Time Series Analysis using R

In a Nutshell

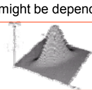
dr. J.J.M. Rijpkema
Eindhoven University of Technology,
dept. Mathematics & Computer Science
P.O.Box 513, 5600 MB Eindhoven, NL

 j.j.m.rijpkema@tue.nl

Stochastic Processes:

$$\mathbf{X}_{\{t\}} = \{\cdots, X_{t_1}, \cdots, X_{t_2}, \cdots\}$$

Stochastic Process | Individual stochasts, that might be dependent



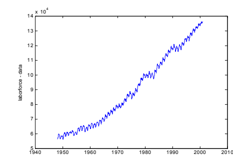
Data Collection

Model Fit

- Model Identification
- Parameter Estimation
- Model Validation

Model Use

Realization



Overview

- **Introduction & Preliminaries**
- **Exploratory Data Analysis**
  - Time Sequence Plot
  - (Partial) Autocorrelation Function
- **Exponential Smoothing Methods**
  - Simple Exponential Smoothing
  - Holt and Holt-Winters Models
- **ARMA Models**
  - The Bare Essentials
  - Model Fitting
- **More Time Series Modeling**

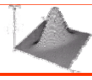
Introduction & Preliminaries

Stochastic Processes:

$$\mathbf{X}_{\{t\}} = \{\cdots, X_{t_1}, \cdots, X_{t_2}, \cdots\}$$

Stochastic Process

Individual stochasts, that might be dependent



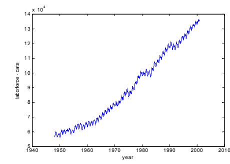
Data Collection

Model Fit

- Model Identification
- Parameter Estimation
- Model Validation

Model Use

Realization

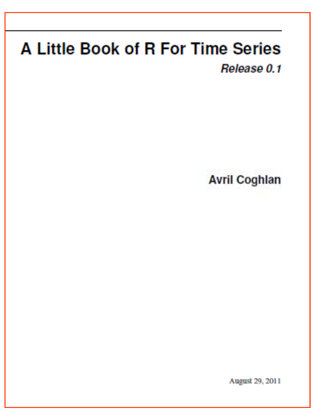


Relevant Models?

2IS55 Restriction

Exploratory Data Analysis?

- References:

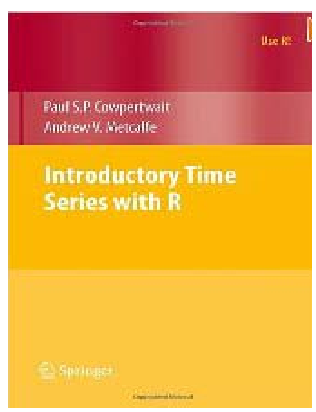


A Little Book of R For Time Series

Release 0.1

Avril Coghlan

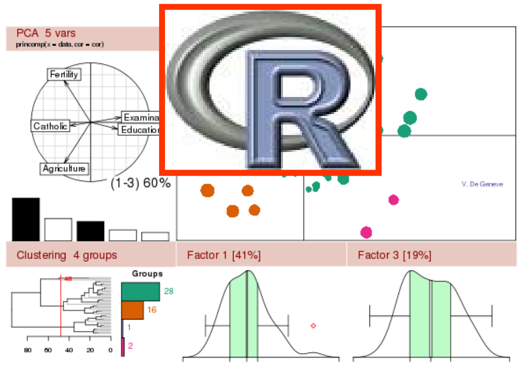
August 29, 2011



http://a-little-book-of-r-for-time-series.readthedocs.org/

Coghlan, Avril, *A Little Book of R for Time Series*,
Cowpertwait, Paul S.P. et al., *Introductory Time Series with R*, chapters 1& 2

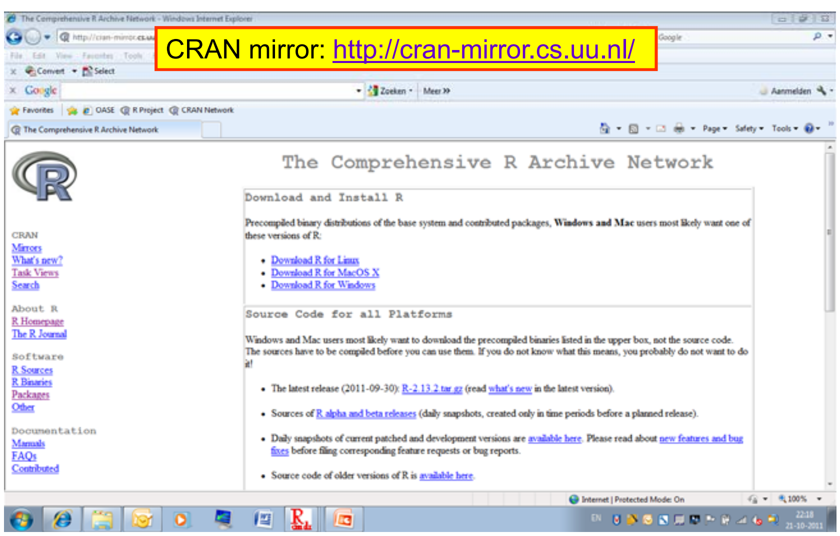
- Software:



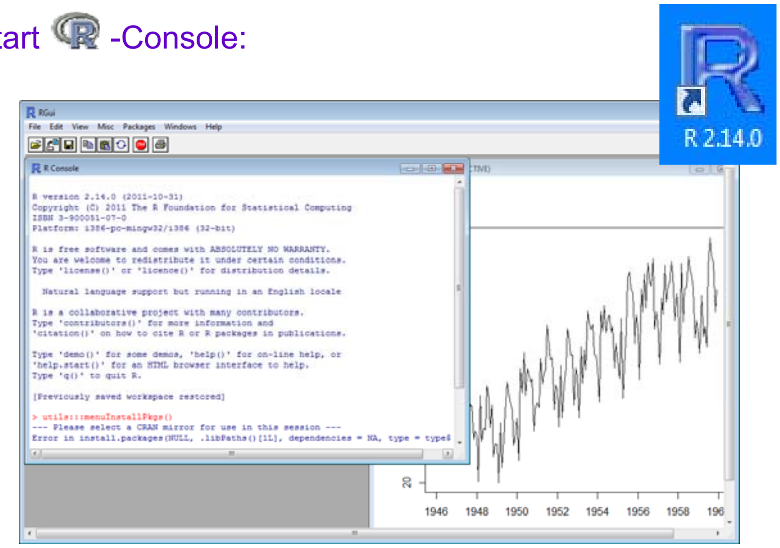
http://www.r-project.org/

Metaphor:

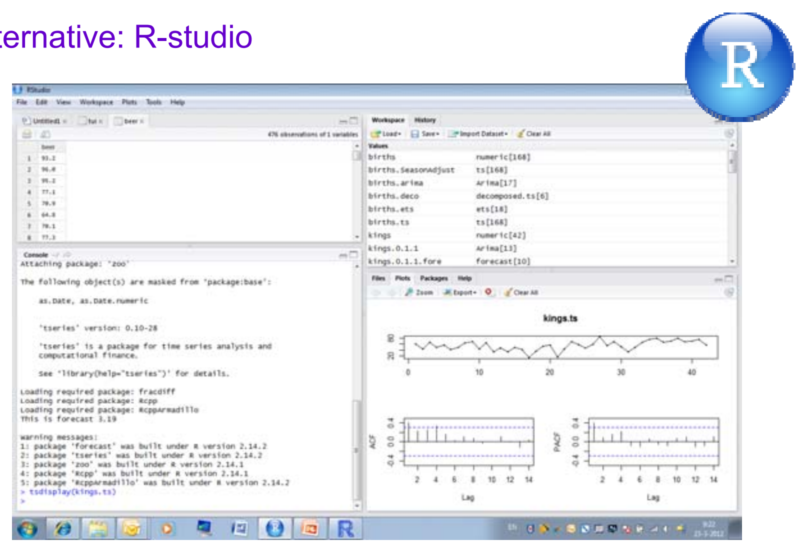
- Download and Install R :



- Start R -Console:

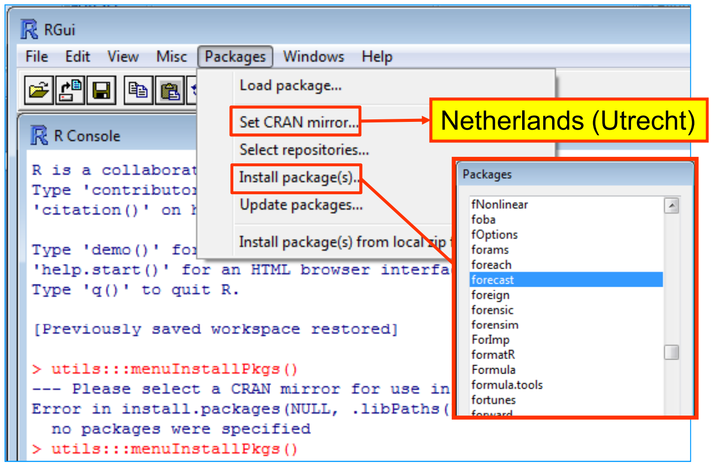


- Alternative: R-studio

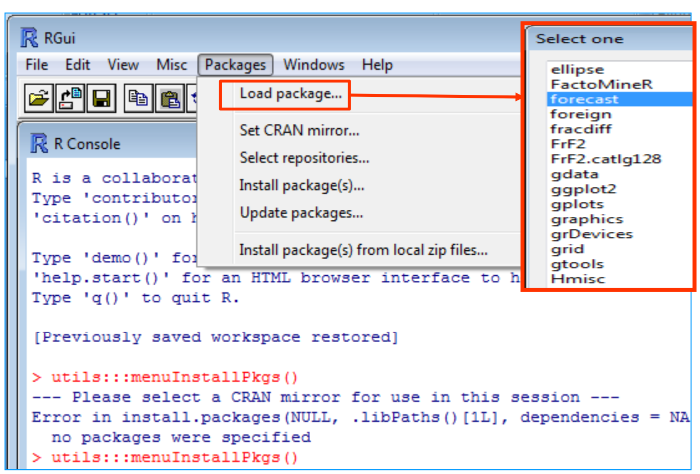


http://rstudio.org/

- Install package 'forecast' (& 'tseries'):



- Load package 'forecast' & 'tseries':




```
> library(forecast)
```

Exploratory Data Analysis

- Example: Age of Death of 42 Successive Kings of England


```
> load("D:/..../kings.RData")
```

- **General Properties:**

```
> class(kings)
[1] "numeric"
```

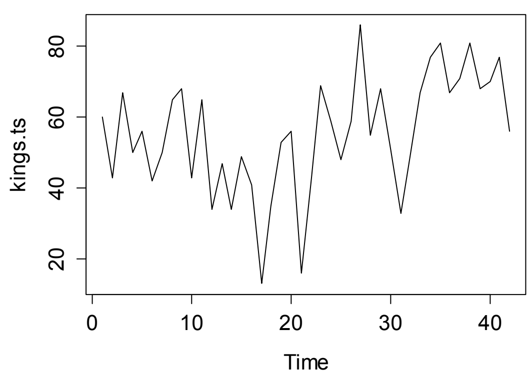
```
> kings
[1] 60 43 67 50 56 42 50 65 68 43 65 34 47 34 49 41 13 35 53 56 16 43
[23] 69 59 48 59 86 55 68 51 33 49 67 77 81 67 71 81 68 70 77 56
```

- **Conversion to time series object:**

```
> kings.ts <- ts(kings)
> kings.ts

Time Series: Start = 1 End = 42 Frequency = 1
[1] 60 43 67 50 56 42 50 65 68 43 65 34 47 34 49 41 13 35 53 56 16 43
[23] 69 59 48 59 86 55 68 51 33 49 67 77 81 67 71 81 68 70 77 56
```


- Time Sequence Plot:


```
> plot(kings.ts)
```



Exploratory Data Analysis
- Main Properties:
  - Trend?
  - Cyclic Variation?
  - Irregular Variation?
- Sudden Changes?
- Outliers?
- Missing Values?

- Autocorrelation Function:

Are successive observations related ?

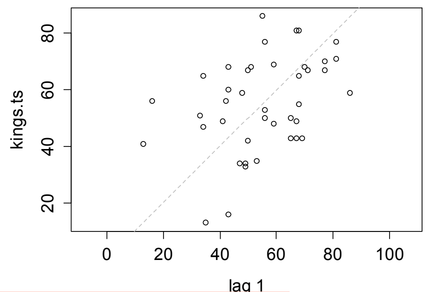
- Scatterplot & Sample Correlation Coefficient of $\{(x_t, x_{t-1})\}$

```
> lag.plot(kings.ts,do.lines=F)
```

$$r_1 \approx \frac{\sum (x_t - \bar{x})(x_{t-1} - \bar{x})}{\sum (x_t - \bar{x})^2}$$

Approximation !

