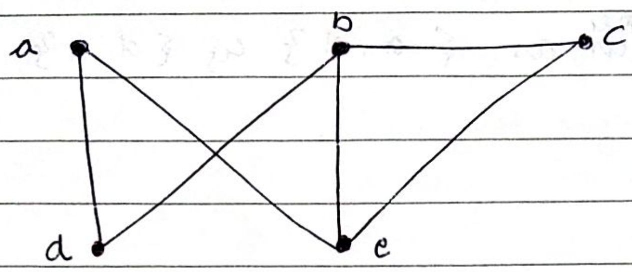
Graph - Theory - 1

Q1



(a) a, e, b, c, b

It is not a simple path as {b, c} edge is traversed twice.

Length = 4

(b) a, e, a, d, b, c, a

It is not a path as {a, e} edge traversed twice and edge {c, a} doesn't exist.

(c) e, b, a, d, b, e

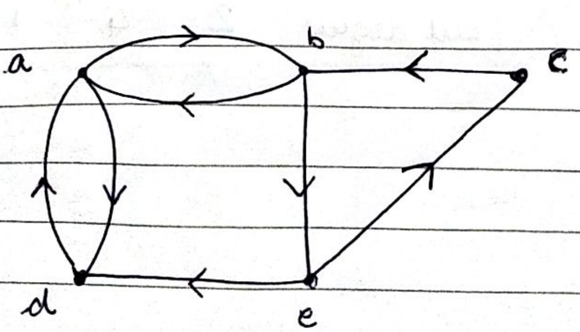
It is not a path as {b, a} edge does not exist

(d) c, b, d, a, e, c

It is a closed walk / circuit

Length = 5

Q2



(a) a, b, e, c, b

It is a simple path with a circuit {b, e, c, b}.

Length = 4

(b) a, d, a, d, a

It is not a simple path as {a, d} & {d, a} are repeating.

Length = 4

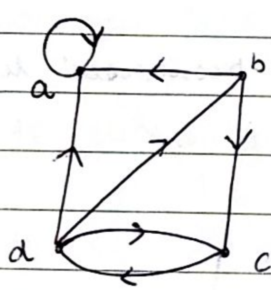
(c) a, d, b, e, a

It is not a path as {d, b} does not exist.

(d) a, b, e, c, b, d, a

It is not a path as {b, d} does not exist.

Q3 (a)

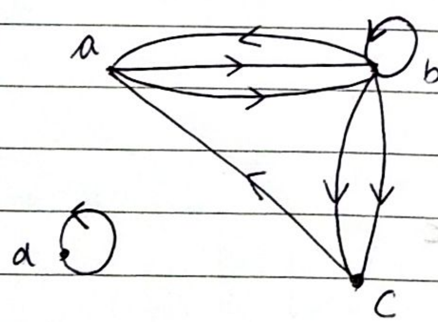


vertex	a	b	c	d
In-degree	3	1	2	1
out-degree	1	2	1	3

vertices = 4

edges = 7

(b)

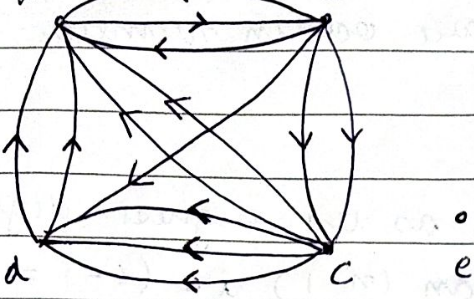


vertex	a	b	c	d
In-degree	2	3	2	1
out degree	2	4	1	1

vertices = 4

edges = 8

(c)

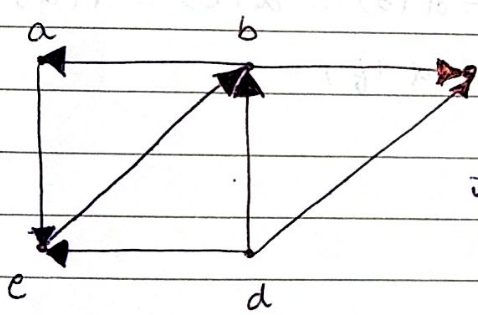


vertex	a	b	c	d	e
In-degree	5	1	2	4	0
Out-degree	1	5	5	2	0

vertices = 5
edges = 13

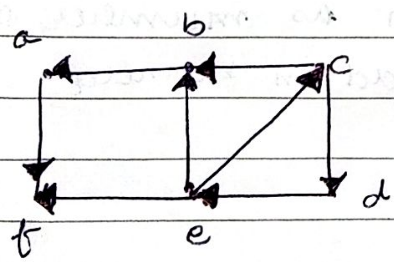
Q4 Find the strongly connected components of each of these graphs

