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Asingment 1.

Ques. by bisection method find approx root of $\sin x - 1/x$ up to 7^{th} stages ?

Ans.

we have $\sin x = 1/x$

$\sin x - 1/x = 0$

let.

$f(x) = \sin x - 1/x$

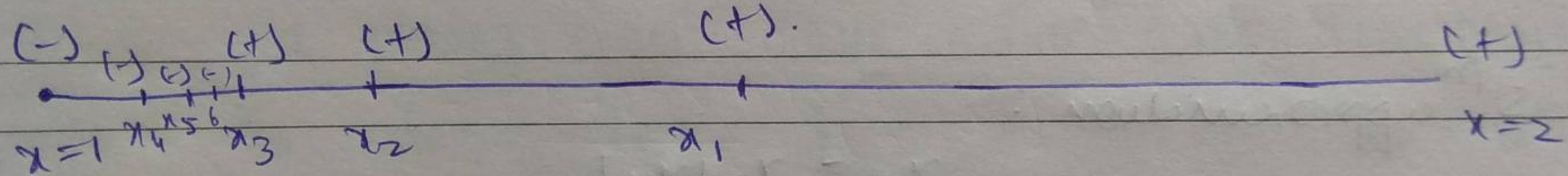
Now put value

$f(0) = -\infty < 0$

$f(1) = \sin(1) - 1 = -0.159 < 0$

$f(2) = 0.409 > 0$

root lies between 1 & 2.



1^{st} approximation:

$$x_1 = \frac{x_a + x_b}{2}$$

$$= 3/2 = 1.5$$

$f(x_1) = 0.331 > 0$

2^{nd} approximation:

$$x_2 = \frac{x_a + x_1}{2}$$

$$= 1.25$$

$f(x_2) = 0.149 > 0$

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3^{rd} approximation:

$$x_3 = \frac{x_a + x_2}{2}$$

$$= \frac{2.25}{2} = 1.125$$

$f(x_3) = 0.013 > 0$

4^{th} approximation:

$$x_4 = \frac{x_a + x_3}{2}$$

$$= \frac{2.125}{2}$$

$$= 1.063$$

$f(x_4) = -0.0669$

$= -0.067.$

5^{th} approximation:

$$x_5 = x_3 + x_4 / 2$$

$$= 1.094$$

$f(x_5) = -0.026 < 0$

6^{th} approximation:

$$x_6 = x_5 + x_3 / 2$$

$$= 1.11$$

$f(x_6) = -0.005 < 0$

7^{th} approximation:

$$x_7 = \frac{x_6 + x_3}{2} = \frac{1.1175}{2}$$

Ans

root of $\sin x - 1/x$ is around $\frac{1.1175}{2} \approx \frac{1.118}{2}$.

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Assingment 2.

Ques-1 use method of false position to find 4^{th} root of 32 upto 4 decimal?

Ans

let $\sqrt[4]{32} = x$

$32 = x^4$

$x^4 - 32 = 0$

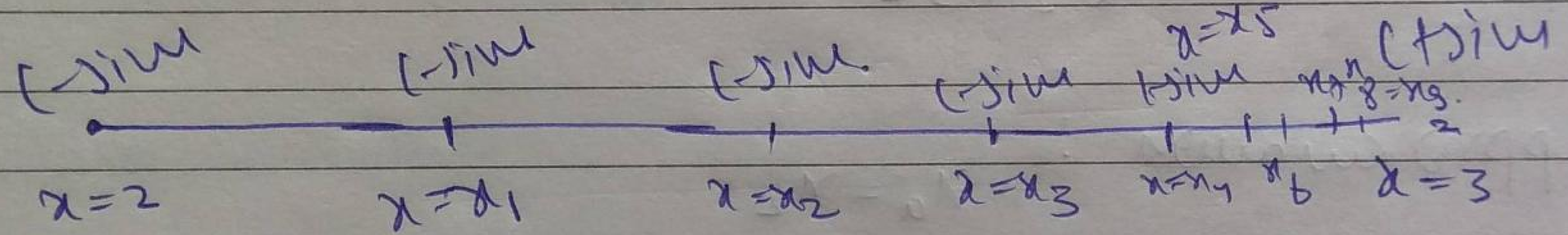
let $f(x) = x^4 - 32$ Now we have find value of f where $f = 0$;

$f(1) < 0 = -31$

$f(2) < 0 = -16$

$f(3) > 0 = 49$

Root lies between 2 and 3.



$a = 2$

$b = 3$

1^{st} iteration approximation :

$$x_1 = x_a - \frac{(x_b - x_a)}{(f(x_b) - f(x_a))} f(x_a)$$

$$= 2 - \frac{(2-3)}{(-16-49)}(-16) = 2 + \frac{16}{65}$$

$$= 2.2462.$$

$$f(x_1) = (2.2462)^2 - 32 = -6.5438.$$

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2^{nd} approximation:

$$x_a = x_1$$
$$x_b = 3$$

$$x_2 = 2.2462 - \frac{(2.2462-3)}{(-6.5438-49)} \times (-6.5438)$$

$$= 2.2462 + \frac{(0.7538)(6.5438)}{(55.5438)}$$

$$x_2 = 2.335$$

$$f(x_2) = (2.335)^4 - 32$$
$$= -2.2732$$

3^{rd} approximation:

$$x_a = x_2$$
$$x_b = 3$$

$$x_3 = 2.335 - \frac{(2.335-3)}{(-2.2732-49)} \times (-2.2732)$$

$$= 2.335 + \frac{(0.665)(2.2732)}{51.2732}$$

$$= 2.3645$$

$$f(x_3) = -0.7423$$

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4^{th} approximation :

$$x_a = x_3$$

$$x_b = 3.$$

$$x_4 = 2.3645 - \frac{(2.3645 - 3)}{(\cancel{-2.2732} - 49)} (\cancel{-2.2732})$$

-0.7423 -0.7423

$$= 2.3645 + \frac{(+0.6355)(+0.7423)}{(+49.7423)}$$

$$= 2.3740$$

$$f(x_4) = -0.237$$

5^{th} approximation :