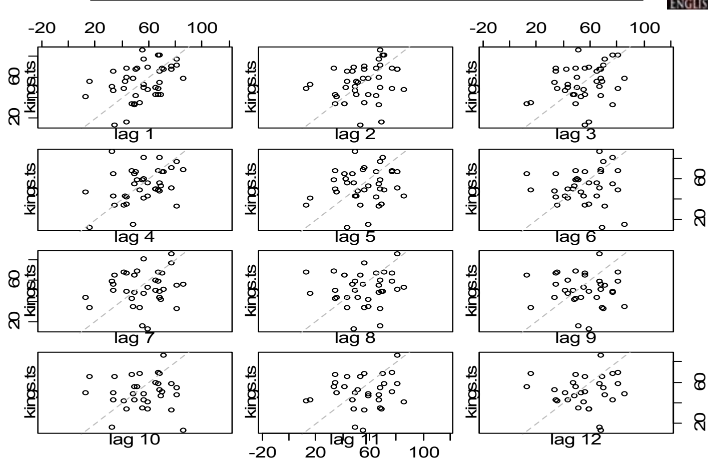
r=0.4



Autocorrelation Coefficient at time-lag 1

- Generalisation: $r_k \approx \frac{\sum (x_t - \bar{x})(x_{t-k} - \bar{x})}{\sum (x_t - \bar{x})^2}$

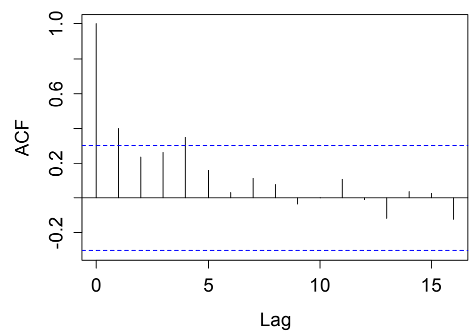

```
> lag.plot(kings.ts,lags=12,do.lines=F)
```



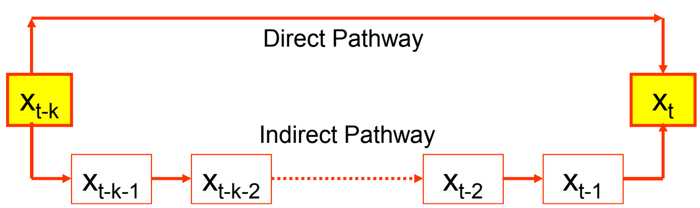
- Autocorrelation Function:


```
> acf(kings.ts)$acf

         [,1]
[1,] 1.000000000
[2,] 0.400584149
[3,] 0.238081338
[4,] 0.259547258
[5,] 0.347780525
[6,] 0.160923687
[7,] 0.031201729
[8,] 0.115411950
[9,] 0.078385052
[10,] -0.036221926
[11,] -0.001138564
[12,] 0.110712654
[13,] -0.010138296
[14,] -0.116699180
[15,] 0.034735266
[16,] 0.025070163
[17,] -0.119576411
```

```
> acf(kings.ts)
```



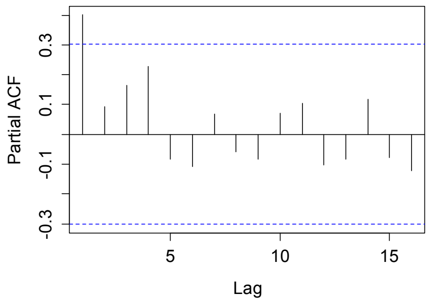
- Partial Autocorrelation:



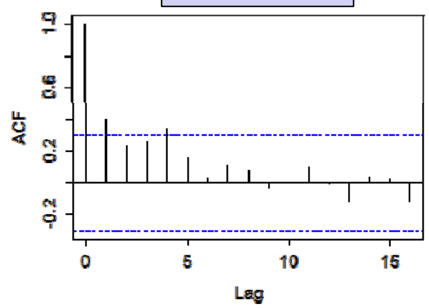
- Autocorrelation Function:
  - Measure for the **overall** correlation between $\{x_{t-k}\}$ and $\{x_t\}$
  - Both through the direct and the indirect pathway !
- Partial Autocorrelation Function:
  - Measure for the **direct** correlation between $\{x_{t-k}\}$ and $\{x_t\}$
  - Indirect pathway corrected for !

- (Partial) Autocorrelation Function:

```
> pacf(kings.ts)
```

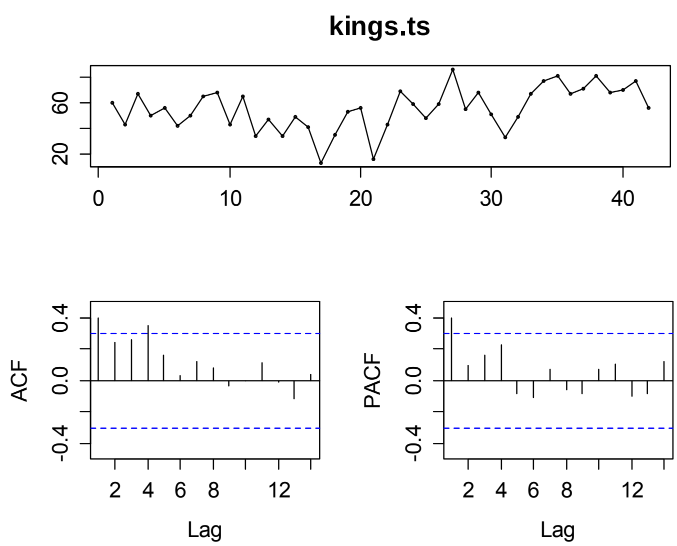


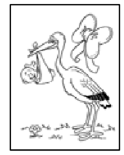

```
> acf(kings.ts)
```



- EDA-Summary:

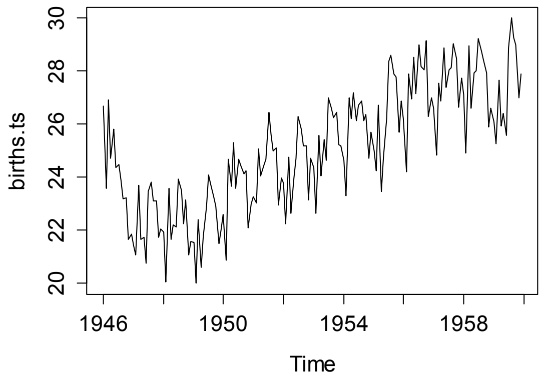
```
> tsdisplay(kings.ts)
```





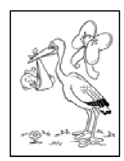
- Example: Births per Month in New York City

```
> load("D:/..../births.RData")
> births.ts <- ts(births,start=c(1946,1),frequency=12)
> plot(births.ts)
```



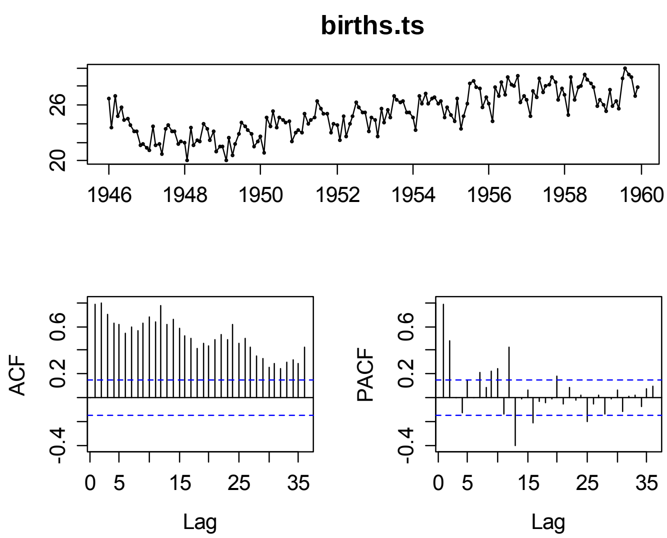
Exploratory Data Analysis

- Main Properties:
  - Trend?
  - Seasonal Variation?
  - Cyclic Variation?
  - Irregular Variation?
- Sudden Changes?
- Outliers?
- Missing Values?



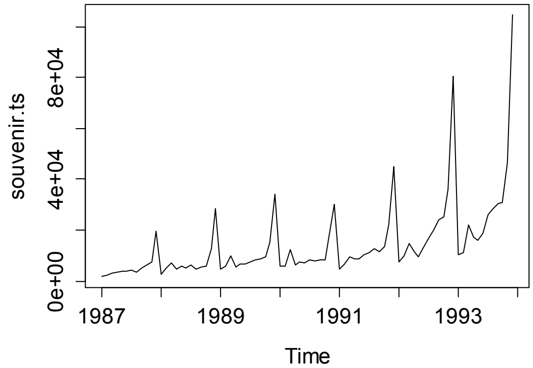
- EDA-Summary:

```
tsdisplay(births.ts)
```



• Example: Monthly Sales for Australian Souvenir Shop

```
> load("D:/..../souvenir.RData")
> souvenir.ts <- ts(souvenir,start=c(1987,1),frequency=12)
> plot(souvenir.ts)
```

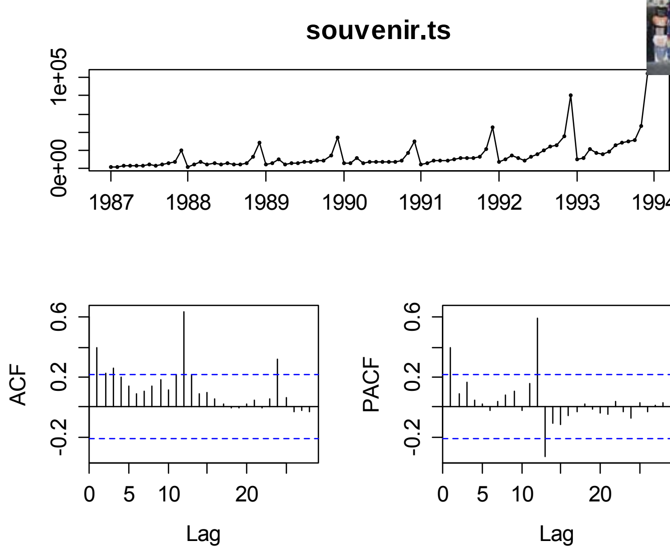


Exploratory Data Analysis

- Main Properties:
  - Trend?
  - Seasonal Variation?
  - Cyclic Variation?
  - Irregular Variation?
- Sudden Changes?
- Outliers?
- Missing Values?

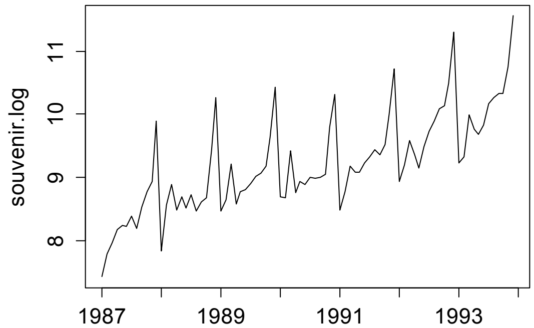
• EDA-Summary:

```
tsdisplay(souvenir.ts)
```



- Time Sequence Plot after Log-Trafo:

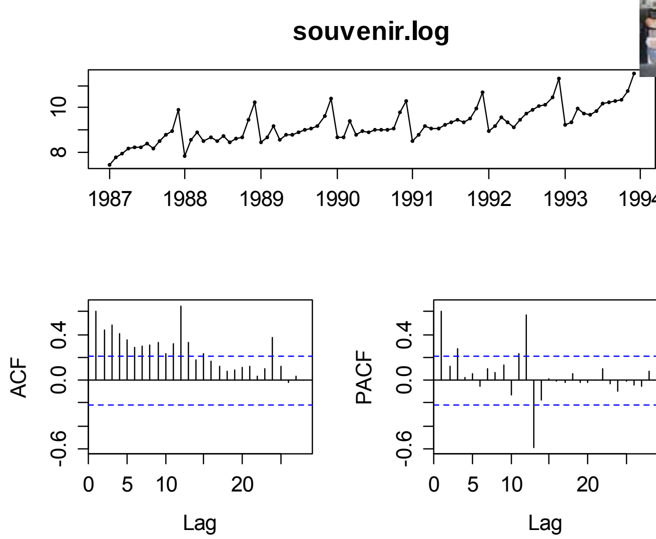
```
> souvenir.log <- log(souvenir.ts)
> plot(souvenir.log)
```



Exploratory Data Analysis
- Main Properties:
  - Trend?
  - Seasonal Variation?
  - Cyclic Variation?
  - Irregular Variation?
- Sudden Changes?
- Outliers?
- Missing Values?

