Spacecraft Dynamics and Control

Skyhook

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1. Introduction

Getting to space is hard! The major mode of space transportation available today are rockets and they are extremely inefficient. Rocket's inefficiency is due the fact that 90% of the payload during space transportation is fuel. Hence, we seek ideas for non-conventional modes of space transportation which primarily require less fuel and hence allow for more payloads to be transported when fully operational. The current project explores the possibility of using tethered satellite systems for transferring space shuttles from one orbit to another. To that end we consider a simplest candidate for a tethered satellite system, a dumbbell satellite 1. The satellite consists of three point masses m_1, m_2, and m_b all connected co-linearly with a rigid mass-less rod. The rigid mass-less rod along with the end masses m_1 and m_2 for the tether of the satellite where payloads are connected.

2. Dynamics Modeling

The system state is described by a six element vector comprising the skyhook centroid location $\mathbf{r} = (r, \theta)$ and orientation angle φ and their derivatives. For the purpose of modelling, we assume the tether to be massless and ideal.The endpoint locations are specified by the vectors $\mathbf{r}_1$ and $\mathbf{r}_2$, which are functions of the state vector and may be written as follows, where $\hat{\mathbf{b}}_1 = (\cos(\varphi), \sin(\varphi))$ is the skyhook orientation unit vector:

$$\mathbf{r}_i = \mathbf{r} \mp L_i \hat{\mathbf{b}}_1 \tag{1}$$

The $\hat{\mathbf{b}}_1$ is the unit vector normal to the axis joining all the masses.

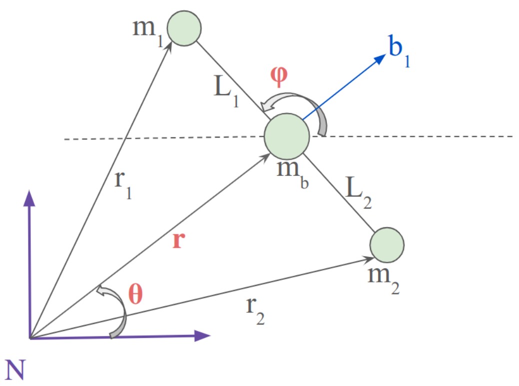

Figure 1. Schematic of the Dumbbell satellite.

In polar cordinates the acceleration of the satellite is given by:

$$\ddot{\mathbf{r}} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathbf{r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta} \tag{2}$$

$$F_r = -\left(\frac{GM_E}{r_1^2} m_1\right)\hat{\mathbf{r}}_1 \cdot \hat{\mathbf{r}} - \left(\frac{GM_E}{r_2^2} m_2\right)\hat{\mathbf{r}}_2 \cdot \hat{\mathbf{r}} - \left(\frac{GM_E}{r^2} m_b\right) \tag{3}$$

$$F_\theta = -\left(\frac{GM_E}{r_1^2} m_1\right)\hat{\mathbf{r}}_1 \cdot \hat{\theta} - \left(\frac{GM_E}{r_2^2} m_2\right)\hat{\mathbf{r}}_2 \cdot \hat{\theta} \tag{4}$$

$$\tau = GM_E\left[\frac{m_2 L_2}{r_2^3} - \frac{m_1 L_1}{r_1^3}\right] r \sin(\varphi - \theta) \tag{5}$$

Governing system of equations

$$\ddot{r} = r\dot{\theta}^2 - \frac{GM_E}{m_1 + m_2 + m_b}\left[D_1 \cos(\varphi - \theta) + D_2 + \frac{m_b}{r^2}\right] \tag{6}$$

$$\ddot{\theta} = \frac{1}{r}\left[-2\dot{r}\dot{\theta} + \frac{GM_E}{m_1 + m_2 + m_b}\left[\frac{m_1 L_1}{r_1^3} - \frac{m_2 L_2}{r_2^3}\right]\sin(\varphi - \theta)\right] \tag{7}$$

