

# ASSIGNMENT

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**Question 20.2023** The probability of a person telling the truth is  $4/6$ . An unbiased die is thrown by the same person twice and the person reports that the numbers appeared in both the throws are same. Then the probability that actually the numbers appeared in both the throws are same is ?

**Solution:** Random variables on  $i \in \{1, 2\}$  defined as

Random Variable	Values	Description
$X_i$	$1 \leq X_i \leq 6$	number appeared on $i$ th die
$Y$	$\{0, 1\}$	person telling the truth or lie
$Z$	$-5 \leq Z \leq 5$	$X_1 - X_2$

$p_{X_1 X_2}(k, m)$  = Probability of getting a number  $k$  and  $m$  on  $X_1$  and  $X_2$  die respectively

$p_Y(0)$  = Probability that person telling the lie

$p_Y(1)$  = Probability that person telling the truth  
Consider

$$Z = X_1 - X_2 \quad (1)$$

$$p_{X_1 X_2}(k, m) = \begin{cases} \frac{1}{6} & \text{if } k = m \\ \frac{5}{6} & \text{otherwise} \end{cases} \quad (2)$$

$$p_Z(k) = \begin{cases} \frac{1}{6} & \text{if } k = 0 \\ \frac{5}{6} & \text{otherwise} \end{cases} \quad (3)$$

$$p_Y(i) = \begin{cases} \frac{2}{3} & \text{if } i = 1 \\ \frac{1}{3} & \text{if } i = 0 \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

Calculating the probability that actually the numbers appeared in both the throws are same i.e.,  $p(Y_i/X_1)$

$$\text{Let, } \Pr(s) = p_Z(0) \quad (5)$$

$$= \frac{1}{6} \quad (6)$$

$$\Pr(s') = 1 - \Pr(s) \quad (7)$$

$$= \frac{5}{6} \quad (8)$$

By Baye's theorem,

$$\Pr(Y = 1/s) = \frac{\Pr(s \cap Y = 1)}{\Pr(s \cap Y = 1) + \Pr(s' \cap Y = 0)} \quad (9)$$

Since  $X_i$  and  $Y$  are independent events

$$\Pr(Y = 1/s) = \frac{\Pr(s)p(Y_1)}{\Pr(s)p(Y_1) + \Pr(s')p(Y_0)} \quad (10)$$

$$= \frac{\left(\frac{1}{6}\right)\left(\frac{2}{3}\right)}{\left(\frac{1}{6}\right)\left(\frac{2}{3}\right) + \left(\frac{5}{6}\right)\left(\frac{1}{3}\right)} \quad (11)$$

$$= \frac{\left(\frac{2}{18}\right)}{\left(\frac{7}{18}\right)} \quad (12)$$

$$= \frac{2}{7} \quad (13)$$

$$\approx 0.286 \quad (14)$$