Revealing economic facts: LLMs know more than they say*

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Abstract

We investigate whether the hidden states of large language models (LLMs) can be used to estimate and impute economic and financial statistics. Focusing on county-level (e.g. unemployment) and firm-level (e.g. total assets) variables, we show that a simple linear model trained on the hidden states of open-source LLMs outperforms the models' text outputs. This suggests that hidden states capture richer economic information than the responses of the LLMs reveal directly. A learning curve analysis indicates that only a few dozen labelled examples are sufficient for training. We also propose a transfer learning method that improves estimation accuracy without requiring any labelled data for the target variable. Finally, we demonstrate the practical utility of hidden-state representations in super-resolution and data imputation tasks.

1 Introduction

During training, generative large language models (LLMs) are exposed to vast amounts of information, including data relevant to economic modelling, such as geospatial statistics and firm-level financial metrics. If LLMs can effectively retrieve and utilise this knowledge, they could reduce dependence on external data sources that are time-consuming to access, clean, and merge, or that incur financial costs. Moreover, if LLMs accurately represent data, they could support downstream tasks like data imputation and outlier detection. In this study, we evaluate if and how LLMs can be used for typical economic data processes.

Not all knowledge within an LLM may be explicit and retrievable in natural language by prompting the model. Instead, we hypothesise that LLMs have latent knowledge about entities such as firms or regions, which is only retrievable with access to the hidden states – also referred to as *embeddings* – of the LLM. For example, suppose we prompt an LLM to estimate the proportion of mortgages in arrears in Orange County, California. If this

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exact data point was not present in the training corpus, the model may be unable to provide an accurate written answer, despite having a generalised economic understanding of US counties that could inform an estimate. Furthermore, it is plausible that extensive post-training aimed at reducing hallucinations may have diminished a model's inclination or ability to make educated guesses. We tap into the LLM's generalised knowledge using its hidden states and find that these allow us to produce better estimates of economic and financial variables than the text output. While proprietary LLMs typically do not expose their hidden states, they are accessible in open-source models.

We test the usefulness of LLMs for data tasks with a large array of empirical analyses on regional economic data of the US, UK, EU, and Germany and financial data on US listed firms. First, we show that a regularised linear model learned on the hidden states often provides substantially better estimates of the economic and financial variables than the text output of the model, particularly for less common statistics. This result holds across open-source LLMs of varying size (1–70 billion parameters). A learning curve analysis finds that usually small samples of labelled data suffice to learn an accurate linear model on the embeddings. We also suggest a simple transfer learning algorithm that allows the model to estimate a statistical variable without any labelled data for that variable. We achieve this by both learning from other labelled variables and using the text output of the LLMs as noisy labels on the variable of interest.

Another way to leverage an LLM's economic or financial knowledge is by using a reasoning paradigm that allows the model to break down the estimation task into steps, reflect on its knowledge and reasoning process, and refine its approach before arriving at an answer. While we find that a reasoning LLM outperforms the direct text output of a comparable model not tuned for reasoning, our approach—applying a linear model to the prompt's embeddings—remains superior. It not only delivers better results but is also orders of magnitude more computationally efficient.

Finally, we investigate the practical relevance of our findings for two data processes. Firstly, we show that we can exploit the embeddings to consistently improve the imputation of economic and financial variables, a common challenge in both research and industrial data pipelines. Secondly, we provide evidence that hidden states of LLMs can help in super-resolution tasks, where we want to estimate economic statistics for a finer-grained geographical level given data at a coarser level.

Our findings strongly support the value of LLMs in data processing tasks, which warrants attention, given that tasks such as identifying data sources, extracting numeric information, merging data series, and handling missing values are time-consuming and errorprone. This is evident from the numerous data providers that sell packaged, cleaned, and well-structured datasets, even when a large proportion of the underlying data is publicly accessible online.

This study is structured as follows. Section 1.1 discusses the relevant literature, Section 2.1 describes the dataset and Section 2 outlines our empirical approach. The main results on the accuracy of our approach to estimate economic variables from embeddings

are shown in Section 3. Section 4 presents the methodology and results for two data applications: imputation and super-resolution. Section 5 concludes.

1.1 Literature review

Our study is most closely related to papers that investigate how well LLMs can recall geospatial statistics. Manvi et al. (2023) show that both GPT3.5 and fine-tuned Llama 2 can be prompted to reproduce geospatial information, such as population density, mean income, education, or house prices. The LLMs recall these variables more accurately than supervised machine learning methods, which inferred the information from other data sources such as satellite images of luminosity at night. Note that the authors only consider the text output of the LLM and do not exploit the hidden states as we do.

Similarly, Li et al. (2024) test how well LLMs can reproduce a large set of variables about cities and regions such as population density, life expectancy, average travel time, and number of patents. As in our study, this paper tests the accuracy of both the text output and a linear model on the hidden layer activations to estimate the statistics. However, they focus on a third approach: they ask the LLM to identify relevant features that can help to predict a variable of interest and then ask the LLM to generate values for these variables. After feeding these variables into a supervised ML models to predict the variable of interest, this approach performs on par with a linear model on the hidden states in 5-fold cross-validation. In contrast to our work, this paper purely focuses on how well the LLMs can recall statistics and does not consider transfer learning or downstream tasks.

Our main empirical approach of learning a linear model on hidden states of a language model is known as *linear probing* in the literature (Belinkov, 2022; Alain and Bengio, 2016) and has been used to understand the inner workings and capabilities of LLMs. Our study builds in particular on the findings by Gurnee and Tegmark (2023), showing that hidden states of LLMs represent space and time. Concretely, a linear model trained on the hidden states of Llama 2 accurately infers the death of historical figures, the release year of artworks, or the geographical location of landmarks. Godey et al. (2024) investigate the scaling laws of these results, showing that larger models represent space and time more accurately. Further, Chen et al. (2023) perturb activations in the LLM to show that there is a causal connection between the representations and the text output.

