

## Mathematical Induction:

Q.1:

Conjecture:  $1 + 3 + 5 + 7 + 9 + \dots + (2n-1) = n^2$

$$\sum_{k=1}^n (2k-1) = n^2 \quad n \geq 1$$

Basis of Induction for  $n=1$

$$\Rightarrow 1 = 1$$

Induction Step: let for  $n=k$  it is true.

$$\text{Hence, } 1 + 3 + 5 + \dots + (2k-1) = k^2$$

Now, adding next term to both sides.

$$1 + 3 + 5 + \dots + (2k-1) + 2(k+1)-1 = k^2 + 2(k+1) - 1 \\ = (k+1)^2$$

Hence, we can see it is true for  $n=1$ .

then it is also true for  $n=k+1$  whenever  $n=k$  case was true.  $\therefore$  It is true  $\forall n \geq 1$

Q.2:

Given to show  $1 + 2 + 2^2 + \dots + 2^n = 2^{n+1} - 1$

I. for  $n=0$ , we have  $2^0 = 2^{0+1} - 1$   
 $\Rightarrow 1 = 1$

II. for  $n=k$ , we have

$$1 + 2 + 2^2 + \dots + 2^k = 2^{k+1} - 1$$

III. Adding next term on both sides.

$$1 + 2 + 2^2 + \dots + 2^k + 2^{k+1} = 2^{k+1} - 1 + 2^{k+1} \\ = 2^{k+2} - 1 = 2^{(k+1)+1} - 1$$

We saw its true for:  $n=0$   
 $n=k$  and then ~~for~~ for  $n=k+1$   
Hence, true for all  $n \geq 0$  by mathematical induction.

Q.3

$$\text{conjecture: } 1^2 + 3^2 + 5^2 + \dots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$

$$\sum_{k=0}^{k=n} (k+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$$

B

I. Checking for left side limit.  $n=0$ 

$$1^2 = 1 \quad \checkmark$$

II. Checking for  $n=k$ .

$$\sum_{k=0}^{k=n} 2k+1$$

$$1^2 + 3^2 + 5^2 + \dots + (2k+1)^2 = \frac{(k+1)(2k+1)(2k+3)}{3}$$

III. Adding next term on both sides.

$$1^2 + 3^2 + 5^2 + \dots + (2k+1)^2 + (2(k+1)+1)^2 = \frac{(k+1)(2k+1)(2k+3) + (2k+3)^2}{3}$$

$$= \frac{(2k+3)}{3} \left[ \frac{(k+1)(2k+1)}{3} + 3(2k+3) \right]$$

$$= \frac{(2k+3)}{3} [2k^2 + 9k + 10]$$

$$= \frac{(2k+3)}{3} (k+2)(2k+5)$$

$$= \frac{[(k+1)+1][2(k+1)+1][2(k+1)+3]}{3}$$

Hence by mathematical induction, it's true  
for all  $n \geq 0$

Q.4

$n^3 + 2n$  divisible by 3 for all  $n \geq 1$

let  $n^3 + 2n = 3m$  (where  $m \in \mathbb{Z}$ )

I. Putting for  $n=1$

$$1+2 = 3m \Rightarrow m \in \mathbb{Z}$$

Hence its a multiple of three for  $n=1$ .

II. let  $n=k$

$$k^3 + 2k = 3m$$

III. Checking for next term  $n=k+1$

$$\begin{aligned} (k+1)^3 + 2(k+1) &= k^3 + 3k^2 + 3k + 1 + 2k + 2 \\ &= (k^3 + 2k) + 3(k^2 + k + 1) \\ &= 3m + 3(k^2 + k + 1) \\ &= 3(m + k^2 + k + 1) \\ &= 3m' = 3m' \end{aligned}$$

Hence its divisible by 3 for  $n=k+1$

By mathematical induction its true  $\forall n \geq 1$

Q.5:

$2^n \times 2^n - 1$  divisible by 3  $\forall n \geq 1$

let  $2^n \times 2^n - 1 = 3m$  (where  $m \in \mathbb{Z}$ )

I. Putting for  $n=1$

$$4-1 = 3m$$

$$m=1 \in \mathbb{Z}$$

Hence, a multiple of three for  $n=1$ .

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II. If true for a value  $k$

$$2^k \times 2^{k-1} = 3m$$

$$m \in \mathbb{Z}$$

III.

Checking for the next value  $k+1$

$$2^{k+1} \times 2^{k+1-1} = 4 \times (2^k \times 2^k) - 1$$

$$= 4 \times (3m + 1) - 1$$

$$= 3m' + 3 \quad (m' = 4m)$$

$$= 3(m' + 1)$$

$$= 3m''$$

So, next value is also

$$m'' \in \mathbb{Z}$$

divisible by 3

Hence, by mathematical induction,  
it is true for all  $n \geq 1$ .

Q. 6

$$1/1! + 2/2! + 3/3! + 4/4! + 5/5! + \dots n/n! = (n+1-1)$$

for  $n \geq 1$

I. Checking for basis of induction:  $n=1$

$$1/1! = (1+1-1)$$

$\therefore$  true for  $n=1$

II. If true for  $n=k$

$$1/1! + 2/2! + 3/3! + \dots k/k! = (k+1-1)$$

III. Checking for next value by adding:

$$1/1! + 2/2! + \dots + k/(k+(k+1)) = (k+1-1 + (k+1)/k+1)$$

$$= (k+2-1)$$

Hence, by mathematical induction its true for all  $n \geq 1$

Q.7 Same as Q.2

Q.8  $n^5 - n$  divisible by 5 for  $n \geq 0$ .

let  $n^5 - n = 5m$  (where  $m \in \mathbb{Z}$ )

I. Putting for left hand limit,  $n=0$ .

$$0 = 5m$$

$$m = 0 \quad (m \in \mathbb{Z})$$

Hence true for  $n=0$

II for induction let  $n=k$ ,  $k \in \mathbb{Z}$

$$k^5 - k = 5m$$

III. for next term,  $n=k+1$

$$(k+1)^5 - (k+1) = \cancel{k^5} + \cancel{n} + 2kn + \cancel{n} - 1$$

~~$$= (k+1)[(k+1)^4 - 1]$$~~

~~$$= (k+1)[\{(k+1)^2 + 1\} \times (k+1)^2 - 1]$$~~

~~$$= (k+1)[(k^2 + 2k + 2)(k^2 + 2k)]$$~~

~~$$= (k+1)[ \quad \quad \quad \text{(using binomial)} ]$$~~

$$= k^5 + 5k^4 + 10k^3 + 10k^2 + 5k + 1 - (k+1)$$

$$= k^5 - k + 5(k^4 + 2k^3 + 2k^2 + k)$$

$$= 5(m + k^4 + 2k^3 + 2k^2 + k)$$

$$= 5m'$$

Hence valid for  $k+1$ .

Its true for  $n \geq 0$  by mathematical induction.

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Q.9  $n^2 - n$  is divisible by 2 for  $n \geq 1$  (where  $n \in \mathbb{Z}$ )

$$\text{Let } n^2 - n = 2m$$

I. Induced Basis of induction for  $n=1$

$$1^2 - 1 = 2m$$

$$m=0$$

$$m \in \mathbb{Z} \checkmark$$

II. Induction step, let it true for  $n=k$

$$k^2 - k = 2m \checkmark$$

III for next term,  $n=k+1$

$$\begin{aligned}(k+1)^2 - (k+1) &= k^2 + 2k + 1 - k - 1 \\ &= k^2 - k + 2k \\ &= 2(m+k)\end{aligned}$$

Hence divisible by 2  $\checkmark$

By mathematical induction its true  $\forall n \geq 1$ .

Q.10.

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

I. for left side limit,  $n=1$

$$1 \cdot 2 \cdot 3 = 1 \cdot 2 \cdot 3 \cdot 4$$

Hence, true for  $n=1$

II. Induction step: for  $n=k$

$$1 \cdot 2 \cdot 3 + 2 \cdot 3 \cdot 4 + \dots + k \cdot k+1 \cdot k+2 = \frac{k(k+1)(k+2)(k+3)}{4}$$

IV. Now for next term we add on both sides.

$$\begin{aligned}
 1 \cdot 2 \cdot 3 + \dots + k(k+1)(k+2) + (k+1)(k+2)(k+3) &= \frac{k(k+1)(k+2)(k+3)}{4} \\
 &\quad + (k+1)(k+2)(k+3) \\
 &= \frac{(k+1)(k+2)(k+3)[k+4]}{4} \\
 &= \frac{(k+1)(k+2)(k+3)(k+4)}{4} \\
 &= \frac{(k+1)(k+1+1)(k+1+2)(k+1+3)}{4}
 \end{aligned}$$

Hence it's true for  $n=k+1$  when it was true for  $n=k$ .  
also it was true for  $n=1$ .