$$\ddot{\varphi} = \frac{GM_E}{I}\left[\left[\frac{m_1 L_1}{r_1^3} - \frac{m_2 L_2}{r_2^3}\right] r \sin(\varphi - \theta)\right] \tag{8}$$

$$D_1 = \left(\frac{m_1}{r_1^3} + \frac{m_2}{r_2^3}\right) \tag{9}$$

$$D_2 = \left(\frac{m_1 L_1}{r_1^3} + \frac{m_2 L_2}{r_2^3}\right) \tag{10}$$

$$I = m_1 L_1^2 + m_2 L_2^2 \tag{11}$$

3. Simulation

The governing system of equations are a three nonlinear, coupled, second order O.D.E. They are numerically integrated using "RK45" method for adaptive time stepping in Python. Various initial conditions were chosen for analyzing stability of the satellite. The rate of change along the radial direction ($\dot{r}$) was taken as zero for the initial condition.

The angular-rate of orbital motion ($\dot{\theta}$) is calculated considering the orbit to be a circular. Hence,

$$V_{c(0)} = \sqrt{\frac{GM_E}{r}} \tag{12}$$

$$\dot{\theta}_{(0)} = \frac{V_{c(0)}}{r} \tag{13}$$

The spin rate ($\dot{\varphi}$) was scaled based on the orbital angular rate. Below are some of the parameters used for our simulation studies.

Table 1. Simulation parameter data

Parameter	Values
m_1	500 kg
m_2	500 kg
L_1	10 m
L_2	10 m
$r_{(0)}$	$1.5R_E$
$\varphi_{(0)}$	$\left(\frac{\pi}{2}\right)$
$\dot{\varphi}_{(0)}$	1000 $\dot{\theta}_{(0)}$

Note: Parameters not mentioned in the table in by default considered to be zero.

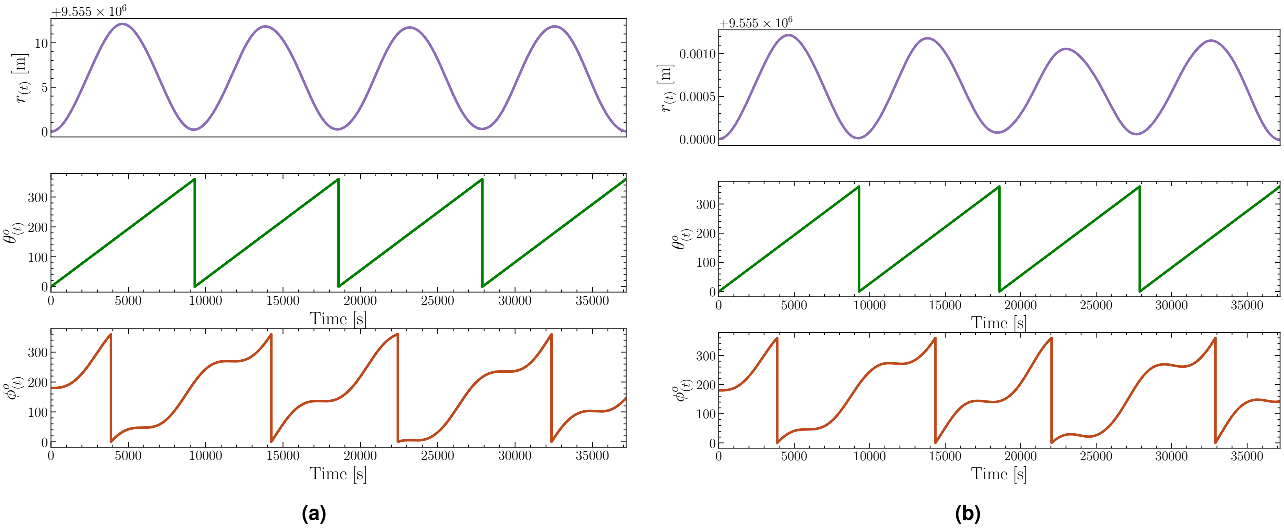

Figure 2. State of the dumbbell satellite for two different geometric parameter.**(a)** Length of satellite's tether is 1mm, **(b)** Length of satellite's tether is 1m.