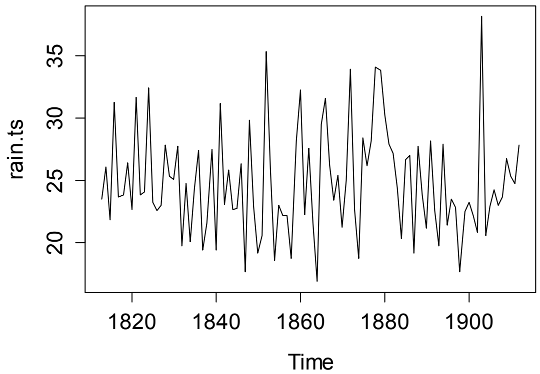
- EDA-Summary:

```
> tsdisplay(souvenir.log.ts)
```



• Example: Annual Rainfall in London

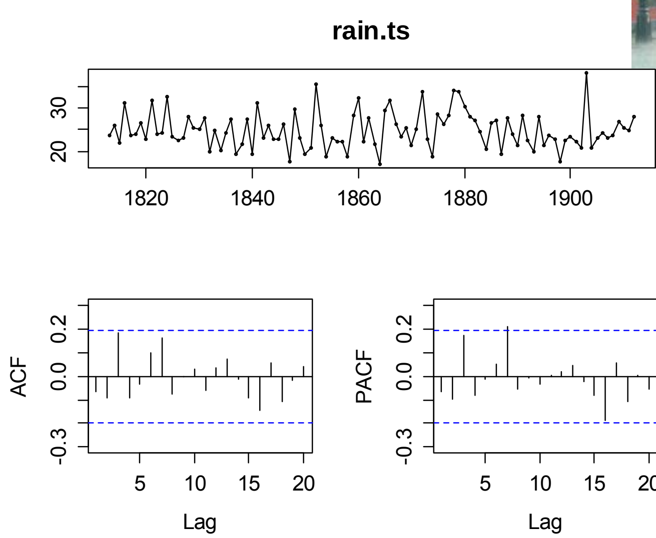
```
> load("D:/..../rain.RData")
> rain.ts <- ts(rain,start=1813)
> plot(rain.ts)
```



Exploratory Data Analysis
- Main Properties:
  - Trend?
  - Cyclic Variation?
  - Irregular Variation?
- Sudden Changes?
- Outliers?
- Missing Values?

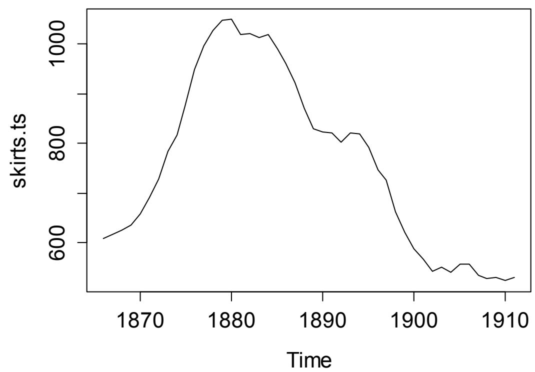
• EDA-Summary:

```
> tsdisplay(rain.ts)
```



• Example: Annual Diameter of Women's Skirts

```
> load("D:/..../skirts.RData")
> skirts.ts <- ts(skirts,start=1866)
> plot(skirts.ts)
```

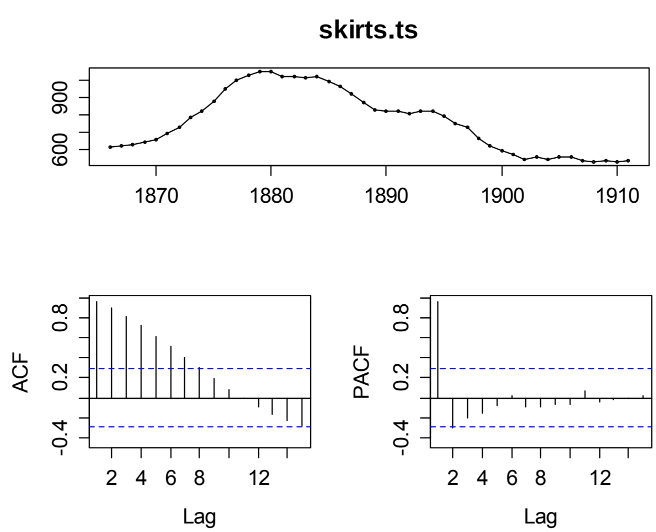


Exploratory Data Analysis

- Main Properties:
  - Trend?
  - Cyclic Variation?
  - Irregular Variation?
- Sudden Changes?
- Outliers?
- Missing Values?

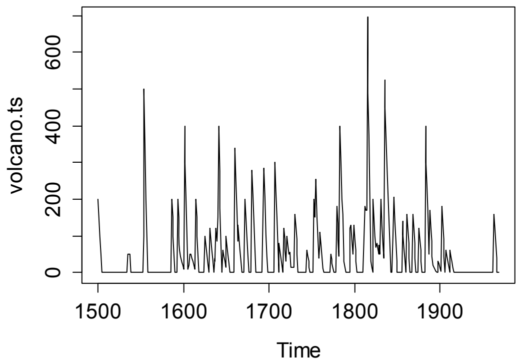
• EDA-Summary:

```
> tsdisplay(skirts.ts)
```



• Example: Volcanic Dust Veil Index

```
> load("D:/..../volcano.RData")
> volcano.ts <- ts(volcano, start=1500)
> plot(volcano.ts)
```

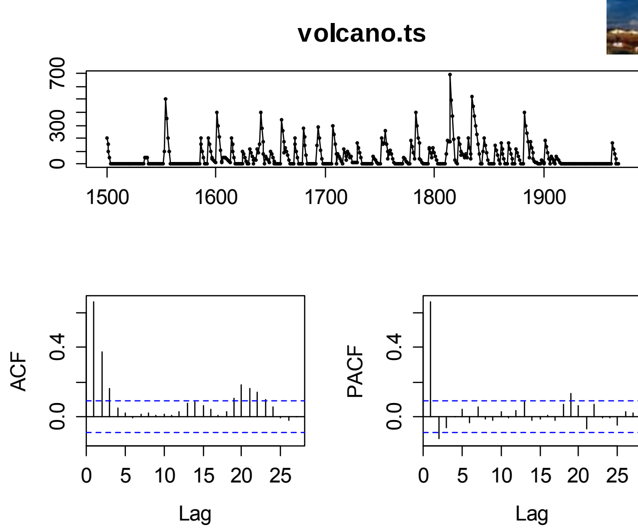


Exploratory Data Analysis

- Main Properties:
  - Trend?
  - Cyclic Variation?
  - Irregular Variation?
- Sudden Changes?
- Outliers?
- Missing Values?

• EDA-Summary:

```
> tsdisplay(volcano.ts)
```



• Types of Variation:

- Trend
  - Long term changes in mean
  - Can be estimated & modeled
  - Can be corrected for

- Seasonal Variation
  - Periodic variations over time
  - Can be estimated & modeled
  - Can be corrected for

- **Stationary Time Series**
  - No systematic change in mean and variance
  - No strictly periodic variations

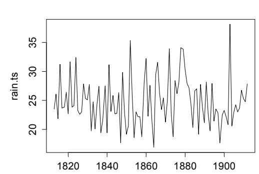
- Other Cyclic Variation
  - Business cycle (~7 year)

- Other Irregular Fluctuations
  - Random or with structure ?

Simple Exponential Smoothing

• Series with NO Trend and No Seasonality

Example: Annual Rainfall in London



Stationary Series
- No trend (?☺?)
- No Seasonality

• Basic Idea: one-step ahead prediction

$$\hat{x}_T(1) = c_0 x_T + c_1 x_{T-1} + c_2 x_{T-2} + \cdots$$

Geometric Weights: $c_i = \alpha(1-\alpha)^i$

• Remark: mainly used for short-term forecasts !

- Interpretation:

$$\hat{x}_T(1) = \alpha x_T + (1-\alpha)\left[\alpha x_{T-1} + \alpha(1-\alpha)x_{T-2} + \cdots\right]$$

$$\boxed{\hat{x}_T(1) = \alpha x_T + (1-\alpha)\hat{x}_{T-1}(1)}$$

Weighted average of past and present

$$\boxed{= \hat{x}_{T-1}(1) + \alpha\left(x_T - \hat{x}_{T-1}(1)\right)}$$

α =0 → past
α =1 → present

- Overview:

Simple Exponential Smoothing

$\boxed{x_{t-1} \mid \hat{L}_{t-1}} \longrightarrow \boxed{\hat{x}_{t-1}(1) = \hat{L}_{t-1} \quad \hat{L}_t = \alpha x_t + (1-\alpha)\hat{L}_{t-1}} \longrightarrow \boxed{x_t \mid \hat{L}_t}$

- Remark:
  - Simple Exponential Smoothing allows for updates of level estimates

- Example: fixed parameter α = 0.2
  - Adequate *initial* value for the level estimate needed $\longrightarrow$ $\hat{L}_1 \approx x_1$

```
> rain.ses1 <- HoltWinters(rain.ts,alpha=0.2,beta=FALSE,gamma=FALSE)
```

- Available Information:

```
> names(rain.ses1)
[1] "fitted" "x" "alpha" "beta" "gamma"
[6] "coefficients" "seasonal" "SSE" "call"
```

- Smoothing Parameters:

```
> rain.ses1

Holt-Winters exponential smoothing without
trend and without seasonal component.

Smoothing parameters: alpha: 0.2
beta : FALSE gamma: FALSE
```

- Final Component Estimate:

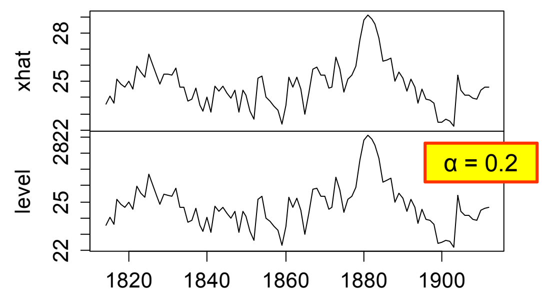
```
Coefficients: [,1]
a 25.30941
```

- ‘Running’ Component Estimates:

```
> rain.ses1$fitted

        xhat    level
1814 23.56000 23.56000
1815 24.06200 24.06200
1816 23.62160 23.62160
1817 25.14528 25.14528
1818 24.84622 24.84622
1819 24.65298 24.65298
1820 25.00438 25.00438
1821 24.53751 24.53751
1822 25.96801 25.96801
1823 25.54640 25.54640
       .........
```

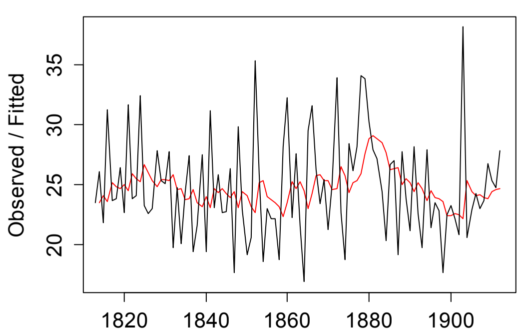
$$\hat{x}_{t-1}(1) = \hat{L}_{t-1}$$

```
> plot(rain.ses1$fitted)
```



- In-Sample Prediction vs. Realization:

```
> plot(rain.ses1)
```



Running SSE

$$\sum_{t=s_0}^{T} \left(x_t - \hat{x}_{t-1}(1) \right)^2$$

```
> rain.ses1$SSE

[1] 1972.197
```

- Remark:
  - In-sample comparison uses the same data for fitting and validation.
  - Independent hold-out sample for validation would be preferred!

