DIFFERENTIATION
C.E.T. QUESTIONS
WITH SOLUTION

MATHEMATICS

KEA

1) If $y = \cos^2\sqrt{x}$ then $\frac{dy}{dx} = ---$

a) $\frac{-\sin\sqrt{x}}{2\sqrt{x}}$ b) $\frac{\cos\sqrt{x}}{2\sqrt{x}}$ c) $\frac{-\sin 2\sqrt{x}}{2\sqrt{x}}$ d) $\frac{\cos 2\sqrt{x}}{2\sqrt{x}}$

$$Sol: \quad \frac{dy}{dx} = 2\cos\sqrt{x} \times -\sin\sqrt{x} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{-\sin 2\sqrt{x}}{2\sqrt{x}}$$

MATHEMATICS

Ans c)

2) If $y = \log\left(\cos\sqrt{x}\right)$ then $\dfrac{dy}{dx} = ---$

a) $\dfrac{1}{\cos\sqrt{x}}$ b) $-\tan\sqrt{x}$ c) $\dfrac{-\tan\sqrt{x}}{2\sqrt{x}}$ d) $\dfrac{\tan\sqrt{x}}{2\sqrt{x}}$

$$Sol: \quad \frac{dy}{dx} = \frac{1}{\cos\sqrt{x}} \times -\sin\sqrt{x} \times \frac{1}{2\sqrt{x}}$$

$$= \frac{-\tan\sqrt{x}}{2\sqrt{x}}$$

MATHEMATICS

Ans c)

3) If $y = sec^{-1}\left(\frac{\sqrt{x}-1}{x+\sqrt{x}}\right) + \sin^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right)$

where $\left|x + \sqrt{x}\right| \le \left|\sqrt{x} - 1\right|$ then $\frac{dy}{dx} = ---$

$a)\ x$ $\quad$ $b)\ 1$ $\quad$ $c) -1$ $\quad$ $d)\ 0$

Sol : $y = sec^{-1}\left(\frac{\sqrt{x}-1}{x+\sqrt{x}}\right) + \sin^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right)$

$$= \cos^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{x+\sqrt{x}}{\sqrt{x}-1}\right) = \frac{\pi}{2}$$

$$\therefore \frac{dy}{dx} = \frac{d\frac{\pi}{2}}{dx} = 0$$

MATHEMATICS

Ans d)

4) If $y = \dfrac{\sec x + \tan x}{\sec x - \tan x}$ then $\dfrac{dy}{dx} = - - -$

a) $2\sec x(\sec x + \tan x)$

b) $2\sec^2 x(\sec x + \tan x)^2$

c) $2\sec x(\sec x + \tan x)^2$

d) $\sec x(\sec x + \tan x)^2$

Sol : $y = \dfrac{\sec x + \tan x}{\sec x - \tan x} \times \dfrac{\sec x + \tan x}{\sec x + \tan x}$

$y = (\sec x + \tan x)^2 \qquad (\because \sec^2 x - \tan^2 x = 1)$

$\dfrac{dy}{dx} = 2(\sec x + \tan x) \cdot (\sec x \tan x + \sec^2 x)$

$\therefore \dfrac{dy}{dx} = 2\sec x(\sec x + \tan x)^2$

MATHEMATICS

Ans c)

5) If $f^1(x) = \sin(x^2)$ and $y = f(x^2+1)$ then $\frac{dy}{dx} = ---$

$a)\ 2\sin(x^2+1)$

$b)\ 2\sin(x^2+1)^2$

$c)\ 2x\sin(x^2+1)$

$d)\ 2x.\sin(x^2+1)^2$

Sol : $y = f(x^2+1)$

$$\Rightarrow \frac{dy}{dx} = f^1(x^2+1).\,2x$$

$$\Rightarrow \frac{dy}{dx} = 2x.\sin(x^2+1)^2$$

MATHEMATICS

Ans d)

6) If $f(x)=\log_x(\log x)$ then $f^1(x)$ at $x=e=---$

$a)\ \dfrac{1}{e}$ $\quad$ $b)\ \dfrac{1}{e^2}$ $\quad$ $c)\ e$ $\quad$ $d)\ e^2$

$$Sol: \quad f(x)=\frac{\log_e(\log x)}{\log_e x}$$

$$\Rightarrow \quad f^1(x)=\frac{\log_e x\dfrac{1}{(\log x)}\cdot\dfrac{1}{x}-\log_e(\log x)\cdot\dfrac{1}{x}}{(\log_e x)^2}$$

$$\Rightarrow \quad f^1(e)=\frac{\dfrac{1}{e}-0}{(1)^2}=\frac{1}{e}$$

MATHEMATICS

Ans a)

KEA

7) If g is the inverse of f and $f^1(x)=\dfrac{1}{1+x^n}$ then $g^1(x)=---$

$a)1-[g(x)]^n$

$b)\ 1+[g(x)]^n$

$c)1-n[g(x)]$

$d)1+n[g(x)]$

$Sol:$ g is inverse of $f=(fog)(x)=x$

$$\Rightarrow \quad f(g(x))=x \Rightarrow f^1(g(x)).g^1(x)=1$$

$$\Rightarrow \quad g^1(x)=\frac{1}{f^1(g(x))}=1+[g(x)]^n$$

MATHEMATICS

Ans b)

