Mathematical Analysis (old)

DCs-AC Model

→ $x_D(t) = x_{D0}(1 - e^{-kt^{b}})$ (using Weibull fn)

↓

Transmit waveform of biomarkers as a fn of time

→ $y_{AC,m}[n] = \text{Poiss}(p_r * \lambda_{AC,m}[n] + p_r \bar{\eta}_m)$

↓

The received no. of biomarkers in n^{th} time slot at m^{th} AC

→ $$\lambda_{AC,m}[n] = \int_{(n-1)T_s}^{nT_s} \int_{t-L^{(hm)}T_s}^{t} h_m(t;\tau)\, x_D(\tau)\, d\tau\, dt$$

↓

Avg. No. of bio-marker molecules received at m^{th} AC.

Consequentially,

$$h_m[n,l] = \begin{cases} F_{W,m}(nT_s, lT_s) - F_{W,m}(nT_s, (l-1)T_s) & l < L^{hm} \\ 0 & l \geq L^{hm} \end{cases}$$

↓

The propogation probability of receiving molecules on n^{th} time slot from transmitted molecules in $n l$ prev. time slot

$$F_{w,m} = \Phi\left(\sqrt{\frac{\lambda_m(t)}{w}}\left(\frac{w}{\mu_m(t)} - 1\right)\right) +$$

$$e^{\frac{2\lambda_m(t)}{\mu_m(t)}}\left(-\sqrt{\frac{\lambda_m(t)}{w}}\left(\frac{w}{\mu_m(t)} + 1\right)\right)$$

Cumulutal dist CDF of propagation time which is obtained using an additive inverse Gaussian (AIG) analysis.

$\Phi(.)$ → CDF of standard gaussian Random Variable

We assume that, $V_m(t)$ & $S_m(t)$ are constant over at least $L^{(nm)}$ Ts seconds

ACs-BC System Model

$$x_{AC,m}(t) = x_{AC_0}\, e^{-k' t^{b'}}$$ (using weibull f^n)

The molecules transmitted by ACs m^{th} AC at a given time.

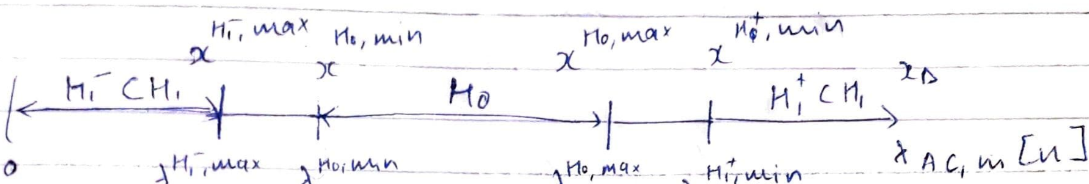
$$y_{BC,m} = \sum_{M=1}^{M} \text{Poiss}(q_n \lambda_{BC,m}) + q_n \bar{\varepsilon}$$

The no. of received molecules at the BC through the molecular ACs-BC channel.

where,

→ $$\lambda_{BC,m} = \int_{0}^{T_{BC}} \int_{t-L^{g_m}T_s}^{t} g_m(t;\tau)\, x_{AC,m}(\tau)\, d\tau\, dt$$

Hypothesis Test for Anomaly Detection:



→ ACs identify anomaly as:

$$H_0 : x^{H_0,min} \leq x_{D_0} \leq x^{H_0,max}$$

$$H_1 : \begin{cases} H_1^- : & x_{D_0} \leq x_0^{H_1,max} \\ H_1^+ : & x_{D_0} \geq x^{H_1^+,min} \end{cases}$$

$$\therefore \quad H_0 : \lambda_{AC,m}^{H_0,min} \leq \lambda_{AC,m}[n] \leq \lambda_{AC,m}^{H_0,max}$$

$$H_1 : \begin{cases} H_1^- : & \lambda_{AC,m}[n] \leq \lambda_{AC,m}^{H_1^-,max} \\ H_1^+ : & \lambda_{AC,m}[n] \geq \cancel{x}\, \lambda_{AC,m}^{H_1,min} \end{cases}$$