(a)



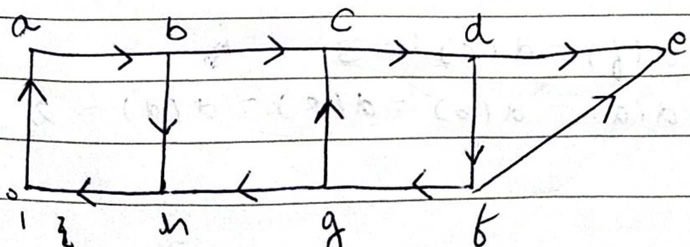
{ a, e, b }
There is only one set of strongly connected component

(b)



{ a, f, e, c, b }, { e, c, d }

(c)



{ a, b, c, d, f, g, h, i }

Q5 Determine whether its a valid graph or no

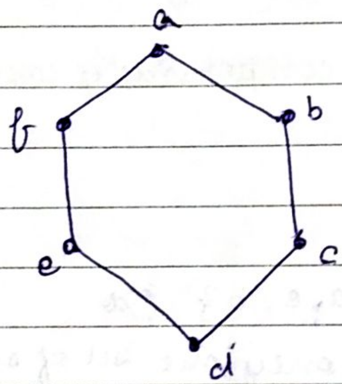
(a) 5 4 3 2 1 0

No it is not a valid graph as number of vertices with odd degree {5, 3, 1} are odd in number.

(b) 6, 5, 4, 3, 2, 1

It is not a valid graph as the highest degree i.e 6 is greater than (n - 1) i.e (6 - 1 = 5)
(n = no. of vertices)

(c) 2 2 2 2 2 2



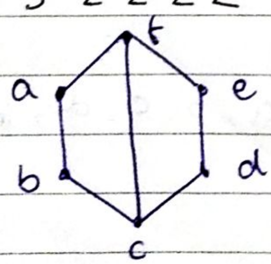
each with degree 2

d(a) = d(b) = d(c) = d(d) =
d(e) = d(f) = 2

(d) 3 3 3 2 2 2

no it is not a valid graph as number of odd degree vertices are odd in number
{3, 3, 3}

e) 3 3 2 2 2 2

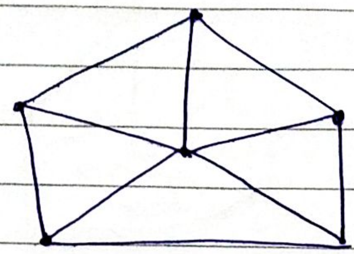


d(f) = d(c) = 3
d(a) = d(b) = d(e) = d(d) = 2

(f) 1 1 1 1 1 1

not possible as it can never be closed.

(g)



h) 5, 5, 4, 3, 2, 1 (Not possible to make a graph)

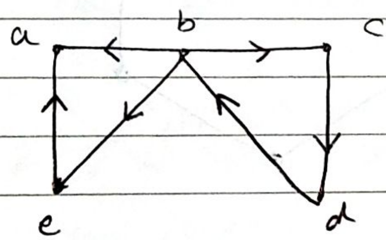
By Havel Hakimi Theorem

~~5~~ 5 4 3 2 1

~~4~~ 3 2 1 0

2 1 0 -1

Q6(a)

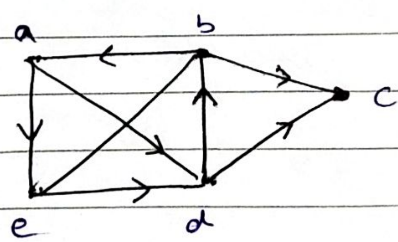


{b, c, d} strongly connected

{a, b, e) weakly connected

Hence weakly connected

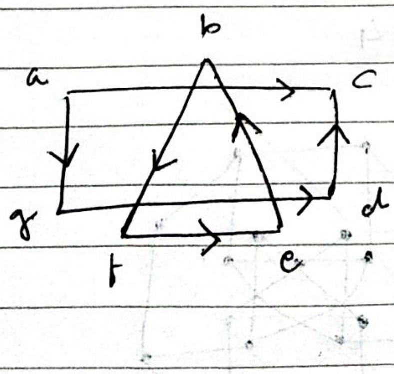
(b)