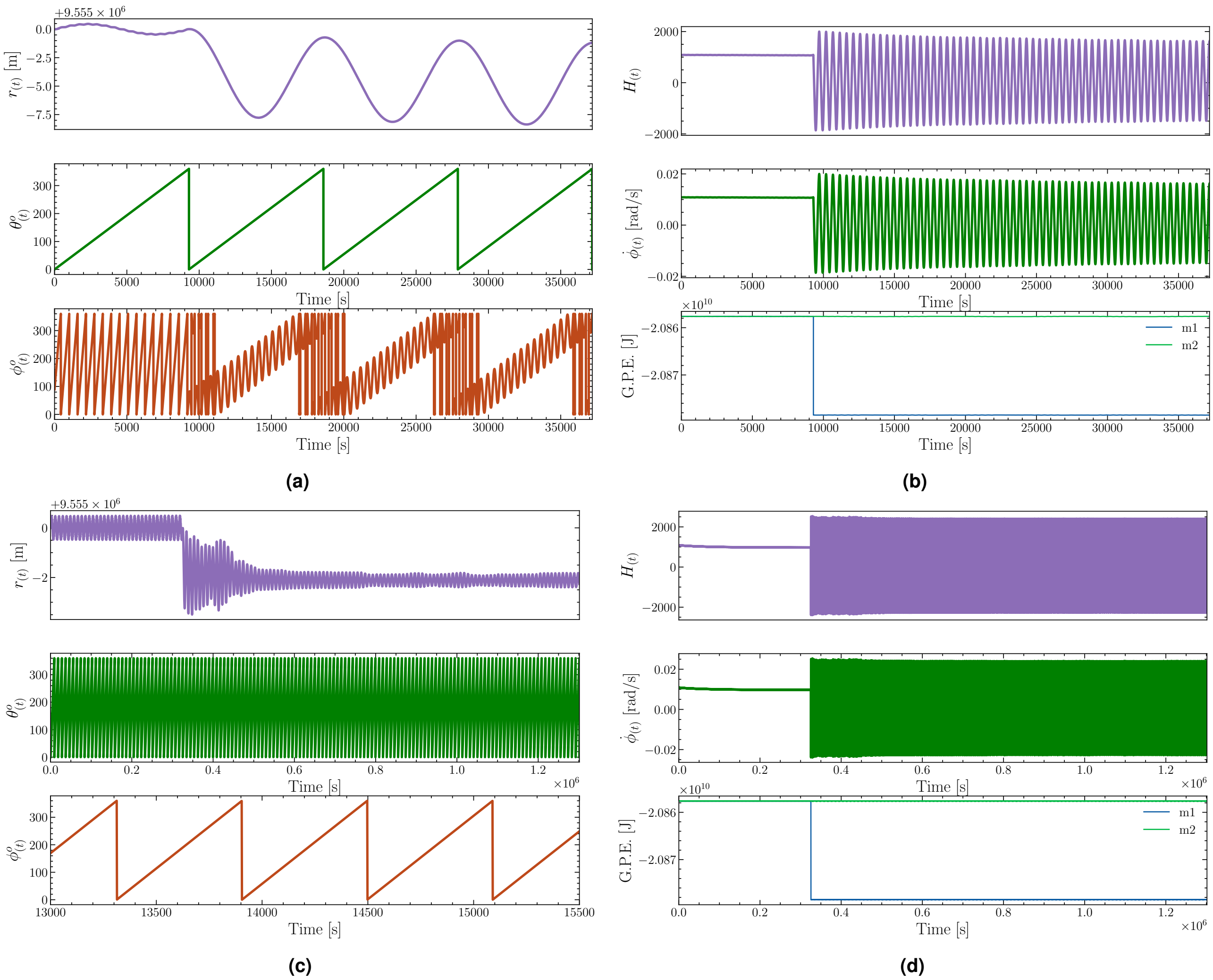

Figure 3. Unbalanced hooking: dumbbell satellite when a single mass is loaded/tethered to m_1.**(a)** Time series of each of the state variable, **(b)** The topmost plot represents the angular momentum of the satellite about the initial center of mass. The middle plot represents the spin rate of the satellite. The bottom plot represents the Gravitaional Potential Engergy of each mass m_1 and m_2. Only m_1 mass has decreased G.P.E. due to addition of mass.