8) If $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$ then $\dfrac{dy}{dx} = ---$

a) $\begin{vmatrix} f^1(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

b) $\begin{vmatrix} f^1(x) & g(x) & h^1(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

c) $\begin{vmatrix} f^1(x) & g^1(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

d) $\begin{vmatrix} f^1(x) & g^1(x) & h^1(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

Sol : On expansion

$$y = f(x)(mc - nb) - g(x)(lc - an) + h(x)(lb - ma)$$

$$\Rightarrow \frac{dy}{dx} = f^1(x).(mc - nb) - g^1(x)(lc - an) + h^1(x)(lb - ma)$$

$$\Rightarrow \frac{dy}{dx} = \begin{vmatrix} f^1(x) & g^1(x) & h^1(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

MATHEMATICS

Ans d)

9) If $y = e^{\log_e (1+x+x^2+\ldots\ldots)}$ where $(|x| < 1)$ then $\dfrac{dy}{dx} =$

a) $\dfrac{-1}{(1-x)^2}$ b) $\dfrac{1}{(1+x)^2}$ c) $\dfrac{1}{(1-x)^2}$ d) $\dfrac{-1}{(1+x)^2}$

Sol : Given $y = 1 + x + x^2 + \ldots\ldots \left(\because y = e^{\log_e x} = x\right)$

$$y = \frac{a}{1-r} = \frac{1}{1-x} = (1-x)^{-1}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1-x)^2} \times -1 = \frac{1}{(1-x)^2}$$

MATHEMATICS

Ans c)

10) If $y = \log\left(\dfrac{1-x^2}{1+x^2}\right)$ then $\dfrac{dy}{dx} =$

$a)\ \dfrac{4x^3}{1-x^4}$ $\quad b)\ \dfrac{4}{1-x^4}$ $\quad c)\ \dfrac{-4x^3}{1-x^4}$ $\quad d)\ \dfrac{-4x}{1-x^4}$

Sol : $y = \log(1-x^2) - \log(1+x^2)$

$$\Rightarrow \frac{dy}{dx} = \frac{-2x}{1-x^2} - \frac{2x}{1+x^2} \text{(on simplicati on)}$$

$$\Rightarrow \frac{dy}{dx} = -2x\left(\frac{-2}{1-x^4}\right) = \frac{-4x}{1-x^4}$$

MATHEMATICS

Ans d)

11) If $y = \tan^{-1}\left(e^{2x}\right)$ then $\dfrac{dy}{dx} =$

a) $\dfrac{2e^{2x}}{1+e^{4x}}$

b) $\dfrac{1}{1+e^{4x}}$

c) $\dfrac{2}{e^{2x}+e^{-2x}}$

d) $\dfrac{-2e^{2x}}{1+e^{4x}}$

$$Sol: \quad \frac{dy}{dx} = \frac{1}{1+\left(e^{2x}\right)^2} \times e^{2x} \times 2 \qquad = \frac{2e^{2x}}{1+e^{4x}}$$

MATHEMATICS

Ans a)

12) If $y = \sin^{-1}(\cos x)$ where $0 \le x \le \pi$ then $\frac{dy}{dx} =$

$a) 1$ $\quad b) \cos^{-1} x$ $\quad c) -1$ $\quad d) \frac{1}{2}$

$Sol:$ $\quad w.k.t.\ \sin^{-1}(\cos x) + \cos^{-1}(\cos x) = \frac{\pi}{2}$

$$\Rightarrow y = \frac{\pi}{2} - \cos^{-1}(\cos x)$$

$$= \frac{\pi}{2} - x \qquad \Rightarrow \frac{dy}{dx} = -1$$

MATHEMATICS

Ans c)

13) If $f(x)$ an even function and $f^1(x)$ exists then $f^1(e)+f^1(-e)=$

$a)\,0$ $b)>0$ $c)<0$ $d)\geq 0$

$Sol:$ $f(x)$ is even $\Rightarrow f(-x)=f(x)$

$$\Rightarrow f^1(-x)\times -1=f^1(x)$$

$$\Rightarrow -f^1(-e)=f^1(e)$$

$$\Rightarrow f^1(e)+f^1(-e)=0$$

MATHEMATICS

Ans a)

KEA

14) If $f(x) = \sin[\pi^2]x + \cos[-\pi^2]x$ then $f^1(x) =$

(where $[\pi^2]$ and $[-\pi^2]$ are the greatest integer functions not greater than its value.)

a) $\sin 9x + \cos 9x$ b) $9\cos 9x - 10\sin 10x$

c) 0 d) -1

Sol : $\pi^2 \approx 9.87$

$\therefore [\pi^2] = 9$ *and* $[-\pi^2] = -10$

Now $f(x) = \sin 9x + \cos(-10x)$

$\Rightarrow f(x) = \sin 9x + \cos 10x$

$\Rightarrow f^1(x) = 9\cos 9x - 10\sin 10x$

MATHEMATICS

Ans b)