Zhu et al. (2024) use linear probing to show that LLM embeddings represent the results of simple addition problems. While the exact number cannot be reconstructed using linear probes, the correlation between the actual number output and the inferred number (via the probes) is high. Linear probing is also used to gauge the truthfulness of statements (Marks and Tegmark, 2023; Orgad et al., 2024; Bürger et al., 2024) and Liu et al. (2024) show that hidden states can also be used to estimate uncertainty of LLM outputs in different tasks.

Our work also relates to studies that use hidden states of LLMs for downstream tasks

such as classification. Particularly, Buckmann and Hill (2024) showed how a linear model trained on hidden states of small LLMs after task-specific prompting can outperform GPT-4 when trained on only a few dozen samples per class (see also (Cho et al., 2023)). Hidden states of LLMs have also proven to perform well as general text embedding Lee et al. (2024); Rui Meng (2024) as evidenced by their competitive performance on the Massive Text Embeding Benchmark (MTEB) (Muennighoff et al., 2022), which includes tasks such as clustering, re-ranking, retrieval and classification.

Our approach to transfer learning is inspired by work on deep learning from noisy labels (see Song et al., 2022, for a review). Oyen et al. (2022) show that neural networks are surprisingly robust to label noise and Liu et al. (2020) find that deep learning models learn to predict correct labels in the early stage of learning and overfit to noisy labels (memorisation) only in later stages.

The literature on the use of LLMs for data imputation and super-resolution tasks – the two practical applications of our methodology that we explore in this study – is scarce. Hayat and Hasan (2024) fine-tuned Llama 2 to impute missing values given a textual description of the instance's features. Ding et al. (2024) use a similar approach for data imputation in recommender systems. By contrast, we do not rely on text output but use the embeddings of the LLM as additional features when employing standard imputation methods.

While there exists a rich statistical and machine learning literature on the estimation of geospatial statistics at a granular level (Tzavidis et al., 2018; Chi et al., 2022; Aiken et al., 2022; Viljanen et al., 2022), we are not aware of any study that explorse the use of LLMs for this purpose.

2 Methodology

2.1 Datasets

We use four public regional datasets covering US counties, EU regions, UK districts and German districts. Additionally, we use a dataset containing public financial information about US listed firms. The datasets are listed in Table 1. We assembled the datasets using different sources, which are described in detail in Appendix A. We provide more extensive results for our main datasets, US counties and US listed firms, which contain a larger number of observations than the other datasets.

Key variables that we use in the regional datasets are population, GDP per capita, unemployment rate, a measure of the income per capita, and a measure of life expectancy. Key variables we observe in the US firms dataset are total assets, market capitalisation, profitability metrics such as return on equity and return on assets. For the super-resolution analysis in Section 4.2 we use additional data that we describe in that Section.

Name	Observations	Variables	Regional level	Grouping variable	Year
US counties	3142	9	counties	51 states	2019
German districts*	401	13	districts	38 gov. districts	2019
UK districts	374	8	districts (LADs)	12 regions	2019
EU regions (NUTS2)	244	7	NUTS-2	27 countries	2019
US listed firms	1986	9	N/A	N/A	2022

^{*}In German, the regional level is referred to as Kreise und kreisfreie Städte and the grouping level as Regierungsbezirke.

Table 1: Datasets.

Name	Source	Quantisation
Main models		
Llama 3.2 1B-Instruct	huggingface.co/meta-llama	no
Phi-3-Mini-4K-Instruct (3.8B)	huggingface.co/microsoft	no
Llama 3 8B-Instruct	huggingface.co/meta-llama	no
Llama 3 70B-Instruct	Text: replicate.com/meta	no
	Embeddings: huggingface.co/bartowski	$IQ2_M.gguf$
Reasoning experiments		
Qwen QwQ-32B	Text: groq.com: qwen-qwq-32b	
Qwen 2.5 32B	Text: groq.com: qwen-2.5-32b	
	Embeddings: huggingface.co/Qwen	q4_k_m.gguf

Table 2: LLMs used in this study.

2.2 Experimental set-up

We exclusively use open-source LLMs, as, to the best of our knowledge, none of the providers of the leading proprietary models provide access to the hidden states of their models. Furthermore, we focus on smaller models because these are easier and cheaper to use. The models we use are listed in Table 2. In the main experiments, we test three models of the Llama 3 family (1B, 8B and 70B parameters) and the Phi-3-mini (3.8B parameters) model. Apart from Llama 3 70B, we accessed the models through the transformers library (Wolf et al., 2020) on an NVIDIA Tesla V100 GPU. Due to its large size, we use a quantised version of Llama 3 70B on 2 NVIDIA T4 GPUs and obtain the embeddings with the llama-cpp-python library. To obtain the text outputs, we use the non-quantised model hosted on replicate.com.

We also consider a reasoning model: Qwen QwQ (32B parameters). To assess whether the reasoning paradigm improves the estimation of regional and financial statistics we pit the model directly against Qwen 2.5 32B, which is the general-purpose LLM on which Qwen QwQ is based. To obtain the text output of both models, we use the instances hosted on groq.com. We obtain the embeddings of Qwen 2.5 32B from a quantised version using the llama-cpp-python.

We prompt the models using a *completion prompt* as follows:

The {variable} in {region} in {year} was

For example, we use the prompt: "The population in Orange County, California in 2019 was". We also tested several other prompting strategies including a question-answering prompt, a few-shot prompt, and a chain-of-thought prompt. Appendix B.1 shows how the completion prompt delivered the most accurate results, on average.

When learning a linear model on the embeddings, we also test a *generic prompt*, which does not vary with the variable of interest but just embeds the name of the entity (e.g. "Orange County, California").

2.3 Linear model on embeddings (LME)

We calibrate a linear model on the embeddings (LME) of the last token of the prompt. Our baseline model has 32 layers, with a 4096-dimensional embedding vector. If not stated otherwise, we use the embeddings of the 25th layer because LME performs better on this layer than on others (Figure AV in the appendix).