→ BCs identify anomaly as

$$W_0 : \sum_{m=1}^{M} \lambda_{BC,m} < k\bar{\lambda}_{BC}$$

$$W_1 : \sum_{m=1}^{M} \lambda_{BC,m} \geq k\bar{\lambda}_{BC}$$

where $\bar{\lambda}_{BC} = \frac{1}{M} \sum_{m=1}^{M} \lambda_{BC,m}$

- Anomaly Detection Performance of ACs

→ The decision rule of hypothesis test based on GLRT for the m^{th} AC with N independent Poisson observat^n:

$$\sum_{n=n_{ss}+1}^{n_{ss}+N} \hat{\lambda}^{H_0}_{AC,m}[n] - \hat{\lambda}^{H_1}_{AC,m}[n] + y_{AC,m}[n]\log(\hat{\lambda}^{H_1}_{AC,m}[n] + P_r\bar{\eta})$$

$$- y_{AC,m}[n]\log(\hat{\lambda}^{H_0}_{AC,m}[n] + P_r\bar{\eta}) \overset{H_1}{\gtrless} \log\gamma$$

where $n_{ss} = \lceil t_s / T_s \rceil$ → min sampling index for steady state

$\forall n \in n_{ss}+1, \ldots, n_{ss}+N$

& ML estimator of $\lambda_{AC,m}[n]$

$= \hat{\lambda}_{AC,m}[n] = \max(y_{AC,m}[n] - P_r\bar{\eta})$

→ The simplified decision rule from bounding LR for the n^{th} AC with N independent Poisson observat^n & limited false alarm probability $P_F^{AC,m} < \xi_1$,

$$H_0: \max(\gamma_L, 0) \le \frac{1}{N}\sum_{n=n_{ss}+1}^{n_{ss}+N} y_{AC,m}[n] \le \gamma_u$$

$$H_1: \begin{cases} \frac{1}{N}\sum_{n=n_{ss}+1}^{n_{ss}+N} y_{AC,m}[n] > \gamma_u & \text{or} \\ \frac{1}{N}\sum_{n=n_{ss}+1}^{n_{ss}+N} y_{AC,m}[n] < \max(\gamma_L, 0) \end{cases}$$

where,

$$\gamma_L = \lambda^{H_0,min}_{AC,m} + P_r\bar{\eta} - \gamma'$$

$$\gamma_u = \lambda^{H_0,max}_{AC,m} + P_r\bar{\eta} + \gamma'$$

→ $P_D^{AC,m} = \min\left(\frac{\Gamma\left(\lceil \max(N\gamma_l - 1, 0) + 1\rceil, N\left(\lambda_{AC,m}^{H_1,max} + P_s\bar{\eta}\right)\right)}{\lceil \max(N\gamma_l - 1, 0)\rceil !}, \right.$

$\left. 1 - \frac{\Gamma\left(\lfloor N\gamma_u + 1\rfloor N\left(\lambda_{AC,m}^{H_1,min} + P_s\bar{\eta}\right)\right)}{\lfloor N\gamma_u \rfloor !} \right)$

lower bound of detection probability

→ The decision rule for the BC with Poisson observatⁿ, where y_{BC}^{THR} is decision threshold at BC,

$H_0 : y_{BC} < y_{BC}^{THR}$

$H_1 : y_{BC} \geq y_{BC}^{THR}$

→ The lower bound of P_D & the upper bound of P_F at BC are:

$$P_D = \sum_{m=0}^{M} P_m' \left(1 - \frac{\Gamma\left(\lceil y_{BC}^{THR}\rceil, m\lambda_{BC} + q_n\bar{E}\right)}{\lceil y_{BC}^{THR} - 1\rceil !} \right)$$

$$P_F = \sum_{m=0}^{M} P_m'' \left(1 - \frac{\Gamma\left(\lceil y_{BC}^{THR}\rceil, m\lambda_{BC} + q_n E\right)}{\lceil y_{BC}^{THR} - 1\rceil !} \right)$$

P_m' & P_m'' → Probability of detecting anomaly at m ACs under H_1 & H_0 respectively.

$$P_m' = \binom{M}{m}\left(1 - P_D^{AC}\right)^{M-m}\left(P_D^{AC}\right)^m$$

$$P_m'' = \binom{M}{m}\left(1 - P_F^{AC}\right)^{M-m}\left(P_F^{AC}\right)^m$$