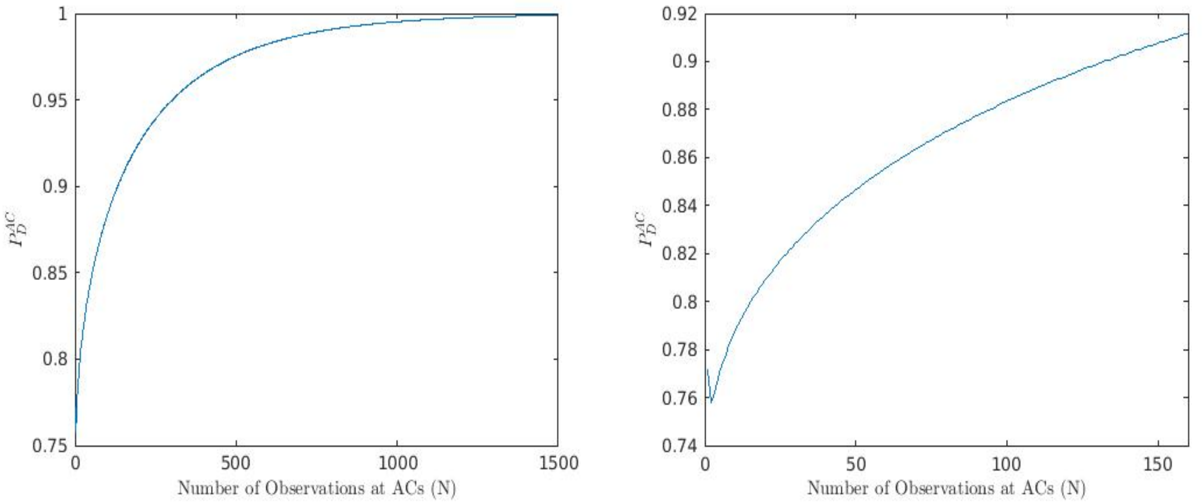
2.1 Simulation Framework

Parameter used in Reproduced Figures

Symbol	**Value**
M	30
N	[1 - 160]
$\bar{\epsilon}_r$	100
γ_l	9.65
γ_u	22.35
$\gamma^{'}$	1.35
q_r	0.05
$\overline{\eta}$	20
p_r	0.05
Δ_λ	1
$P_F^{AC,m}$	$[10^{-4} - 10^{-1}]$
$\lambda_{AC,m}^{H_0,min}$	10
$\lambda_{AC,m}^{H_0,max}$	20
y_{BC}^{THR}	[0 - 30]

2.2 Description of Figures

- **Figure 1 and Figure 2**



(a) F1 (b) F2

Inference:

We can see that as we increase the number of observations, P_D^{AC} increases. At around N = 1500, the value approaches unity. But as we can see in F2, it approaches 0.9 in 150 observations itself. This is a functional value and a fair compromise, so we can deduce that 150-200 observations are enough to give us a workable output.

- **Figure 3**

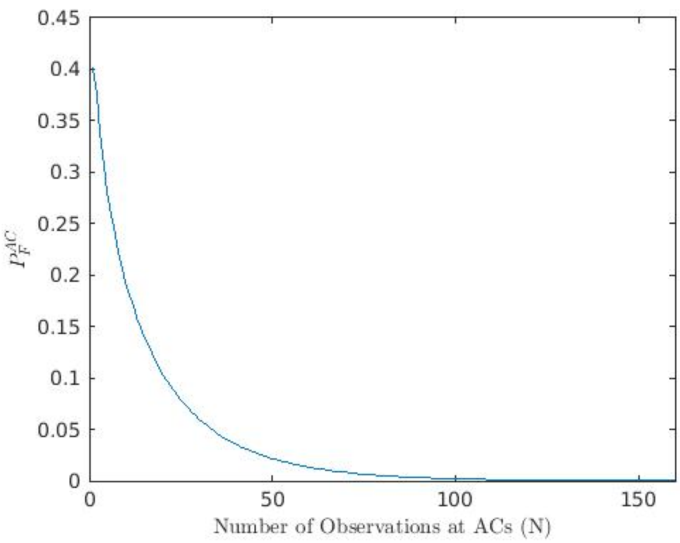


Figure 2: F3

Inference:

We can see that as we increase the number of observations, P_F^{AC} decreases and approaches 0. It gives us a workable output in the similar setup for 150-200 observations as well.