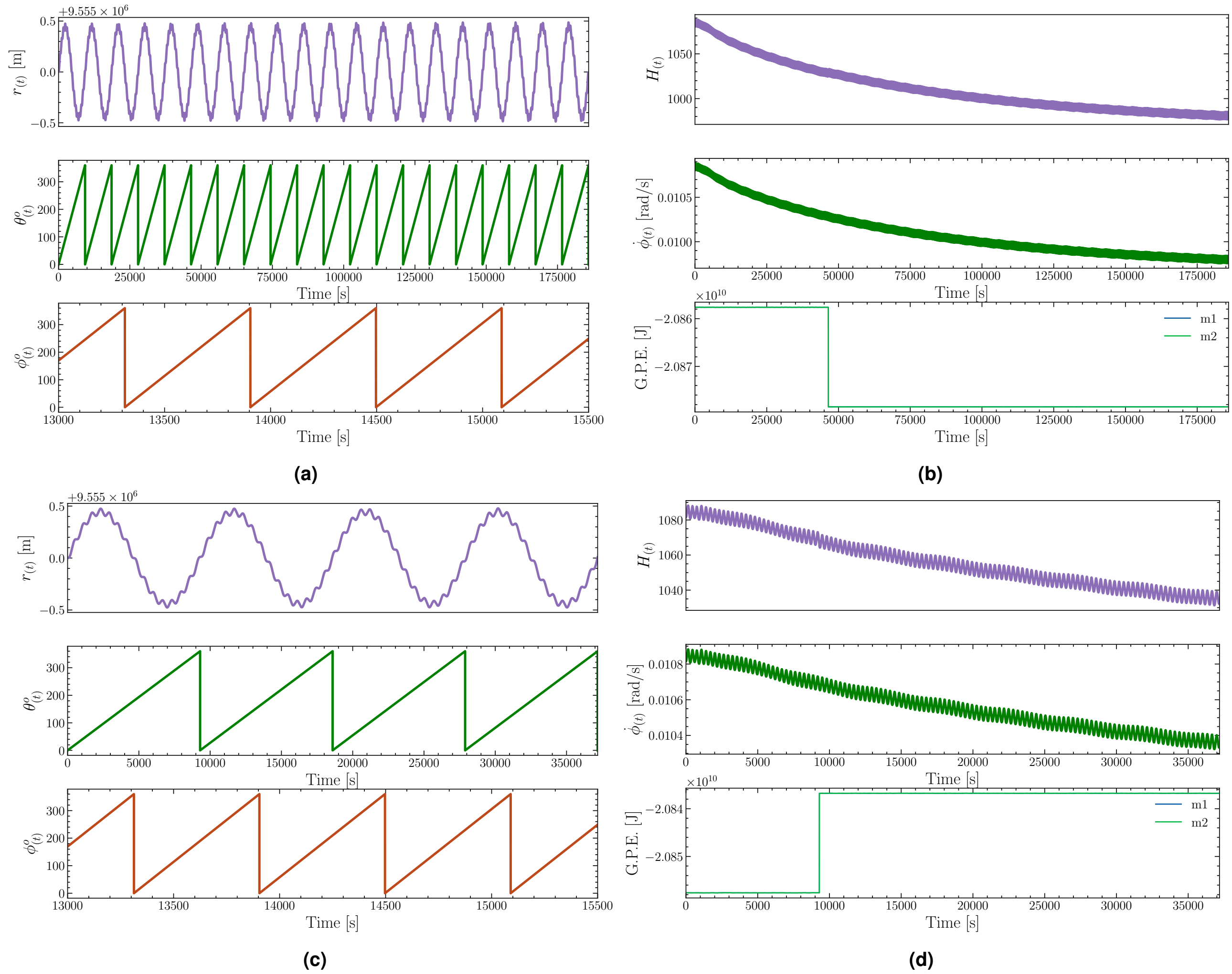

Figure 4. Balanced hooking: dumbbell satellite when two equal masses are loaded/unloaded to the satellite.**(a)** Time series of each of the state variable when two equal masses were added to the satellite, **(b)** The topmost plot represents the angular momentum of the satellite about the initial center of mass. The middle plot represents the spin rate of the satellite. The bottom plot represents the Gravitaional Potential Engergy of each mass m_1 and m_2. The G.P.E. of both the masses have decreased due to simultaneous addition, **(c)** Time series of