$$x_a = x_4$$

$$x_b = 3$$

$$x_5 = 2.374 - \frac{(2.374 - 3)}{(-0.237 - 49)} (-0.237)$$

$$= 2.374 + \frac{(0.626)}{(49.237)} (0.237)$$

$$= 2.3770.$$

$$f(x_5) = -0.0760$$

6^{th} iteration: $x_a = x_5$
$x_b = 3$

$$x_6 = 2.377 - \frac{(2.377 - 3)}{(-0.076 - 49)}(-0.076)$$

$$= 2.377 + \frac{(0.623)}{(49.076)}(0.076)$$

$$= 2.378$$

$$f(x_6) = -0.0223$$

7^{th} iteration: $x_a = x_6$
$x_b = 3$

$$x_7 = 2.378 - \frac{(2.378 - 3)}{(-0.0223 - 49)}(-0.0223)$$

$$= 2.3783$$

$$f(x_7) = -0.0061$$

8^{th} iteration: $x_a = x_7$
$x_b = 3$

$$x_8 = 2.3783 - \frac{(3 - 2.3783)}{(49 + 0.0061)}(-0.0061)$$

$$= 2.3784$$

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$f(x_8) = -0.0008$

9^{th} iteration : $x_a = x_8$
$x_b = 3$

$$x_9 = 2.3784 + \frac{(3-2.3784)}{(49+0.0008)}(0.0008)$$

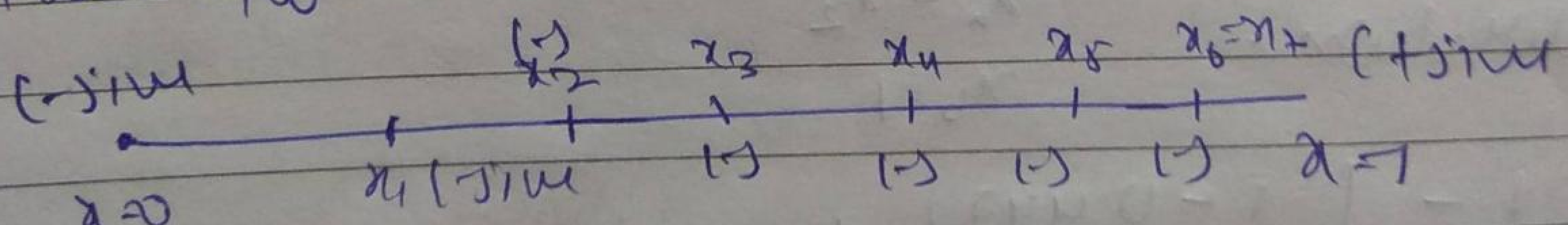
$= 2.3784$

$x_8 = x_9$ hence $\sqrt[4]{32} = x_8$
$= 2.384$

Ques. 2 use secant method to determine root of equation $xe^x = \cos x$?

Ans let
$f(x) = xe^x - \cos x.$

Put value:
$f(0) = 0 - 1 < 0$
$f(1) = 2.177 > 0$
root lies b/w 0 & 1



1^{st} iteration : $x = a = 0$
$x = b = 1$

$$x_1 = x_b - \frac{(x_b - x_a)}{(f(x_b) - f(x_a))} f(x_b).$$

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$$x_1 = 1 - \frac{(1-0)}{(2.178+0.1)}(2.178)$$

$$= 0.3145$$

$$f(x_1) = -0.519.$$

→ 2^{nd} iteration: $x = a = x_1$

$x_b = 1$

$$x_2 = 1 - \frac{(1-0.315)}{(2.178+0.519)}(2.178)$$

$$= 1 - \frac{(0.685)}{2.697}(2.178)$$

$$= 0.447$$

$$f(x_3) = -0.203$$

→ 3^{rd} iteration: $x_a = x_2$

$x_b = 1$

$$x_3 = 1 - \frac{(1-0.447)}{(2.178+0.203)}(2.178)$$

$$= 1 - \frac{0.553\ (2.178)}{2.381} = 0.506$$

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$f(x_3) = -0.035$

→ 4 iteration : $x_a = x_3$
$x_b = 1$

$$x_4 = 1 - \frac{(1-0.506)(2.178)}{(2.178+0.035)}$$

$$= 1 - \frac{(0.494)(2.178)}{(2.213)} = 0.514$$

$f(x_4) = -0.011$

→ 5^{th} iteration : $x_a = x_4$
$x_b = 1$

$$x_5 = 1 - \frac{(1-0.514)}{(2.189)}(2.178)$$

$= 0.516$

$f(x_5) = -0.005$

→ 6^{th} iteration : $x_a = x_5$
$x_b = 1$

$$x_6 = 1 - \frac{(1-0.516)}{(2.183)}(2.178) = 0.517.$$

$f(x_6) = -0.002.$

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→ 7^{th} iteration : $x_a = x_6$
$x_b = 1.$

$$x_7 = 1 - \frac{(1-0.517)}{(2.180)}(2.178)$$

$= 0.517$

here $x_6 = x_7$ so xe^x = corr root is 0.517

Z. [illegible]

assignment 3.

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Ques. Solve $\sqrt{28}$ upto 4 decimal places using newton raphson method

Ans

let $\sqrt{28} = x$. | square on both side

$x^2 = 28$

$x^2 - 28 = 0$

let $f(x) = x^2 - 28$ Now find root of f.

$f(x) = x^2 - 28$ | $f(5) = -3 < 0$

$f'(x) = 2x$ | $f(6) = 8 > 0$ let $x_0 = \frac{5+6}{2}$

$= 5.5$

we know that $x_{n+1} = x_n - \frac{f(n)}{f'(x_n)}$

$= x_n - \frac{(x_n)^2 - 28}{2(x_n)}$

$$\boxed{x_{n+1} = \frac{x_n^2 + 28}{2x_n}}$$

1st iteration : $x_1 = \frac{(5.5)^2 + 28}{2(5.5)}$

$x_1 = 5.2955$

$f(x_1) = 0.037 > 0$

2nd iteration : $x_1 = \cancel{\frac{x_1 + x_0}{2}} = 5.2915$

$x_2 = 5.2915$

3rd iteration

$$x_3 = \frac{x_2^2 + 28}{2x_2}$$

$$= 5.2915$$

Ans

$x = 52.915$ value of $\sqrt{28}$

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assignment : 4

Ques. $f(x) = x^3 - 3x - 2$ apply modified Newton's method to approximate the multiple root $x = -1$?

Ans. we know that

$$f(x) = x^3 - 3x - 2$$

$$f'(x) = 3x^2 - 3.$$

let $H(x) = \dfrac{f(x)}{f'(x)} = \dfrac{x^3 - 3x - 2}{3x^2 - 3}$

$$H'(x) = \frac{(3x^2-3)^2 - (x^3-3x-2)(6x)}{(3x^2-3)^2}$$

$$= \frac{3x^4 + 3 - \cancel{6x^2} - 2x^4 + \cancel{6x^2} + 4x}{3x^4 + 3 - 6x^2}$$

$$= \frac{x^4 + 4x + 3}{3(x^2-1)^2}$$

$$P(x) = \frac{H(x)}{H'(x)} = \frac{x^3 - 3x - 2}{\cancel{3(x^2-1)}} \quad \frac{\cancel{3}(x^2-1)^{\cancel{2}}}{x^4 + 4x + 3}$$

$$P(x) = \frac{(x^3 - 3x - 2)(x^2 - 1)}{x^4 + 4x + 3}$$

we know that

$$P_{n+1} = P_n - \frac{H(x)}{H'(x)}$$

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let $P_0 = -1.2$

$$P_1 = P_0 - P(P_0)$$

$$x_{n+1} = x_n - P(x)$$

$$= x - \frac{(x^3-3x-2)(x^2-1)}{x^4+4x+3}$$

$$= \frac{\cancel{x^5} + 4x^2 + 3\cancel{x} - (\cancel{x^5} - x^3 - 3x^3 + \cancel{3x} - 2x^2 + 2)}{x^4+4x+3}$$

$$x_{n+1} = \frac{4x^3 + 6x^2 - 2}{x^4+4x+3}$$

1^{st} : iterative method.

$x_0 = -1.2 \Rightarrow$

$$x_1 = \frac{4(-1.2)^3 + 6(-1.2)^2 - 2}{(-1.2)^4 + 4(-1.2) + 3}$$

$$= \frac{-4(1.728) + 6(1.44) - 2}{(2.0736) + 3 - 4.8} = \frac{-0.272}{0.2736}$$

$$= -0.994152046$$

2^{nd} iterative method

$$P_1 = -0.994152046$$

$$P_2 = \frac{-11.86026387 8}{0.000204393} = \frac{\cancel{-15.785}}{\cancel{0.}} = -0.99999427$$

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3^{rd} iteration approx:

$$P_3 = -1.000000000$$

4^{th} iteration approx:

$$P_4 = -1.000000000$$

Ans

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assignment: 5.