To estimate the performance of LME, we employ grouped cross-validation on the regional datasets. Specifically, we ensure that the training and test sets do not share any values of the grouping variable. The grouping variable for each dataset is shown in Table 1. For US counties, for example, the grouping variable is US states. Our baseline results are based on 5-fold cross-validation but we also report learning curves, where, adhering to the grouping, we sample training sets of increasing size.

We either train the linear model on the embeddings directly or on the first k PCA components, with $k \in \{5, 10, 25, 50, 100, 200\}$. We fit the PCA to all observations and not only on the training set.

In the following, we denote the ground truth values of the variable of interest as y and the embedding matrix as E. The prediction model, usually a linear model, is denoted as m and the predicted values $\hat{y} = m(E)$. The LLM's text output is denoted as y. In some analyses, we train a model on data of several variables or several datasets. In these cases we use subscripts y_v, E_v or y_d, E_d to respectively denote variables as v and datasets as d.

2.4 Data transformations and performance metrics

To avoid a strong influence of outliers on the performance metrics and on the estimation of the LME, we transform some of the dependent variables. Specifically, across all datasets, we applied a log transformation to non-negative variables with a skewed distribution. For skewed variables with negative values, we use a cubic transformation $(f(x) = sgn(x) \times |x|^{\frac{1}{3}})$. The full list of variables in all datasets and the transformations applied are shown in Table AIII.

When assessing the performance of the text output and LME, we test both models on exactly the same observations. The text outputs of the LLMs sometimes fall outside the

range of expected values (e.g. unemployment rate > 50%). In this case, we decide to remove observations to reduce noise. Parsing numeric values from LLMs' textual answers is challenging, due to the inconsistent format and inconsistent use of units. We went to considerable effort to correctly parse the text output and conducted comprehensive manual checks on all variables to ensure the accuracy of our parsing approach. Our parsing approach is described in detail in Appendix B.2.

Despite the use of data transformations and the removal of obvious outliers, we still observe some outliers in the values parsed from the text output. To minimise the influence of these outliers on performance, we employ Spearman correlation as our main performance metric, measuring the rank correlation between the ground truth and estimated values of the variable of interest. We also report our key results for Pearson correlation and do not observe a qualitative difference in our conclusions.

3 Using LLM embeddings to infer statistics

We start our analyses with a cross-validation exercise comparing the performance of LME and text output. Next, we run a learning curve analysis to understand how many training samples are required for LME to perform well. Finally, we test different transfer learning approaches, where we try to estimate a variable without access to any labelled data on that variable. In this section, we focus on the performance of the *baseline LME*, a ridge regression model trained on all 4096 embedding dimensions (i.e. without PCA) of the 25th layer of LLama 3 8B.

3.1 Cross-validation

Figure 1 compares the performance (Spearman correlation) of the text output to the baseline LME learned on the embeddings of the completion prompt and the generic prompt. The variables are ordered by decreasing performance of the text output. LME based on the completion prompt performs better than LME on the generic prompt in all datasets except the US firms data, where both perform equally well on most variables. We therefore focus on the completion prompt in the following analyses.

LME beats the text output most of the time. The performance advantage of LME increases with the decreasing performance of the text output. On more common variables, such as population, or unemployment rate, the text output performs as well as, or better than, LME. On less common statistics, however, such as the proportion of mortgages that are at least 90 days delinquent (US counties) or the number of government employees (German districts), LME performs substantially better. The results are similar when choosing Pearson correlation as the performance metric, as shown in Figure AII in the Appendix.

Next we test whether we can improve the performance of LME by reducing the dimensionality of the embeddings using a PCA before training LME. For our two key datasets,

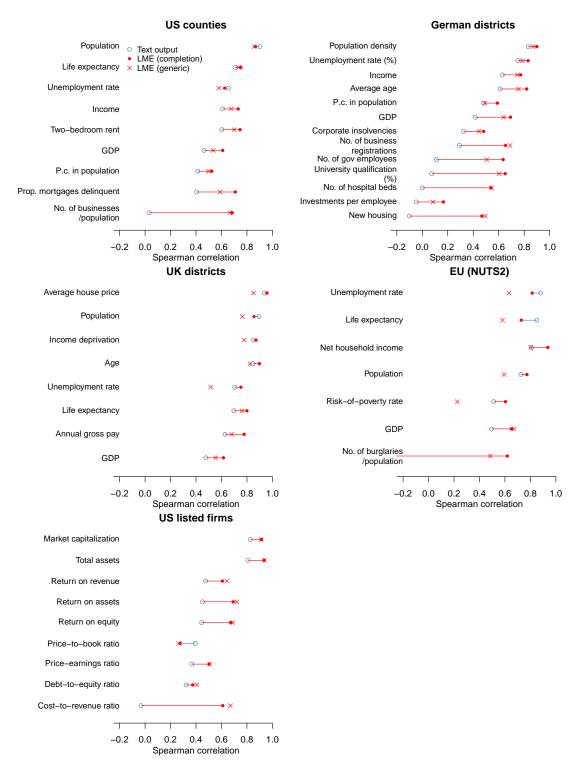


Figure 1: Cross-validation performance. The variable labels are shortened. The full variable names used when querying the LLM are shown in Table AIII

US counties and US firms, Figures AIII and AIV in the appendix respectively show that LME usually performs best without applying a PCA.

We also compare the performance by layer and see that across the key datasets, embeddings from the 25th layer perform better on average than those from earlier or later layers (Figure AV in the appendix). However, the differences in performance are small with the exception of the lower performance when using layer 5, the earliest layer we examine. Table AIII presents the cross-validation results numerically for all of the datasets.