- Goodness-of-Fit Measures:
  - **Bias:**
    - Mean Error
      $$ME = \frac{1}{n}\sum_{t=1}^{n}(x_t - \hat{x}_t)$$
    - Mean Percentage Error
      $$MPE = \frac{1}{n}\sum_{t=1}^{n}\left(\frac{x_t - \hat{x}_t}{x_t}\right)$$

The closer to 0, the ☺

  - **Variability:**
    - Mean Absolute Error
      $$MAE = \frac{1}{n}\sum_{t=1}^{n}\left|x_t - \hat{x}_t\right|$$
    - Mean Absolute Percentage Error
      $$MAPE = \frac{100}{n}\sum_{t=1}^{n}\left|\frac{x_t - \hat{x}_t}{x_t}\right|$$
    - Root Mean Squared Error
      $$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}(x_t - \hat{x}_t)^2}$$
    - Mean Absolute Scaled Error
      $$MASE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{x_t - \hat{x}_t}{q}\right|$$

The smaller the ☺

- In-Sample Accuracy:

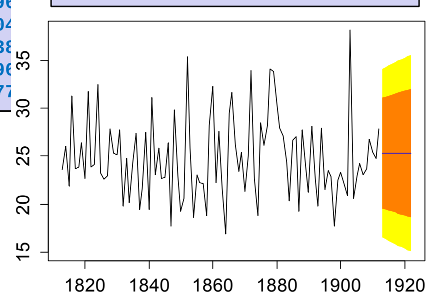
```
> accuracy(rain.ses1)

        ME       RMSE        MAE         MPE        MAPE       MASE
0.08835385 4.46331492 3.41468131 -2.41358228 13.79285103 0.70334023
```


- Holt-Winters Forecasts:

```
> rain.ses1.fore <- forecast.HoltWinters(rain.ses1,h=10)
```

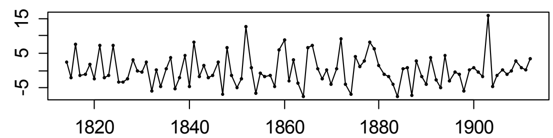
```
> rain.ses1.fore
Point Forecast  Lo 80    Hi 80    Lo 95    Hi 95
1913 25.30941 19.56146 31.05736 16.51867 34.10014
1914 25.30941 19.44762 31.17119 16.34458 34.27423
1915 25.30941 19.33596 31.28285 16.17381 34.44500
1916 25.30941 19.22635 31.39247 16.00617 34.61264
1917 25.30941 19.11867 31.50014 15.8415
1918 25.30941 19.01284 31.60597 15.6796
1919 25.30941 18.90876 31.71005 15.5204
1920 25.30941 18.80634 31.81247 15.3638
1921 25.30941 18.70551 31.91330 15.2096
1922 25.30941 18.60620 32.01261 15.0577
```

```
> plot.forecast(rain.ses1.fore)
```



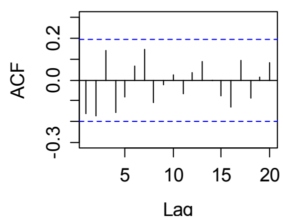
- Diagnostics: In-Sample Forecast Errors

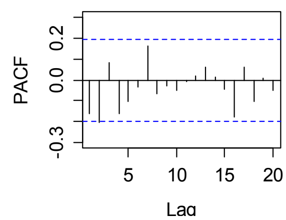
Constant over time?




```
> tsdisplay(rain.ses1.fore$residuals)
```

Non-zero Autocorrelations?





$$X_{BL}^2 = N(N+2)\sum_{i=1}^{k}\frac{r_i^2}{N-i} \sim \chi^2_{N-\#p}$$

```
> Box.test(rain.ses1.fore$residuals,lag=20,type="Ljung-Box")

Box-Ljung test data: rain.ses1.fore$residuals
X-squared = 22.5621, df = 20, p-value = 0.3108
```


- Example: optimal α estimated
  - Principle:
    - Optimal value for the parameter α to be determined from 'running' 1-step ahead prediction:

$$\min_{\alpha} \sum_{t=0}^{T}\left(x_t - \hat{x}_{t-1}(1)\right)^2 \qquad \text{(or related, eg. AIC or BIC)}$$

```
> rain.ses2 <- HoltWinters(laborforce.ts,beta=FALSE,gamma=FALSE)
```

- Smoothing Parameters:

```
> rain.ses2

Smoothing parameters: alpha: 0.02412151
beta : FALSE gamma: FALSE
```

$\hat{x}_T(1) = \alpha x_T + (1-\alpha)\hat{x}_{T-1}(1)$ $\longrightarrow$ More emphasis on the past!

- Final Component Estimate:

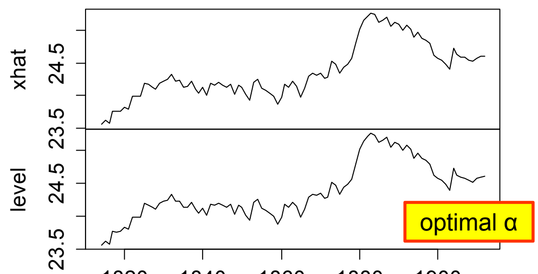
```
Coefficients: [,1]
[,1] a 24.67819
```

- 'Running' Component Estimates:

```
> rain.ses2$fitted

         xhat      level
1814 23.56000 23.56000
1815 23.62054 23.62054
1816 23.57808 23.57808
1817 23.76290 23.76290
1818 23.76017 23.76017
1819 23.76306 23.76306
1820 23.82691 23.82691
1821 23.79900 23.79900
1822 23.98935 23.98935
1823 23.98623 23.98623
.........
```

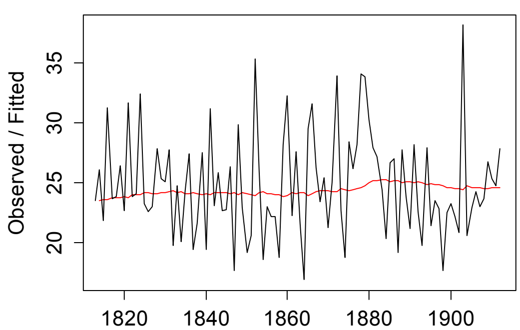
$$\hat{x}_{t-1}(1) = \hat{L}_{t-1}$$

```
> plot(rain.ses2$fitted)
```



- In-Sample Prediction vs. Realization:

```
> plot(rain.ses2)
```



Running SSE

$$\sum_{t=s_0}^{T}\left(x_t - \hat{x}_{t-1}(1)\right)^2$$

```
> rain.ses2$SSE

[1] 1828.855
```

- In-Sample Accuracy:

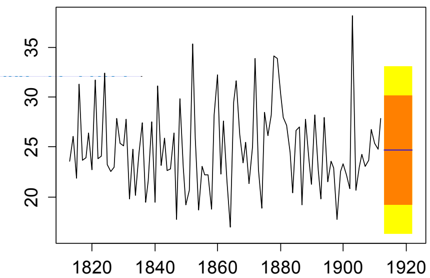
```
> accuracy(rain.ses2)

       ME      RMSE       MAE        MPE      MAPE      MASE
0.4682496 4.2980556 3.3501092 -0.8688727 13.4015953 0.6900400
```

- Holt-Winters Forecasts:

```
> rain.ses2.fore <- forecast.HoltWinters(rain.ses2,h=10)
```

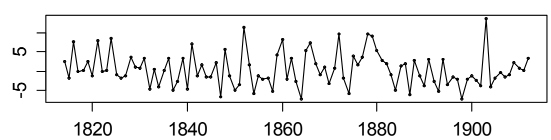
```
> rain.ses2.fore
Point Forecast  Lo 80    Hi 80    Lo 95    Hi 95
1913 24.67819 19.17493 30.18145 16.26169 33.09470
1914 24.67819 19.17333 30.18305 16.25924 33.09715
1915 24.67819 19.17173 30.18465 16.25679 33.09960
1916 24.67819 19.17013 30.18625 16.25434 33.10204
1917 24.67819 19.16853 30.18785 16.25
1918 24.67819 19.16694 30.18945 16.24
1919 24.67819 19.16534 30.19105 16.24
1920 24.67819 19.16374 30.19265 16.24
1921 24.67819 19.16214 30.19425 16.24
1922 24.67819 19.16054 30.19584 16.23
```

```
> plot.forecast(rain.ses2.fore)
```



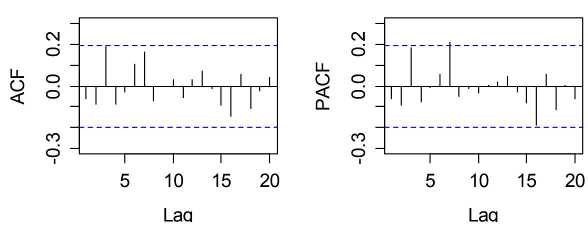
- Diagnostics: In-Sample Forecast Errors

Constant over time?




```
> tsdisplay(rain.ses2.fore$residuals)
```

Non-zero Autocorrelations?



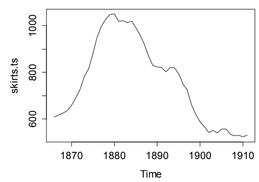
$$X_{BL}^2 = N(N+2)\sum_{i=1}^{k}\frac{r_i^2}{N-i} \sim \chi^2_{N-\#p}$$

```
> Box.test(rain.ses2.fore$residuals,lag=20,type="Ljung-Box")

Box-Ljung test data: rain.ses2.fore$residuals
X-squared = 17.4008, df = 20, p-value = 0.6268
```

Holt's Exponential Smoothing

- Series with Trend but No Seasonality

Example: Annual Diameter of Women's Skirts



- No Seasonality (annual data!)

- Principle:

Holt's Exponential Smoothing

$$\boxed{x_{t-1}^T}\boxed{\hat{L}_{t-1}}\boxed{\hat{T}_{t-1}} \rightarrow \boxed{\begin{array}{l}\hat{x}_{t-1}^T(1) = \hat{L}_{t-1} + \hat{T}_{t-1} \\ \hat{L}_t = \alpha x_t^T + (1-\alpha)\left(\hat{L}_{t-1} + \hat{T}_{t-1}\right) \\ \hat{T}_t = \beta\left(\hat{L}_t - \hat{L}_{t-1}\right) + (1-\beta)\hat{T}_{t-1}\end{array}} \rightarrow \boxed{x_t^T}\boxed{\hat{L}_t}\boxed{\hat{T}_t}$$

- Remarks:
  - Holt's ES allows for updates of level and trend estimations
    - Two parameter version of Exponential Smoothing
  - Special: Brown's Exponential Smoothing
    - Both parameters are equal: $\alpha = \beta$
  - Similar to ARIMA(0,2,1) model (to be discussed)

- Practical:
  - Adequate *initial* values for the level and trend estimates needed

$$\longrightarrow \hat{L}_1 \approx x_1 \qquad \longrightarrow \hat{T}_1 \approx x_2 - x_1$$

  - Optimal value for the parameters α and β to be determined from 'running' 1-step ahead prediction:

$$\min_{\alpha,\beta} \sum_{t=0}^{T} \left(x_t - \hat{x}_{t-1}(1)\right)^2 \qquad \text{(or related, eg. AIC or BIC)}$$

• Example: optimal α and β estimated

```
> skirts.hes <- HoltWinters(x=skirts.ts,gamma=FALSE)
```

• Smoothing Parameters:

```
> skirts.hes

Smoothing parameters: alpha: 0.8383481 beta : 1 gamma: FALSE
```

$$\hat{L}_t = \alpha x_t^T + (1-\alpha)\left(\hat{L}_{t-1} + \hat{T}_{t-1}\right)$$
$$\hat{T}_t = \beta\left(\hat{L}_t - \hat{L}_{t-1}\right) + (1-\beta)\hat{T}_{t-1}$$

→ Main emphasis on recent values!