$\therefore$  It's true for all  $n \geq 1$  by mathematical induction.

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Set Theory:Q.1

- a)  $\phi \in \{\phi\}$  is True
- b)  $\phi \in \{\phi, \{\phi\}\}$  is True
- c)  $\{\phi\} \in \{\phi\}$  is false
- d)  $\{\phi\} \in \{\{\phi\}\}$  is true
- e)  $\{\phi\} \subset \{\phi, \{\phi\}\}$  is true
- f)  $\{\{\phi\}\} \subset \{\phi, \{\phi\}\}$  is true
- g)  $\{\{\phi\}\} \subset \{\{\phi\}, \{\phi\}\}$  is false

Q.2

List all the members of these sets.

a)  $\{x | x \in \mathbb{R}, x^2 = 1\} = x \in [-1, 1]$   
 $\{-1, \dots, 0, \dots, 1\}$

a)  $\{x | x \in \mathbb{R}, x^2 = 1\} = \{1, -1\}$

b)  $\{x | x \in \mathbb{Z}^+, x < 12\} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

c)  $\{x | x \in \mathbb{Z} \text{ and } x \text{ is a square of an integer } x < 100\}$   
 $= \{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$

d)  $\{x | x \text{ is an integer such that } x^2 = 2\} = \{\}$

Q.3

a) To show,  $\phi \times A = A \times \phi = \phi$

we can show using cardinality.

we know,  $|A \times B| = |A| \times |B|$

Hence,  $|\phi \times A| = |A \times \phi| = |A| \cdot |\phi| = |A| \cdot 0 = 0$

Hence, ~~empty~~ as empty set cardinality is zero.

$$\phi \times A = A \times \phi = \phi$$

b)  $A \times B \times C$  and  $(A \times B) \times C$  are not same.

for cartesian product we know,

$$A \times B = \{(x, y) \mid x \in A, \text{ and } y \in B\}$$

while

$$A \times B \times C = \{(x, y, z) \mid x \in A \text{ and } y \in B \text{ and } z \in C\}$$

it contains triplets.

$$\text{But } (A \times B) \times C = \{(x, y), z \mid (x \in A \text{ and } y \in B) \text{ and } z \in C\}$$

here we can see it is a collection doublets with first value themselves being doublets.

Therefore  $A \times B \times C$  is not same as  $(A \times B) \times C$ .

Q.4:  $A = \{a, b, c\}$   $B = \{x, y\}$   $C = \{0, 1\}$

$$a) A \times B \times C = \{(a, x, 0), (a, x, 1), (a, y, 0), (a, y, 1), (b, x, 0), (b, x, 1), (b, y, 0), (b, y, 1), (c, x, 0), (c, x, 1), (c, y, 0), (c, y, 1)\}$$

$$b) C \times B \times A = \{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c), (1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\}$$

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c)  $C \times A \times B = \{(0,a,x), (0,a,y), (0,b,x), (0,b,y), (0,c,x), (0,c,y), (1,a,x), (1,a,y), (1,b,x), (1,b,y), (1,c,x), (1,c,y)\}$

d)  $B \times B \times B = \{(x,x,u), (u,u,y), (x,y,x), (x,y,y), (y,x,x), (y,x,y), (y,y,x), (y,y,y)\}$

Q.5: a) Power set.

a)  $\{a\} \Rightarrow P = \{\emptyset, \{a\}\}$

b)  $\{a, b\} \Rightarrow P = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

c)  $\{\emptyset, \{\emptyset\}\} \Rightarrow P = \{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\{\emptyset\}\}\}$

Q.5 b) Cardinality:

a)  $\emptyset \quad |S| = 0$

b)  $\{\emptyset\} \quad |S| = 1$

c)  $\{\emptyset, \{\emptyset\}\} \quad |S| = 2$

d)  $\{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\} \quad |S| = 3$

Q.6:

Given, A : set of sophomores

B : set of students in discrete maths.

a) Sophomores taking discrete

$$A \cap B$$

b) Sophomores not taking discrete

$$A - B$$

c) Sophomores or taking discrete

$$A \cup B \quad (\text{or } A \text{ or } B)$$

d) Not sophomore or not taking discrete

$$\bar{A} \cup \bar{B}$$

$$V - (A \cap B)$$

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Q.7:

$$\text{a) } A = \{1, 2, 3, 4, 5\} \quad B = \{0, 3, 6\}$$

$$\text{a) } A \cup B = \{0, 1, 2, 3, 4, 5, 6\}$$

$$\text{b) } A \cap B = \{3\}$$

$$\text{c) } A - B = \{1, 2, 4, 5\}$$

$$\text{d) } B - A = \{0, 6\}$$

$$\text{b) } A = \{a, b, c, d, e\} \quad B = \{a, b, c, d, e, f, g, h\}$$

$$\text{a) } A \cup B = \{a, b, c, d, e, f, g, h\}$$

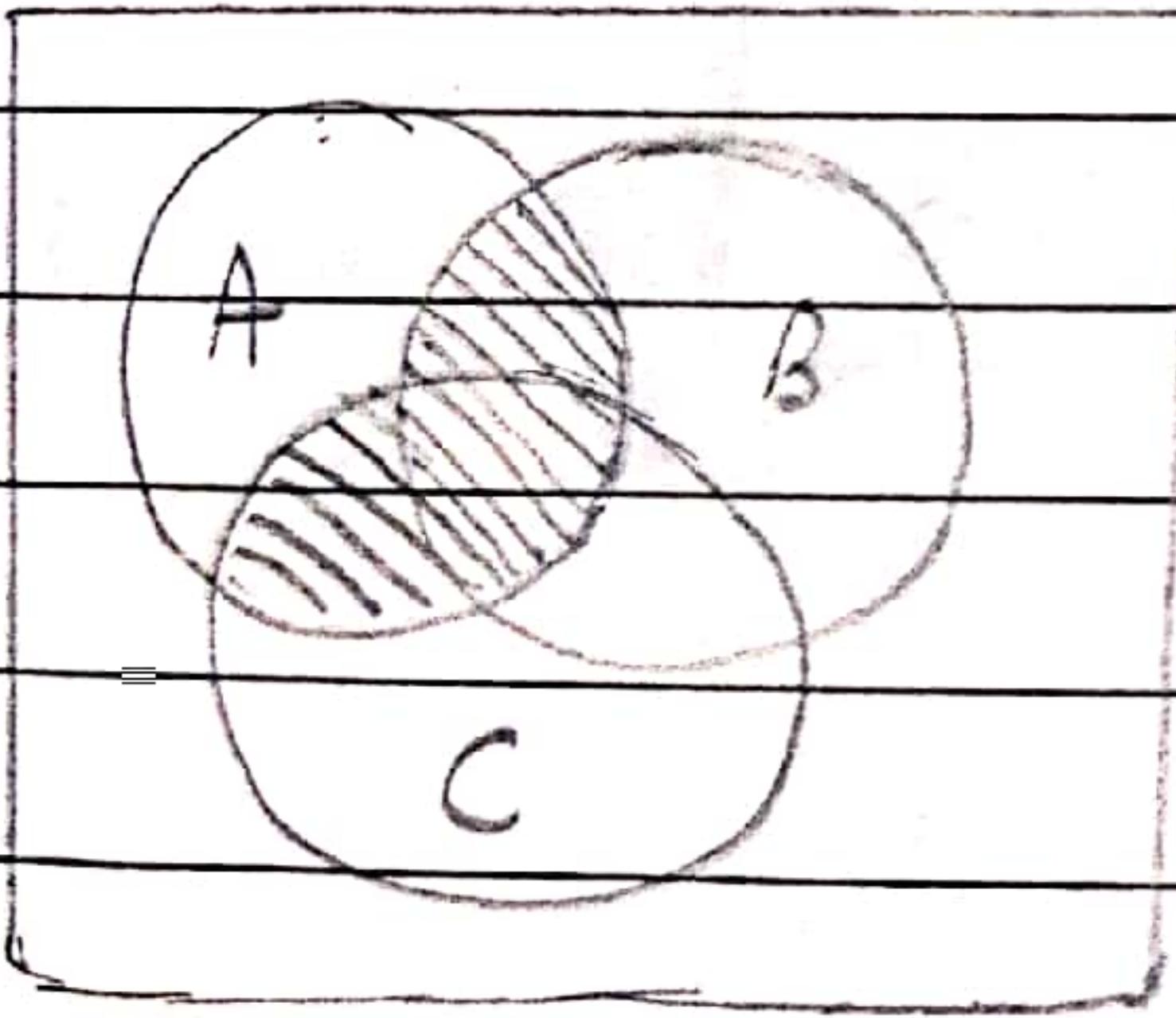
$$\text{b) } A \cap B = \{a, b, c, d, e\}$$