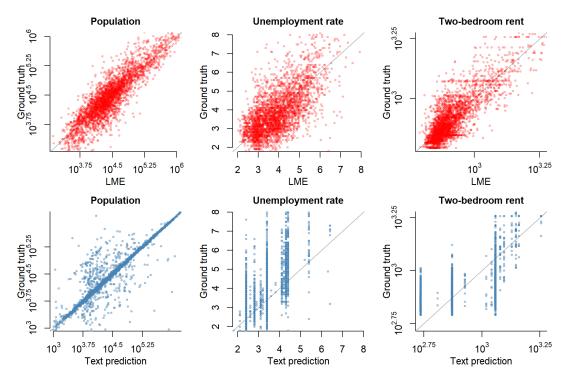


Figure 2: Comparing actual values (ground truth) to the predicted values by LME (top row) and the text output (bottom row).

Figure 2 plots the actual values against both the inferred values of LME (completion prompt, top row) and the text output (bottom row) on a few variables of the US counties dataset. The text output exactly recalls the population in a large number of counties, particularly for those counties with a large population. For the other variables, however, the text output is clustered on a few distinct values.^{1,2} By contrast, the distribution of the LME predictions, does not cluster but also does not reproduce the exact values.

For the US states dataset, we also evaluate the performance of the text output and

¹Clustering of values in the text outputs has also been observed by Li et al. (2024) in their analysis of LLMs' knowledge of regional statistics.

²Note that the values are not just clustered by US state. We typically observe several unique values within states as shown by Figure AVII which depicts the number of unique ground-truth and text output values for each state.

LME within states. To obtain stable estimates we exclude states that have less than 50 counties. Figure AVI in the appendix compares the Spearman correlation between the ground truth and the text output and LME, respectively. Except for population, we observe a performance advantage of LME over the text output in most states for the remaining seven variables.³

3.1.1 Comparing base models

To test whether our finding that LME tends to beat the text output generalises across model families and model sizes, we run the cross-validation analysis on our two largest datasets (US counties and US listed firms) for all four LLMs.

Figure 3 (left panel) shows how the performance of LME improves consistently with increasing model size.⁴ The middle panel of the figure shows that the accuracy of the text output also increases with larger models, but less consistently. The right panel shows the difference in Spearman correlation of the two approaches (LME - text output). While the mean difference (in red) is larger for the small models, it is still large – on average 16 percentage points – for the 8B and 70B model.

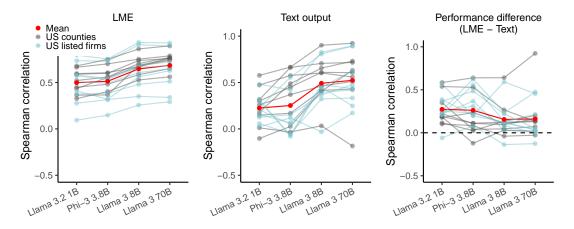


Figure 3: Performance of LLMs of different sizes.

3.1.2 LME vs. reasoning paradigm

Since 2024, it has emerged that the performance of language models does not only scale with training-time compute (i.e. larger models and more training data) but also test-time compute. Allowing an LLM more time to generate responses, break down tasks into

³We cannot show the variable proportion of mortgages being delinquent because it has a low coverage in the raw data, which is why we do not observe at least 50 non-missing observations in any state.

⁴As the models differ in their number of layers, we train LME on the embeddings of the last layer of each model. Furthermore, while LME on the embeddings of the Llama models work best when no dimensionality reduction is applied, Phi generally performs better when training LME on 25 PCA components. Thus, we applied the dimensionality reduction for Phi.

multiple steps, and reflect on its reasoning process significantly enhances its performance on reasoning tasks. We test whether this reasoning paradigm is also useful for our specific task: Can an LLM accurately reason about its internal knowledge of regions and firms, perform calculations based on this knowledge, and ultimately provide precise estimates of relevant statistics?

We use the open-source reasoning model Qwen QwQ-32B (Qwen Team, 2025). It is based on Qwen 2.5 32B (Yang et al., 2024), an LLM not tuned to reason using test-time compute. By comparing these two models, we can directly measure the effect of the reasoning paradigm on the accuracy.⁵ We pit the text output of the two models against LME trained on the embeddings of the final token of the prompt of the Qwen base model (Qwen 2.5 32B). Due to the high computational costs of the reasoning model, we only test it on a subsample of 500 entities per variable.

A closer look at the reasoning process reveals that the LLM tries to consider different information to provide the best estimates based on its knowledge base.⁶. However, despite the efforts, we do not observe a consistent increase in performance.

Figure 4 shows the results for our two main datasets. Overall, we do not see a consistent improvement in performance when using the reasoning model to generate the text output. However, the reasoning model performs substantially better on the three US county variables on which the non-reasoning model performs worst. LME beats the text output of both models on most variables. The difference in computational costs is significant. The reasoning model uses more tokens to answer the question, a median of 1767 output tokens, whereas Qwen 2.5 answers the question with median of 8 tokens.

3.1.3 Learning curves

The cross-validation results show that we can infer statistical variables more accurately from the embeddings than from the text output. However, the 5-fold cross-validation setting assumes that 80% of the values on the variable we are predicting are available to the modeller. If LME can reach a high accuracy with only a small number of labelled data points, its relevance for data processing tasks will be much higher. To test this we conduct a learning curve and estimate the performance as a function of the number of training samples. Figure 5 compares the text output (red line) against the LME when trained on different numbers of observations in training samples for some variables in

⁵In contrast to the previous experiments where we use a completion prompt, we use the questionanswering prompt (see Section B.1) for the two Qwen models.

⁶For example, when asked for the proportion of delinquent mortgages in Atlantic County, New Jersey in 2019 one of many points the reasoning model made was "Wait, I think that in 2019, the national delinquency rate was around 3-4%, but that's the national average. Atlantic County might be different, especially if it's a coastal area that might have been affected by Hurricane Sandy in 2012, but that's a few years prior. Maybe the recovery from that could affect 2019 numbers? Not sure." Another sentence from the same answer: "Hmm, I'm not entirely sure, but I think the national average was around 1.8%, and New Jersey might be a bit higher. Since Atlantic County is a county with some economic challenges, maybe it's a bit higher than the state average."