• Final Component Estimate:

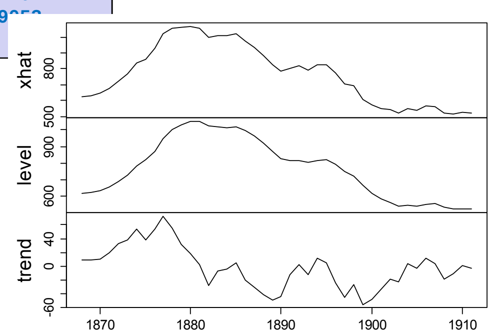
```
Coefficients: [,1]
a 529.308585
b 5.690464
```


• 'Running' Component Estimates:

```
> skirts.hes$fitted

        xhat     level     trend
1868 626.0000 617.0000 9.0000000
1869 633.3233 625.1617 8.1616519
1870 645.9730 635.5673 10.4056551
1871 674.8676 655.2175 19.6501514
1872 721.5669 688.3922 33.1747101
1873 765.5280 726.9601 38.5679...
... ...
```

$$\hat{x}_{t-1}^T(1) = \hat{L}_{t-1} + \hat{T}_{t-1}$$

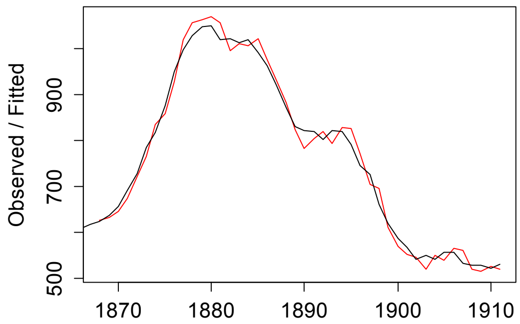
```
> plot(skirts.hes$fitted)
```



optimal α and β

- In-Sample Prediction vs. Realization:

```
> plot(skirts.hes)
```



Running SSE

$$\sum_{t=s_0}^{T}\left(x_t - \hat{x}_{t-1}(1)\right)^2$$

```
> skirts.hes$SSE

[1] 16954.18
```

- In-Sample Accuracy:

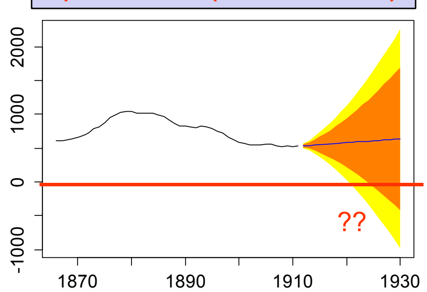
```
> accuracy(skirts.hes)

      ME      RMSE       MAE       MPE      MAPE      MASE
255.25079 454.78440 269.86059 33.42056 35.34162 11.21304
```


- Holt-Winters Forecasts:

```
> skirts.hes.fore <- forecast.HoltWinters(skirts.hes,h=19)
```

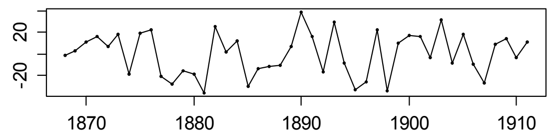
```
> skirts.hes.fore

Point Forecast    Lo 80     Hi 80     Lo 95     Hi 95
1912 534.9990 509.55210 560.4460 496.08130 573.9168
1913 540.6895 491.01052 590.3685 464.71204 616.6670
1914 546.3800 465.36129 627.3987 422.47258 670.2874
1915 .... .....
```

```
> plot.forecast(skirts.hes.fore)
```



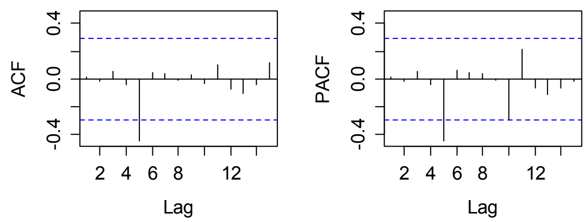
- Diagnostics: In-Sample Forecast Errors

Constant over time?




```
> tsdisplay(skirts.hes.fore$residuals)
```

Non-zero Autocorrelations?



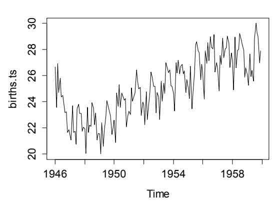
$$X^2_{BL} = N(N+2)\sum_{i=1}^{k}\frac{r_i^2}{N-i} \sim \chi^2_{N-\#p}$$

```
> Box.test(skirts.hes.fore$residuals,lag=20,type="Ljung-Box")

Box-Ljung test data: skirts.fore$residuals
X-squared = 19.7312, df = 20, p-value = 0.4749
```

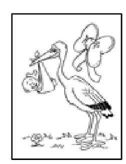

Holt-Winters Exponential Smoothing

(**Additive** Seasonality)

- Example: Births per Months in New York City



- Trend ?
- Additive Seasonality ?



- Principle:

Additive Holt-Winter's ES

$$x^{TS}_{t-1} \quad \hat{L}_{t-1} \quad \hat{T}_{t-1} \quad \hat{I}_{t-1}$$

$$\hat{x}^{TS}_{t-1}(1) = \hat{L}_{t-1} + \hat{T}_{t-1} + \hat{I}_{t-s}$$

$$\hat{L}_t = \alpha\left(x^{TS}_t - \hat{I}_{t-s}\right) + (1-\alpha)\left(\hat{L}_{t-1} + \hat{T}_{t-1}\right)$$

$$\hat{T}_t = \beta\left(\hat{L}_t - \hat{L}_{t-1}\right) + (1-\beta)\hat{T}_{t-1}$$

$$\hat{I}_t = \gamma\left(x^{TS}_t - \hat{L}_t\right) + (1-\gamma)\hat{I}_{t-s}$$

$$x^{TS}_t \quad \hat{L}_t \quad \hat{T}_t \quad \hat{I}_t$$

- Remarks:
  - Holt-Winter's ES allows for updates of level, trend and seasonality estimations
    - Three parameter version of Exponential Smoothing
  - Similar to SARIMA $(0,1,1)x(0,1,1)_s$ model (to be discussed)
  - Special: Simple Seasonal ES → No trend (T_t=0), only seasonality !

Simple Additive Seasonal ES

$$x_{t-1}^S \quad \hat{L}_{t-1} \quad \hat{I}_{t-1} \rightarrow \begin{aligned} \hat{x}_{t-1}^S(1) &= \hat{L}_{t-1} + \hat{I}_{t-s} \\ \hat{L}_t &= \alpha\left(x_t^S - \hat{I}_{t-s}\right) + (1-\alpha)\hat{L}_{t-1} \\ \hat{I}_t &= \gamma\left(x_t^{TS} - \hat{L}_t\right) + (1-\gamma)\hat{I}_{t-s} \end{aligned} \rightarrow x_t^S \quad \hat{L}_t \quad \hat{I}_t$$

- Practical:
  - Adequate *initial* values for the level, trend and seasonality estimates needed
  - Optimal values for the parameters α, β and γ to be determined from:

$$\min_{\alpha,\beta,\gamma} \sum_{t=0}^{T}\left(x_t - \hat{x}_{t-1}(1)\right)^2$$ (or related, eg. AIC or BIC)

- Example: optimal α, β and γ estimated

```
births.hw <- HoltWinters(births.ts,seasonal="additive")
```

- Smoothing Parameters:

```
births.hw

Smoothing parameters:
alpha: 0.4823655
beta : 0.02988495
gamma: 0.563186
```

$$\begin{aligned} \hat{L}_t &= \alpha x_t^T + (1-\alpha)\left(\hat{L}_{t-1} + \hat{T}_{t-1}\right) \\ \hat{T}_t &= \beta\left(\hat{L}_t - \hat{L}_{t-1}\right) + (1-\beta)\hat{T}_{t-1} \\ \hat{I}_t &= \gamma\left(x_t^{TS} - \hat{L}_t\right) + (1-\gamma)\hat{I}_{t-s} \end{aligned}$$

→ Emphasis for trend on the past!

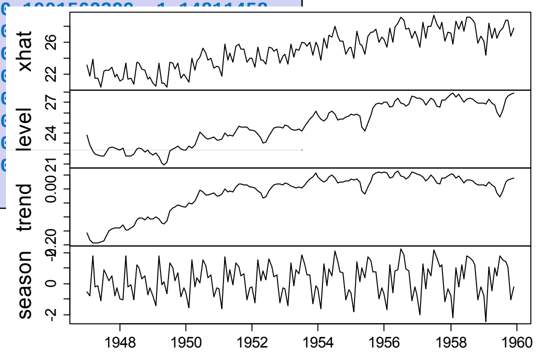
- Final Component Estimates:

```
Coefficients: [,1]

a 28.04366357
b 0.04199921

s1 -0.78546221
s2 -2.19944507
s3 0.87813012
s4 -0.65164728
s5 0.63427267
s6 0.21182821
s7 2.23177191
s8 2.17167733
s9 1.52077678
s10 1.16900861
s11 -0.97500043
s12 -0.18636055
```

- 'Running' Component Estimates: $\hat{x}_{t-1}^{TS}(1) = \hat{L}_{t-1} + \hat{T}_{t-1} + \hat{I}_{t-s}$

```
> births.hw$fitted

            xhat    level         trend        season
Jan 1947 23.13579 23.81055 -0.1567618007 -0.51798958
Feb 1947 21.83089 22.83531 -0.1812218860 -0.82319792
Mar 1947 23.90724 22.29623 -0.1919165635  1.80292708
Apr 1947 21.58463 22.00869 -0.1947742244 -0.2292
May 1947 21.51602 21.85461 -0.1935580066 -0.1450
Jun 1947 20.43661 21.77488 -0
Jul 1947 22.44490 21.74120 -0
Aug 1947 22.51935 22.05453 -0
Sep 1947 22.50969 22.51328 -0
Oct 1947 22.96787 22.64867 -0
Nov 1947 21.63717 22.57404 -0
Dec 1947 22.07360 22.49168 -0
Jan 1948 21.19997 22.35201 -0
Feb 1948 .......
```

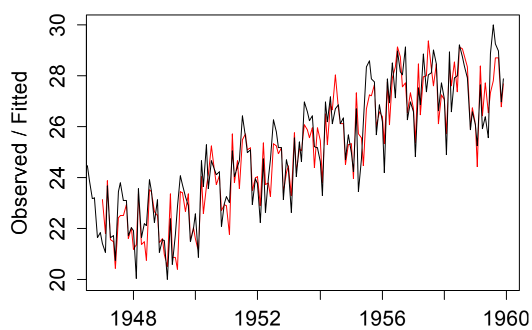
```
> plot(births.hw$fitted)
```



optimal α, β and γ

- In-Sample Prediction vs. Realization:

```
> plot(births.hw)
```



Running SSE

$$\sum_{t=s_0}^{T} \left(x_t - \hat{x}_{t-1}(1) \right)^2$$

```
> births.hw$SSE

[1] 90.94058
```

- In-Sample Accuracy:

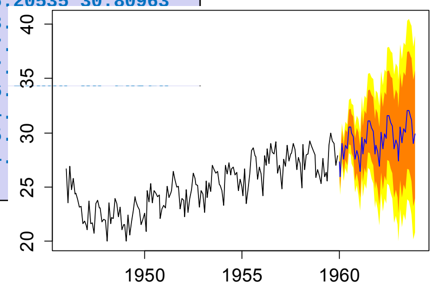
```
> accuracy(births.hw)

      ME     RMSE      MAE      MPE     MAPE      MASE
12.60682 17.77135 12.93276 50.15378 51.52523 10.81909
```

• Holt-Winters Forecasts:

```
> births.hw.fore <- forecast.HoltWinters(births.hw,h=48)
```

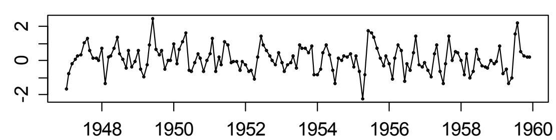
```
> births.hw.fore

  Point  Forecast  Lo 80    Hi 80    Lo 95    Hi 95
Jan 1960 27.30020 26.32517 28.27523 25.80902 28.79139
Feb 1960 25.92822 24.83950 27.01694 24.26316 27.59327
Mar 1960 29.04779 27.85040 30.24518 27.21654 30.87905
Apr 1960 27.56001 26.25756 28.86247 25.56808 29.55195
May 1960 28.88793 27.48307 30.29280 26.739
Jun 1960 28.50749 27.00220 30.01278 26.20535 30.80963
Jul 1960 30.56943 28.96521 32.17365 28.
Aug 1960 30.55133 28.84929 32.25338 27.
Sep 1960 29.94243 28.14338 31.74148 27.
Oct 1960 29.63266 27.73720 31.52813 26.
Nov 1960 27.53065 25.53918 29.52213 24.
Dec 1960 28.36129 26.27407 30.44852 25.
Jan 1961 27.80419 25.52190 30.08648 24.
Feb 1961 26.4322 .......
```

```
> plot.forecast(births.hw.fore)
```



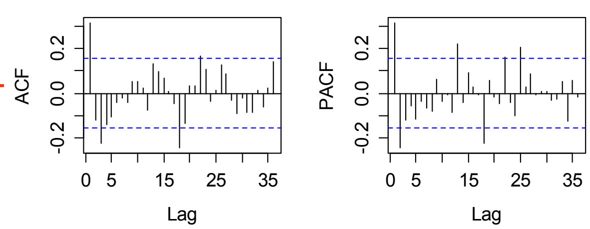
• Diagnostics: In-Sample Forecast Errors

Constant over time?




```
> tsdisplay(births.hw.fore$residuals)
```

Non-zero Autocorrelations?



$$X_{BL}^2 = N(N+2)\sum_{i=1}^{k}\frac{r_i^2}{N-i} \sim \chi^2_{N-\#p}$$

```
> Box.test(births.hw.fore$residuals,lag=20,type="Ljung-Box")

data: births.hw.fore$residuals
X-squared = 70.9555, df = 35, p-value = 0.0003082
```

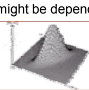
ARMA Models: the Bare Essentials

Stochastic Processes:

$$\mathbf{X}_{\{t\}} = \{\cdots, X_{t_1}, \cdots, X_{t_2}, \cdots\}$$

Stochastic Process

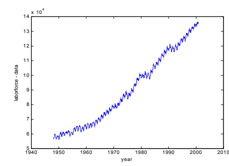
Individual stochasts, that might be dependent