each of the state variable when two equal masses were removed from the satellite, **(d)** The G.P.E. of both the masses have increased due to simultaneous addition.

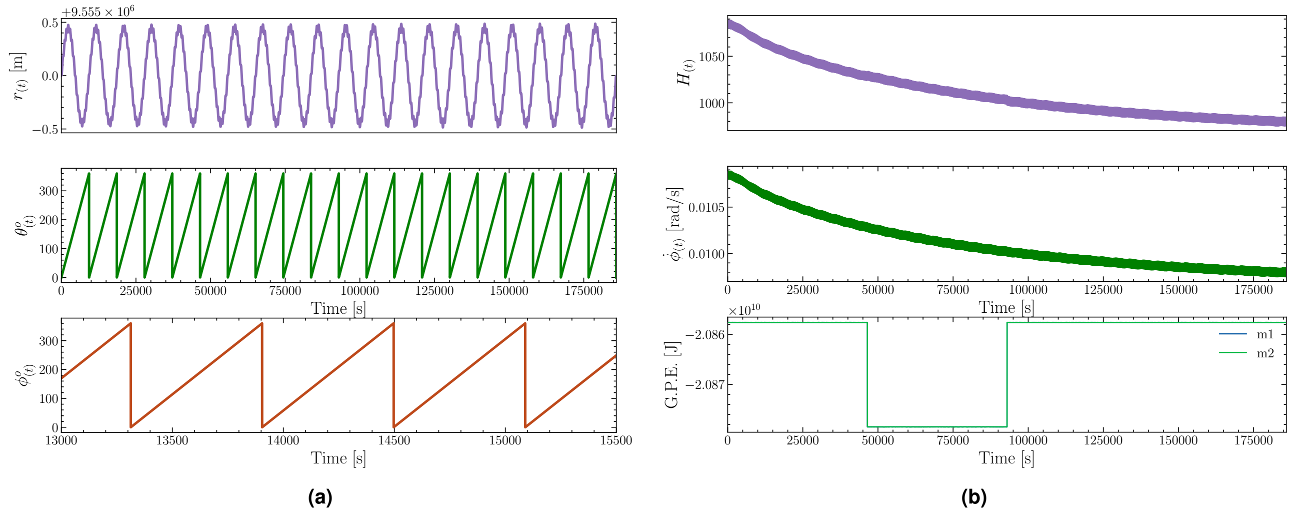

Figure 5. Balanced hooking with both addition and removal of mass: The satellite is added with two equal mass simultaneously and then the same amount of mass is removed after certain period of time.**(a)** State variables of the satellite system **(b)** The G.P.E. represents not only the time at which masses are added and removed but also how the satellite fully recovers after mass removal.