ASSIGNMENT

EE22BTECH11016-Chinthalapudi Yashwanth*

Question 20.2023 The probability of a person telling the truth is 4/6 . An unbiased die is thrown by the same person twice and the person reports that the numbers appeared in both the throws are same. Then the probability that actually the numbers appeared in both the throws are same is ?
Solution: Random variables on $i \in \{1, 2\}$ defined as

Random Variable	Values	Description
X_i	$1 \le X_i \le 6$	number appeared on ith die
Y	$\{0, 1\}$	person telling the truth or lie

$p_Y(0)$ = Probability that person telling the lie
$p_Y(1)$ = Probability that person telling the truth
Consider

$$Z = X_1 - X_2 \tag{1}$$

Z	0	$\neq 0$
$\Pr(Z)$	$\frac{1}{6}$	$\frac{5}{6}$

$$p_Y(i) = \begin{cases} \frac{2}{3} & \text{if } i = 1 \\ \frac{1}{3} & \text{if } i = 0 \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

Given in the question that the person reports that the numbers appeared in both the throws are same. the probability that actually the numbers appeared in both the throws are same, that is simply, the probability of person's truth given that numbers on both dice are same that is, $\Pr(Y = 1/Z = 0)$.

$$\Pr(Y = 1/Z = 0) = \frac{\Pr((Z = 0).(Y = 1))}{\Pr(Z = 0)} \tag{3}$$

Since X_i and Y are independent events

$$\Pr(Y = 1/Z = 0) = \frac{\Pr(Z = 0)p_Y(1)}{\Pr(Z = 0)} \tag{4}$$

$$= \frac{\left(\frac{1}{6}\right)\left(\frac{2}{3}\right)}{\left(\frac{1}{6}\right)} \tag{5}$$

$$= \frac{2}{3} \tag{6}$$

$$\approx 0.667 \tag{7}$$