Model Fit

- Model Identification
- Parameter estimation
- Model Validation

Realization:



- Purely Random Processes
- Moving Average Processes: MA(q)
- Autoregressive Processes: AR(p)
- ARMA(p,q) & ARIMA(p,d,q) Processes

- Purely Random Process: (white noise)

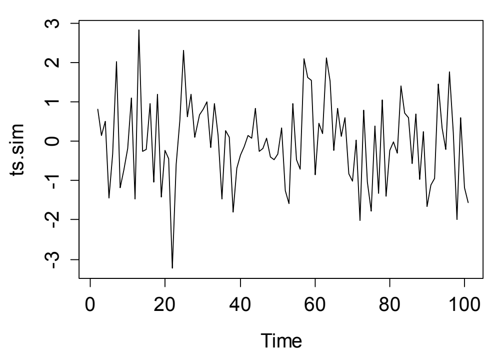
$$\mathbf{Z}_{\{t\}} = \{\cdots, Z_{t_1}, \cdots, Z_{t_2}, \cdots\}$$

$\{Z_t\} \sim N(0, \sigma_Z^2)$ and mutually independent

Basic Building Block

- Simulation:

```
> ts.sim <- arima.sim(list(order=c(0,0,0)),n=100)
> plot(ts.sim)
```

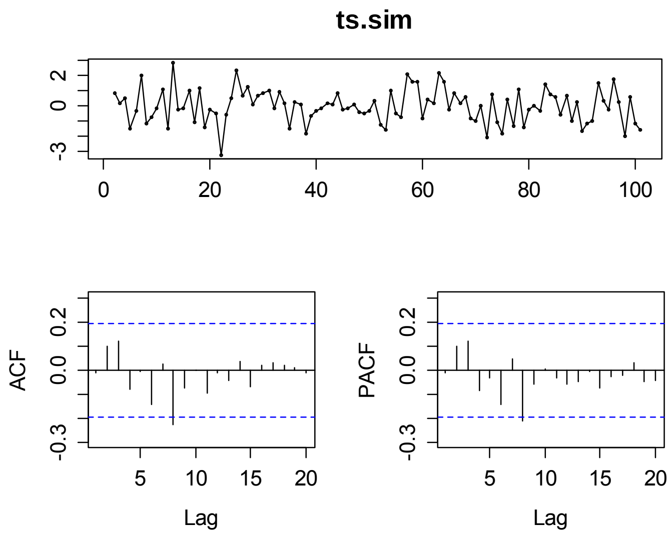


Exploratory Data Analysis

- Main Properties:
  - Trend?
  - Seasonal Variation?
  - Cyclic Variation?
  - Irregular Variation?

- EDA-Summary:

```
> tsdisplay(ts.sim)
```



- Moving Average Process: MA(q)

$$X_t = Z_t + \beta_1 Z_{t-1} + \cdots + \beta_q Z_{t-q}$$

- Interpretation: Present Value = Moving Average of Past Disturbances (=shock)
  - Process is mainly influenced by 'random' events from the past: Economics ??
  - MA-models are often used to model time series which show short term dependencies between successive observations

- Operator Notation: $\mathbf{B} \bullet X_t = X_{t-1}$

backshift operator **B**

$$X_t = \theta_q(\mathbf{B}) \bullet Z_t \qquad \text{with:} \quad \theta_q(\mathbf{B}) = 1 + \beta_1 \mathbf{B} + \cdots + \beta_q \mathbf{B}^q$$

- Example: MA(2) Process

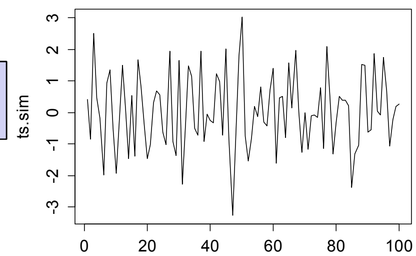
$$X_t = Z_t - 0.3Z_{t-1} - 0.4Z_{t-2}$$

- Operator Notation:

$$X_t = \theta_q(\mathbf{B}) \bullet Z_t \qquad \text{with:} \qquad \theta_q(\mathbf{B}) = 1 - 0.3\mathbf{B} - 0.4\mathbf{B}^2$$

- Simulation:

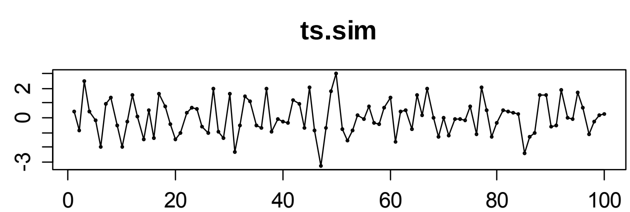
```
> ts.sim <- arima.sim(list(order=
+ c(0,0,2), ma=c(-0.3,-0.4)),n=100)
> plot(ts.sim)
```



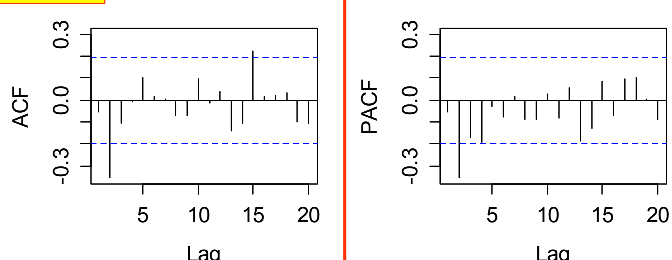
- EDA-Summary:

```
> tsdisplay(ts.sim)
```

$X_t = Z_t - 0.3Z_{t-1} - 0.4Z_{t-2}$



MA-specific



• Autoregressive Process: AR(p)

$$X_t = \alpha_1 X_{t-1} + \cdots + \alpha_p X_{t-p} + Z_t$$

- Interpretation: Present Value = Moving Average of Past Values + Disturbance
  - Process is mainly influenced by past values of the process !
  - AR-models are often used to model time series which show longer term dependencies between successive observations

- Operator Notation: $\mathbf{B} \bullet X_t = X_{t-1}$

backshift operator **B**

$$\varphi_p(\mathbf{B}) \bullet X_t = Z_t \qquad \text{with:} \quad \varphi_p(\mathbf{B}) = 1 - \alpha_1 \mathbf{B} - \cdots - \alpha_p \mathbf{B}^p$$

• Example: AR(2) Process

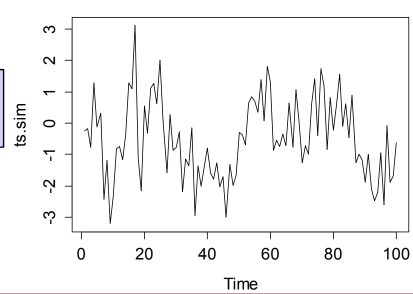
$$X_t = 0.3 X_{t-1} + 0.4 X_{t-2} + Z_t$$

- Operator Notation:

$$\varphi_p(\mathbf{B}) \bullet X_t = Z_t \qquad \text{with:} \quad \varphi_p(\mathbf{B}) = 1 - 0.3\mathbf{B} - 0.4\mathbf{B}^2$$

- Simulation:

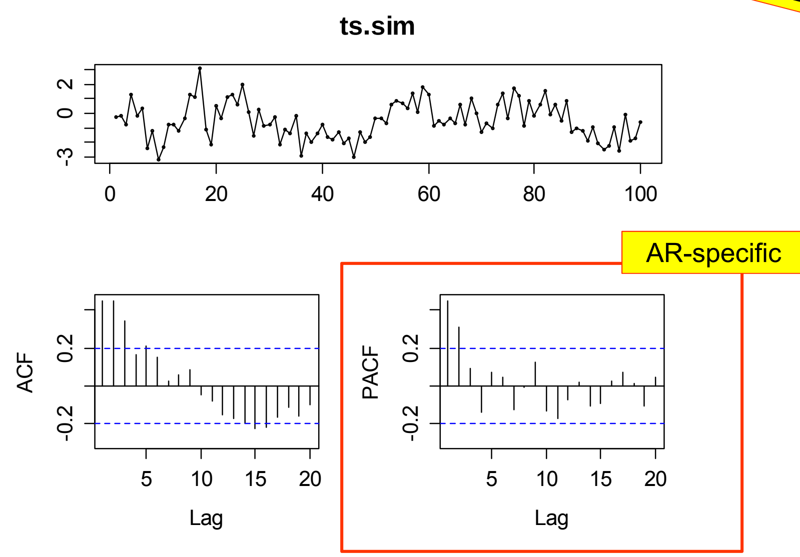
```
> ts.sim <- arima.sim(list(order=
+ c(2,0,0), ar=c(0.3,0.4)),n=100)
> plot(ts.sim)
```



- EDA-Summary:

```
> tsdisplay(ts.sim)
```

$X_t = 0.3X_{t-1} + 0.4X_{t-2} + Z_t$



- Autoregressive Moving Average Process: ARMA(p,q)

$$X_t = \left(\alpha_1 X_{t-1} + \cdots + \alpha_p X_{t-p}\right) + \left(Z_t + \beta_1 Z_{t-1} + \cdots + \beta_q Z_{t-q}\right)$$

- Interpretation: Process is influenced both by levels and by disturbances from the past!

- Operator Notation: $\varphi_p(\mathbf{B}) \bullet X_t = \theta_q(\mathbf{B}) \bullet Z_t$

backshift operator **B**

$$\varphi_p(\mathbf{B}) = 1 - \alpha_1 \mathbf{B} - \cdots - \alpha_p \mathbf{B}^p \qquad \theta_q(\mathbf{B}) = 1 + \beta_1 \mathbf{B} + \cdots + \beta_q \mathbf{B}^q$$

• Example: ARMA(1,1) Process

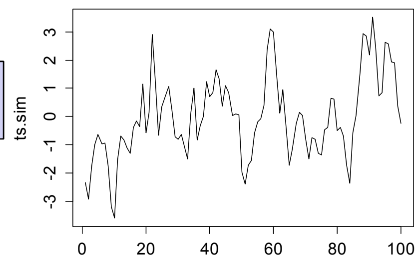
$$X_t = 0.6X_{t-1} + Z_t + 0.4Z_{t-1}$$

- Operator Notation: $\varphi_p(\mathbf{B}) \bullet X_t = \theta_q(\mathbf{B}) \bullet Z_t$

$$1 - 0.6\mathbf{B} = \varphi_1(\mathbf{B}) \qquad \theta_q(\mathbf{B}) = 1 + 0.4\mathbf{B}$$

- Simulation:

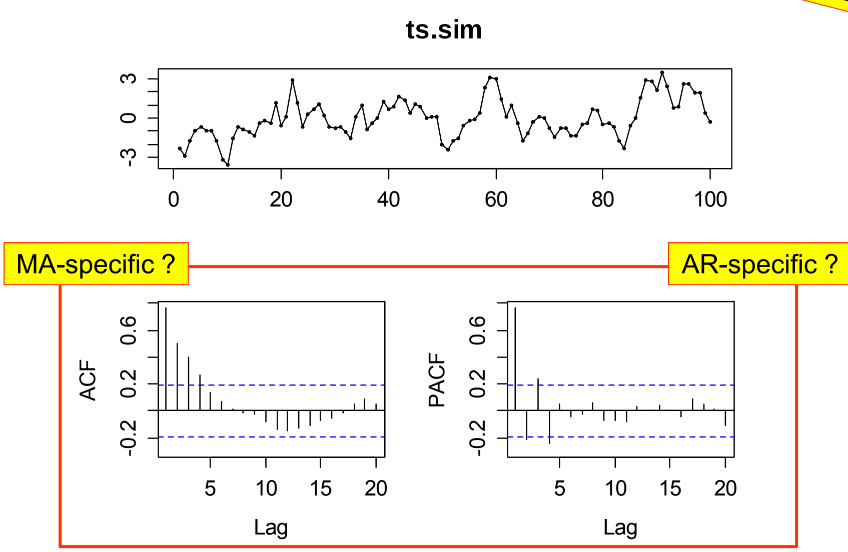
```
> ts.sim <- arima.sim(list(order=
+ c(1,0,1),ar=c(0.6),ma=c(0.4)),n=100)
> plot(ts.sim)
```



• EDA-Summary:

```
> tsdisplay(ts.sim)
```

$X_t = 0.6X_{t-1} + Z_t + 0.4Z_{t-1}$



ARMA Model Fitting

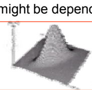
stationary series without seasonality

Stochastic Processes:

$$\mathbf{X}_{\{t\}} = \{\cdots, X_{t_1}, \cdots, X_{t_2}, \cdots\}$$

Stochastic Process

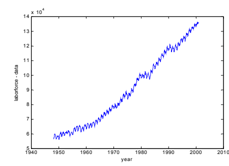
Individual stochasts, that might be dependent



Model Fit

- Model Identification
- Parameter estimation
- Model Validation

Realization:



Theoretical Finger Print

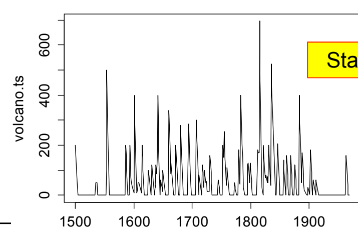
- Autocorrelation
- Partial Autocorrelation

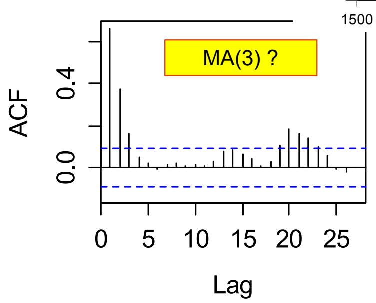
EDA - Finger Print

- Autocorrelation
- Partial Autocorrelation

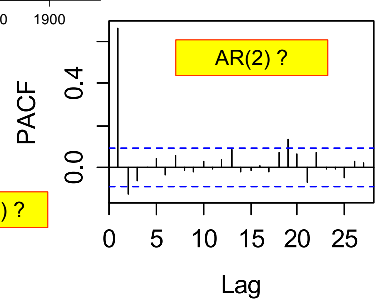
- Example: Volcanic Dust Veil Index

```
> plot(volcano.ts); tsdisplay(volcano.ts)
```





ARMA(1,1) ?