$$\text{c) } A - B = \{\}$$

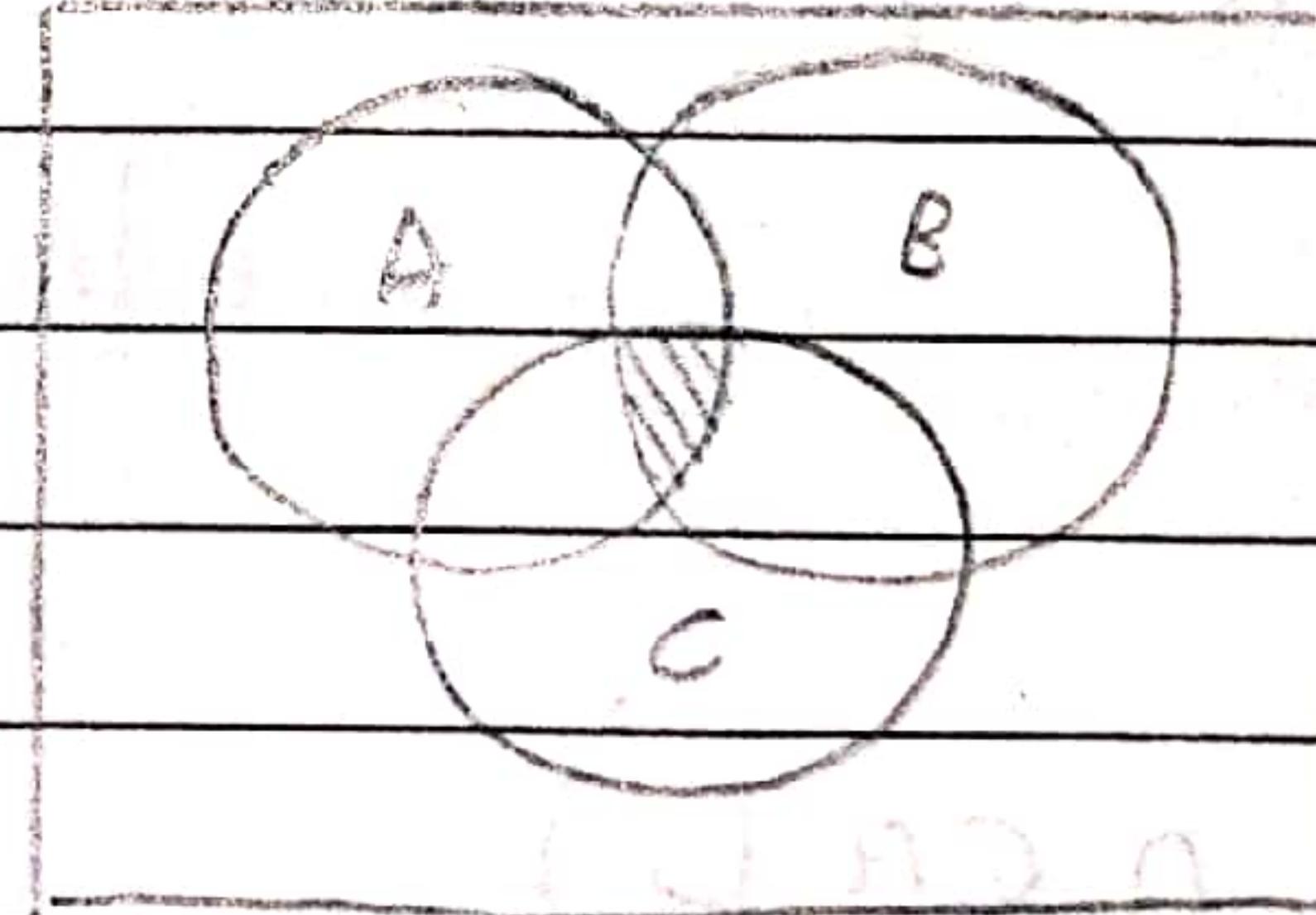
$$\text{d) } B - A = \{f, g, h\}$$

Q.8: Venn

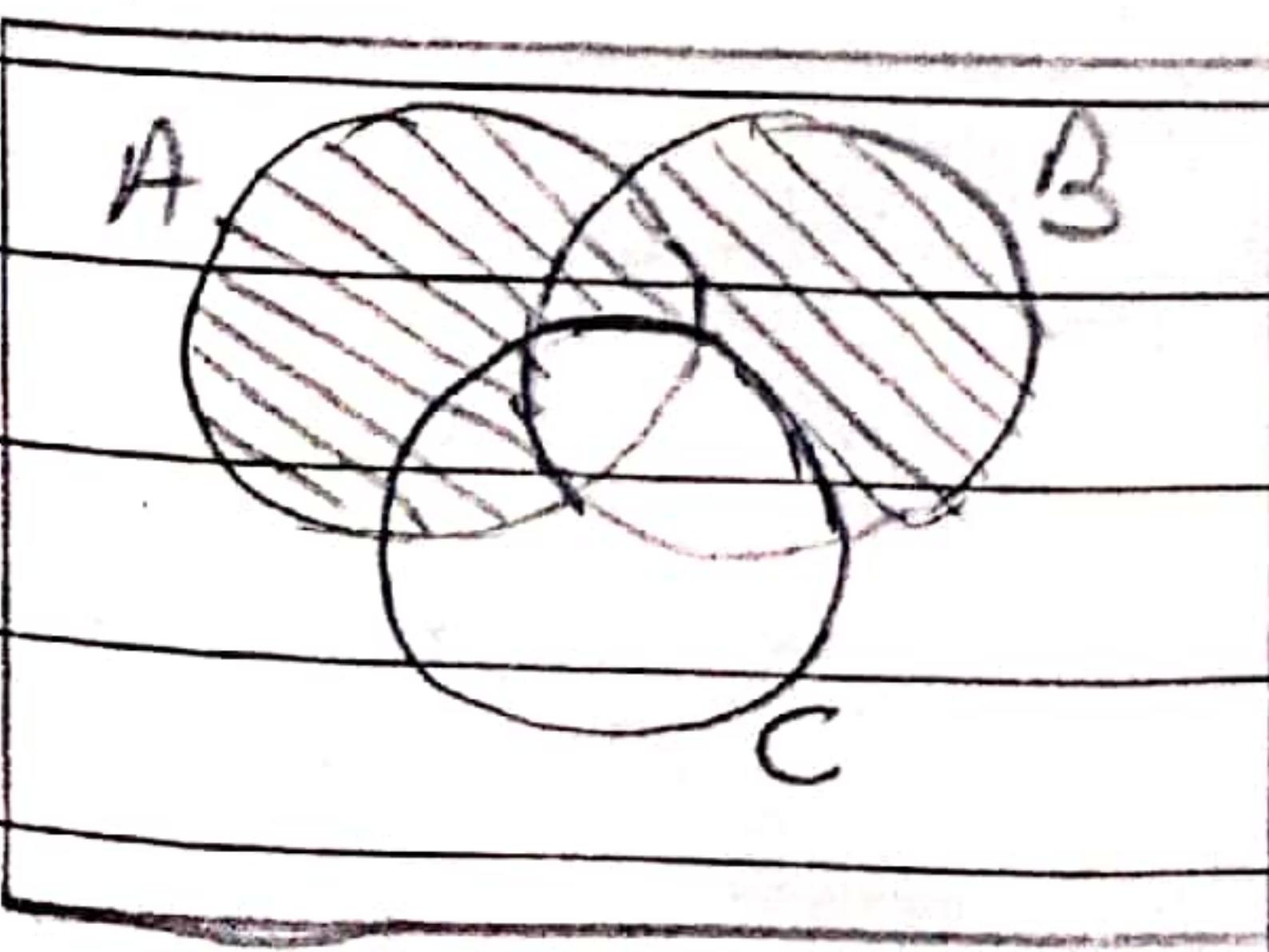
$$\text{a) } A \cap (B \cup C)$$