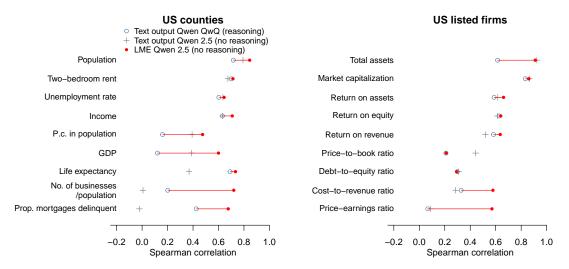


Figure 4: Cross-validation performance comparing reasoning model Qwen QwQ (text output) to Qwen 2.5 (text output + LME).

the US counties and US firm datasets. LME often reaches its maximum performance with only a small number of training samples. Dimensionality reduction via PCA only helps the performance on very small sample sizes and when using only two components. Figure 6 shows the learning curve results for all datasets comparing the text output to baseline LME at a training sample size of 25, 50, and 100 observations. With only 50 samples, LME is superior on nearly all of the variables.

3.2 Transfer learning

3.2.1 Learning from embeddings of other variables

The previous analyses assume that we have at least some labelled instances of a variable v to estimate its values on other instances using the embeddings. Here, we test whether we can reliably estimate a regional or firm variable without access to any values of this variable. If the embeddings encode numbers in a consistent way across different variables, we should be able to predict variable v using a model learned from other variables. More formally, given all variables v of a dataset, we train a regression model on embeddings and outcome values of variables v and test the models on variable v. Training observations are described as v and test the models on variable v. Note that v is v and test observations as v and test observations as v in the embeddings, which also contains the name of the variable of interest v.

To ensure our model is flexible enough to learn generalised embeddings, we train a neural network on the embeddings. We chose a neural network with two layers (128 neurons and 32 neurons, Leaky ReLU activation function) as our transfer learning model. We use the Adam optimiser with an initial learning rate of 10^{-5} . To avoid overfitting, we only learn for 20 epochs and apply a strong dropout of 0.5 on the first layer. We test this

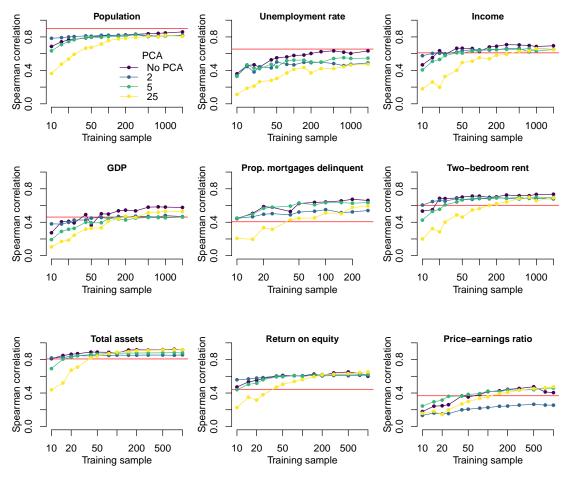


Figure 5: Learning curve analysis.

approach on all five datasets. Here, and in the following transfer learning analyses, all variables are transformed as described in Section 2.1 before they are normalised (mean of zero, standard deviation of one).

The left panel of Figure 7 compares the performance (Spearman correlation) of this approach to the text prediction. Each dot reflects one variable, for which a separate model was trained on all other variables of the same dataset. This approach to transfer learning does not beat the text prediction. While transfer learning is superior in some cases, it falls behind the text output most of the time. We obtain similar results, when using a linear model instead of the neural network. We also test another simple transfer learning approach, where we predict variable y in Dataset A from the same variable of different datasets. For example, we train a model on the unemployment rate of the UK and Europe to predict the unemployment rate in the US. However, this transfer learning approach is also inferior to the text prediction on average, and sometimes by a large

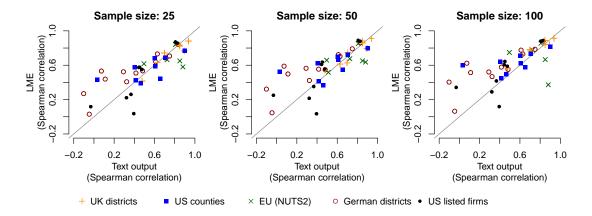


Figure 6: Comparison between text output and baseline LME with different training samples

margin.

Collectively, these transfer learning results show that embeddings do not generalise reliably to other variables.

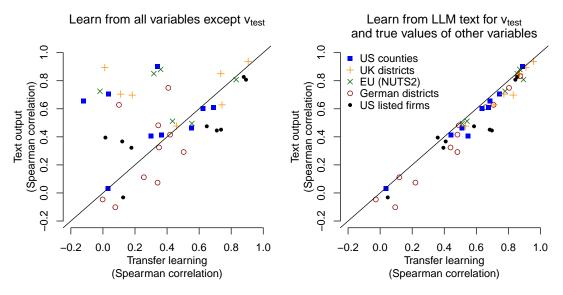


Figure 7: Transfer learning analysis.

3.2.2 Learning from text output

Given the poor performance of the simple transfer learning approaches, we refine our method to exploit information about the variable we predict (v_{test}) , even without access to ground truth values. Concretely, we use the text outputs of the LLM as noisy or

biased labels of the variable of interest in the training set (y_{test}^{txt}) . For the remaining variables we use the ground truth labels, as above. Formally, let the training set be $\{([y_v, \overset{txt}{y}_{v_{test}}], [E_v, E_{v_{test}}]) : v \in \mathbb{V} \setminus v_{test}\}$ and the test set be $\{(y_v, E_v) : v = v_{test}\}$. Even though the text labels are often inaccurate, they might provide enough information for the model to learn from the embeddings of the specific prompt of v_{test} .