Ques. 1 solve by gauss elimination method:

$$2x + 4y + z = 3$$
$$3x + 2y - 2z = -2$$
$$x - y + z = 6.$$

Ans we have equation let convert them into A & B

$$A = \begin{bmatrix} 2 & 4 & 1 \\ 3 & 2 & -2 \\ 1 & -1 & 1 \end{bmatrix} \quad \& \quad B = \begin{bmatrix} 3 \\ -2 \\ 6 \end{bmatrix}$$

we know aquimented matrix = [A:B]

$$[A:B] = \left[\begin{array}{ccc:c} 2 & 4 & 1 & 3 \\ 3 & 2 & -2 & -2 \\ 1 & -1 & 1 & 6 \end{array}\right]$$

$$R_1 \rightleftharpoons R_3$$

$$= \left[\begin{array}{ccc:c} 1 & -1 & 1 & 6 \\ 3 & 2 & -2 & -2 \\ 2 & 4 & 1 & 3 \end{array}\right]$$

$$R_2 \to R_2 - 3R_1$$
$$R_3 \to R_3 - 2R_1$$

$$= \left[\begin{array}{ccc:c} 1 & -1 & 1 & 6 \\ 0 & 5 & -5 & -20 \\ 0 & 6 & -1 & -9 \end{array}\right]$$

$$R_2 \to \left(\frac{1}{5}\right) R_2 \quad \& \quad R_3 \to R_3 - 6R_2$$

$$= \left[\begin{array}{ccc:c} 1 & -1 & 1 & 6 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 5 & 15 \end{array}\right]$$

let get equation from $AX = B$.

$$x - y + z = 6$$
$$y - z = -4$$
$$5z = 15.$$

from this equation : $z = 3$;
$y - z = -4 \Longrightarrow y = -1$
$x - y + z = 6 \Longrightarrow 2$

$(x, y, z) = (2, -1, 3)$

Ques. 2 Solve using gauss jordan method.

$$10x + y + z = 12$$
$$2x + 10y + z = 13$$
$$x + y + 5z = 7.$$

Ans. let right equation in matrix from.

$$A = \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad B = \begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix}$$

Write Augmented matrix = $[A:B]$

$$[A:B] = \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix} \vdots \begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix}$$

$R_1 \rightleftharpoons R_3$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 5 \\ 2 & 10 & 1 \\ 10 & 1 & 1 \end{bmatrix} \vdots \begin{bmatrix} 7 \\ 13 \\ 12 \end{bmatrix}$$

$R_2 \to R_2 - 2R_1$
$R_3 \to -R_3 + 10R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 5 \\ 0 & 8 & -9 \\ 0 & +9 & +49 \end{bmatrix} \vdots \begin{bmatrix} 7 \\ -1 \\ +58 \end{bmatrix}$$

$\cancel{C_2 \to C_2 - C_1 \ \& \ C_3 \to C_3 - 5C_1}$

$$\cancel{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 8 & -9 \\ 0 & -9 & -49 \end{bmatrix}} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -14 \\ -58 \end{bmatrix}$$

$49 + \frac{81}{8}$

$58 + \frac{9}{8}$

$R_3 \to R_3 - \frac{9}{8} R_2$

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & 8 & -9 \\ 0 & 0 & \frac{473}{8} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ \frac{473}{8} \end{bmatrix}$$

$R_3 \to R_3 \left(\frac{8}{473}\right)$

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & 8 & -9 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 1 \end{bmatrix}$$

$R_2 \to R_2 + 9R_3$

$$\begin{bmatrix} 1 & 1 & 5 \\ 0 & 8 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 8 \\ 1 \end{bmatrix}$$

$R_2 \to \left(\frac{1}{8}\right) R_2$

$R_1 \to R_1 - R_2 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 1 \end{bmatrix}$$

$x = 1$

$y = 1$

$z = 1$

$= A$

Ques 3 Solve equation by crout | cholesky | LU factorization method ?

$x + 2y + 3z = 14$
$2x + 3y + 4z = 20$
$3x + 4y + z = 14$
Step (i)
Ans: writing matrix form where

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 14 \\ 20 \\ 14 \end{bmatrix} \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

step (ii) find $LU = A$ where $L = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix}$ &

$$U = \begin{bmatrix} U_{11} & 0 & 0 \\ U_{21} & U_{22} & 0 \\ U_{31} & U_{32} & U_{33} \end{bmatrix} \Rightarrow \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix}$$

soln.

$$LU = \begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 1 \end{bmatrix}$$

multiply and compare.

$\boxed{U_{11} = 1}$ $\boxed{U_{12} = 2}$ $\boxed{U_{13} = 3}$

$\boxed{L_{21} = 2}$

$L_{21} U_{12} + U_{22} = 3$
$2 * 2 + U_{22} = 3$
$\boxed{U_{22} = -1}$

$L_{21} U_{13} + U_{23} = 4$
$\boxed{U_{23} = -2}$

$L_{31}U_{11} = 3$ $\quad$ $L_{31}U_{12} + L_{32}U_{22} = 4$ $\quad$ $L_{31}U_{13} + L_{32}U_{23} + U_{33} = 1$

$\boxed{L_{31} = 3}$ $\quad$ $\boxed{L_{32} = 2}$ $\quad$ $\boxed{U_{33} = -4}$

Step (iii) $V = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$ by $LV = B$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 14 \\ 20 \\ \cancel{20}\, 14 \end{bmatrix}$$

$V_1 = 14$

$2V_1 + V_2 = 20 \implies V_2 = -8$

$3V_1 + 2V_2 + V_3 = 14 \implies V_3 = -12$

$$V = \begin{bmatrix} 14 \\ -8 \\ -12 \end{bmatrix}$$

Step (iv) find X using $U\cancel{X}=V$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & -2 \\ 0 & 0 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -8 \\ -12 \end{bmatrix}$$

$-4z = -12 \implies \boxed{z = 3}$

$-y - 2z = -8 \implies \boxed{y = 2}$

$x + 2y + 3z = 14$ $\quad$ Ans

$\Downarrow$

$\boxed{x = 1}$

assignment 6

Ques. Solve following equation by jacobi iterative method correct to 3 decimal places :-

$$10x - 5y - 2z = +3$$
$$4x - 10y + 3z = -3$$
$$x + 6y + 10z = -3.$$

Ans: rearrange the given equation :

$$x = \frac{(3 + 5y + 2z)}{10} \quad \text{——(i)}$$

$$y = -\frac{1}{10}(-3 - 4x - 3z) = \frac{1}{10}(3 + 4x + 3z) \quad \text{——(ii)}$$

$$z = -\frac{1}{10}(3 + x + 6y) \quad \text{——(iii)}$$

1^{st} iteration : Put $x_0 = y_0 = z_0 = 0$ in equation (i) | (ii) | (iii)

$$x_1 = \frac{1}{10}(3+0+0) = 0.3$$

$$y_1 = \frac{1}{10}(3+0+0) = 0.3$$

$$z_1 = -\frac{1}{10}(3+0+0) = -0.3$$

2^{nd} iteration : Put $x_1 = y_1 = 0.3$ & $z_1 = -0.3$ in (i) | (ii) | (iii)

$$x_2 = \frac{1}{10}(3 + 5(0.3) + 2(-0.3)) = \frac{1}{10}(3 + 0.9)$$
$$= 0.39$$

$$y_2 = \frac{1}{10}(3 + 4(0.3) + 3(-0.3)) = \frac{1}{10}(3 + 0.3)$$
$$= 0.33$$

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$z_3 = -\frac{1}{10}(3 + (0.3) + 6(0.3))$

$= -\frac{1}{10}(3 + 2.1) = -0.51$

3^{rd} iteration : $x_3 = 0.39, y_3 = 0.33, z_3 = -0.51$ in (i) / (ii) / (iii)

$x_4 = \frac{1}{10}(3 + 5(0.33) + 2(-0.51))$

$= \frac{1}{10}(3 + 1.65 - 1.02) = \underline{0.363}$.