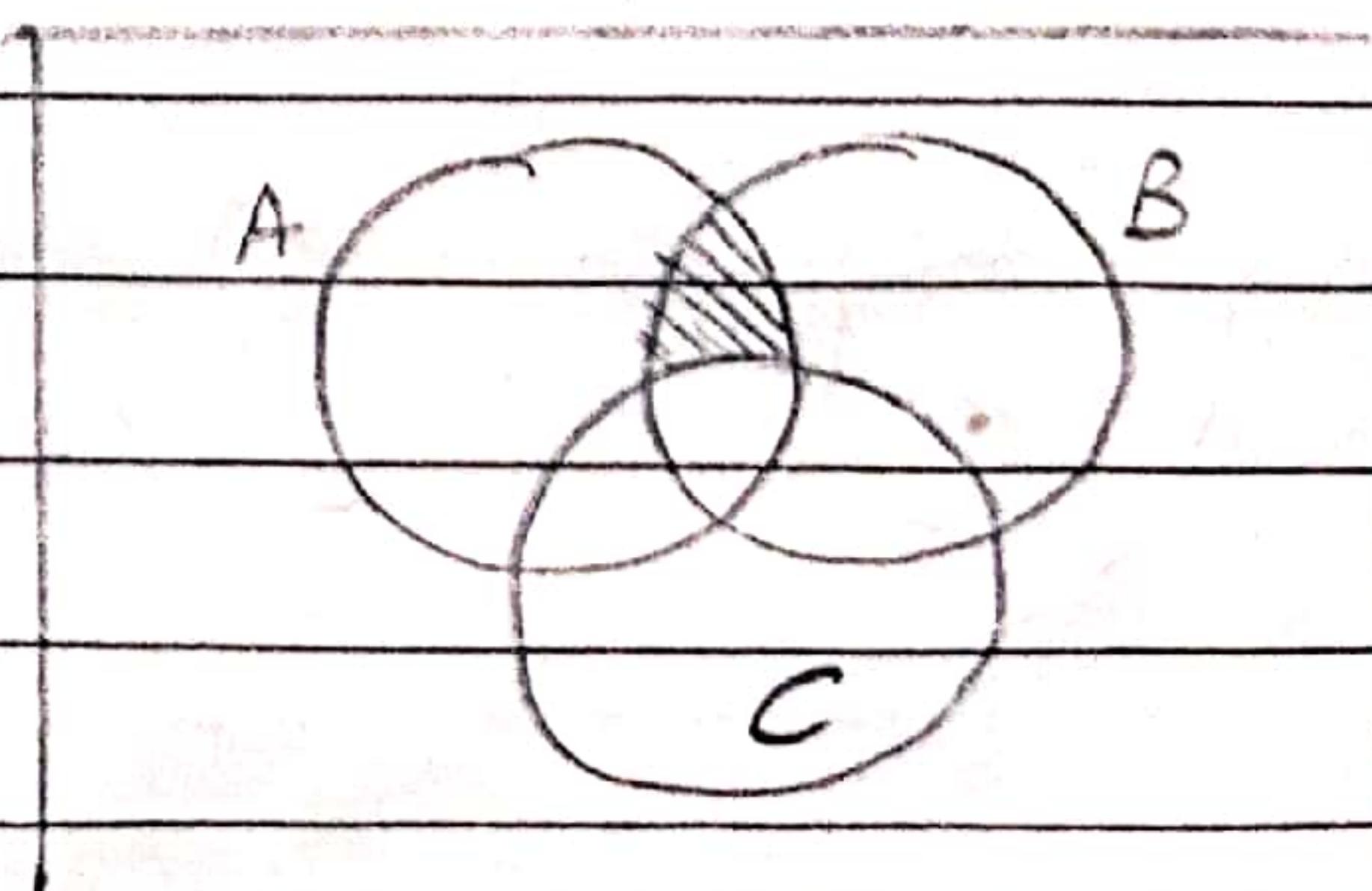
$$\text{b) } A \cap B \cap C$$



$$\text{c) } (A - B) \cup (A - C) \cup (B - C)$$

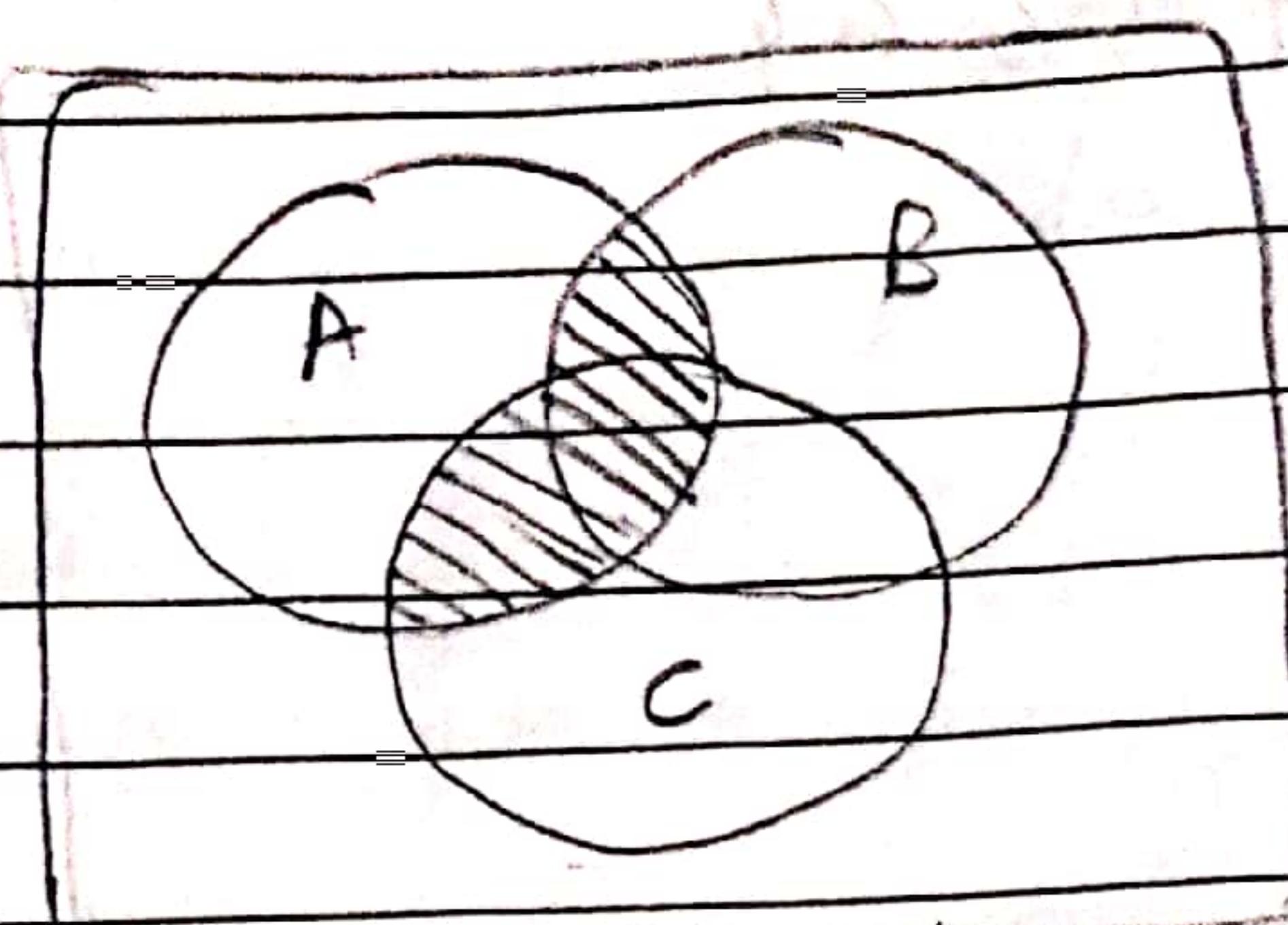


$$\text{d) } A \cap (B - C)$$

