ASSIGNMENT

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Question 12.13.6.11 In a game, a man wins a rupee for a six and loses a rupee for any other number when a fair die is thrown. The man decided to throw a die thrice but to quit as and when he gets a six. Find the expected value of the amount he wins / loses.

Solution: For $i \in \{1, 2, 3\}$. Random variables defined as

Random Variable	Values	Description
X_i	$\{-1, 1\}$	Money recieved on "i^{th}" roll

$$p_{X_i}(k) = \begin{cases} \frac{1}{6} & \text{if } k = 1 \\ \frac{5}{6} & \text{if } k = -1 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

$$E(X_i) = \sum_{-1}^{1} k p_{X_i}(k) \tag{2}$$

$$= (-1)\frac{5}{6} + 0 + (1)\frac{1}{6} \tag{3}$$

$$= -\frac{4}{6} \tag{4}$$

Calculating the expected value

$$E(X) = \text{Expected value of } X_1(\text{if } p_{X_1}(k) = \frac{1}{6}) + \text{Expected value of } X_2(\text{if } p_{X_1}(k) = \frac{5}{6}) + \text{Expected value of } X_3(\text{if } p_{X_1}(k) = p_{X_2}(k) = \frac{5}{6}) \tag{5}$$

$$E(X) = E(X_1) + p_{X_1}(-1) \cdot E(X_2) + p_{X_1}(-1) \cdot p_{X_2}(-1) \cdot E(X_3) \tag{6}$$

$$E(X) = E(X_1) + \frac{5}{6} \cdot E(X_2) + \frac{25}{36} \cdot E(X_3) \tag{7}$$

$$= \left(-\frac{4}{6}\right) + \frac{5}{6}\left(-\frac{4}{6}\right) + \frac{25}{36}\left(-\frac{4}{6}\right) \tag{8}$$

$$= \left(-\frac{4}{6}\right)\left(\frac{36 + 30 + 25}{36}\right) \tag{9}$$

$$= \left(-\frac{364}{216}\right) \tag{10}$$

$$\approx -1.685 \tag{11}$$