$y_4 = \frac{1}{10}(3 + 4(0.39) + 3(-0.51))$

$= \frac{1}{10}(3 + 0.05) = \underline{0.305}$

$z_4 = \frac{-1}{10}(3 + (0.39) + 6(0.33))$

$= -\frac{1}{10}(3 + 2.37) = -0.537$

4^{th} iteration: $x_4 = 0.363, y_4 = 0.305, z_4 = -0.537$ in equation.

$x_5 = \frac{1}{10}(3 + 5(0.305) + 2(-0.537))$

$= 0.3484$

$y_5 = \frac{1}{10}(3 + 4(0.363) + 3(-.537))$

$= 0.2841$

$$z_5 = -\frac{1}{10}(3 + (0.363) + 6(0.303))$$

$$= -0.578$$

5th iteration: $x_5 = 0.344,\ y_5 = 0.284,\ z_5 = -0.578.$

$$x_6 = \frac{1}{10}(3 + 5(0.284) + 2(-0.578))$$

$$= \underline{\underline{0.338}}$$

$$y_6 = \frac{1}{10}(3 + 4(0.344) + 3(-0.578))$$

$$= \underline{\underline{0.282}}$$

$$z_6 = -\frac{1}{10}(3 + (0.344) + 6(0.284))$$

$$= \underline{\underline{-0.505}}$$

6th iteration: $x_6 = 0.338,\ y_6 = 0.282,\ z_6 = -.505.$

$$x_7 = \frac{1}{10}(3 + 5(0.282) + 2(-0.505))$$

$$= 0.34$$

$$y_7 = \underline{\underline{0.2837}}.$$

$$z_7 = -0.503.$$

7^{th} iteration : $x_7 = 0.34$

$y_7 = 0.2884$

$z_7 = -0.503.$

$x_8 = 0.341$

$y_8 = 0.285$

$z_8 = -0.504$

8^{th} iteration : $x_8 = 0.341$

$y_8 = 0.285$

$z_8 = -0.504$

$x_9 = 0.342$

$y_9 = 0.285$

$z_9 = -505$

9^{th} iteration : $x_9 = 0342$

$y_9 = 0.285$

$z_9 = -505$

$x_{10} = 0.342$

$y_{10} = 0285$

$z_{10} = 0.505$

where 9 iteration where $x = 0.342$

$y = 0.285$

$z = -0.505$

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Ques 2. apply gauss seidal iterative method to solve equation :-

$$10x_1 - 2x_2 - x_3 - x_4 = 3$$
$$-2x_1 + 10x_2 - x_3 - x_4 = 15$$
$$-x_1 - x_2 + 10x_3 - 2x_4 = 27$$
$$-x_1 - x_2 - 2x_3 + 10x_4 = -9.$$

Ans.

rearrange equation :

$$x_1 = \frac{1}{10}(3 + 2x_2 + x_3 + x_4)$$

$$x_2 = +\frac{1}{10}(15 + 2x_1 + x_3 + x_4)$$

$$x_3 = \frac{1}{10}(27 + x_1 + x_2 + 2x_4)$$

$$x_4 = \frac{1}{10}(-9 + x_1 + x_2 + 2x_3)$$

let start first iteration : $x_2 = x_3 = x_4 = 0$ put in equation

$$x_1 = \frac{1}{10}(3 + 0 + 0 + 0) = \underline{\underline{0.3}}$$

$$x_2 = \frac{1}{10}(15 + 2(0.3) + 0 + 0) = \underline{\underline{0.156 \times 10}}$$

$$x_3 = \frac{1}{10}(27 + 0.3 + 1.56 + 2(0)) = \underline{\underline{2.886}}$$

$$x_4 = \frac{1}{10}(-9 + 0.3 + 1.56 + 2(2.886)) = \underline{\underline{-0.137}}$$

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2nd iteration : $x_2 = 1.56$

$x_3 = 2.886$

$x_4 = -0.137$

$$x_1 = \frac{1}{10}\left(3 + 2(1.56) + 2.886 - 0.137\right)$$

$$= 0.887$$

$$x_2 = \frac{1}{10}\left(15 + 2(0.887) + 2.886 - 0.137\right)$$

$$= 1.952$$

$$x_3 = \frac{1}{10}\left(27 + 0.887 + 1.952 + 2(-0.137)\right)$$

$$= \frac{1}{10}(29.565) = 2.957$$

$$x_4 = \frac{1}{10}\left(-9 + 0.887 + 1.952 + 2(2.957)\right)$$

$$= \cancel{1.208}\ -0.025$$

3rd iteration: $x_2 = 1.952$

$x_3 = 2.957$

$x_4 = -0.025$

$x_1 = 0.984$

$x_2 = 1.990$

$x_3 = 2.992$

$x_4 = -0.004$

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4^{th} iteration : $x_2 = 1.99$
$x_3 = 2.992$
$x_4 = -0.004$

$x_1 = 0.997$
$x_2 = 1.998$
$x_3 = 2.999$
$x_4 = -0.001$

5^{th} iteration : $x_2 = 1.998$
$x_3 = 2.999$
$x_4 = -0.001$

$x_1 = 0.999$
$x_2 = 1.9996 = 2.000$
$x_3 = 2.9997 = 3$
$x_4 = -0. = 0$

6 iteration : $x_2 = 2$
$x_3 = 3$
$x_4 = 0$

$x_1 = 1$
$x_2 = 2$
$x_3 = 3$
$x_4 = 0$

6 iteration $x = 1$
$y = 2$
$z = 3$
$k = 0$

assignment : 7.

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Ques. 1 evaluate :

(i) $\Delta^2 \left(\frac{5x+12}{x^2+5x+16} \right)$

Ans. We know that:

$$\Delta f = f(x+h) - f(x).$$

$$\Rightarrow \Delta \left[\left(\frac{5(x+h)+12}{(x+h)^2+5(x+h)+16} \right) \right] - \Delta \left[\frac{5x+12}{x^2+5x+16} \right]$$

$$\Rightarrow \Delta [\sim].$$

Simplication $f(x) = \frac{5x+12}{x^2+5x+16} = \frac{5x+12}{(x+\frac{5}{2})^2+16-\frac{25}{4}}$

$$f(x) = \frac{4(5x+12)}{(2x+5)^2+39}$$

$$\Rightarrow \Delta = E-1$$
$$\Delta^2 = E^2-2E+1$$

$$\Rightarrow (E^2-2E+1) f(x)$$

$$\Rightarrow f(x+2h) - 2f(x+h) + f(x)$$

$$\Rightarrow f(x+2h) + f(x) - 2f(x+h)$$

$$\Rightarrow \frac{4(5x+10h+12)}{(2x+4h+5)^2+39} + \frac{4(5x+12)}{(2x+5)^2+39} - \frac{8(5x+5h+12)}{(2x+2h+5)^2+39}$$

Ans

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Ques 2. $\Delta\left(\frac{x^3}{\cos 2x}\right)$ evaluate?