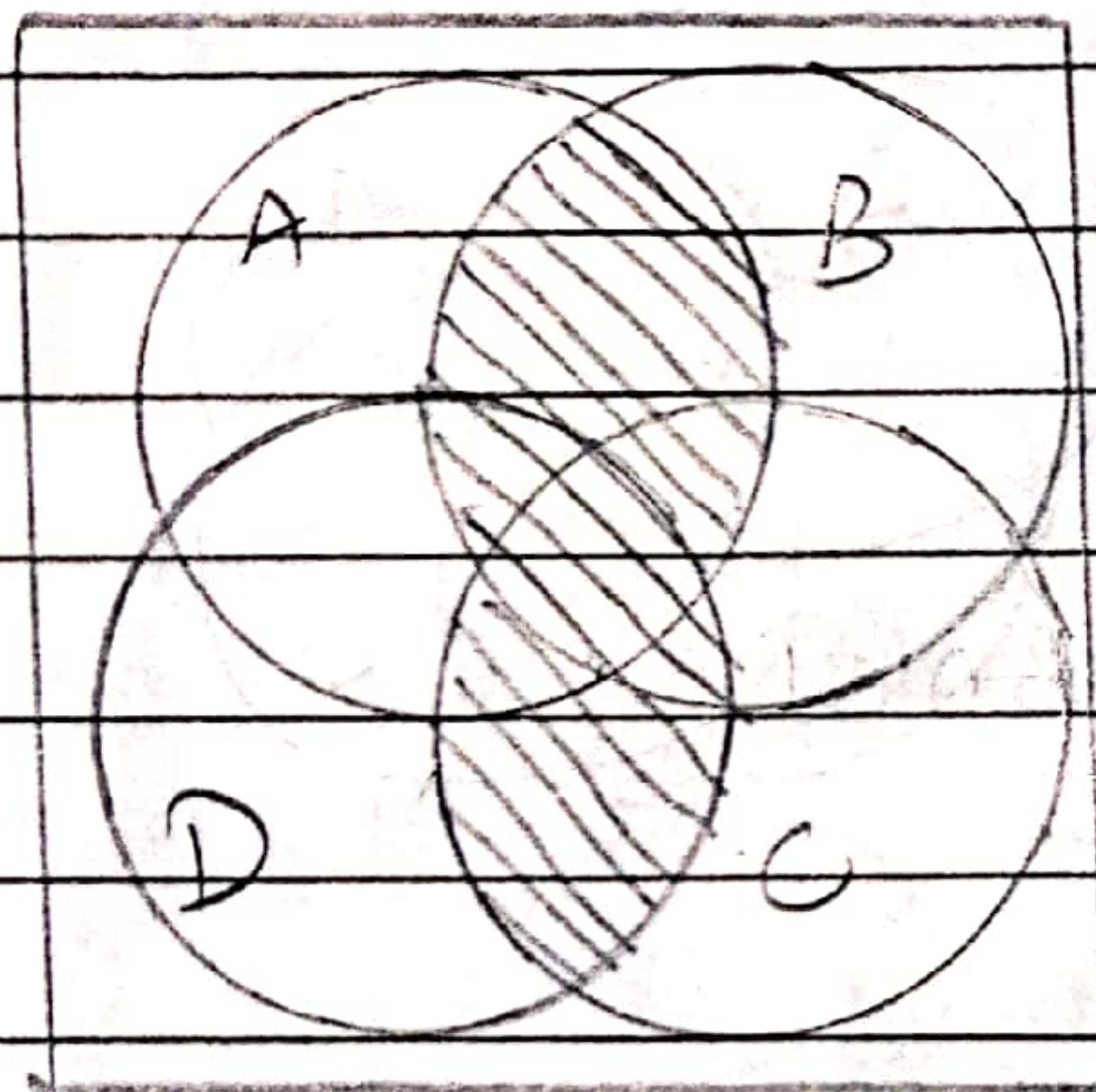


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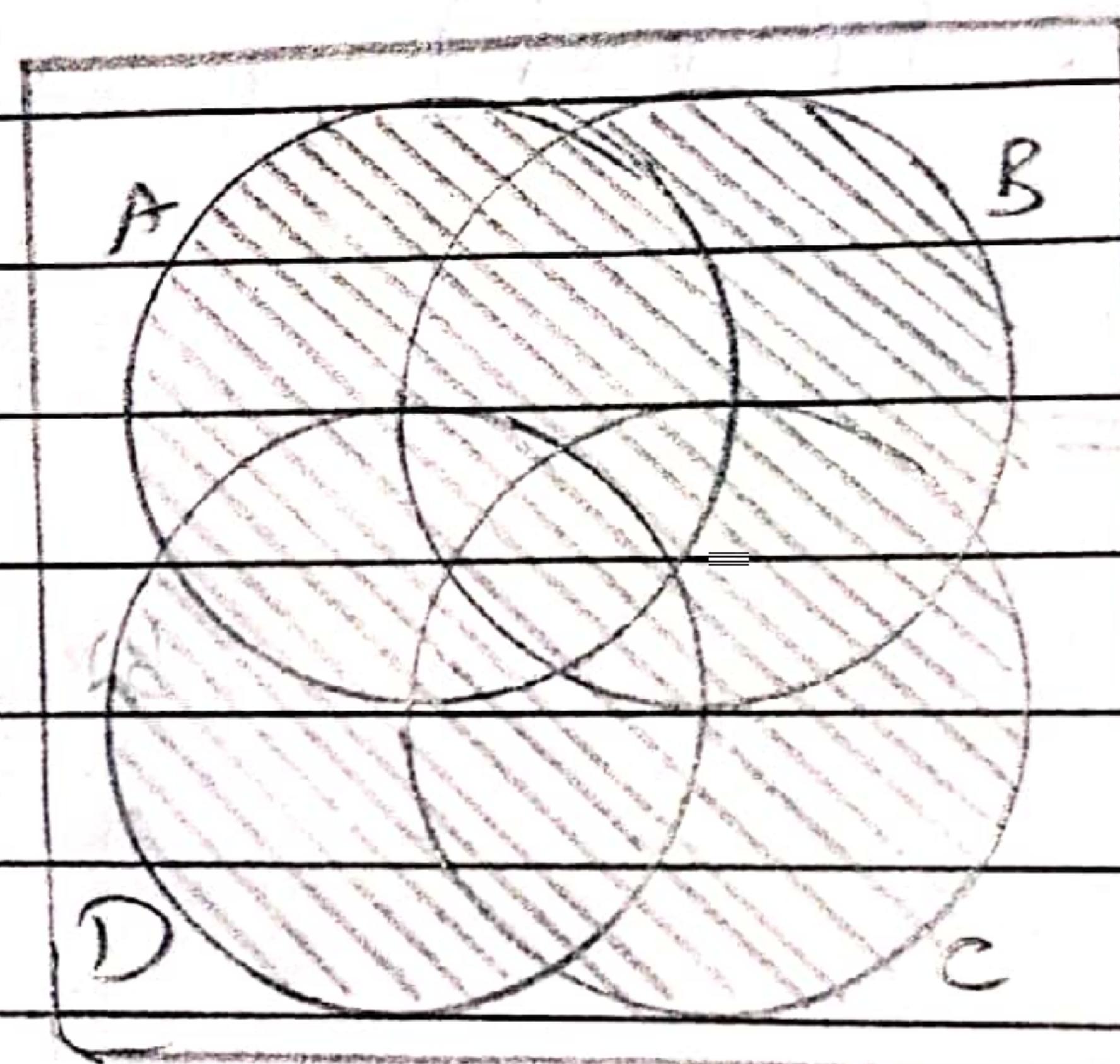
e) f)  $(A \cap B) \cup (A \cap C)$



