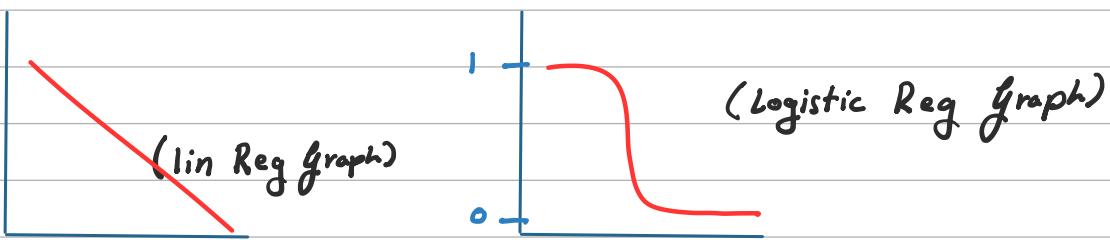


04 - Logistic Regression



To solve Classification problem $\xrightarrow{\quad}$ Binary Classification
Multiclass Classification

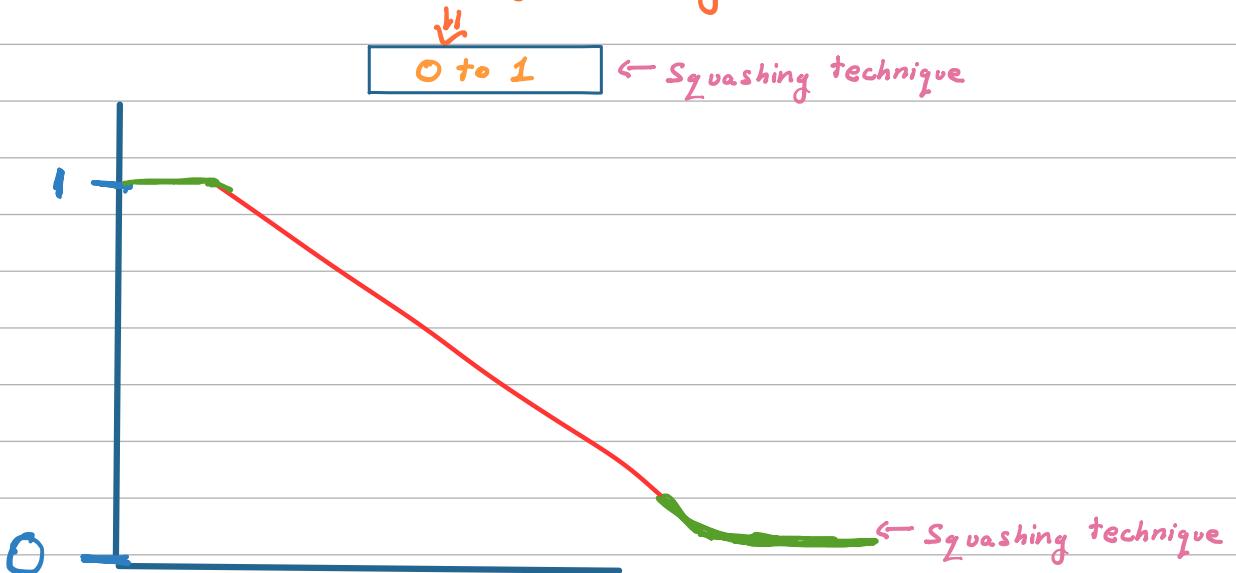
Can we solve this classification problem using Regression?

No, Because when new outliers is added to the data the Best fit line changes and the value might give >1 or <0 which may be inaccurate

Why?

- ① Best fit line changes because of outliers \leftarrow Prediction goes wrong
- ② The outcomes >1 and <0

To solve this problem we use logistic Regression



$$z = \boxed{h_{\theta}(x) = \theta_0 + \theta_1 x_1} \rightarrow \text{Best fit Line}$$

↓

[sigmoid Activation function]

↓

0 to 1

$$\sigma = \frac{1}{1 + e^{-z}} \Rightarrow 0 \text{ to } 1$$

$$h_{\theta}(x) = \sigma(\theta_0 + \theta_1 x_1)$$

We use sigmoid function so that we can squash the line.

