Log-level and Log-log transformations in Linear Regression Models

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Level-Level

A "Level-level" Regression Specification.

$$y = \beta_0 + \beta_1 x_1 + \epsilon$$

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- How do we interpret β_1?
- Differentiate w.r.t. x_1 to find the marginal effect of x on y. In this case, β IS the marginal effect.

$$\frac{dy}{dx} = \beta$$

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A “Log-level” Regression Specification.

$$\log(y) = \beta_0 + \beta_1 x_1 + \epsilon$$

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- You might want to run this specification if you think that increases in x lead to a constant *percentage* increase in y. (wage on education? forest lumber volume on years?)

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- First solve for y.

$$\log(y) = \beta_0 + \beta_1 x_1 + \epsilon$$
$$\Rightarrow y = e^{\beta_0 + \beta_1 x_1 + \epsilon}$$

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- Then differentiate to get the marginal effect:

$$\frac{dy}{dx_1} = \beta e^{\beta_0 + \beta_1 x_1 + \epsilon} = \beta_1 y$$

So the marginal effect depends on the value of y, while β itself represents the *growth rate*.

$$\beta_1 = \frac{dy}{dx_1}\frac{1}{y}$$

For example, if we estimated that β_1 is .04, we would say that another year increases the volume of lumber by 4%.

Log-Log

A “Log-Log” Regression Specification.

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- To caculate marginal effects. Solve for y ...

$$\log(y) = \beta_0 + \beta_1 \log(x_1) + \epsilon y = e^{\beta_0 + \beta_1 \log(x_1) + \epsilon}$$

... and differentiate w.r.t. x

$$\frac{dy}{dx_1} = \frac{\beta_1}{x_1} e^{\beta_0 + \beta_1 \log(x_1) + \epsilon} = \beta_1 \frac{y}{x_1}$$

Log-Log Continued.

'Log-Log" Regression Specification Continued...

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- From previous slide the marginal effect is

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- From previous slide the marginal effect is

$$\frac{dy}{dx_1} = \beta_1 \frac{y}{x_1}$$

- Solving for β_1 we get

$$\beta_1 = \frac{dy}{dx_1}\frac{x_1}{y}$$

Hence β_1 is an *elasticity*. If x_1 is price and y is demand and we estimate $\beta_1 = -.6$, it means that a 10% increase in the price of the good would lead to a 6% decrease in demand.