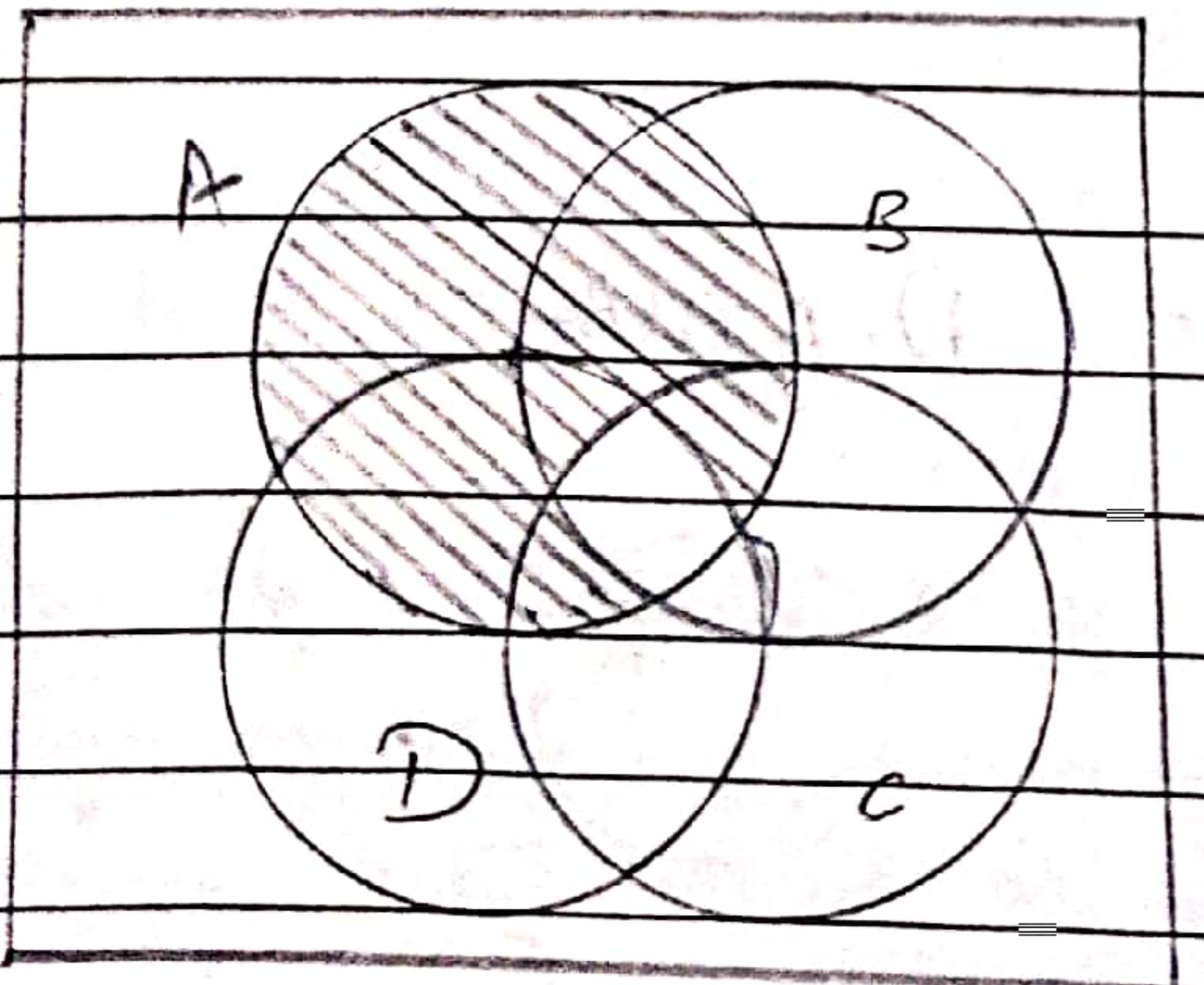
Q.9  
a)  $(A \cap B) \cup (C \cap D)$



b)  $A \cup B \cup C \cup D$



c)  $A - (B \cap C \cap D)$



Q. 10:

$$a) (A \cup B) \subseteq (A \cup B \cup C)$$

Sol: Let  $x \in A \cup B$

Acc. to the definition of union,

$x \in A$  or  $x \in B$  or in both.

Now, using addition of prepositions: ( $\frac{P}{\therefore p \vee q}$ ) ✓

$x \in A$  or  $x \in B$  or  $x \in C$ .

By union,  $x \in A \cup B \cup C$

$\therefore$  By definition of subset  $(A \cup B) \subseteq (A \cup B \cup C)$

$$b) (A \cap B \cap C) \subseteq (A \cap B)$$

Sol: Let  $x \in A \cap B \cap C$ .

Using definition of intersection,  $x$  belongs to all of them

$\Rightarrow x \in A$  and  $x \in B$  and  $x \in C$ .

Using simplification of prepositions ( $\frac{P \wedge Q}{\therefore P}$ ) ✓

$x \in A$  and  $x \in B$

Using definition of intersection  $x \in A \cap B$

$\therefore$  by definition of subset  $(A \cap B \cap C) \subseteq (A \cap B)$

$$c) (A - B) - C \subseteq A - C$$

Sol: Let  $x \in (A - B) - C$ .

Using definition of difference,

$\Rightarrow x \in A$  and  $x \notin B$  and  $x \notin C$

Using simplification of proposition ( $\frac{P \wedge Q}{\therefore P}$ ) ✓

$x \in A$  and  $x \notin C$ .

Using definition of diff.  $x \in (A - C)$

By definition of subset,  $(A - B) - C \subseteq (A - C) \subseteq$

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$$d) (A - C) \cap (C - B) = \emptyset$$

Sol: Let  $x \in (A - C) \cap (C - B)$

So, we can write by definition of intersection  
 $x \in A - C$  and  $x \in C - B$

Now using definition of difference

$x \in A$  and  $x \notin C$  and  $x \in C$  and  $x \notin B$   
 cannot be True

$\Rightarrow x \in A$  and False and  $x \notin B$

By domination law  $\Rightarrow$  False.

$\therefore$  the set cannot have any element  $\Rightarrow x \in \emptyset$

By definition of subset  $(A - C) \cap (C - B) \subseteq \emptyset$

as we know empty set is always a subset of any set

$$\emptyset \subseteq (A - C) \cap (C - B)$$

Hence, as both are subset of each other

$$(A - C) \cap (C - B) = \emptyset$$

✓

$$e) (B - A) \cup (C - A) = (B \cup C) - A$$

Sol: Let  $x \in (B - A) \cup (C - A)$

By definition of union  $x \in (B - A)$  or  $x \in (C - A)$

$\Rightarrow$  By definition of difference

$$(x \in B \wedge \neg(x \in A)) \vee (x \in C \wedge \neg(x \in A))$$

By distributive law,  $(x \in B \vee x \in C) \wedge \neg(x \in A)$

By definition of union,  $(x \in B \cup C) \wedge \neg(x \in A)$

By definition of difference  $x \in (B \cup C) - A$

Now, by definition of subset  $(B - A) \cup (C - A) \subseteq (B \cup C) - A$

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for the other side.  
let  $x \in (B \cup C) - A$ .

By definition of difference  $\Rightarrow (x \in B \cup C) \wedge \neg(x \in A)$

By definition of union  $\Rightarrow (x \in B \text{ or } x \in C) \text{ and } x \notin A$

By distributive law  $\Rightarrow x \notin B$

$\Rightarrow (x \in B \text{ and } x \notin A) \text{ or } (x \in C \text{ and } x \notin A)$

We can write now,  $x \in (B - A) \text{ or } x \in (C - A)$

and by union,  $x \in (B - A) \cup (C - A)$

By definition of subset  $\Rightarrow (B - A) \cup (C - A) \subseteq (B \cup C) - A$

$$(B \cup C) - A \subseteq (B - A) \cup (C - A)$$

Since, both are subset of each other

$$\Rightarrow [(B - A) \cup (C - A) = (B \cup C) - A] \quad \checkmark$$

## Countability:

Q.1 :

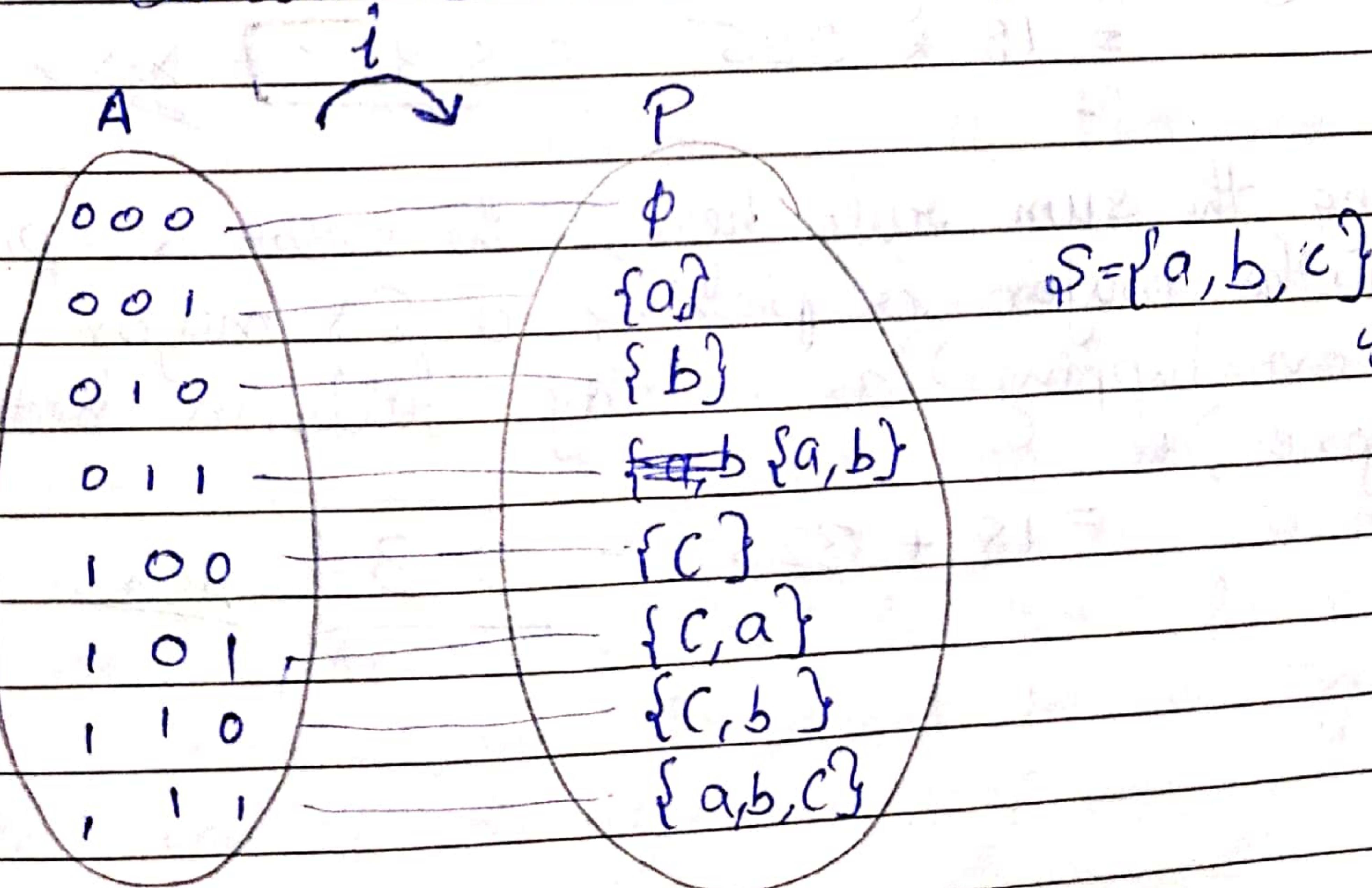
Let us consider, a one-to-one correspondence between subsets of  $S$  and bit strings (with each character either 0 or 1) of length  $|S|$ . A subset of  $S$  is associated with the bit string with '1' in the  $i$ th position of the  $i$ th element in the list in the subsets. So, by the Multiplication rule, there are  $2^{|S|}$  bit strings of length  $|S|$ .