Ans we know that.

$\Delta f(x) = f(x+h) - f(x)$

$$= \frac{(x+h)^2}{\cos(2x+2h)} - \frac{x^2}{\cos 2x}$$

$$= \frac{\cos 2x\,(x^2+h^2+2xh) - x^2[\cos 2x \cos 2h - \sin 2x \sin 2h]}{\cos 2x \cos(2x+2h)}$$

$$= \frac{\cos 2x\,[x^2 - \cos 2h] + [\cos 2x\,(h^2+2xh) + \sin 2x + \sin 2h]}{\cos 2x \cos(2x+2h)}$$

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assignment : 8

Ques. 1 Represent the function $f(x) = x^4 - 12x^3 + 24x^2 - 30x + 9$ and its successive differences in factorial notation?

Ans : we have

$$y = x^4 - 12x^3 + 24x^2 - 30x + 9.$$

we know that

$$y = A[x]^4 + B[B]^3 + C[x]^2 + D[x] + E$$

	1	-12	24	-30	9 = E.
1.	–	1	-11	13	
	1	-11	13	-17 = D	
2.	–	2	-18		
	1	-9	-5 = C		
3.	–	3			
	1	-6 = B			
4	–				
	1 = A.				

factorial notation : $[x]^4 - 6[x]^3 - 5[x]^2 - 17[x] + 9$

$$\Delta y = 4x^3 - 36x^2 + 48x - 30.$$

we know that

$$y = A[x]^3 + B[x]^2 + C[x] + D.$$

	4	-36	48	-30 = D.
1	–	4	-32	
	4	-32	16 = C	
2	–	8		
	4	-24 = B		
3	–			
	4 = A			

factorial notation : $4[x]^3 - 24[x]^2 + 16[x] - 30$

$\Delta^2 y = 12x^2 - 72x + 48$

we know that

$y = A[x]^2 + B[x] + C$

	12	-72	48 = C
1	—	12	
	12	-60 = B	
2	—		
	12 = A		

factorial notation : $12[x]^2 - 160[x] + 48$

$\Delta^3 y = 24x - 72$

we know that

$y = A[x] + B$

	24	-72 = B
1	—	
	24 = A	

factorial notation : $24[x] - 72$.

$\Delta^4 y = 24$

we know that

$y = B$.

factorial notation : 24.

Ans.

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Ques 2 find the term of first term of series whose second & Subsequent term are : 8, 3, 0, -1, 0.?

Ans

we convert question in tabular form :

x :	1	2	3	4	5	6
y :	–	8	3	0	-1	0

form difference table :

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1	y_1					
		$8-y_1$				
2	8		y_1-13			
		-5		$15-y_1$		
3	3		2		y_1-15	
		-3		0		$15-y_1$
4	0		2		0	
		-1		0		
5	-1		2			
		1				
6	0					

by Newton binomial formula :

$$\Delta^5 y = 0$$

$$\Rightarrow 15 - y_1 = 0$$

$$\Rightarrow y_1 = 15$$

Ans.

Ques. 3 find missing value in following table:

x :	0	5	10	15	20	25
y :	6	10	–	17	–	31

Ans.

let missing terms y_1 & y_2 and form difference table.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
0	6				
		4			
5	10		y_1-14		
		y_1-10		$41-3y_1$	
10	y_1		$27-2y_1$		$6y_1+y_2-108$
		$17-y_1$		$3y_1+y_2-67$	
15	17		y_1+y_2-34		$149-4y_1-4y_2$
		y_2-17		$82-y_1-3y_2$	
20	y_2		$48-2y_2$		
		$31-y_2$			
25	31				

by newton binomial formula: $\Delta^4 y = 0$

$$6y_1 + y_2 = 108$$
$$4y_1 + 4y_2 = 149$$

$$20y_1 = 283$$
$$y_1 = 14.15 \implies y_2 = 23.1$$
$$y_1 = 14.15$$

Ans.

assignment 9.

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Ques. 1 In the table below the value of y are consecutive terms of a series of which 23.6 is the 6^{th} term. find 1^{st} & 10^{th} term of series?

x :	3	4	5	6	7	8	9
y :	4.8	8.4	14.5	23.6	36.2	52.8	73.9

Ans. difference table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
3	4.8						
		3.6					
4	8.4		2.5				
		6.1		0.5			
5	14.5		3		0		
		9.1		0.5		0	
6	23.6		3.5		0		0
		12.6		0.5		0	
7	36.2		4		0		
		16.6		$\boxed{0.5}$			
8	52.8		$\boxed{4.5}$				
		$\boxed{21.1}$					
$\boxed{9}$	$\boxed{73.9}$						

$x \longrightarrow$

Newton back ward formula:

$$y = y_0 + \frac{p}{1!}\Delta y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \ldots$$

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let $P = \frac{x - x_n}{h}$

where. $x = x$

$x_n = 9$

$h = 1$

$P = (x-9)$

$y = (x-9)(21.1) + 73.9 + \frac{(x-9)(x-8)}{2}(4.5) + \frac{(x-9)(x-8)(x-7)}{6} \times 0.5$

$= 73.9 + (21.1x - 189.9) + (2.25x^2 - 38.25x + 162) +$

$(0.083x^3 - 1.992x^2 + 15.853x - 41.832)$

$y(x) = 0.083x^3 + 0.258x^2 - 1.297x + 4.168$

$9 \times 8 + 8 \times 7 + 9 \times 7$

NOW. Put Value

$72 + 56 + 63$

$x = 1 \;\&\; 10$

$x = 1$

$y(x) = 3.212$ Ans

$x = 10$

$y(x) = 99.998$ Ans

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assignment 10.

Ques. 1 given $y_{20} = 512$, $y_{30} = 439$, $y_{40} = 346$, $y_{50} = 243$, find y_{35} by bessels formula ?