- **Figure 4**

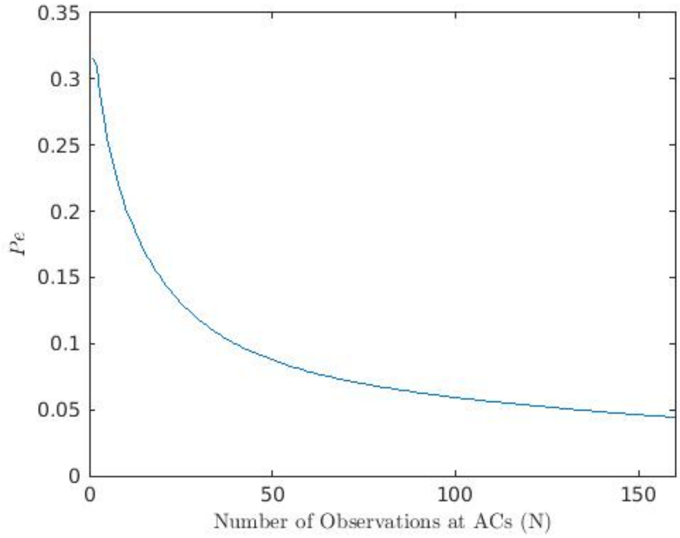
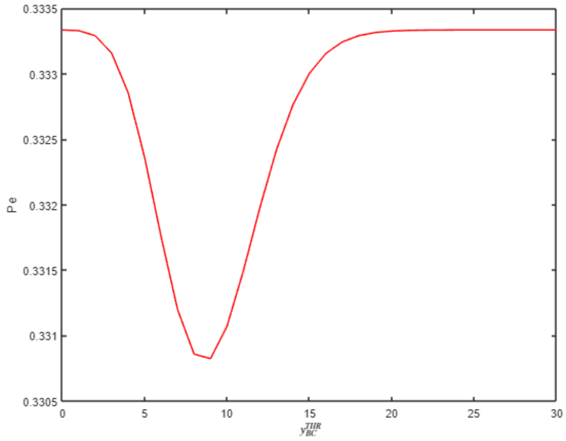


Figure 3: F4

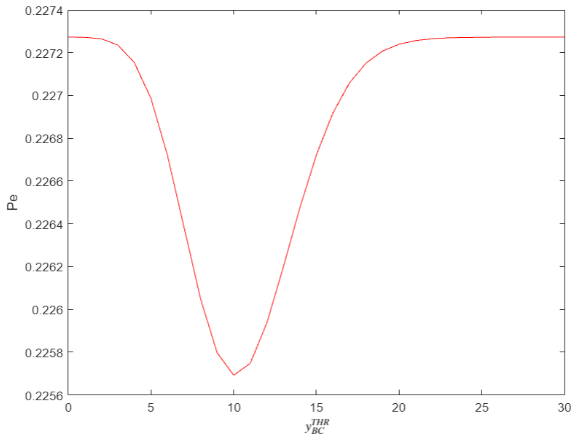
Inference:

Much dependent on behaviour or P_F^{AC} and P_D^{AC}, we can observe that as the former approaches 0, and the latter approaches 1, P_e^{AC} too approaches 0. We can conclude that by limiting probability of false alarm and probability of detection, we can successfully reduce the probability of error within the same set up.

- **Figure 5 and Figure 6**



(a) F5



(b) F6

Inference:

For AC-BC model,the probability of detection and false alarm from the previous channel is a direct parameter where for F5, $P_F^{AC} = 0.375$ and $P_D^{AC} = 0.757$ and in case of F6, $P_F^{AC} = 0.225$ and $P_D^{AC} = 0.8$. Using average optimum values for both of those in the equation of probability of error in the AC-BC model, we can see that an optimum threshold value can be deduced. Around $y_{BC}^{THR} = 10$, we can observe a dip.

3 Contribution of team members

3.1 Technical contribution of all team members

Tasks	Kesha Bagadia	Yashvi Pipaliya	Yashvi Gandhi	Manal Shah
Plotting	Yes	Yes	Yes	Yes
Mathematical model	Yes	Yes	Yes	Yes
Video Making	Yes	Yes	Yes	Yes

3.2 Non-Technical contribution of all team members

Tasks	Kesha Bagadia	Yashvi Pipaliya	Yashvi Gandhi	Manal Shah
Report Writing	Yes	Yes	Yes	Yes
Research	Yes	Yes	Yes	Yes
Conceptual analysis (MIRO)	Yes	Yes	Yes	Yes

References

[1] S. Ghavami, "Anomaly detection in molecular communications with applications to health monitoring networks," *IEEE Transactions on Molecular, Biological and Multi-Scale Communications*, vol. 6, no. 1, pp. 50–59, 2020.