Mathematical Analysis (New)

1 DC - ACs Model.

→ DCs release biomarker cells & they are received by ACs

No of received cells in n^{th} time slot at m^{th} AC is:

$$y_{AC,m}[n] = \text{Poiss}\left(p_r \cdot \lambda_{AC,m}[n] + p_r \cdot \bar{\eta}_m\right)$$

probability of reception

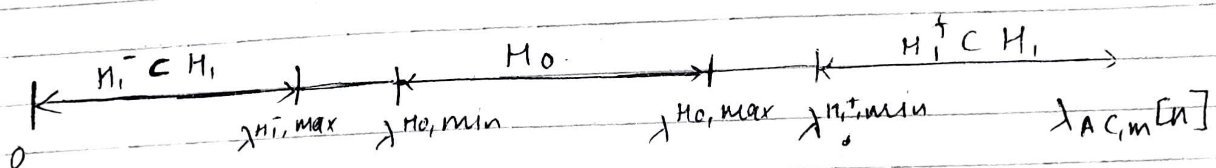
avg of no. of biomarker molecules received at m^{th} AC, n^{th} time slot

mean of Poisson Noise.

→ w.k.t.,

$$p(y_{AC,m}; \lambda_{AC,m}) = \prod_{n=1}^{N} \exp\left(-\lambda_{AC,m}[n] - p_r\bar{\eta}\right) \cdot \frac{\left(\lambda_{AC,m}[n] + p_r\bar{\eta}\right)^{y_{AC,m}[n]}}{y_{AC,m}[n]!}$$

→ We consider,



$\therefore \forall n,$

$$H_0 : \lambda_{AC,m}^{H_0,min} \leq \lambda_{AC,m}[n] \leq \lambda_{AC,m}^{H_0,max}$$

$$H_1 : \begin{cases} H_1^- : & \lambda_{AC,m}[n] \leq \lambda_{AC,m}^{H_1^-,max} \\ H_1^+ : & \lambda_{AC,m}[n] \geq \lambda_{AC,m}^{H_1^+,min} \end{cases}$$

The likelihood ratio of hypothesis test based on CGLRT can be given as,

$$L_{AC,m} = \frac{p(y_{AC,m}; H_1)}{p(y_{AC,m}; H_0)} \overset{H_1}{>} \gamma$$

$$= \prod_{n=1}^{N} \exp\left(-\lambda^{H_1}_{AC,m}[n] - p_{e1}\bar{\eta}\right)(\lambda$$

$$L_{AC,m} = \frac{\prod_{n=1}^{N} \exp\left(-\lambda^{H_1}_{AC,m}[n] - p_{e1}\bar{\eta}\right)\left(\lambda^{H_1}_{AC,m}[n] + p_{e1}\bar{\eta}\right)^{y_{AC,m}[n]}}{\prod_{n=1}^{N} \exp\left(-\lambda^{H_0}_{AC,m}[n] - p_{e1}\bar{\eta}\right)\left(\lambda^{H_0}_{AC,m}[n] + p_{e1}\bar{\eta}\right)^{y_{AC,m}[n]}} \overset{H_1}{>} \gamma$$

∴ The simplified decision rule deduced by bounding likelihood ratio,

$$H_0 : \gamma_l \leq \frac{1}{N} \sum_{n=1}^{N} y_{AC,m}[n] \leq \gamma_u$$

$$H_1 : \begin{cases} \frac{1}{N} \sum_{n=1}^{N} y_{AC,m}[n] > \gamma_u & \text{or} \\ \frac{1}{N} \sum_{n=1}^{N} y_{AC,m}[n] < \gamma_l \end{cases}$$

where,

$$\gamma_l = \lambda^{H_0,\min}_{AC,m} + p_{e1}\bar{\eta} - \gamma'$$

$$\gamma_u = \lambda^{H_0,\max}_{AC,m} + p_{e1}\bar{\eta} + \gamma'$$

∴ The simplified test statistic can be given as,

$$\boxed{Y_{AC,m} = \frac{1}{N} \sum_{n=1}^{N} y_{AC,m}[n]}$$

Using this, we can define P_D & P_F

→ Probability of false Alarm.

$$P_F^{AC,m} = P_r\{\gamma_l \le Y_{AC,m} \le \gamma_u ; H_0\}$$

$$= P_r\{0 \le Y_{AC,m} < \gamma_u ; H_0\} -$$

$$P_r\{0 \le Y_{AC,m} < \gamma_l ; H_0\}$$

$$\therefore P_F^{AC,m} = P_r\{0 \le Poiss(N\lambda_{AC,m}^{H_0,max} + NP_r\bar{\eta}) \le N\gamma_u\} -$$

$$P_r\{0 \le Poiss(N\lambda_{AC,m}^{H_0,min} + NP_r\bar{\eta}) < N\gamma_u\}$$

$$= Poisscdf(N\lambda_{AC,m}^{H_0,max} + NP_r\bar{\eta}, N\gamma_u) -$$

$$Poisscdf(N\lambda_{AC,m}^{H_0,min} + NP_r\bar{\eta}, N\gamma_l)$$

→ Probability of Detection:

$$P_D^{AC,m} = P_r\{0 < Y_{AC,m} < \gamma_l ; H_1^-\} \cdot P_r\{H_1^- | H_1\} +$$

$$P_r\{Y_{AC,m} > \gamma_u ; H_1^+\} \cdot P_r\{H_1^+ | H_1\}$$

Assumming, symmetric distribution,

$$P_r\{H_1^- | H_1\} = P_r\{H_1^+ | H_1\} = 0.5$$

$$\therefore P_D^{AC,m} = 0.5\left[P_r\{Poiss(N\lambda_{AC,m}^{H_1^+,min} + NP_r\bar{\eta}) > N\gamma_u\} + P_r\{Poiss(N\lambda_{AC,m}^{H_1^-,max} + NP_r\bar{\eta}) < N\gamma_l\}\right.$$

$$= \frac{1 - Poisscdf(N\lambda_{AC,m}^{H_1^+,min} + NP_r\bar{\eta}, N\gamma_u)}{2} + \frac{Poisscdf(N\lambda_{AC,m}^{H_1^-,max} + NP_r\bar{\eta}, N\gamma_l)}{2}$$

Now, w.k.t error can be generated in two cases i.e,

1. We detect the signal, even though it's not sent (P_F - False Alarm)
2. We don't detect the signal, even though it's sent (P_{MD} - Miss Detection)

$\therefore$ Probability of error will be summation of both, with β (prior probability) as a controlling parameter.

$$P_e^{AC} = P_{MD}^{AC} \cdot \beta_{AC} + P_F^{AC} (1-\beta_{AC})$$

$$= (1 - P_D^{AC}) \cdot \beta_{AC} + P_F^{AC} (1-\beta_{AC})$$

Assuming, symmetric distribution, $\beta_{AC} = 0.5$

ACs - BC Model

→ Anomaly in ACs,

$$H_0 : x^{H_0,min} \le x_{\Delta_0} \le x^{H_0,max}$$

$$H_1 : \begin{cases} H_1^- : x_{\Delta_0} \le x^{H_1^-,max} \\ H_1^+ : x_{\Delta_0} \ge x^{H_1^+,min} \end{cases}$$

→ Anomaly in BC using hypothesis test,

$$W_0 : \sum_{m=1}^{M} \lambda_{BC,m} < k \bar{\lambda}_{BC}$$

$$W_1 : \sum_{m=1}^{M} \lambda_{BC,m} \ge k \bar{\lambda}_{BC}$$

where, W_0 & W_1 are events corresponding to H_0 & H_1 in BC

K is min no of ACs which detect anomaly in BC - ACm link.

$\bar{\lambda}_{BC}$ is avg. no. of received molecules at BC using.

$$Y_{AC,m} = \frac{1}{N} \sum_{n=1}^{N} y_{AC,m}[n]$$

It can be given for $\forall m \in 1, \ldots, M$

→ If ACs-BC link will alarm the presence of anomaly when atleast K no. of ACs alarm hypothesis H_1 by transmitting $x_{AC,m}(t)$ molecules.

The Decision rule, for the BC with Poisson observation, where y_{BC}^{THR} is decision threshold at BC,

$$H_0 : y_{BC} < y_{BC}^{THR}$$

$$H_1 : y_{BC} \geq y_{BC}^{THR}$$

For perfect ACs-BC link, we derive lower bound for prob. of detection & upper bound for prob. of false alarm.

$$Q_D = P_r \left\{ \sum_{m=1}^{M} \lambda_{BC,m} \geq k \bar{\lambda}_{BC} ; H_1 \right\} = \sum_{m=k}^{M} p'_m$$

$$Q_F = P_r \left\{ \sum_{m=1}^{M} \lambda_{BC,m} \geq K \bar{\lambda}_{BC} ; H_0 \right\} = \sum_{m=k}^{M} p''_m$$

where,

Min. no. of ACs which detect the anomaly in order to enable BC

→ Probability of Detection:

$$P_D = Pr\{ y_{BC} \geq y_{BC}^{THR} ; H_1 \}$$

$$= Pr\{ y_{BC} \geq y_{BC}^{THR} \mid W_0 ; H_1 \} Pr\{ W_0 ; H_1 \} +$$

$$Pr\{ y_{BC} \geq y_{BC}^{THR} \mid W_1 ; H_1 \} Pr\{ W_1 ; H_1 \}.$$

(using Total Prob. Theorem)