Ans.

x :	20	30	40	50
y :	512	439	346	243.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	512			
		-73		
$\boxed{30}$	$\boxed{439}$		-20	
		$\boxed{-93}$		10
35 → 40	346		$\boxed{-10}$	
		-103		
50	243.			

from table: $x_0 = 30$

$y_0 = 439$

$\Delta y_0 = -93$

So. $\Delta^2 y_0 = -10$ $\quad \Delta^2 y_{-1} = -20$ $\quad \Delta^3 y_{-1} = 10$

$x = 35$

$$p = \frac{x - x_0}{h} = \frac{35-30}{10} = 0.5$$

We know that bessel formula:

$$y_x = y_0 + \frac{p}{1!}\Delta y_0 + \frac{p(p-1)}{2!}\left(\frac{\Delta^2 y_0 + \Delta^2 y_{-1}}{2}\right) + \frac{p(p-1)(p-1/2)}{3!}\Delta^3 y_{-1}$$

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$$y_{35} = 439 + (0.5)(-93) + \frac{(0.5)(0.5-1)}{2!}\left(\frac{-10-20}{2}\right) + \frac{(0.5)(0.5-1)(0.5-1/2)}{3!}(10)$$

$$= 439 + (-46.5) + (1.875) + 0$$

$y_{35} = 483.625$ Ans of $y(35)$

Ques. 2 apply Stirling formula, to compute $y_{14.2}$ from the following table:

x	10	12	14	16	18
y	0.24	0.281	0.318	0.352	0.384

Ans. difference table:

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
10	0.240				
		0.041			
12	0.281		-0.004		
		0.037		0.001	
$\boxed{14}$	$\boxed{0.318}$		-0.003		0
14.2 →		$\boxed{0.034}$		0.001	
16	0.352		$\boxed{-0.002}$		–
		0.032		–	
18	0.384				–

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form table: $x_0 = 14$

$y_0 = 0.318$

$\Delta y_0 = 0.034$ $\quad \Delta y_{-1} = 0.037$ $\quad \Delta y_{-2} = 0.041$

$\Delta^2 y_0 = -0.002$ $\quad \Delta^2 y_{-1} = -0.003$ $\quad \Delta^2 y_{-2} = -0.004$

$\Delta^3 y_0 = -$ $\quad \Delta^3 y_{-1} = 0.001$ $\quad \Delta^3 y_{-2} = 0.001$

form stirling formula:

$$y_x = y_0 + \frac{p}{1!}\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2-1)}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right)$$

$$+ \frac{p^2(p^2-1)}{4!}\Delta^4 y_{-2}$$

Put value:

$$p = \frac{x - x_0}{h} = \frac{14.2 - 14}{2} = \frac{0.1}{2.}$$

$$y_{14.2} = 0.318 + (0.1)\left(\frac{0.034 + 0.037}{2}\right) + \frac{(0.01)(-0.003)}{2}$$

$$+ \frac{(0.5)(0.25-1)}{5\times 25\ \ 3!}\left(\frac{0.001 + 0.001}{2}\right)$$

$$= 0.318 + (0.004) + (0) + (-0)$$

$$y_{14.2} = 0.322$$

Ans.

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assignment : 11

Ques. 1. Using lagrange's interpolation formula, find $f(5)$ from the following data:

x	1	3	4	6	9
$f(x)$	-3	9	30	132	156

Ans.

According to lagrange interpolation formula:

$$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} f(x_0) +$$

$$f(x_1)\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} + f(x_2)\frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)(x_2-x_4)}$$

we have value from table:

$x = 5.$ $\quad x_0 = 5-4 = 1 \qquad y_0 = f(x_0) = -3$

$x_1 = 3 \qquad y_1 = f(x_1) = 9$

$x_2 = 4 \qquad y_2 = f(x_2) = 30$

$x_3 = 6 \qquad y_3 = f(x_3) = 132$

$x_4 = 9 \qquad y_4 = f(x_4) = 156.$

Putting value in formula:

$$f(5) = \frac{(5-3)(5-4)(5-6)(5-9)}{(1-3)(1-4)(1-6)(1-9)}(-3) + \frac{(5-1)(5-4)(5-6)(5-9)}{(3-1)(3-4)(3-6)(3-9)}(9)$$

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$$\frac{(5-1)(5-3)(5-6)(5-9)}{(4-1)(4-3)(4-6)(4-9)}(30) + \frac{(5-1)(5-3)(5-4)(5-9)}{(6-1)(6-3)(6-4)(6-9)}(132)$$

$$+ \frac{(5-1)(5-3)(5-4)(5-6)}{(9-1)(9-3)(9-4)(9-6)}(156).$$

$$= (-0.1) + (-4) + (32) + (46.933) + (-1.733)$$
$$= 73.1$$

we have answer of $f(5) = \underline{\underline{73.1}}$.

Ques. 2 Using newton divided difference formula find $f(0.5)$ & $f(3.1)$ from following data:

x	0	1	2	3	4
$f(x)$	3	6	11	18	27

Ans: Newton divided difference table:

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	$\boxed{3}$				
		$\boxed{3}$			
1	6		$\boxed{1}$		
		5		0	
2	11		1		0
		7		0	
3	18		1		
		9			
4	27				

we Know That: NDDF.

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)$$
$$(x-x_1)(x-x_2)\Delta^3 y_0 \cdots\cdots \infty.$$

we have find $f(0.5)$ & $f(3.1)$

$$f(0.5) = 3 + (0.5-0)(3) + (0.5-0)(0.5-1)(1) + \cdots\infty$$
$$= 3 + (1.5) + (-0.25)$$
$$= 3 + (1.25)$$
$$f(0.5) = 4.25$$

$$f(3.1) = 3 + (3.1-0)(3) + (3.1-0)(3.1-1)(1) + 0 \cdots$$
$$= 3 + (9.3) + (6.51)$$
$$f(3.1) = 15.81$$

Ques. 3 The following table of x & y. find value of x corresponding to y = 12 using lagrange formula?

x:	1.2	2.1	2.8	4.1	4.9	6.2
y:	4.2	6.8	9.8	13.4	15.5	19.6

Ans. from table we have value of:

$x_0 = 1.2$ $\quad$ $y_0 = 4.2$

$x_1 = 2.1$ $\quad$ $y_1 = 6.8$

$x_2 = 2.8$ $\quad$ $y_2 = 9.8$

$x_3 = 4.1$ $\quad$ $y_3 = 13.4$

$x_4 = 4.9$ $\quad$ $y_4 = 15.5$

$x_5 = 6.2$ $\quad$ $y_5 = 19.6$

we know that: NDDf.

$$f(x) = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)$$
$$(x-x_1)(x-x_2)\Delta^3 y_0 \dots\dots \infty$$

we have find $f(0.5)$ & $f(3.1)$

$$f(0.5) = 3 + (0.5-0)(3) + (0.5-0)(0.5-1)(1) + \dots\dots \infty$$
$$= 3 + (1.5) + (-0.25)$$
$$= 3 + (1.25)$$
$$f(0.5) = 4.25$$

$$f(3.1) = 3 + (3.1-0)(3) + (3.1-0)(3.1-1)(1) + 0 \dots \infty$$
$$= 3 + (9.3) + (6.51)$$
$$f(3.1) = 15.81$$

Ques. 3 The following table of x & y. find value of x corresponding to $y = 12$ using lagrange formula?

x:	1.2	2.1	2.8	4.1	4.9	6.2
y:	4.2	6.8	9.8	13.4	15.5	19.6

Ans. from table we have value of:

$x_0 = 1.2$ $\quad$ $y_0 = 4.2$

$x_1 = 2.1$ $\quad$ $y_1 = 6.8$

$x_2 = 2.8$ $\quad$ $y_2 = 9.8$

$x_3 = 4.1$ $\quad$ $y_3 = 13.4$

$x_4 = 4.9$ $\quad$ $y_4 = 15.5$

$x_5 = 6.2$ $\quad$ $y_5 = 19.6$

from lagrange formula:

$$y = \frac{(x-x_5)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)(x_0-x_5)} f(x_0) +$$

$$\frac{(x-x_0)(x-x_2)(x-x_3)(x-x_4)(x-x_5)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)(x_1-x_5)} f(x_1) \text{ ------}$$

we have $y = 12$. so put values. we get.

$$12 = \frac{(x-6.2)(x-2.1)(x-2.8)(x-4.1)(x-4.9)}{(1.2-2.1)(1.2-2.8)(1.2-4.1)(1.2-4.9)(1.2-6.2)} \times (4.2). +$$

$$\frac{(x-1.2)(x-2.8)(x-4.1)(x-4.9)(x-6.2)}{(2.1-1.2)(2.1-2.8)(2.1-4.1)(2.1-4.9)(2.1-6.2)} (6.8) + \cdots$$

we have to more difficulty so we use inverse interpolation method.