We use the same neural network set-up as above. The right panel of Figure 7 compares this performance of this transfer learning approach to the text output. This transfer learning approach almost always outperforms the text prediction even when the text prediction itself is accurate. On average, transfer learning beats the text output by 7.2 percentage points. This is an important finding. We can extract the knowledge about a specific economic or financial variable from LLM embeddings without any labelled instances of that variable. The result is robust to the choice of the hyperparameters of the neural network and we observe qualitatively similar, but slightly inferior performance, when using a linear model instead of the neural network.

4 Leveraging LLM embeddings for imputation and super-resolution

4.1 Imputation

Our analyses have established that embeddings contain rich knowledge about regions and firms. Here we test whether this generalised knowledge can be exploited to improve imputation accuracy. We randomly mask values of variables in our dataset and use the LLM embeddings to inform standard imputation methods. To obtain a general numeric representation of entities, we use generic prompts (see Section 2.2), which only contain the name of the entity (e.g. region or firm) and the year.

To test whether embeddings help the accuracy of off-the-shelf imputation algorithms, we compare two imputation strategies. The *baseline* strategy estimates imputed values using only the K numeric features (e.g. population and unemployment rate) in the dataset. The *embedding* approach extends the feature matrix by appending the first c PCA components of the embeddings.

Our approach to introduce missing values is as follows: Given a dataset with N observations and K variables, all variables are transformed as described in Section 2.1 before they are normalised (mean of zero, standard deviation of 1). First, we sub-sample k features, for each of which we randomly mask p% of its values, creating the incomplete data D_{miss} . We then impute the missing values in D_{miss} using imputation algorithm m.

We test three imputation methods m: using a low-rank Gaussian copula model (Zhao and Udell, 2020) and iterative multivariate imputation using (i) Bayesian Ridge regression and (ii) random forest (Wilson, 2020). We assess the accuracy of the imputation with the mean absolute difference between ground truth and reconstructed values.

For each imputation method, we conduct a large array of experiments to ensure robust-

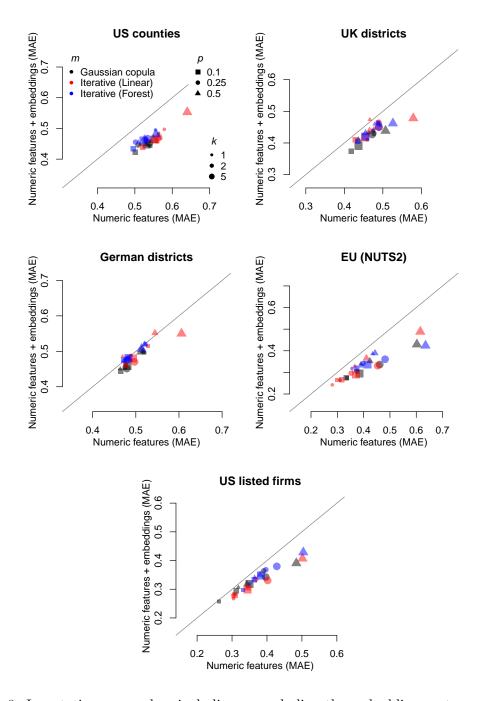


Figure 8: Imputation error when including or excluding the embedding vectors as features in the imputation method.

ness of our results. Specifically, we consider all possible combinations of the parameters settings $k \in \{1, 2, 5\}, p \in \{0.1, 0.25, 0.5\}$. To obtain stable results, we replicate the ex-

	Regional level				
Region	lower	higher			
US	3142 counties	51 states			
EU (NUTS3)	1165 NUTS-3	244 EU NUTS-2			
Germany	401 districts	38 gov. districts			

Table 3: Regional levels of super-resolution datasets

periments for each parameter combination 25 times, randomly choosing the instances and features which are imputed. We use the first c=25 PCA components of the $25^{\rm th}$ layer of LLama 3 8B-Instruct.

Figure 8 presents the results across all five datasets and parameter settings m, k, and p. Overall, the embedding strategy outperforms the baseline approach, which relies solely on the numeric features. The improvement is particularly notable in some datasets. For instance, in the US counties dataset, the embedding strategy reduces the mean error by 14% on average across parameter settings.

4.2 Super-resolution

We define a super-resolution problem as the estimation of a statistic at a finer-grained geographical level where actual statistics are not reported or are expected to be imprecise, given values of that statistic at a coarser level. Similarly to imputation, this task has high practical relevance. We can think of many key economic variables that are only published at a higher regional level but that have systematic lower-level regional variation that we want to estimate. LME, exploiting an LLM's generalised knowledge about regions, could provide accurate estimates in this situation. Unfortunately, we cannot test super-resolution in those scenarios where lower geographical level data do not exist because we require these data to assess the accuracy of our approach.

We approach the super-resolution task by training an LME on a higher regional level (e.g. unemployment rate of US states) to obtain estimates at the lower regional level (e.g. unemployment rate of US counties). We use the completion prompt for both geographical levels.

We test this super-resolution approach on three of our datasets (US, EU, Germany) for which we were able to find data on two regional levels, as shown in Table 3.

We pit this super-resolution approach against the text output for the lower level and against a *naive* method, which projects the higher region values onto the lower level. For example, the unemployment rate of all counties in Alabama would be set to the unemployment rate reported for the state Alabama. Figure 9 presents the results, ordering the variables by decreasing text performance.