$$h_{\theta}(x) = \frac{1}{1 + e^{-z}}$$

$$z = \theta_0 + \theta_1 x_1$$

logistic Regression

Linear Regression Cost function

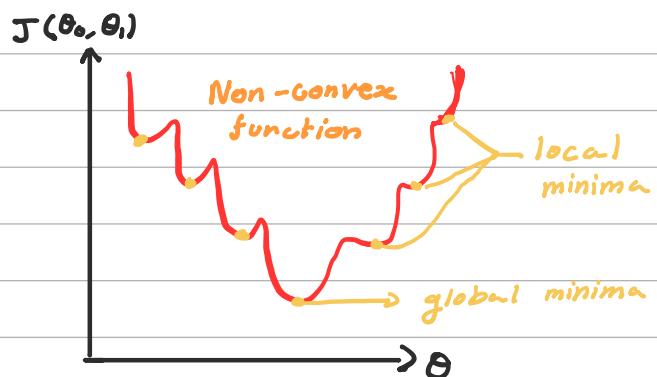
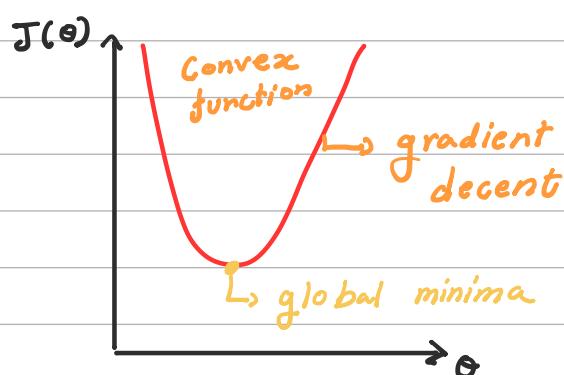
$$J(\theta_0, \theta_1) = \frac{1}{n} \sum_{i=1}^n (y_i - h_{\theta}(x_i))^2$$

Logistic Regression Cost function

$$J(\theta_0, \theta_1) = \frac{1}{n} \sum_{i=1}^n (y_i - h_{\theta}(x_i))^2$$

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1$$

$$h_{\theta}(x) = \frac{1}{1 + e^{-z}} = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x_1)}}$$



Non-Convex function: Function which may have many local minima is known as Non-Convex function.

Note: Due to many Local minima present we don't use the the Logistic Regression cost function so we use Log Loss cost fun

Log Loss

$$J(\theta_0, \theta_1) = \begin{cases} -\log(h_{\theta}(x)) & \text{if } y=1 \\ -\log(1-h_{\theta}(x)) & \text{if } y=0 \end{cases}$$

$$h_{\theta}(x) = \frac{1}{1+e^{-z}} = \frac{1}{1+e^{-(\theta_0 + \theta_1 x)}}$$

$$J(\theta_0, \theta_1) = -y \log(h_{\theta}(x)) - (1-y) \log(1-h_{\theta}(x)) \Rightarrow \text{convex function}$$

condition 1:

if $y=1$

$$= -\log(h_{\theta}(x)) \Rightarrow y=1$$

if $y=0$

$$\Rightarrow -\log(1-h_{\theta}(x))$$

Logistic Regression with Regularization Parameters

cost function

$$J(\theta_0, \theta_1) = -y \log(h_{\theta}(x)) - (1-y) \log(1-h_{\theta}(x))$$

$$h_{\theta}(x) = \frac{1}{1 + e^{-(\theta_0 + \theta_1 x)}}$$

$$J(\theta_0, \theta_1) = \begin{cases} -\log(h_{\theta}(x)) & \text{if } y=1 \\ -\log(1-h_{\theta}(x)) & \text{if } y=0 \end{cases}$$

Reduce Overfitting
↓

$$J(\theta_0, \theta_1) = -y \log(h_{\theta}(x)) - (1-y) \log(1-h_{\theta}(x)) + \lambda_2 \text{ Regularization}$$

$$J(\theta_0, \theta_1) = -y \log(h_{\theta}(x)) - (1-y) \log(1-h_{\theta}(x)) + \lambda_1 \text{ Regularization} \Leftarrow \text{feature selection}$$

$$J(\theta_0, \theta_1) = -y \log(h_{\theta}(x)) - (1-y) \log(1-h_{\theta}(x)) + \lambda_2 \text{ Reg} + \lambda_1 \text{ Reg} \quad (\text{Elastic net})$$

λ_2 Regularization \Rightarrow Reduce Overfitting

$$J(\theta_0, \theta_1) = -y \log(h_{\theta}(x)) - (1-y) \log(1-h_{\theta}(x)) + \lambda \sum_{i=1}^n (\text{slope})^2$$

λ_1 Regularization \Rightarrow feature Selection $\lambda = \text{hyperplane}$

$$J(\theta_0, \theta_1) = -y \log(h_{\theta}(x)) - (1-y) \log(1-h_{\theta}(x)) + \lambda \sum_{i=1}^n |\text{slope}|$$

Elastic Net

$$J(\theta_0, \theta_1) = -y \log(h_{\theta}(x) - (1-y)) + \log(1 - h_{\theta}(x)) + \lambda_1 \sum_{i=1}^n (slope_i)^2 + \lambda_2 \sum_{i=1}^n |slope_i|$$