15) If $y = \sin^{-1}\left[\frac{5x + 12\sqrt{1-x^2}}{13}\right]$ then $\frac{dy}{dx} =$

a) $\frac{5}{\sqrt{1-x^2}}$ b) $\frac{12}{\sqrt{1-x^2}}$

c) $\frac{-1}{\sqrt{1-x^2}}$ d) $\frac{1}{\sqrt{1-x^2}}$

Sol : $y = \sin^{-1}\left[\frac{5}{13}x + \frac{12}{13}\sqrt{1-x^2}\right]$

Put $x = \sin\theta$ and $\frac{5}{13} = \cos\alpha$

$\therefore \sqrt{1-x^2} = \cos\theta \; and \sin\alpha = \sqrt{1-\frac{25}{169}} = \frac{12}{13}$

$\therefore y = \sin^{-1}[\sin\theta.\cos\alpha + \cos\theta.\sin\alpha]$

$\Rightarrow y = \sin^{-1}(\sin(\theta+\alpha)) \Rightarrow y = \theta + \alpha$

$\Rightarrow y = \sin^{-1}x + \cos^{-1}\frac{5}{13}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$

MATHEMATICS

Ans d)

16) If $y = \tan^{-1}\left[\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right]$ then $\frac{dy}{dx} =$

a) $\frac{x^2}{\sqrt{1+x4}}$ b) $\frac{x^2}{\sqrt{1-x^4}}$ c) $\frac{x}{\sqrt{1-x^4}}$ d) $\frac{x}{\sqrt{1+x^4}}$

Sol: Put $x^2 = \cos\theta$

$\therefore \sqrt{1+x^2} = \sqrt{2}\cos\theta/2 \ and \sqrt{1-x^2} = \sqrt{2}\sin\theta/2$

onsimplicatin

$$y = \tan^{-1}\left[\frac{\cos\theta/2 - \sin\theta/2}{\cos\theta/2 + \sin\theta/2}\right] = \tan^{-1}\left[\frac{1-\tan\theta/2}{1+\tan\theta/2}\right]$$

$$y = \tan^{-1}\left(\tan\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\right) \Rightarrow y = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(x^2\right) \quad \Rightarrow \frac{dy}{dx} = \frac{-1}{2}\times\frac{-1}{\sqrt{1-x^4}}\times 2x = \frac{x}{\sqrt{1-x^4}}$$

MATHEMATICS

Ans c)

17) If $y=\sqrt{\sin x+\sqrt{\sin x+\sqrt{\sin x+\ldots\ldots.to\infty}}}$ then $\frac{dy}{dx}=$

a) $\frac{\cos x}{2y+1}$

b) $\frac{\cos x}{2y-1}$

c) $\frac{-\cos x}{2y-1}$

d) $\frac{-\cos x}{2y+1}$

Sol : In General

$$If\ y=\sqrt{f(x)+\sqrt{f(x)+\sqrt{f(x)+\ldots\ldots to\infty}}}$$

then $\frac{dy}{dx}=\frac{f^{1}(x)}{2y-1}$

$$\therefore \frac{dy}{dx}=\frac{\cos x}{2y-1}\left(where\ f(x)=\sin x\right)$$

MATHEMATICS

Ans b)

18) If $y=\sqrt{x+\sqrt{x+\sqrt{x+........to\,\infty}}}$ then $\frac{dy}{dx}=$

$a)\ \frac{1}{2y-1}$

$b)\ \frac{-1}{2y-1}$

$c)\ \frac{1}{2y+1}$

$d)\ \frac{-1}{2y+1}$

$$Sol: \quad \therefore \frac{dy}{dx}=\frac{f^1(x)}{2y-1}\left(where\, f(x)=x\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{1}{2y-1}$$

MATHEMATICS

Ans a)

19) If $2x^2 + 3xy + 4y^2 - 24 = 0$ then $\frac{dy}{dx}_{(1,2)} =$

a) $\frac{11}{19}$ b) $\frac{-11}{19}$ c) $\frac{-10}{19}$ d) $\frac{10}{19}$

Sol: In general $\frac{dy}{dx} = \frac{-f_x^1}{f_y^1}$ where f_x^1 = derivative of $f(x, y)$ w.r.t. 'x' by keeping the other variable 'y' as constant and f_y^1 = derivative of $f(x, y)$ w.r.t 'y' by keeping the other variable 'x' as constant.