The super-resolution approach performs well, surpassing the text prediction on most variables. A notable outlier is population, where the super-resolution performs worse

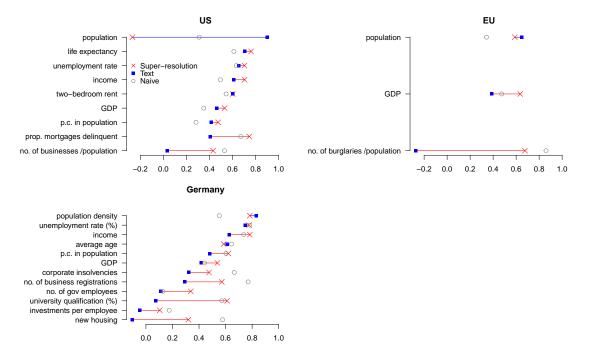


Figure 9: Super-resolution analysis.

than random. This result is unique to models in the Llama family, whereas results from Microsoft's Phi models indicate that super-resolution outperforms the naive approach (Figure AVIII). The naive approach performs well on many of the variables in Germany but falls behind the other two approaches on the US data. Given the small training sample size, in particular for the US and Germany (51 and 38 observations at the higher geographical level, respectively), the performance of the super-resolution approach is impressive. Super-resolution is a particular transfer learning approach and thus supports our previous finding (Section 3.2) that the embeddings can be used for accurate estimation outside the domain of the training set.

5 Conclusion

We have shown that LLMs know more than they say. A linear model trained on the LLM's hidden states and only a few dozen labelled samples often outperforms the LLM's text output in the estimation of statistics – in particular when the target variable is less common statistic. A key contribution of our work is the transfer learning approach which shows that we can estimate target variables more accurately than the text output without any labelled data on that target variable. Finally, we provide robust evidence that the hidden states can be used for data processing tasks. In particular, adding a low-rank representation of the hidden states to the feature matrix consistently improves the performance of standard imputation approaches in all of our datasets. Our super-resolution

results show that, in most cases, we can beat the text output in estimating statistics on a lower geographical level by learning from the higher geographical level.

We considered several LLMs with 1 to 70 billion parameters and show that our result that LLMs know more than they say holds across all of them, including a reasoning model that uses extensive test-time compute. We leave it for future work to test whether this finding also holds for the largest and most accurate open source models such LLama 3 405B or DeepkSeek V3 with 671 billion parameters. In this study we exclusively worked with public data sources; it would be interesting to examine the extent to which our results hold for proprietary datasets, which are less likely to be observed in the training corpus of open-source LLMs. Finally, future research could investigate how useful the hidden states representing firms or geographic entities can be for other data processing tasks, such as outlier detection.

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Appendix

A Datasets

A.1 US counties

The data on US counties for the year 2019 is one of our two main datasets and we used a variety of sources to assemble it. We obtained income and GDP data from the website of the Bureau of Economic analysis⁷. The unemployment data was sourced from the Economic Research Service of the US Department of Agriculture⁸. Life expectancy data is provided by the Institute for Health Metrics and Evaluation in the Global Health Data Exchange database. We obtained mortgage delinquency rates from the Consumer Financial Protection Bureau.⁹ Data on the number of businesses was downloaded from the County Business Patterns Tables from the United States Census Bureau.¹⁰ Information on the median rent is available from the website of the Office of Policy Development and research.¹¹ The remaining variables we use come from the R package usdata (Çetinkaya Rundel et al., 2021) which lists the US Census and the American Community Survey as sources.

A.2 UK regions

We gathered data for the year 2019 for the 374 local authority districts (LADs) in the United Kingdom. Most variables were sourced from the Office of National Statistics (ONS) using API access via the R package onsr (Vasilopoulos, 2023). Some series are not directly accessible via API and were downloaded from the official websites of the ONS, Northern Ireland Statistics and Research Agency (NISRA) and the Scottish and Welsh government websites.

A.3 EU regions

All the EU series for the year 2019 are collected from eurostat, an official database of the the European Union. We access the series at the NUTS2 and NUTS3 level via API calls using the R package eurostat (Lahti et al., 2017, 2023). Note that fewer series are available on the NUTS3 level.

A.4 German districts

All series for the year 2019 are downloaded directly from the Regionalatlas Deutschland¹² by the Federal Statistical Office of Germany.

⁷https://www.bea.gov/data

 $^{^8}$ https://www.ers.usda.gov/data-products/county-level-data-sets

⁹https://www.consumerfinance.gov/data-research/mortgage-performance-trends/download-the-data

 $^{^{10}\}mathrm{https://www.census.gov/programs-surveys/cbp/data/tables.html}$

¹¹https://www.huduser.gov/portal/datasets/50per.html

¹²https://regionalatlas.statistikportal.de/

Prompt type	System prompt
Completion prompt	You are a helpful assistant.
Question-answer prompt	You are a helpful assistant. If you do not know the
	answer to the question, provide your best estimate.
	Answer shortly like this. 'My answer: {number}
5-shot prompt	You are a helpful assistant. Answer the question. Use
	the format provided in the examples.
Chain-of-thought	You are a helpful assistant. If you do not know the
	answer to the question, provide your best estimate.
	Think carefully before giving an answer. Once you
	have the answer just state it like this. 'My final an-
	swer: {number}'.

Table AI: System prompts

A.5 US listed firms

We collected financial information as of end 2022^{13} , including balance sheet, income and share price data, of US listed firms using yahoo!finance's ¹⁴ free API access. We excluded values on the variables $return\ on\ revenue$ and $return\ on\ equity$ when firms have negative values on stockholders' equity or total revenue, respectively.

B Methodology

B.1 Prompting

Additionally to the baseline completion prompting strategy (see Section 2.2), we also tested three alternative strategies. First, we apply the standard prompting template that is provided via the apply_chat_template function in Transformers package (Wolf et al., 2020), referred as question-answer prompting in this paper. For example, What was the unemployment rate in Orange County, California in 2019? This led to a lower response rate for US listed firms, with the model occasionally refusing to give an answer. Second, we test 5-shot prompting, where we provide 5 randomly chosen Q&A pairs answers to the model to learn from. Finally, we use chain-of-thought prompting, adding phrase "think carefully before giving an answer" to the system prompt. The system prompt of the different prompting strategies are shown in Table AI.

Table AII indicates the response rates and figure AI compares the performance of different prompting strategies. On average, the completion strategy has the highest response rate and performs best.