$$\text{Therefore } |P(S)| = 2^{|S|}$$

A = list of bit strings of length  $|S|$

$P$  = Power set of  $S$

$x$  = a subset of  $S$  is associated with a 1 in  $i$ th position if  $i$ th element is in the subset  $x$



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Q.2.

Let  $x$  and  $y$  be 2 sets having  $m$  &  $n$  elements respectively. In a function from  $x$  to  $y$ , every element from  $x$  must be mapped to an element of  $y$ .  $\Rightarrow$  each element of  $x$  has ' $n$ ' elements to choose from.  
Therefore total no. of function will be  
 $n \times n \times n \times \dots n$   <sup>$m$  times</sup> =  $n^m$   
 $\Rightarrow$  Total no. of function =  $n^m$  [Ans]

Q.3 Given Maths major = 18

C.S. majors = 325.

a) Using the product rule here, first event is picking a maths major and second event is picking a C.S. major.

$$= 18 \times 325 = 5850$$
 [Ans]

b) Using the sum rule here, the event is picking a maths major or picking a C.S. major.  
(Non overlapping) (as nobody falls in both categories).

$$= 18 + 325 = 343$$
 [Ans]

Q.4. Given, 12 colors, 2 genders and 3 diff. sizes for each gender.

Using the product rule here, because the first event is picking the color, second event is choosing gender & third is picking the size.

$$= 12 \times 2 \times 3 = \underline{\underline{72 \text{ ways}}}$$

Q.5:

A student can choose a project by selecting one from any of the three lists. Since, no project is on more than one list, by the sum rule we can say there are  $23 + 15 + 19 = \underline{\underline{57 \text{ ways}}}$  to choose.

Q.8

Assuming on the contrary, that there is no integer in the set that divides another integer in the set.

If  $a$  is in the set such that  $a \leq n$  then  $2a$  cannot be in the set.

Thus, if there are  $k$  elements in the set not exceeding  $n$ , then there are  $k$  ~~elements~~ integers greater than  $n+1$ , but less than  $2n$  which cannot be in the set.

Thus on maximum ' $n$ ' elements can be there in the set.

Thus, ~~there~~ Contradicting the fact that  $S$  has  $n+1$  elements. Hence, in ' $n+1$ ' integers, there must be an integer that  $\alpha$  divides one of the other integers.

Genius  
ASV

Game 1 - - -  
Game 2 - - -  
Game 3 - - -  
Game 4 - - -  
Game 5 - - -  
Best of out of 3 at most 5 play-offs.

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Q. 6:

Team 1

Team 2

Game 4

Team 1

Team 2

Team 1

Game 3

Team 1

Team 2

Team 1

Team 2

Team 1

Team 2

Team 1

Team 2

Game 2

Team 1

Team 2

Team 1

Team 2

Team 1

Team 2

Team 1

Team 2

Team 1

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We can see 20 different ways are possible

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~~Q. 7:~~

- a). Suppose for each suit, we have a box that contains card of that suit.

The no. of boxes is 4, and to have at least 3 cards  $\Rightarrow \left[ \frac{N}{4} \right]$  cards in the same box.

The total no. of cards to be chosen.

$$N = 2 \times 4 + 1 = 9 \text{ (minimum).}$$

- b). In worst case scenario, we may select all other suites ( $13 \times 3$ ) [spades, clubs and diamonds] and then any hearts.

So, to guarantee we need to select take minimum  $39 + 3 = 42$  cards to ensure 3 hearts.

- ~~Q. 9~~ There are 30 students in class. Assume that each student has at least a first name and a last name. Each last name has to begin with a letter of the alphabet.

Objects = first letter of each last name  
 $= 30 = N$

Holes = letter of alphabets =  $k = 26$

$$\left[ \frac{30}{26} \right] = 2$$

So, by Pigeonhole principle, there are atleast two students who have a last name that begin with same letter.

Q. 10 :

Total 20 balls are present, half of them is red and the other half is blue.

a) The pigeon holes are the colors.

$\lceil \frac{x}{2} \rceil$  must be equal to ~~3~~ 3 & and least +ve integer satisfying this integer is equation is

$$\underline{\underline{5}} =$$

b) The first 10 balls she selects could be all red, so, she needs to select minimum of  $10 + 3 = 13$  balls to be sure of having at least three blue balls.

Date / /

## Principle of Inclusion-Exclusion

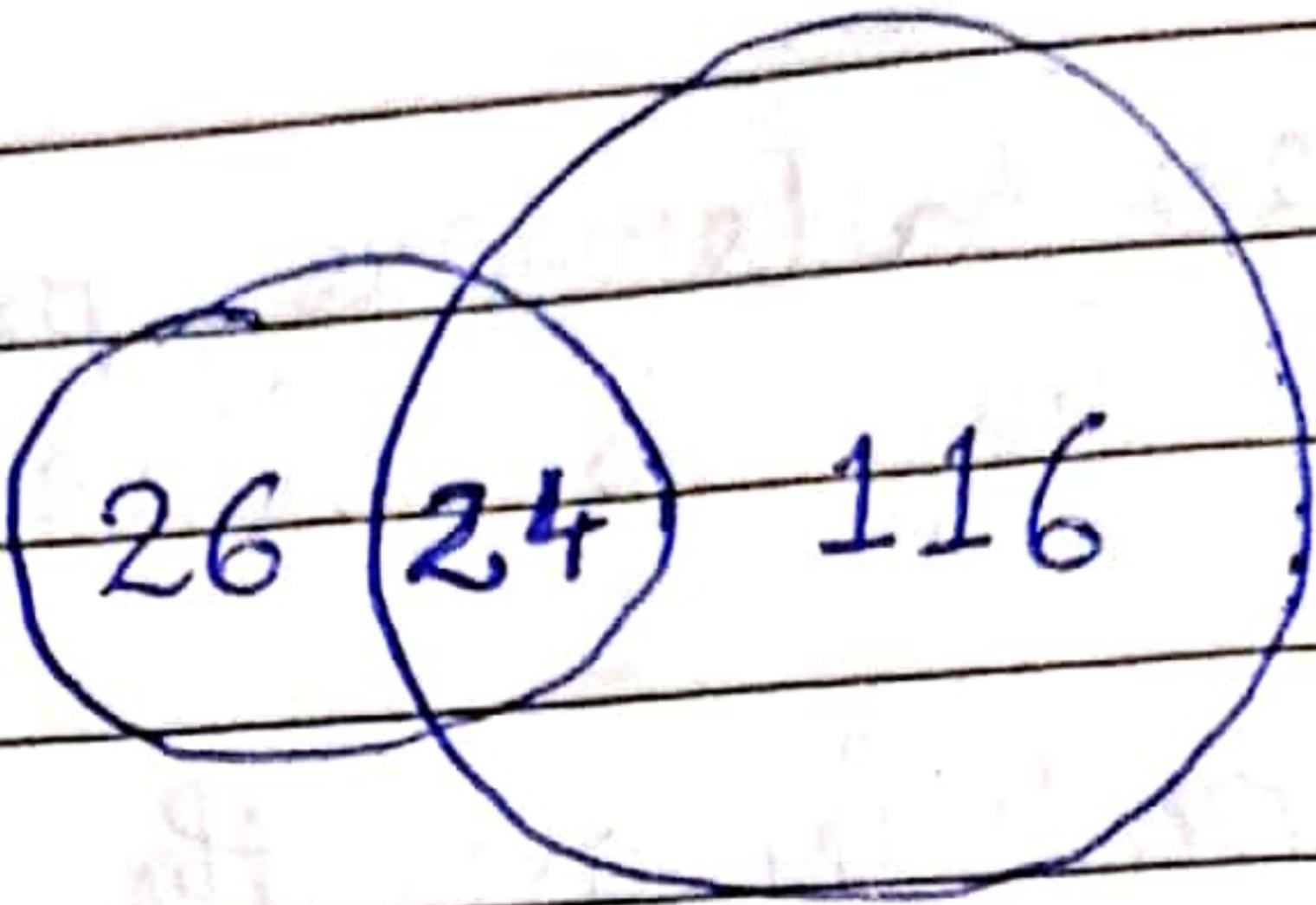
Q.1

$$U = 200$$

$$M = 50$$

$$E = 140$$

$$\text{we know } n(M \cap E) = 24$$



for ~~MUE~~:  $MUE = M + E -$

$$n(MUE) = n(M) + n(E) - n(M \cap E)$$

$$= 166$$

Hence, people at the party will total minus ones giving exam that is ~~is~~  $n(\overline{MUE})$