- Conjecture 1: MA(3)-model ?

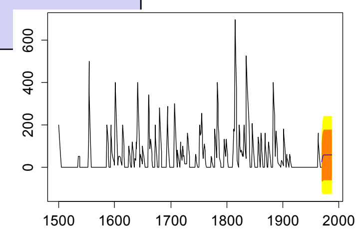
```
> volcano.arma.0.3 <- Arima(volcano.ts,order=c(0,0,3))
```

```
> volcano.arma.0.3

Series: volcano.ts
ARIMA(0,0,3) with non-zero mean

Coefficients:
         ma1     ma2     ma3 intercept
      0.7438  0.4513  0.1916  57.4559
s.e.  0.0455  0.0502  0.0442   7.6534

sigma^2 estimated as 4852:  log likelihood=-2661.69

AIC=5333.39 AICc=5333.52 BIC=5354.15
```

$$\tilde{X}_t = X_t - 57.5$$
$$\tilde{X}_t = Z_t + 0.74Z_{t-1} + 0.45Z_{t-2} + 0.19Z_{t-3}$$

- Model Forecasts:

```
> volcano.arma.0.3.fore <-
+ forecast(volcano.arma.0.3,h=19)
> plot.forecast(volcano.arma.0.3.fore)
```



- In-Sample Accuracy:

```
> accuracy(volcano.arma.0.3)
```

```
        ME       RMSE        MAE  MPE  MAPE      MASE
-0.2205417 69.7825327 37.8638920 -Inf   Inf 0.9926308
```

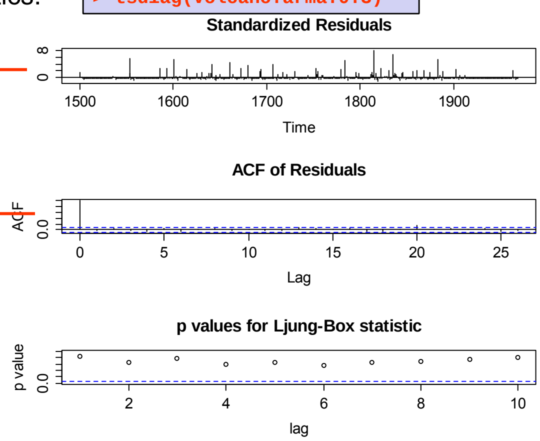
- In-Sample Diagnostics:

```
> tsdiag(volcano.arma.0.3)
```

Constant over time?

Non-zero Autocorrelations?

$$X^2_{BL} = N(N+2)\sum_{i=1}^{k}\frac{r_i^2}{N-i} \sim \chi^2_{N-\#p}$$



- Conjecture 2: AR(2)-model ?

```
> volcano.arma.2.0 <- Arima(volcano.ts,order=c(2,0,0))
```

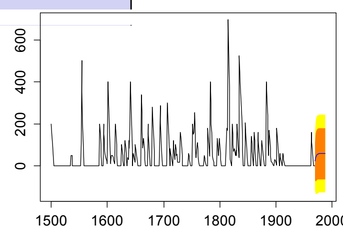
```
> volcano.arma.2.0

Series: volcano.ts
ARIMA(2,0,0) with non-zero mean

Coefficients:
         ar1      ar2  intercept
      0.7533  -0.1268    57.5274
s.e.  0.0457   0.0458     8.5958

sigma^2 estimated as 4870:  log likelihood=-2662.54

AIC=5333.09   AICc=5333.17   BIC=5349.7
```

$\tilde{X}_t = X_t - 57.5$

$\tilde{X}_t = 0.75\tilde{X}_{t-1} - 0.12\tilde{X}_{t-2} + Z_t$

- Model Forecasts:

```
> volcano.arma.2.0.fore <-
+ forecast(volcano.arma.2.0,h=19)
> plot.forecast(volcano.arma.2.0.fore)
```



- In-Sample Accuracy:

```
> accuracy(volcano.arma.2.0)
```

```
        ME       RMSE        MAE  MPE  MAPE      MASE
-0.2205417 69.7825327 37.8638920 -Inf   Inf 0.9926308
```

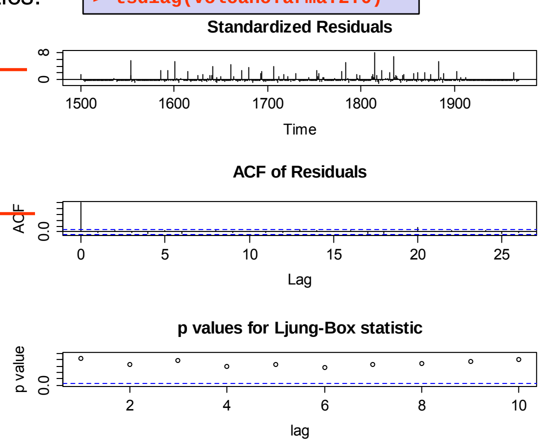
- In-Sample Diagnostics:

```
> tsdiag(volcano.arma.2.0)
```

Constant over time?

Non-zero Autocorrelations?

$X_{BL}^2 = N(N+2)\sum_{i=1}^{k}\frac{r_i^2}{N-i} \sim \chi^2_{N-\#p}$



- Conjecture 3: ARMA(1,1)-model ?

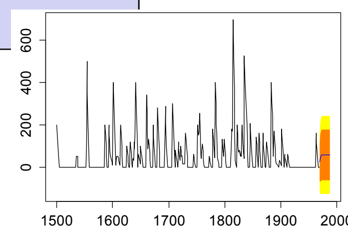
```
> volcano.arma.1.1 <- Arima(volcano.ts,order=c(1,0,1))
```

```
> volcano.arma.1.1

Series: volcano.ts
ARIMA(1,0,1) with non-zero mean

Coefficients:
         ar1     ma1 intercept
      0.5848  0.1555   57.5473
s.e.  0.0526  0.0613    8.9412

sigma^2 estimated as 4883:  log likelihood=-2663.21

AIC=5334.42  AICc=5334.5  BIC=5351.03
```

$$\tilde{X}_t = X_t - 57.5$$
$$\tilde{X}_t = 0.3\tilde{X}_{t-1} + 0.4\tilde{X}_{t-2} + 0.4\tilde{X}_{t-3} + Z_t$$

- Model Forecasts:

```
> volcano.arma.1.1.fore <-
+ forecast(volcano.arma.1.1,h=19)
> plot.forecast(volcano.arma.1.1.fore)
```



- In-Sample Accuracy:

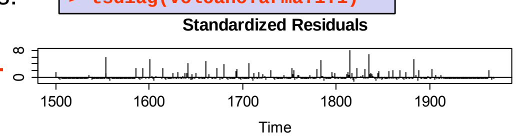
```
> accuracy(volcano.arma.1.1)
```

```
        ME       RMSE        MAE   MPE   MAPE      MASE
-0.2374427 69.8819979 37.7767429  -Inf    Inf 0.9903461
```

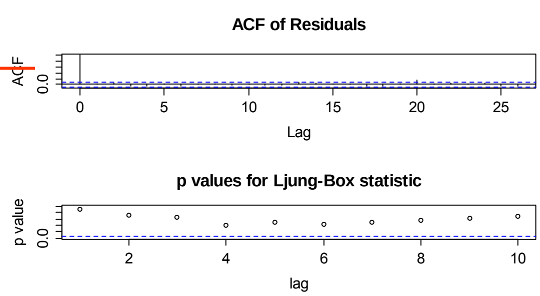
- In-Sample Diagnostics:

```
> tsdiag(volcano.arma.1.1)
```

Constant over time?



Non-zero Autocorrelations?

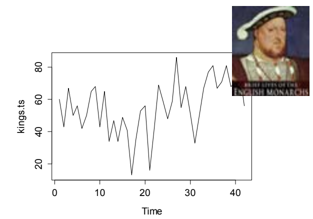


$$X_{BL}^2 = N(N+2)\sum_{i=1}^{k}\frac{r_i^2}{N-i} \sim \chi^2_{N-\#p}$$

ARIMA Model Fitting

non-stationary series

Stationary ?
- Level / Variation ?

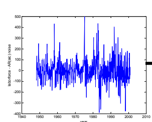


- Remedy:
  - Remove dominant non-stationarity through finite differencing:

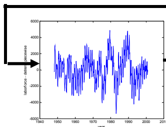
$$\nabla X_t = X_t - X_{t-1} = (1-\mathbf{B}) \bullet X_t$$

  - If successful then fit ARMA(p,q) on ∇X_t ➔ ARIMA(p,1,q) on X_t

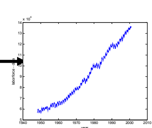
ARMA(p,q) model on W_t:



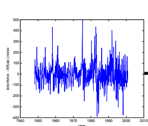
$$\varphi_p(\mathbf{B}) \bullet W_t = \theta_q(\mathbf{B}) \bullet Z_t$$



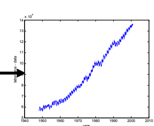
$$W_t = \nabla^d X_t$$



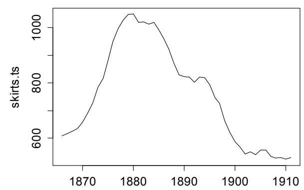
ARIMA(p,d,q)-model on X_t:



$$\varphi_p(\mathbf{B}) \bullet (1-\mathbf{B})^d \bullet X_t = \theta_q(\mathbf{B}) \bullet Z_t$$

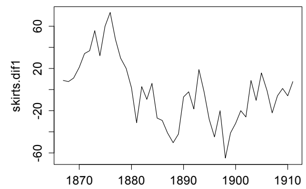


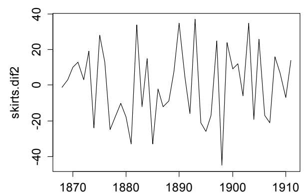
• Example: Annual Diameter of Women's Skirts




```
> plot(skirts.ts)
```

```
> skirts.d1 <- diff(skirts.ts,differences=1)
> plot(skirts.d1)
```

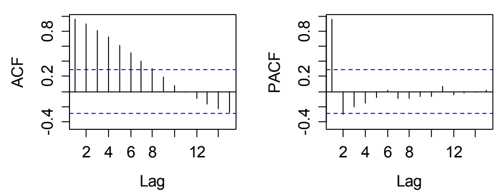


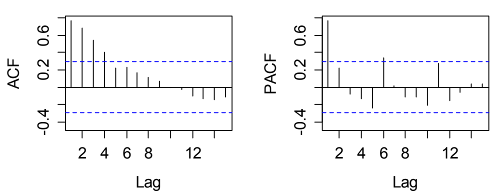



```
> skirts.d2 <- diff(skirts.ts,differences=2)
> plot(skirts.d2)
```


• EDA-Summary:

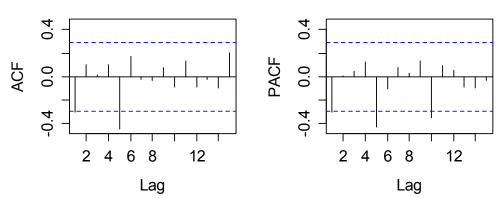
```
> tsdisplay(skirts.ts)
```





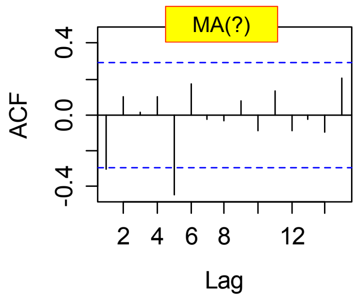

```
> tsdisplay(skirts.d1)
```

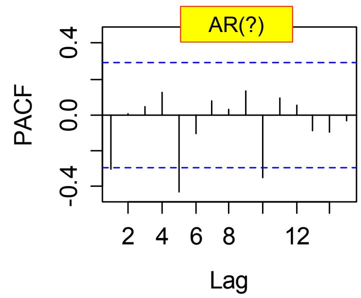
```
> tsdisplay(skirts.d2)
```



- Model: $W_t = \nabla^2 X_t = \nabla\left(X_t - X_{t-1}\right) = X_t - 2X_{t-1} + X_{t-2}$

```
> tsdisplay(skirts.d2)
```





ARMA(p,q) on W_t ⟷ ARIMA(p,2,q) on X_t

- Conjecture: ARIMA(1,2,1)-model ?

```
> skirts.arima.1.2.1 <- Arima(skirts.ts,order=c(1,2,1))
```

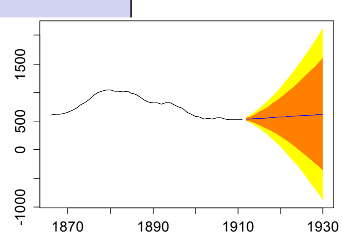
```
> skirts.arima.1.2.1

Series: skirts.ts
ARIMA(1,2,1)

Coefficients:
          ar1     ma1
      -0.3123  0.0139
s.e.   0.4265  0.4449

sigma^2 estimated as 388.7: log likelihood=-193.66

AIC=393.33 AICc=393.93 BIC=398.68
```

$W_t = \nabla^2 X_t$

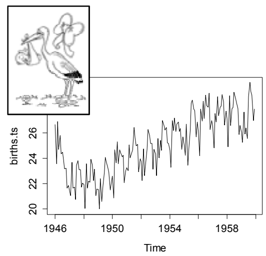
$W_t = -0.31 W_{t-1} + Z_t + 0.01 Z_{t-1}$

- Model Forecasts:

```
> skirts.arima.1.2.1.fore <-
+ forecast(skirts.arima.1.2.1,h=19)
> plot(skirts.arima.1.2.1.fore)
```

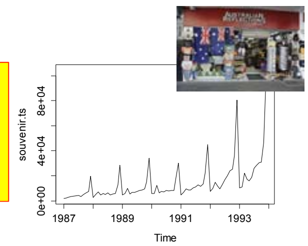


SARIMA Model Fitting



Stationary ?
- Level / Variation ?

Seasonality ?
- Additive / Multiplicative ?



- Remedy:
  - Remove dominant non-stationarity through finite differencing:

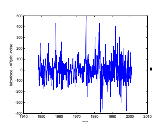
$$\nabla X_t = X_t - X_{t-1} = (1-\mathbf{B}) \bullet X_t$$

  - Remove dominant seasonality through seasonal differencing:

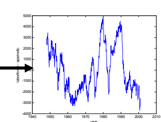
$$\nabla_S X_t = X_t - X_{t-s} = \left(1-\mathbf{B}^s\right) \bullet X_t$$

Combined adjustment: $W_t = \nabla^d \bullet \nabla_s^D \bullet X_t = (1-\mathbf{B})^d \bullet \left(1-\mathbf{B}^s\right)^D \bullet X_t$

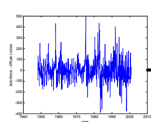
ARMA(p,q) for W_t:



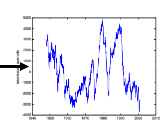
$$\boxed{\phi_p(\mathbf{B}) \bullet W_t = \theta_q(\mathbf{B}) \bullet Z_t}$$



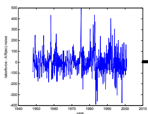
Generalisation to account for changes in seasonality:



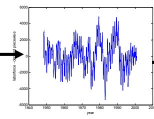
$$\boxed{\begin{aligned} \phi_p(\mathbf{B}) \bullet \Phi_P\left(\mathbf{B}^s\right) \bullet W_t &= \\ \theta_q(\mathbf{B}) \bullet \Theta_Q\left(\mathbf{B}^s\right) \bullet Z_t & \end{aligned}}$$



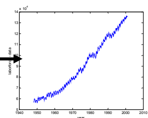
Equivalent:



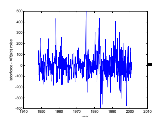
$$\phi_p(\mathbf{B})\bullet\Phi_P(\mathbf{B}^s)\bullet W_t = \theta_q(\mathbf{B})\bullet\Theta_Q(\mathbf{B}^s)\bullet Z_t$$



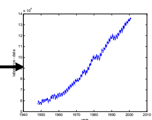
$$W_t = \nabla^d \nabla_s^D X_t$$



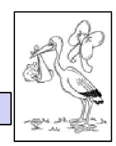
SARIMA (p,d,q)x(P,D,Q)$_s$ -model:



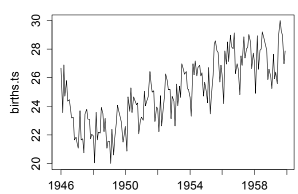
$$\phi_p(\mathbf{B})\bullet\Phi_P(\mathbf{B}^s)\bullet\nabla^d\bullet\nabla_s^D\bullet X_t = \theta_q(\mathbf{B})\bullet\Theta_Q(\mathbf{B}^s)\bullet Z_t$$

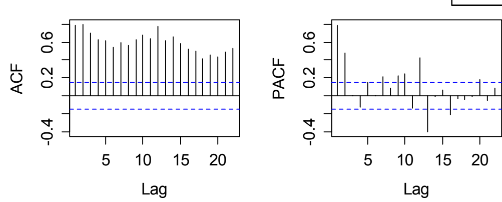


• Example: Births per Month in New York City

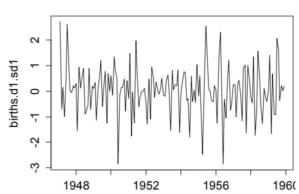


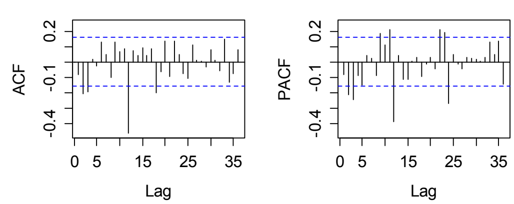
```
> plot(births.ts); tsdisplay(births.ts)
```






```
> births.d1.sd1<-diff(diff(births.ts,differences=1),lag=12,differences=1)
> plot(births.d1.sd1); tsdisplay(births.d1.sd1)
```

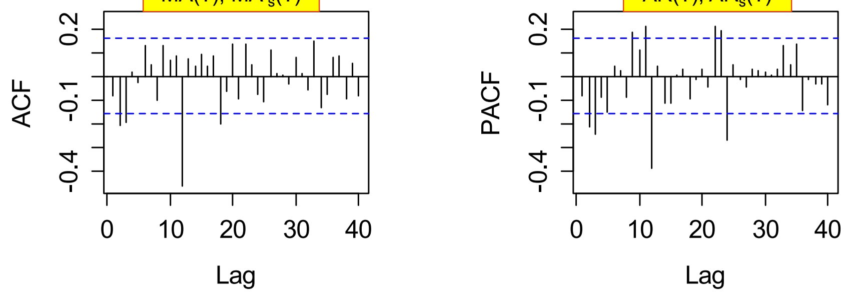




• Model: $W_t = \nabla\nabla^{12} X_t = (1-\mathbf{B})(1-\mathbf{B}^{12})\, X_t$

```
tsdisplay(births.d1.sd1,lag.max=40)
```

MA(?), MA$_s$(?) AR(?), AR$_s$(?)



ARMA(p,q) on W_t ↔ SARIMA(p,1,q)x(P,1,Q)$_{12}$ on X_t

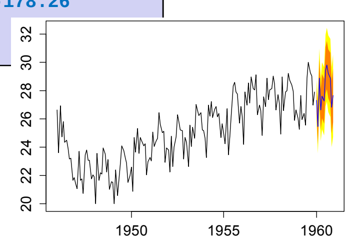
• Conjecture: SARIMA(3,1,3)x(1,1,0)$_{12}$-model ?

```
> births.sarima.3.1.3.1.1.0<-Arima(births.ts,order=c(3,1,3),
+ seasonal=list(order=c(1,1,0),period=12))
```

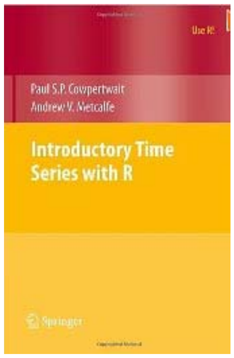
```
> births.sarima.3.1.3.1.1.0

Series: births.ts
ARIMA(3,1,3)(1,1,0)[12]

Coefficients:
          ar1      ar2     ar3     ma1      ma2      ma3     sar1
      -1.0567  -0.0302  0.3607  0.9311  -0.4072  -0.7804  -0.5139
s.e.   0.1441   0.2055  0.1497  0.1211   0.1565   0.1288   0.0767

sigma^2 estimated as 0.5566:  log likelihood=-178.26

AIC=372.51 AICc=373.5 BIC=396.86
```

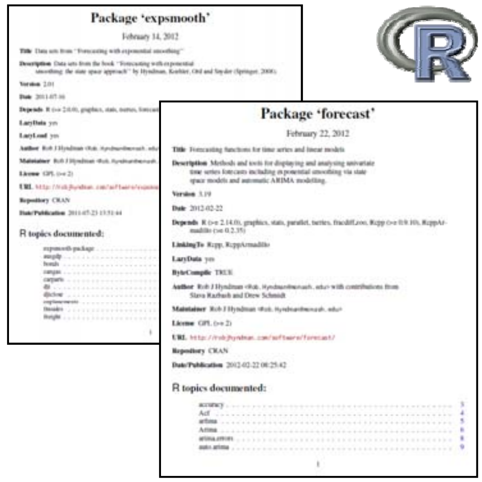
• Model Forecasts:

```
> births.sarima.3.1.3.1.1.0.fore<-
+ forecast(births.sarima.3.1.3.1.1.0,h=12)
> plot(births.sarima.3.1.3.1.1.0.fore)
```



More Time Series Modeling…….




Package 'expsmooth'

February 14, 2012

R topics documented:

Package 'forecast'

February 22, 2012

R topics documented:

Paul S. Cowpertwaith et al.,
Introductory Time Series with R,
ISBN 978-0387886978


Journal of Statistical Software

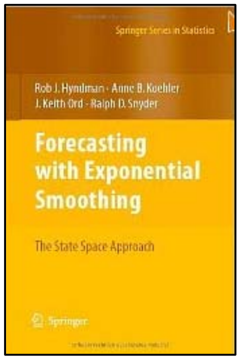
July 2008, Volume 27, Issue 3. http://www.jstatsoft.org/

Automatic Time Series Forecasting: The forecast Package for R

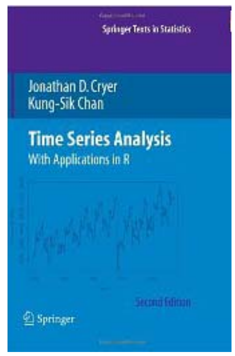
Rob J. Hyndman, Monash University
Yeasmin Khandakar, Monash University

Journal of Statistical Forecasting,
July 2008, Volume 27, Issue 3

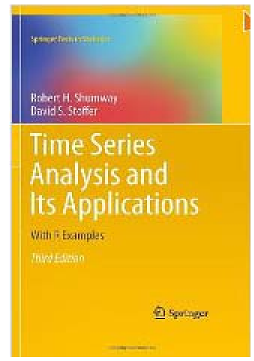
http://www.youtube.com/watch?v=1Lh1HlBUf8k



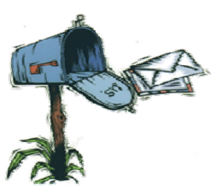
Rob. J. Hyndman et al.,
Forecasting with Exponential Smoothing,
ISBN 978-3540719168



Jonathan D. Cryer et al.,
Time Series Analysis with Applications in R,
ISBN 978-1441926135



Robert M. Shumway et al.,
Time Series Analysis and its Applications
with R examples,
ISBN 978-1441978646



Suggestions for Improvements of TSA^R are welcomed:

j.j.m.rijpkema@tue.nl