Now $f_x^1 = 4x + 3y$ $f_y^1 = 3x + 8y$

$$\therefore \frac{dy}{dx} = \frac{-4x-3y}{3x+8y} \Rightarrow \frac{dy}{dx}_{\substack{at\ (1,\ 2)\\ x,\ y}} = \frac{-4-6}{3+16} = \frac{-10}{19}$$

MATHEMATICS

Ans c)

20) If $y^2(2-x)=x^3$ then $\dfrac{dy}{dx}_{at\,(1,1)}=$

$a)\,3$ $\quad b)\,2$ $\quad c)\,-2$ $\quad d)\,-3$

Sol : Given $y^2(2-x)-x^3=0$

$$\therefore f_x^1 = y^2(-1)-3x^2 = -y^2-3x^2$$

$$f_y^1 = (2-x)2y = 4y-2xy$$

$$\because \frac{dy}{dx} = \frac{-\left(-y^2-3x^2\right)}{4y-2xy} = \frac{y^2+3x^2}{4y-2xy}$$

$$\therefore \frac{dy}{dx}_{at\,(1,\,1)} = \frac{1+3}{4-2} = \frac{4}{2} = 2$$

MATHEMATICS

Ans b)

21) If $\log\left(x^2+y\right)-4xy^2=0$ then $y^1(0)=$

$a)\,0$ $\quad b)\,-1$ $\quad c)\,4$ $\quad d)\,-4$

Sol : Here $f_x^1=\dfrac{1}{x^2+y}\times 2x-4y^2$

$$f_y^1=\frac{1}{x^2+y}\times 1-8xy$$

$$\therefore y^1=\frac{dy}{dx}=\frac{-f_x^1}{f_y^1}=-\left[\frac{\dfrac{2x}{x^2+y}-4y^2}{\dfrac{1}{x^2+y}-8xy}\right]$$

$$\therefore y^1(0)=y^1_{\substack{(0,\ 1)\\ x\quad y}}=-\left(\frac{0-4}{1-0}\right)=4$$

MATHEMATICS

Ans c)

22) If $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$ then $\frac{dy}{dx} =$

a) $-\tan\theta$ b) $\tan\theta$ c) $\cot\theta$ d) $-\cot\theta$

$$Sol: \quad Here\ \frac{dx}{d\theta} = a(-\sin\theta + \theta\cos\theta + \sin\theta) \quad = a\theta\cos\theta$$

$$\frac{dy}{d\theta} = a(\cos\theta - \{\theta(-\sin\theta) + \cos\theta\}) = a\theta\sin\theta$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\theta\sin\theta}{a\theta\cos\theta} = \tan\theta$$

MATHEMATICS

Ans b)

23) If $x=\theta.\sin 2\theta,\ y=\theta\cos 2\theta$ then $\frac{dy}{dx}$ at $\theta=\frac{\pi}{4}$

$a)\ \frac{\pi}{4}$ $\quad b)\ -\frac{\pi}{4}$ $\quad c)\ \frac{\pi}{2}$ $\quad d)\ -\frac{\pi}{2}$

Sol : *Here* $\frac{dx}{d\theta}=\theta\cos 2\theta\times 2+\sin 2\theta\times 1$

$$\frac{dy}{d\theta}=\theta(-\sin 2\theta\times 2)+\cos 2\theta\times 1$$

$$\therefore\ \frac{dy}{dx}=\frac{-2\theta\sin 2\theta+\cos 2\theta}{2\theta\cos 2\theta+\sin 2\theta}$$

$$\therefore\ \left.\frac{dy}{dx}\right|_{\text{at }\theta=\frac{\pi}{4}}=\frac{-\frac{2\pi}{4}\times 1+0}{\frac{2\pi}{4}\times 0+1}=\frac{-\pi}{2}$$

MATHEMATICS

Ans d)

24) If $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ then $\left.\dfrac{dy}{dx}\right|_{\text{at } \theta=\frac{\pi}{2}} =$

$a)\ 1 \qquad b)\ -1 \qquad c)\ 0 \qquad d)\ \pi$

Sol : Here $\dfrac{dx}{d\theta} = a(1 + \cos\theta)$, $\dfrac{dy}{d\theta} = a\sin\theta$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1+\cos\theta)} = \frac{\sin\theta}{1+\cos\theta}$$

$$\therefore \left.\frac{dy}{dx}\right|_{\text{at } \theta=\frac{\pi}{2}} = \frac{\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}} = \frac{1}{1} = 1$$

MATHEMATICS

Ans a)

25) If $3^x + 3^y = 3^{x+y}$ then $\frac{dy}{dx}_{(1,\,1)} =$

$a)\ 2$ $b)\ -2$ $c)\ 1$ $d)\ -1$

$Sol:$ Given $3^x + 3^y = 3^{x+y}$

$\div by\ 3^{x+y}\ we\ get\ 3^{-y} + 3^{-x} = 1$

$$\Rightarrow 3^{-y}.\log_e 3 \times -\frac{dy}{dx} + 3^{-x}.\log_e 3 \times -1 = 0$$

$$\Rightarrow -3^{-y}\frac{dy}{dx} - 3^{-x} = 0 \Rightarrow \frac{dy}{dx} = -3^{y-x}$$

$$\Rightarrow \frac{dy}{dx}_{(1,1)} = -3^0 = -1$$

MATHEMATICS

Ans d)