From definations of Q_D, W_0 & W_1,

$$= Pr\{ y_{BC} \geq y_{BC}^{THR} \mid \sum_{m=1} \lambda_{BC,m} < K\bar{\lambda}_{BC} ; H_1 \} (1-Q_D)$$

$$+ Pr\{ y_{BC} \geq y_{BC}^{THR} \mid \sum_{m=1} \lambda_{BC,m} \geq K\bar{\lambda}_{BC} ; H_1 \} Q_D$$

$$= Pr\{ y_{BC} \geq y_{BC}^{THR} \mid \bigcup_{i=0}^{K-1} \{ \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC} \} ; H_1 \} (1-Q_D)$$

$$+ Pr\{ y_{BC} \geq y_{BC}^{THR} \mid \bigcup_{i=K}^{M} \{ \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC} \} ; H_1 \} Q_D$$

$$= \frac{Pr\{ \bigcup_{i=0}^{K-1} \{ \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC} \} \cap \{ y_{BC} \geq y_{BC}^{THR} \} ; H_1 \}}{Pr\{ \bigcup_{i=0}^{K-1} \{ \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC} \} ; H_1 \}} (1-Q_D)$$

$$+ \frac{Pr\{ \bigcup_{i=K}^{M} \{ \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC} \} \cap \{ y_{BC} \geq y_{B}^{THR} \} ; H_1 \}}{Pr\{ \bigcup_{i=K}^{M} \{ \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC} \} ; H_1 \}} (Q_D)$$

$$= \sum_{i=0}^{K-1} \frac{(Pr\{ y_{BC} \geq y_{BC}^{THR} \mid \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC} ; H_1 \}}{Pr\{ \sum_{m=1}^{M} \lambda_{BC,m} < K\bar{\lambda}_{BC} ; H_1 \}} \times$$

$$\times \Pr\left\{ \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC}, H_1 \right\} \Big) (1 - Q_D)$$

$$+ \sum_{i=K}^{M} \Big(\Pr\left\{ y_{BC} \geq y_{BC}^{THR} \,\Big|\, \sum_{m=1}^{M} \lambda_{BC,m}[n] = i\bar{\lambda}_{BC}; H_1 \right\}$$

$$\frac{}{\Pr\left\{ \sum_{m=1}^{M} \lambda_{BC,m} \geq K\bar{\lambda}_{BC}; H_1 \right\}}$$

$$\times \Pr\left\{ \sum_{m=1}^{M} \lambda_{BC,m} = i\bar{\lambda}_{BC}; H_1 \right\} \Big) Q_D$$

Following the Poisson Distribution of y_{BC},

$$= \frac{1}{\sum_{i=0}^{K-1} P_i} \sum_{i=0}^{K-1} P_i \left(1 - \frac{\Gamma(\lceil y_{BC}^{THR} \rceil, i\bar{\lambda}_{BC}) + q_n \bar{\varepsilon}}{\lceil y_{BC}^{THR} - 1 \rceil !} \right) (1 - Q_D)$$

$$+ \frac{1}{\sum_{i=1}^{M} P_i'} \sum_{i=K}^{M} P_i' \left(1 - \frac{\Gamma(\lceil y_{BC}^{THR} \rceil, i\bar{\lambda}_{BC}) + q_n \bar{\varepsilon}}{\lceil y_{BC}^{THR} - 1 \rceil !} \right) Q_D$$

$$\therefore \boxed{P_D = \sum_{i=0}^{M} P_i' \left(1 - \frac{\Gamma(\lceil y_{BC}^{THR} \rceil, i\bar{\lambda}_{BC}) + q_n \bar{\varepsilon}}{\lceil y_{BC}^{THR} - 1 \rceil !} \right)}$$

FINALLY !

Probability of False Alarm:

$$P_F = \Pr\{ y_{BC} \geq y_{BC}^{THR} ; H_0 \}$$

$$= \Pr\{ y_{BC} \geq y_{BC}^{THR} \mid W_0; H_0 \} \Pr\{ W_0; H_0 \}$$

$$+ \Pr\{ y_{BC} \geq y_{BC}^{THR} \mid W_1; H_0 \} \Pr\{ W_1; H_0 \}$$

Following similar steps as of P_D, we get,

$$= \frac{1}{\sum_{i=0}^{K-1} P_i''} \sum_{i=0}^{K-1} P_i'' \left(1 - \frac{\Gamma\left(\lceil y_{BC}^{THR} \rceil, i\bar{\lambda}_{BC}\right) + q_{\sigma_1}\bar{\Sigma}}{\lceil y_{BC}^{THR} - 1 \rceil !}\right)(1 - Q_F)$$

$$+ \frac{1}{\sum_{i=K}^{M} P_i''} \sum_{i=K}^{M} P_i'' \left(1 - \frac{\Gamma\left(\lceil y_{BC}^{THR} \rceil, i\bar{\lambda}_{BC}\right) + q_{\sigma_1}\bar{\Sigma}}{\lceil y_{BC}^{THR} - 1 \rceil !}\right) Q_F$$

$$= \sum_{i=0}^{M} P_i'' \left(1 - \frac{\Gamma\left(\lceil y_{BC}^{THR} \rceil, i\bar{\lambda}_{BC}\right) + q_{\sigma_1}\bar{\Sigma}}{\lceil y_{BC}^{THR} - 1 \rceil !}\right)$$

$$\therefore P_e = P_{MD} \cdot \beta + P_F (1 - \beta)$$

$$= (1 - P_D)\beta + P_F (1 - \beta)$$

where $\beta = 0.5$