ASSIGNMENT

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Question 20.2023 The probability of a person telling the truth is 4/6 . An unbiased die is thrown by the same person twice and the person reports that the numbers appeared in both the throws are same. Then the probability that actually the numbers appeared in both the throws are same is ?
Solution: Random variables defined as

Random Variable	Values	Description
X_i	$i \in \{1,2\}$	getting a number on ith die
Y_i	$i \in \{0,1\}$	person telling the truth or lie

$p_{X_1X_2}(k,m)$ = Probablity of getting a number k and m on X_1 and X_2 die respectively
$p(Y_0)$ = Probability that person telling the lie
$p(Y_1)$ = Probability that person telling the truth

$$p_{X_1X_2}(k,m) = \begin{cases} \frac{1}{6} = & \text{if } k = m \\ \frac{5}{6} & \text{otherwise} \end{cases} \tag{1}$$

$$p(Y_i) = \begin{cases} \frac{2}{3} & \text{if } i = 1 \\ \frac{1}{3} & \text{if } i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Calculating the probability that actually the numbers appeared in both the throws are same i.e., $p(Y_i/X_1)$

$$\text{Let, } p(s) = p_{X_1X_2}(k,m); \text{when } k = m$$

$$= \frac{1}{6} \tag{3}$$

$$p(s^-) = 1 - p(s)$$

$$= \frac{5}{6} \tag{4}$$

By Baye's theorem,

$$p(Y_i/X_1) = \frac{p(s \cap Y_1)}{p(s \cap Y_1) + p(s^- \cap Y_0)} \tag{5}$$

Since X_i and Y_i are independent events

$$p(Y_i/X_1) = \frac{p(s)p(Y_1)}{p(s)p(Y_1) + p(s^-)p(Y_0)} \tag{6}$$

$$= \frac{\left(\frac{1}{6}\right)\left(\frac{2}{3}\right)}{\left(\frac{1}{6}\right)\left(\frac{2}{3}\right) + \left(\frac{5}{6}\right)\left(\frac{1}{3}\right)} \tag{7}$$

$$= \frac{\left(\frac{2}{18}\right)}{\left(\frac{7}{18}\right)} \tag{8}$$

$$= \frac{2}{7} \tag{9}$$

$$\approx 0.286 \tag{10}$$