26) If $x^2 + y^2 = t + \frac{1}{t}$ and $x^4 + y^4 = t^2 + \frac{1}{t^2}$ then $\frac{dy}{dx} =$

a) $\frac{y}{x}$ b) $-\frac{y}{x}$ c) $-\frac{1}{xy^3}$ d) $\frac{1}{x^3}$

$Sol:$ $$Now\left(x^2+y^2\right)^2 = t^2 + \frac{1}{t^2} + 2$$

$$\Rightarrow x^4 + y^4 + 2x^2y^2 = x^4 + y^4 + 2$$

$$\Rightarrow x^2y^2 = 1$$

$$\Rightarrow \frac{dy}{dx} = \frac{-f_x^1}{f_y^1} = -\frac{2xy^2}{2x^2y} = -\frac{y}{x}$$

MATHEMATICS

Ans b)

27) The derivative of $\cot^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ w.r.t. $\tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}$ is

$a)\ 1$ $\quad b)\ -1$ $\quad c)\ \frac{1}{2}$ $\quad d)\ -\frac{1}{2}$

$$Sol:\ Let\ \ u=\cot^{-1}\left(\frac{\sin x}{1+\cos x}\right)=\cot^{-1}\left(\frac{2\sin x/2\ \ \cos x/2}{2\cos^2 x/2}\right)\Rightarrow \cot^{-1}\left(\tan\frac{x}{2}\right)$$

$$\therefore u=\cot^{-1}\left(\cot\left[\frac{\pi}{2}-\frac{x}{2}\right]\right)=\frac{\pi}{2}-\frac{x}{2}$$

$$v=\tan^{-1}\sqrt{\frac{1+\cos x}{1-\cos x}}=\tan^{-1}\sqrt{\frac{2\cos^2 x/2}{2\sin^2 x/2}}=\tan^{-1}\left(\cot\frac{x}{2}\right)$$

$$\therefore v=\tan^{-1}\left(\tan\left[\frac{\pi}{2}-\frac{x}{2}\right]\right)=\frac{\pi}{2}-\frac{x}{2}\quad \therefore \frac{du}{dv}=\frac{\frac{du}{dx}}{\frac{dv}{dx}}=\frac{-\frac{1}{2}}{-\frac{1}{2}}=1$$

MATHEMATICS

Ans a)

28) The derivative of $\sin^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$ w.r.t. $\sec^{-1}\left[\frac{1-3x^2}{3x-x^3}\right]$ is

$a)1$ $\quad b)-1$ $\quad c)-\frac{1}{3}$ $\quad d)\frac{1}{3}$

Sol : Let $u=\sin^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$

$$v=\sec^{-1}\left[\frac{1-3x^2}{3x-x^3}\right]=\cos^{-1}\left[\frac{3x-x^3}{1-3x^2}\right]$$

$$Now\quad u+v=\frac{\pi}{2}\quad\Rightarrow\frac{du}{dv}+1=0\Rightarrow\frac{du}{dv}=-1$$

MATHEMATICS

Ans b)

KEA

29) If $x^x = y^y$ then $\dfrac{dy}{dx} =$

$a)\ 1+\log\left(\dfrac{x}{y}\right)$ $b)\ \dfrac{1+\log x}{1+\log y}$ $c)\ -\dfrac{y}{x}$ $d)\ \dfrac{-x}{y}$

Sol : $x^x = y^y \Rightarrow x^x - y^y = 0$

$\therefore f_x^1 = x^x(1+\log x)$ $f_y^1 = -y^y(1+\log y)$

$$\therefore \frac{dy}{dx} = \frac{-f_x^1}{f_y^1} = \frac{-x^x(1+\log x)}{-y^y(1+\log y)} = \frac{1+\log x}{1+\log y}$$

MATHEMATICS

Ans b)

30) If $y=(1+x)(1+x^2)\ldots\ldots(1+x^{100})$ then $\frac{dy}{dx}$ at $x=0$ is

a) 100 b) 0 c) 1 d) 5050

Sol : Given $y=(1+x)(1+x^2)\ldots\ldots(1+x^{100})$

$$\Rightarrow \log y=\log(1+x)+\log(1+x^2)+\ldots..+\log(1+x^{100})$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx}=\frac{1}{1+x}+\frac{2x}{1+x^2}+\ldots\ldots+\frac{100x^{99}}{1+x^{100}}$$

$$\Rightarrow \frac{dy}{dx}=y\left[\frac{1}{1+x}+\frac{2x}{1+x^2}+\ldots\ldots+\frac{100x^{99}}{1+x^{100}}\right]$$

$$\Rightarrow \left.\frac{dy}{dx}\right|_{at\ x=0}=1[1+0+\ldots+0]=1$$

MATHEMATICS

Ans c)

31) If $y = x^{\sin x} + \sqrt{x}$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is

