How To Remember Special Values of Sine and Cosine

The following is a special table for remembering the special exact values of the sine and cosine functions in Quadrant I. The key to the following table is just knowing a few simple patterns. The first is to know the important angles in Quadrant I: These are (in degrees) $0°$, $30°$, $45°$, $60°$, $90°$ (note the quadrantal angles $0°$ and $90°$ have been included). The second is to recognize the special values of the sine and cosine functions all have the form $\dfrac{\sqrt{n}}{2}$ as follows:

θ	$\sin\theta$	$\cos\theta$
$0 = 0°$	$\dfrac{\sqrt{0}}{2} = \dfrac{0}{2} = 0$	$\dfrac{\sqrt{4}}{2} = \dfrac{2}{2} = 1$
$\dfrac{\pi}{6} = 30°$	$\dfrac{\sqrt{1}}{2} = \dfrac{1}{2}$	$\dfrac{\sqrt{3}}{2}$
$\dfrac{\pi}{4} = 45°$	$\dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$	$\dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$
$\dfrac{\pi}{3} = 60°$	$\dfrac{\sqrt{3}}{2}$	$\dfrac{\sqrt{1}}{2} = \dfrac{1}{2}$
$\dfrac{\pi}{2} = 90°$	$\dfrac{\sqrt{4}}{2} = \dfrac{2}{2} = 1$	$\dfrac{\sqrt{0}}{2} = \dfrac{0}{2} = 0$

In order to reproduce the above table, you also need to remember that $\sin\theta$ increases from 0 to 1 in Quadrant I, while $\cos\theta$ decreases from 1 to 0 in Quadrant I. There are several ways to keep them straight: One way is you can remember the graphs of the sine and cosine function and thereby recall that $\sin 0 = 0$, $\cos 0 = 1$. Another way is to recall the unit circle picture of the trigonometric functions. Recall that on the unit circle, at angle θ, $(x, y) = (\cos\theta, \sin\theta)$. As $\theta = 0$ is the positive x axis—which hits the unit circle at $(1, 0)$—we have

$$(1, 0) = (x, y) = (\cos 0, \sin 0), \qquad \cos 0 = 1, \qquad \sin 0 = 0.$$

The values of angles outside Quadrant I can be computed using reference angles, and the values of the other trigonometric functions can be computed using the reciprocal and quotient identities.

Example: Compute $\sec\frac{\pi}{4}$ and $\tan\frac{\pi}{3}$.

$$\sec\frac{\pi}{4} = \frac{1}{\cos\frac{\pi}{4}} = \frac{1}{\left(\frac{1}{\sqrt{2}}\right)} = \sqrt{2}, \qquad \tan\frac{\pi}{3} = \frac{\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \sqrt{3}.$$