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)(y-y_4)(y-y_5)}{(y_0-y_1)(y_0-y_2)(y_0-y_3)(y_0-y_4)(y_0-y_5)} x_0 +$$

$$\frac{(y-y_0)(y-y_2)(y-y_3)(y-y_4)(y-y_5)}{(y_1-y_0)(y_1-y_2)(y_1-y_3)(y_1-y_4)(y_1-y_5)} x_1 + \frac{(y-y_0)(y-y_1)(y-y_3)(y-y_4)(y-y_5)}{(y_2-y_0)(y_2-y_1)(y_2-y_2)(y_2-y_3)(y_2-y_4)} \times x_2$$

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Putting value we get.

$$x = \frac{(12-6.8)(12-9.8)(12-13.4)(12-15.5)(12-19.6)}{(4.2-6.8)(4.2-9.8)(4.2-13.4)(4.2-15.5)(4.2-19.6)}(1.2) +$$

$$\frac{(12-4.2)(12-9.8)(12-13.4)(12-15.5)(12-19.6)}{(6.8-4.2)(6.8-9.8)(6.8-13.4)(6.8-15.5)(6.8-19.6)}(2.1) +$$

$$\frac{(12-4.2)(12-6.8)(12-13.4)(12-15.5)(12-19.6)}{(9.8-4.2)(9.8-6.8)(9.8-13.4)(9.8-15.5)(9.8-19.6)}(2.8) +$$

$$\frac{(12-4.2)(12-6.8)(12-9.8)(12-15.5)(12-19.6)}{(13.4-4.2)(13.2-6.8)(13.2-9.8)(13.2-15.5)(13.2-19.6)}(4.1) +$$

$$\frac{(12-4.2)(12-6.8)(12-9.8)(12-13.2)(12-19.6)}{(15.5-4.2)(15.5-6.8)(15.5-9.8)(15.5-13.2)(15.5-19.6)}(4.9) +$$

$$\frac{(12-4.2)(12-6.8)(12-9.8)(12-13.2)(12-15.5)}{(19.6-4.2)(19.6-9.8)(19.6-13.2)(19.6-6.8)(19.6-15.5)}(6.2).$$

$$x = \frac{(5.2)(2.2)(-1.4)(-3.5)(-7.6)(1.2)}{(-2.6)(-5.6)(-9.2)(-11.3)(-15.4)} +$$

$$\frac{(7.8)(2.2)(-1.4)(-3.5)(-7.6)(2.1)}{(2.6)(-3)(-6.6)(-8.7)(-12.8)} +$$

$$\frac{(7.8)(5.2)(-1.4)(-3.5)(-7.6)(2.8)}{(5.6)(3)(-3.6)(-5.7)(-9.8)} +$$

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$$\frac{(7.8)(5.2)(2.2)(-3.5)(-7.6)(4.1)}{(9.2)(6.6)(3.6)(-2.1)(-6.2)} +$$

$$\frac{(7.8)(5.2)(2.2)(-1.4)(-7.6)(4.9)}{(11.3)(8.7)(5.7)(2.1)(-4.1)} +$$

$$\frac{(7.8)(5.2)(2.2)(-1.4)(-3.5)(6.2)}{(15.4)(12.8)(9.8)(6.2)(4.1)}$$

$$x = (0.022) + (-0.298) + (1.252) + (3.419) + (-0.750) + (0.055)$$

$$x = 3.7$$

The value of x for $y = 12$ is $\underline{3.7}$.

assignment : 12.

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Ques: 1 A slider in a machine moves long a fixed straight rod. its distance x cm long the rod is given below for various values of time t second. find
- velocity
- acceleration of slider.

when t = 0.1 second

t :	0	0.1	0.2	0.3	0.4	0.5	0.6
x :	30.13	31.62	32.87	33.64	33.95	33.81	33.24

Ans:

t	x	Δy_0	$\Delta^2 y_0$	$\Delta^3 y_0$	$\Delta^4 y_0$	$\Delta^5 y_0$	$\Delta^6 y_0$
0	30.13						
		1.49					
$\boxed{0.1}$	$\boxed{31.62}$		-0.24				
		$\boxed{1.25}$		-0.24			
0.2	32.87		$\boxed{-0.48}$		0.26		
		0.77		$\boxed{0.02}$		-0.27	
0.3	33.64		-0.46		$\boxed{-0.01}$		0.29.
		0.31		0.01		$\boxed{0.02}$	
0.4	33.95		-0.45		0.01		
		-0.14		0.02			
0.5	33.81		-0.43.				
		-0.57					
0.6	33.24						

$\boxed{h = 0.1}$

I derivative:

$$y'_{0.1} = \frac{1}{0.1}\left[\Delta y_0 - \frac{\Delta^2 y_0}{2} + \frac{\Delta^3 y_0}{3} \cdots\right]$$

$$= 10\left[1.25 - \frac{(-0.48)}{2} + \frac{(0.02)}{3} - \frac{(-0.01)}{4} + \frac{0.02}{5}\right]$$

$$= 10\left[1.25 + 0.24 + \underbrace{\frac{0.02}{3}}_{0.007} + \underbrace{\frac{0.01}{4}}_{0.003} + \underbrace{\frac{0.002}{5}}_{0.004}\right]$$

$$= 10\left[1.504\right] = 15.04$$

= Ans

2 derivative :

$$y''_{x=0.1} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \frac{137}{180}\Delta^6 y_0\right]$$

$$= 100\left[(-0.48) - (0.02) + \frac{11}{12}(-0.01) - \frac{5}{6}(0.02)\right]$$

$$= 100\left[(-0.48) - (0.02) + (-0.092) - (0.017)\right]$$

$$= -60.9$$

= Ans

#. assignment : 13

Ques. evaluate $\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta$ taking nine ordinates?

Ans.

$\theta = 0$ — 1 2 3 4 5 6 7 8 — $\frac{\pi}{2} = \theta$

#. Nine ordinates.

⇓

means eight '8' division of range

let $f(x) = \sqrt{\cos x}$

$$h = \frac{\text{upper limit} - \text{lower limit}}{\text{No. of parts}}$$

$$= \frac{\pi/2 - 0}{8} = \frac{\pi}{16}$$

z. interval.