$a) 1+\frac{1}{\sqrt{2\pi}}$

$b) \frac{1}{\sqrt{2\pi}}$

$c) 1-\frac{1}{\sqrt{2\pi}}$

$d) \frac{\pi^2}{4}+\frac{1}{\sqrt{2\pi}}$

$$Sol: \quad \frac{dy}{dx} = x^{\sin x}\left\{\frac{\sin x}{x} + \log x.\cos x\right\} + \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx}_{at\, x=\frac{\pi}{2}} = \left(\frac{\pi}{2}\right)^1 \left\{\frac{1}{\frac{\pi}{2}} + \log\frac{\pi}{2}.0\right\} + \frac{1}{2\sqrt{\frac{\pi}{2}}}$$

$$\therefore \frac{dy}{dx}_{at\, x=\frac{\pi}{2}} = 1 + \frac{1}{\sqrt{2\pi}}$$

MATHEMATICS

Ans a)

32) If $y = x^{\sin x} + (\sin x)^x$ then $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is

$a)\,1 \qquad b)\,\frac{\pi^2}{4} \qquad c)\,\frac{4}{\pi^2} \qquad d)\,\frac{\pi}{2}\log\frac{\pi}{2}$

$$Sol: \quad \frac{dy}{dx} = x^{\sin x}\left\{\frac{\sin x}{x} + \log x.\cos x\right\}$$

$$+ (\sin x)^x\left\{x.\frac{1}{\sin x}.\cos x + \log(\sin x)\right\}$$

$$\therefore \frac{dy}{dx}_{at\,x=\frac{\pi}{2}} = \left(\frac{\pi}{2}\right)^1\left\{\frac{1}{\frac{\pi}{2}} + \log\frac{\pi}{2}.0\right\} + (1)^{\frac{\pi}{2}}\left\{\frac{\pi}{2}.\frac{1}{1}.0 + \log 1\right\}$$

$$\therefore \frac{dy}{dx}_{at\,x=\frac{\pi}{2}} = 1$$

MATHEMATICS

Ans a)

33) If $x = e^{y+e^{y+........to\infty}}$, $x > 0$ then $\frac{dy}{dx}$ is

a) $\frac{1+x}{x}$ b) $\frac{1}{x}$ c) $\frac{1-x}{x}$ d) $\frac{x}{1+x}$

Sol : Given $x = e^{y+x}$

$$\Rightarrow \log x = (y+x)\log e \Rightarrow y = \log x - x$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{x} - 1 = \frac{1-x}{x}$$

MATHEMATICS

Ans c)

34) If $f(x) = be^{ax} + ae^{bx}$ then $f^{11}(0) =$

a) $2ab$

b) 0

c) ab

d) $ab(a+b)$

$Sol:$ $f^{1}(x) = ab.e^{ax} + ab\,e^{bx}$

$\Rightarrow f^{11}(x) = ab.e^{ax}.a + ab.e^{bx}.b$

$\Rightarrow f^{11}(0) = a^2b + ab^2 = ab(a+b)$

MATHEMATICS

Ans d)

35) If $\sqrt{r} = ae^{\theta.\cot\alpha}$ where 'a' and 'α' are real numbers then $\frac{d^2r}{d\theta^2}$ - $4r\cot^2\alpha$ is

$a)\,0$ $\quad b)\,1$ $\quad c)\,\frac{1}{r}$ $\quad d)\,r$

$Sol:$

$$\sqrt{r} = ae^{\theta\cot\alpha} \Rightarrow r = a^2.e^{2\cot\alpha.\theta}$$

$$\Rightarrow \frac{dr}{d\theta} = a^2.e^{2\cot\alpha.\theta}.2\cot\alpha$$

$$\Rightarrow \frac{d^2r}{d\theta^2} = 2a^2\cot\alpha.e^{2\cot\alpha.\theta}.2\cot\alpha$$

$$\Rightarrow \frac{d^2r}{d\theta^2} = 4\left(a^2.e^{2\cot\alpha.\theta}\right).\cot^2\alpha = 4r\cot^2\alpha$$

$$\Rightarrow \frac{d^2r}{d\theta^2} - 4r\cot^2\alpha = 0$$

Ans a)

MATHEMATICS

36) If $y = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \ldots\ldots\ldots$ then $\frac{d^2y}{dx^2} =$

a) $-y$ b) y c) x d) $-x$

Sol : w.k.t. $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots$

$$\therefore y = e^{-x} \Rightarrow \frac{dy}{dx} = -e^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = e^{-x} = y$$

MATHEMATICS

Ans b)

$$37)\ \text{If } f(x)=\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix} \text{ then } f^{11}(x)=$$

a) $6x^2$ b) $-6x^2$ c) $12x$ d) $-12x$

$$Sol: \quad f^1(x)=\begin{vmatrix} 1 & 2x & 3x^2 \\ 1 & 2x & 3x^2 \\ 0 & 2 & 6x \end{vmatrix}+\begin{vmatrix} x & x^2 & x^3 \\ 0 & 2 & 6x \\ 0 & 2 & 6x \end{vmatrix}+\begin{vmatrix} x & x^2 & x^3 \\ 1 & 2x & 3x^2 \\ 0 & 0 & 6 \end{vmatrix}$$

$$=6\left(2x^2-x^2\right)=6x^2$$

$$\therefore f^{11}(x)=12x$$

MATHEMATICS

Ans c)

