NCERT-2

EE22BTECH11016 - Ch.Yashwanth

Question 12.13.6.11 In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

Solution: To solve this problem using the binomial distribution, we can first consider each throw of the die as a Bernoulli trial. In each trial, the man either wins a rupee (when he rolls a six) or loses a rupee (for any other number). Let's define the random variable:

$$X = \begin{cases} 1 & \text{if he wins a rupee (rolls a six)} \\ -1 & \text{if he loses a rupee (other than six)} \end{cases}$$

The probability of winning a rupee (rolling a six) in a single trial is denoted by p, and the probability of losing a rupee (rolling any other number) is denoted by q. Since it's a fair die,

$$p = \frac{1}{6} \quad and \quad q = \frac{5}{6}$$

Now, the man will continue to throw the die until he rolls a six. This can take 1, 2, or 3 throws (because he decides to quit as soon as he gets a six). We can calculate the expected value of the amount he wins/loses for each of these cases:

1. If he wins on the first throw, then he gets one rupee so overall gain = 1 (probability of success, p):

$$E(X_1) = 1 \cdot p = \frac{1}{6}$$

2. If he wins on the second throw, then he loses a rupee in first throw and gains a rupee in second throw so overall gain = 0 (probability of failure on the first throw, q, and then success on the second throw, p):

$$E(X_2) = (-1+1) \cdot q \cdot p = 0$$

3. If he wins on the third throw, then he loses two rupees in first two throws and gain a rupee in third throw so overall gain = -1 (probability of failure on the first two throws, q^2, and then success on the third throw, p):

$$E(X_3) = (-1-1+1) \cdot q^2 \cdot p = -\left(\frac{5}{6}\right) \cdot \left(\frac{5}{6}\right) \cdot \left(\frac{1}{6}\right) = -\frac{25}{216}$$

4. If he loses in all three throws, then he loses three rupees so overall gain = -3 (probability of failure on the first two throws, q^3):

$$E(X_4) = (-1-1-1) \cdot q^3 = -3 \cdot \left(\frac{5}{6}\right) \cdot \left(\frac{5}{6}\right) \cdot \left(\frac{5}{6}\right) = -\frac{375}{216}$$

Now, let's calculate the expected value of the amount he wins/loses in total:

Overall Expected Value (E) = $E(X_1)$ + $E(X_2)$ + $E(X_3)$ + $E(X_4)$

$$E = \frac{1}{6} + 0 + \left(-\frac{25}{216}\right) + \left(-\frac{375}{216}\right) \approx -1.685$$

So, the expected value of the amount he wins/loses is approximately -1.685 rupees. This means, on average, he is expected to lose about 1.685 rupees when playing this game.