$$= n(U) - n(MUE)$$

$$= 200 - 166$$

$$= 34 \quad \underline{\text{Ans}}$$

Q.2

$$n(C) = 25$$

$$n(M) = 13$$

$$n(M \cap C) = 8$$

$$n(MUC) = n(M) + n(C) - n(M \cap C)$$

$$= 25 + 13 - 8$$

$$= 30$$

Hence, total 30 students are there.

Q.3:

$$n(U) = 1807$$

freshmen

$$n(C) = 453$$

$$n(M) = 567$$

$$n(M \cap C) = 299$$

$$\text{to find } n(\overline{M \cup C}) = n(U) - n(M \cup C)$$

$$= 1807 - (n(M) + n(C) - n(M \cap C))$$

$$= 1807 - 453 - 567 + 299$$

$$= 2106 - 1020$$

$$= 1086 \quad \underline{\text{Ans}}$$

Q.4:

$$n(S) = 1232$$

$$n(F) = 879$$

$$n(R) = 114$$

$$n(S \cap F) = 103$$

$$n(S \cap R) = 23$$

$$n(F \cap R) = 14$$

$$n(S \cup F \cup R) = 2092$$

$$\begin{aligned} n(S \cup F \cup R) &= n(S) + n(F) + n(R) - (n(S \cap F) + \\ &\quad - n(S \cap R) - n(F \cap R) + n(S \cap F \cap R)) \end{aligned}$$

$$\Rightarrow n(S \cap F \cap R) = 2092 - 1232 - 879 - 114 + 103 + 23 + 14$$

$$= 860 - 879 + 26$$

$$= 886 - 879$$

$$= 7$$

Ans

I have taken all three

Date 1/1

~~Q.5~~

$$n(C) = 345$$

$$n(M) = 212$$

$$n(M \cap C) = 188$$

in either M or C  $\Rightarrow n(M \cup C) = n(C) + n(M) - n(M \cap C)$

$$= 212 + 345 - 188$$

$$= 557 - 188$$

$$= 369 \text{ Ans}$$

~~Q.6~~

Let percentage be direct cardinalities with  $n(U) = 100$

$$n(TV) = 96$$

$$n(TP) = 98$$

$$n(TP \cap TV) = 95$$

neither TV nor TP  $\Rightarrow n(\overline{TP} \cap \overline{TV}) = n(\overline{TP \cup TV})$

$$= n(U) - n(TP \cup TV)$$

we have  $= 100 - (n(TP) + n(TV)) - n(TP \cap TV)$

$$= 100 - 96 - 98 + 95$$

$$= 195 - 194$$

$$= 1$$

1% of households have neither TV nor TP.

~~Q.7~~

$$n(A_1) = 100 = n(A_2) = n(A_3)$$

$\Leftrightarrow$  for  $n(A_1 \cup A_2 \cup A_3)$ .

a) pairwise disjoint, total 300 elements.

b) 50 common pairwise, no common in all three:

$$= 100 \times 3 - 50 \times 3 + 0$$

$$= 150$$

c) 50 common pairwise, 25 common in all.

$$= 100 \times 3 - 50 \times 3 + 25 \\ = 175$$

d) Sets are equal : 100.

Q. Q:  $n(A_1) = 100$

$$n(A_2) = 1000$$

$$n(A_3) = 10000$$

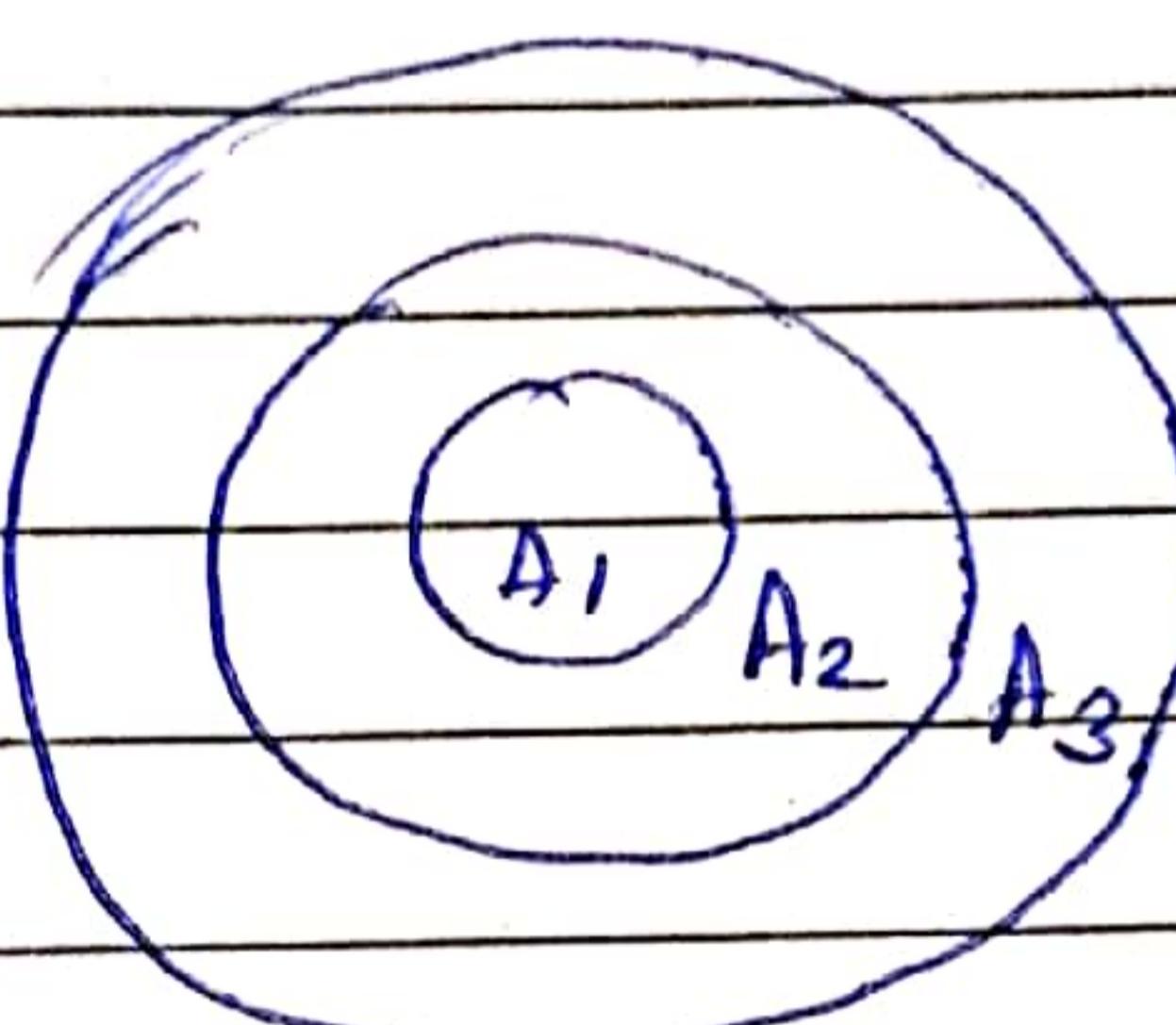
find  $n(A_1 \cup A_2 \cup A_3)$ .

a)  $A_1 \subseteq A_2 \& A_2 \subseteq A_3 \Rightarrow$

Hence,

$$n(A_1 \cup A_2 \cup A_3)$$

$$= 10000$$



b) Pairwise disjoint:  $n(A_1 \cup A_2 \cup A_3) = n(A_1) + n(A_2) + n(A_3)$   
 $= 11100$  ~~Any~~

c) 2 common pairwise, & 1 common in all 3.

$$= 10000 + 1000 + 100 - 2 \times 3 + 1$$

$$= 11093$$

$$= 11095$$