38) If $y = 2^x$ then $\dfrac{d^2y}{dx^2} =$

$a)\ x(x-1)2^{x-2}$ $\qquad$ $b)\ 0$

$c)\ y(\log 2)^2$ $\qquad$ $d)\ -1$

$Sol:\quad Given\ y = 2^x$

$$\Rightarrow \frac{dy}{dx} = 2^x.\log 2$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2^x.(\log 2)^2 = y(\log 2)^2$$

MATHEMATICS

Ans c)

39) If $y = x + \tan x$ then $\cos^2 x \cdot y^{11} + 2x =$

a) x^2 b) $-x^2$ c) $2y^2$ d) $2y$

Sol : $\frac{dy}{dx} = 1 + \sec^2 x$

$$\Rightarrow \frac{d^2 y}{dx^2} = 2\sec x . \sec x . \tan x = \frac{2\tan x}{\cos^2 x}$$

$$\Rightarrow \frac{d^2 y}{dx^2} \times \cos^2 x = 2\tan x$$

$$\Rightarrow \cos^2 x . y^{11} = 2\tan x$$

$$\Rightarrow \cos^2 x . y^{11} + 2x = 2\tan x + 2x = 2y$$

MATHEMATICS

Ans d)

40) If $f(x)$ is function such that $f^{11}(x)+f(x)=0$ and $g(x)=[f(x)]^2+[f^1(x)]^2$ and $g(3)=8$ then $g(8)=$

a) 8 b) 3 c) 0 d) 5

Sol: Clearly $g^1(x)=2f(x).f^1(x)+2f^1(x).f^{11}(x)$

$$\Rightarrow g^1(x)=2f^1(x)\left[f(x)+f^{11}(x)\right]=0$$

$$\Rightarrow g(x)=k\,\forall x$$

$$\Rightarrow g(3)=k\Rightarrow 8=k$$

$$\therefore g(x)=8\,\forall x\Rightarrow g(8)=8$$

MATHEMATICS

Ans a)

41) If $x = 2\cos\theta + 3\sin\theta$, $y = 2\sin\theta - 3\cos\theta$ then $y.\frac{d^2y}{dx^2} + 1 =$

a) $y.\frac{dy}{dx}$ b) $-y.\frac{dy}{dx}$ c) $\left(\frac{dy}{dx}\right)^2$ d) $-\left(\frac{dy}{dx}\right)^2$

Sol : Clearly $x^2 + y^2 = 4 + 9 = 13$

$$\Rightarrow 2x + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow y.\frac{dy}{dx} = -x \Rightarrow y.\frac{d^2y}{dx^2} + \frac{dy}{dx}.\frac{dy}{dx} = -1$$

$$\Rightarrow y.\frac{d^2y}{dx^2} + 1 = -\left(\frac{dy}{dx}\right)^2$$

MATHEMATICS

Ans d)

42) If $(x+y)^{102} = x^{51}.\mathrm{y}^{51}$ then $\dfrac{d^2y}{dx^2} =$

$a)\,1$ $\quad b)\ \dfrac{-x^2}{y^2}$ $\quad c)\ \dfrac{x^2}{y^2}$ $\quad d)\ 0$

$$Sol: w.k.t.\ (x+y)^{m+n} = x^m.y^n \Rightarrow \frac{dy}{dx} = \frac{y}{x}$$

$$\therefore\ (x+y)^{102} = x^{51}.\mathrm{y}^{51} \Rightarrow \frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{d^2y}{dx^2} = \frac{x.\frac{dy}{dx} - y.1}{x^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{x.\frac{y}{x} - y}{x^2} = 0$$

MATHEMATICS

Ans d)

43) If $y = e^{\sqrt{x.\sqrt{x.\sqrt{x.\ldots\ldots\ldots}}}}$ $(x > 1)$ then $\frac{d^2y}{dx^2}$ at $x = \log_e 3 =$

$a)\ 1$ $\quad b)\ 2$ $\quad c)\ 3$ $\quad d)\ 4$

Sol : $\log y = \sqrt{x\sqrt{x\sqrt{x\ldots\ldots\ldots}}}$

$$\Rightarrow \log y = \sqrt{x.\log y}$$

$$\Rightarrow (\log y)^2 = x.\log y \Rightarrow \log_e y = x \Rightarrow y = e^x$$

$$\therefore \frac{dy}{dx} = e^x \Rightarrow \frac{d^2y}{dx^2} = e^x$$

$$\therefore \left.\frac{d^2y}{dx^2}\right|_{at\ x=\log_e 3} = e^{\log_e 3} = 3$$

MATHEMATICS

Ans c)

KEA

44) If $g(x)$ is the inverse of $f(x)$ and $f(\cos^2 x) = x$ then $g^{11}(0) =$

$a)\,1$ $\quad b)\,-1$ $\quad c)\,2$ $\quad d)\,-2$

Sol: $(fog)(x) = x \Rightarrow f(g(x)) = x$ and $f(\cos^2 x) = x$

$$\Rightarrow f^{-1}(x) = \cos^2 x$$

$$\Rightarrow g(x) = \cos^2 x$$

$$\therefore\ g^{1}(x) = 2\cos x.-\sin x = -\sin 2x$$

$$\therefore\ g^{11}(x) = -\cos 2x.2 = -2\cos 2x$$

$$\therefore\ g^{11}(0) = -2\cos 0 = -2$$

MATHEMATICS

Ans d)