{b, a, e, d} strongly connected

{b, c, d} weakly connected

(c)



weakly connected

Q7 (a) K7

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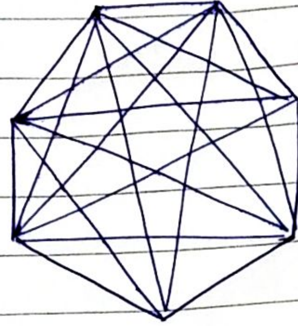
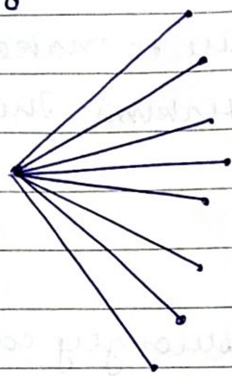
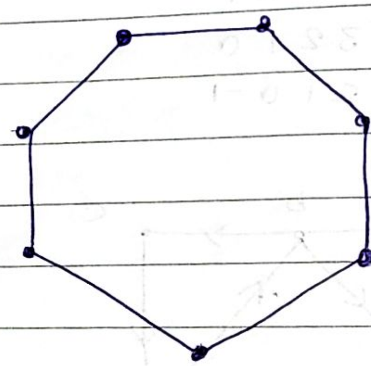


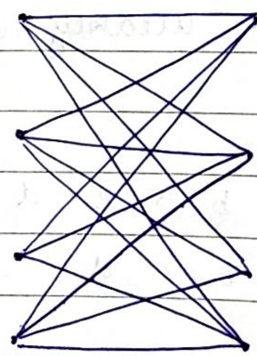
(b) K1, 8



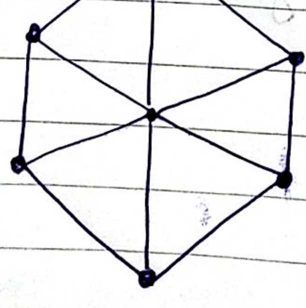
(d) C7



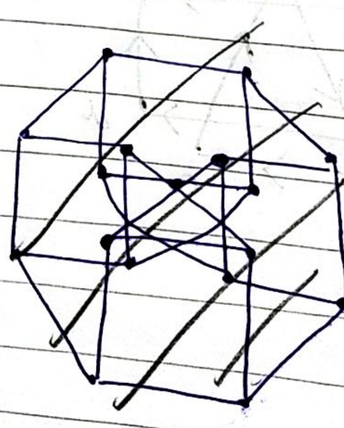
(c) K4, 4



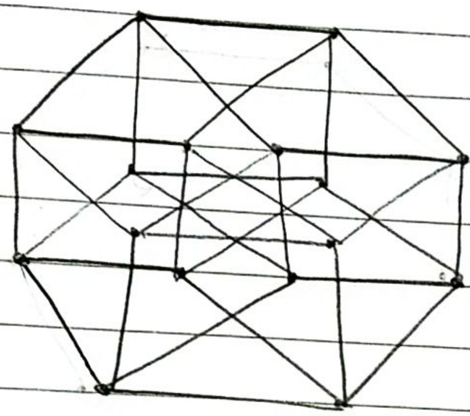
(e) W7



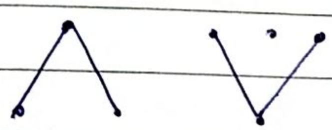
(f) Q4



5) Q_7

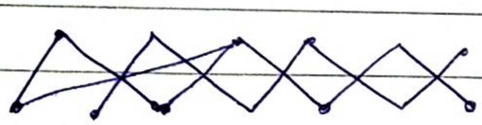


11)



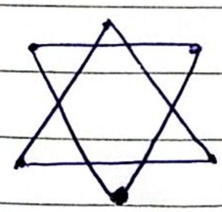
not connected , 3 components

2)



connected

)



not connected , 2 components