Ordinate.

lower limit (L)

	L	L+h	L+2h	L+3h	L+4h	L+5h	L+6h	L+7h	L+8h
x/θ :	0	$\pi/16$	$\pi/8$	$3\pi/16$	$\pi/4$	$5\pi/16$	$3\pi/8$	$7\pi/16$	$\frac{\pi}{2}$
$f(x)$:	1	0.9903	0.9612	0.9118	0.8409	0.7454	0.6186	0.4417	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8

we taking upto 4 decimal places.

whenever in question doesn't have specification about method then by default we use 'Simpson's 1/3 rule'.

$$\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+y_5 \cdots y_{n-1}) + 2(y_2+y_4 \cdots y_{n-2})\right]$$

Substitute Values.

$$\int_0^{\pi/2} \sqrt{\cos\theta}\, d\theta = \frac{\pi/16}{3}\Big[(1+0) + 4(0.9903 + 0.9118 + 0.7454 + 0.4417) + 2(0.9612 + 0.8409 + 0.6186)\Big]$$

$$= \frac{\pi}{48}\Big[1 + 4(3.0892) + 2(2.4907)\Big]$$

$$= \frac{\pi}{48}\Big[1 + 4.8414 + 12.3568\Big]$$

$$\boxed{\int_0^{\pi} \sqrt{\cos\theta}\, d\theta = \text{[redacted]}\ 1.1911}$$

Ans

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assignment : 14

Ques. 1 from taylor equation find value of y for $x = 0.2$. where $y(0) = 0$ & $\frac{dy}{dx} = 2y + 3e^x$?

Ans.

we know that: from taylor equation. when $x_0 = 0$

$$\therefore \quad f(x) = f(0) + \frac{x}{1!} f'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f''''(0) ----$$

given $\Rightarrow$ $\frac{dy}{dx} = 2y + 3e^x$

$y(0) = 0$

let $f'(x) = \frac{dy}{dx}$

$f'(0) = 2y + 3e^x$

$= 2(0) + 3e^{(0)}$

$f'(0) = 3 = y_1$

Now to get y_2 differential y_1

$y_2 = (y_1)'$

$= (2y + 3e^x)'$

$y_2 = 2y_1 + 3e^x$

$(y_2)_0 = 2(3) + 3e^0 \Rightarrow 9.$

to get y_3 differential y_2:

$$y_3 = (y_2)'$$
$$= (2y_1 + 3e^x)'$$
$$= 2y_2 + 3e^x$$

$$(y_3)_0 = 2(9) + 3^0 = 21$$

to get y_4 differential y_3:

$$y_4 = (y_3)'$$
$$= (2y_2 + 3e^x)'$$
$$= 2y_3 + 3e^x$$

$$(y_4)_0 = 2(21) + 3^0 = 45$$

from taylor equation:

$$y = y_0 + x(y_1)_0 + \frac{x^2}{2!}(y_2)_0 + \frac{x^3}{3!}(y_3)_0 + \frac{x^4}{4!}(y_4)_0 \ldots$$

$$= 0 + x(3) + \frac{x^2}{2!}(9) + \frac{x^3}{3!}(21) + \frac{x^4}{4!}(45) \ldots$$

$$y = 3x + \frac{9x^2}{2} + \frac{7x^3}{2} + \frac{15x^4}{8} \ldots$$

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$$y(0.2) = 3(0.2) + 9\frac{(0.04)}{2} + 7\frac{(0.008)}{2} + 15\frac{(0.0016}{8}$$

$$= 0.6 + 0.18 + 0.028 + 0.003$$

$$\boxed{y(0.2) = 0.811}$$ Ans.

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Assignment: 14

Ques. 2 Using euler method solve for $y(0.1)$ from $y_1 = x^2 + y + xy$, $y(0) = 1$ take $h = 0.025$?

Ans: given: $y_0 = 1$
$x_0 = 0$
$h = 0.025$

construct table:

x	y	y_1	$y_{new} = y_{old} + h(y_1)$
0	1	$0+1+(0)(1) = 1$	$y = 1 + (0.025)1 \Rightarrow 1.025$
0.025	1.025	$0.025 + 1.025 + (0.025)(1.025) \Rightarrow 1.07563$	$(1.025) + 0.025(1.07563) \Rightarrow 1.05789$
0.05	1.05789	$0.05 + 1.05789 + (0.05 * 1.05789) \Rightarrow 1.15448$	$(1.05789) + 0.025(1.15448) \Rightarrow 1.08075$
0.075	1.08075	$0.075 + 1.08075 + (0.075 * 1.08075) \Rightarrow 1.23681$	$(1.08075) + 0.025(1.23681) \Rightarrow 1.11167$
$0.01 * 10 \Rightarrow 0.1$	1.11167		

So we have $y(0.1) = 1.11167$ Ans

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Ques 3 Using modified euler method to get value of $y(0.2)$ & $y(0.4)$ given $y_1 = y + e^x$ & $y(0) = 0$?
$h = 0.2$

Ans. given: $y_1 = y + e^x$

make table:

x	y	y_1	mean slope (ms)	new y = old y + h (ms).
$y(0) = 0$.			$ms = \frac{1^{st} y_1 + New\ y_1}{2}$	old y = 0
block = 1		$1^{st} y_1$		
0	0	1	1	⇒ 0 + (1)(0.2) = 0.2
0.2	0.2	1.42140	1.2107	⇒ 0 + (1.2107)(0.2) = 0.2421
0.2	0.2421	1.4635	1.2318	⇒ 0.2464
0.2	0.2464	1.4678	1.2339	⇒ 0.2468
0.2	0.2468	1.4682	1.2341	⇒ 0.2468.

$\boxed{y(0.2) = 0.2468}$ Ans

x	y	y_1	mean slope (ms)	new y
block = 2		$1^{st} y_1$		old y = 0.2468
0.2	0.2468	1.4682	1.4682	⇒ 0.2468 + 0.2 (1.4682) = 0.5404
0.4	0.5404	2.0322	1.7502	⇒ 0.5968
0.4	0.5968	2.0886	1.7784	⇒ 0.6025
0.4	0.6025	2.0943	1.7813	⇒ 0.6031
0.4	0.6031	2.0949	1.7816	⇒ 0.6031

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$$\boxed{y(0.4) = 0.6031}$$

Ans.

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assignment : 15.

Ques. Apply runge kutta method and find $y(0.2)$ in step of 0.1 if $y_1 = x + y^2$ & $y(0) = 1$?

Ans.

Step 1: $y_0 = 1 \quad x_0 = 0$

$h = 0.1$

$y_1 = x + y^2 = f(x, y)$

$k_1 = h f(x_0, y_0)$

$= 0.1 (x_0 + y_0^2) = 0.1 (1) = 0.1$

$k_2 = h (f (x_0 + h/2, y_0 + k_1/2))$

$= 0.1 ((0.05) + (1.05)^2) = 0.1153$

$k_3 = h f(x_0 + h/2, y_0 + k_2/2)$

$= 0.1 ((0.05) + (1.0577)^2)$

$= 0.1 (1.1687) = 0.1169.$

$k_4 = h f (x_0 + h, y_0 + k_3)$

$= 0.1 ((0.1) + (1.1169)^2)$

$= 0.1347.$

$k = \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4) = 0.1165$

Step (ii) $x_0 = 0.1$
$y_0 = 1.1165$
$h = 0.1$
$y_1 = x + y^2$

$k_1 = h f(x_0, y_0)$

$= 0.1 (0.1 + (1.1165)^2) = 0.1347$

$k_2 = h f(x_0 + h/2, y_0 + k_1/2)$

$= 0.1 (0.15 + (1.1839)^2) = 0.1552$

$k_3 = h f(x_0 + h/2, y_0 + k_2/2)$

$= 0.1 (0.15 + (1.1941)^2) = 0.1576$

$k_4 = h f(x_0 + h, y_0 + k_3)$

$= 0.1 (0.2 + (1.274)^2) = 0.1823$

$k = \frac{1}{6} (0.9426) = 0.1571$

$\boxed{y_2 = y_0 + k = 1.2736}$