45) If $y = (\sin^{-1}x)^2 - (\cos^{-1}x)^2$ then $y^{11}(1-x^2) =$

$a)\ 2xy$ $\quad b)\ 2xy^1$ $\quad c)\ xy^1$ $\quad d)\ xy$

$$Sol: \quad y^1 = 2\sin^{-1}x\frac{1}{\sqrt{1-x^2}} + 2\cos^{-1}x.\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}.y^1 = 2.\frac{\pi}{2} = \pi$$

$$\therefore \sqrt{1-x^2}.y^{11} + y^1\frac{1}{2\sqrt{1-x^2}} \times -2x = 0$$

$$\therefore (1-x^2).y^{11} - xy^1 = 0$$

$$\Rightarrow (1-x^2)y^{11} = xy^1$$

MATHEMATICS

Ans c)

46) If $f(x) = |\cos x|, x \in \left(\frac{\pi}{2}, \pi\right)$ then $f^1\left(\frac{3\pi}{4}\right) =$

a) $\frac{-1}{\sqrt{2}}$ b) $\frac{1}{\sqrt{2}}$ c) 1 d) $-\sqrt{2}$

Sol : When $\frac{\pi}{2} < x < \pi$ *then* $\cos x < 0$

$\therefore |\cos x| = -\cos x$

$\therefore f(x) = -\cos x \Rightarrow f^1(x) = \sin x$

$\Rightarrow f^1\left(\frac{3\pi}{4}\right) = \sin 135^0 = \frac{1}{\sqrt{2}}$

MATHEMATICS

Ans b)

47) If $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \sin^{-1}\left(\frac{\sqrt{x}-1}{\sqrt{x}+1}\right), \left|\sqrt{x}+1\right| \le \left|\sqrt{x}-1\right|$ then $\frac{d^2y}{dx^2} =$

a) 0 b) 1 c) $\frac{\pi}{2}$ d) $\frac{\pi^2}{4}$

Sol : Clearly $y = \sec^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right) + \operatorname{cosec}^{-1}\left(\frac{\sqrt{x}+1}{\sqrt{x}-1}\right)$

$$\Rightarrow y = \frac{\pi}{2} \Rightarrow \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{d^2y}{dx^2} = 0$$

MATHEMATICS

Ans a)

48) The value of 'c' in Rolle's theorem for the function $f(x) = x^3 - 3x$ in the interval $[0, \sqrt{3}]$ is

$a)\,1 \qquad b)\,-1 \qquad c)\,\frac{3}{2} \qquad d)\,\frac{1}{3}$

Sol : Clearly $f(x)$ is continuous in $[0, \sqrt{3}]$

and differtiable in $(0, \sqrt{3})$

$\Rightarrow f^1(c) = 0$

$\Rightarrow 3c^2 - 3 = 0$

$\Rightarrow c^2 = 1 \Rightarrow c = \pm 1$

$\therefore\ c = 1 \in [0, \sqrt{3}]$

MATHEMATICS

Ans a)

49) For the function $f(x) = x + \frac{1}{x},\ x \in [1, 3]$

the value of 'c' in M.V.T. is

$a) 1$ $b) \sqrt{3}$ $c) 2$ $d) 3$

Sol : Clearly $f(x)$ is continuous in $[1,3]$ are differtiable in $(1,3)$

$$\Rightarrow \frac{f(3) - f(1)}{3-1} = f^1(c)$$

$$\Rightarrow \frac{\left(3 + \frac{1}{3}\right) - (2)}{2} = 1 - \frac{1}{c^2} \Rightarrow \frac{\left(1 + \frac{1}{3}\right)}{2} = 1 - \frac{1}{c^2}$$

$$\Rightarrow \frac{2}{3} = \frac{c^2 - 1}{c^2} \quad \Rightarrow 2c^2 - 3c^2 + 3 = 0$$

$$\Rightarrow -c^2 + 3 = 0 \Rightarrow c = \sqrt{3} \in [1,3]$$

MATHEMATICS

Ans b)

50) If $f(x)=\frac{\sin x}{1!}-\frac{\sin^3 x}{3!}+\frac{\sin^5 x}{5!}-\ldots\ldots\ldots$ then $f^1(o)=$

$a)\,0$

$b)\,1$

$c)\,-1$

$d)$ does not exists

$Sol:$ $\quad w.k.t\ \sin x=\frac{x}{1!}-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots\ldots\ldots$

$\therefore \mathrm{f}(\mathrm{x})=\sin(\sin x)$

$\Rightarrow f^1(x)=\cos(\sin x).\cos x$

$\Rightarrow f^1(0)=\cos(\sin 0)\times\cos 0=1\times1=1$

MATHEMATICS

Ans b)