 $^{^{13}}$ We could not access data from 2019, as we do for the other datasets, because the free API access is limited to the most recent years.

¹⁴https://finance.yahoo.com/

Variable	Completion	Chain-of- thought	Few-shot	Q&A
US counties				
GDP per capita (in \$)	1.00	1.00	0.99	1.00
life expectancy at birth	1.00	1.00	1.00	1.00
median monthly two-bedroom rent (in \$)	1.00	1.00	0.98	1.00
personal income per capita (in \$)	1.00	1.00	0.99	1.00
population	1.00	1.00	1.00	1.00
unemployment rate	1.00	0.99	0.99	1.00
number of business establishments per 100,000	1.00	1.00	1.00	1.00
people	1.00	1.00	0.00	1.00
percentage change in population compared to	1.00	1.00	0.98	1.00
the previous year	1.00	0.00	0.00	1 00
proportion of mortgages being 90 or more days	1.00	0.99	0.20	1.00
delinquent				
US listed firms				
cost-to-revenue ratio	1.00	0.97	0.69	0.83
debt-to-equity ratio	1.00	0.96	0.99	0.79
market capitalization	1.00	0.94	0.99	0.97
price-earnings ratio	1.00	0.88	0.99	0.91
price-to-book ratio	1.00	0.95	0.99	0.92
return on assets	1.00	0.88	0.99	0.96
return on equity	1.00	0.92	0.82	0.88
return on revenue	1.00	0.98	0.84	0.91
total assets	1.00	0.98	0.99	0.96

Table AII: Response rate when using different prompting strategies

B.2 Extraction of numeric estimates from text output

Extracting numeric values from text outputs of smaller, open-source LLMs is challenging due to inconsistencies in the response structure and the numerical representation. We develop a rich extraction function with regular expressions. Firstly, we remove date strings (as these also contain numbers) and format numeric values by removing digit group separators before we convert answers into consistent units for the different variables. Additionally, with Q&A prompting (see XXX), LLMs typically provide the answer first and then offer an explanation. In contrast, for other strategies, LLMs tend to explain first and return the answer at the end. Therefore, based on the prompting strategy, we apply an additional conditional statement to select either the first or the last numeric value as the final prediction.

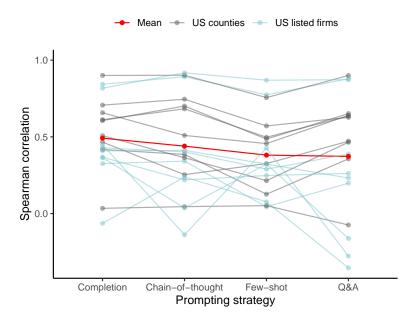


Figure AI: Performance of LME when using different prompting strategies.

C Additional results

Variable	Transfor	No PCA	PCA components			Text	
, and an	mation	1.0 1 011	5	25	100	200	10110
EU (NUTS2)							
GDP per capita (in €)	log	0.65	0.24	0.24	0.25	-0.13	0.50
life expectancy at birth	no	0.73	0.32	0.1	-0.28	-0.43	0.85
net disposable household income (in €)	no	0.94	0.88	0.17	-0.40	-0.16	0.81
population	log	0.77	0.76	0.59	0.34	-0.21	0.72
risk-of-poverty rate	no	0.60	0.29	0.01	-0.19	N/A	0.51
unemployment rate	no	0.81	0.74	0.11	-0.08	-0.16	0.88
number of burglaries per hundred thousand in-	no	0.62	0.55	0.05	0.07	-0.23	-0.43
habitants							
EU (NUTS3)							
GDP per capita (in €)	log	0.70	0.28	0.45	0.36	0.37	0.39
population	log	0.82	0.85	0.78	0.68	0.55	0.65
number of burglaries per hundred thousand in-	no	0.52	0.24	0.14	0.17	0.27	-0.27
habitants							
German districts							
average age of the population	no	0.82	0.71	0.81	0.72	0.51	0.61
disposable income per capita (in €)	no	0.77	0.77	0.73	0.64	0.36	0.63
GDP per capita (in €)	log	0.69	0.55	0.59	0.54	0.42	0.42
population density (per square km)	log	0.90	0.87	0.87	0.81	0.62	0.83
total investments per employee (in €)	no	0.17	0.01	0.05	0.07	0.03	-0.05
unemployment rate (%)	no	0.83	0.80	0.8	0.78	0.54	0.75
number of hospital beds per 1,000 inhabitants	no	0.54	0.45	0.50	0.48	0.31	N/A
number of business registrations per 10,000 in-	no	0.65	0.52	0.64	0.56	0.35	0.29
habitants							
number of government employees per 1,000 in-	no	0.64	0.43	0.61	0.59	0.46	0.11
habitants							
number of completed housing units per 1,000	no	0.47	0.33	0.43	0.31	0.28	-0.10
inhabitants							
number of filed corporate insolvencies per	no	0.48	0.45	0.46	0.40	0.35	0.32
10,000 taxable companies							
percentage change in population compared to	no	0.59	0.57	0.52	0.44	0.39	0.48
the previous year		0.05			0 = 0	0.00	0.0=
percentage of school leavers with a general uni-	no	0.65	0.55	0.65	0.52	0.33	0.07
versity entrance qualification (%)							
UK districts	***	0.06	0.97	0.72	0.00	0.03	0.04
average house price (in £)	no log	0.96	$0.87 \\ 0.42$	0.73	0.09		$0.94 \\ 0.48$
GDP per capita (in £)	log	$0.62 \\ 0.80$	0.42 0.66	$0.54 \\ 0.75$	$0.48 \\ 0.66$	0.21 0.38	0.48 0.70
life expectancy for females at birth	no						
median age	no	$0.90 \\ 0.78$	0.83	0.85	$0.63 \\ 0.54$	0.48	$0.85 \\ 0.63$
median annual gross pay (in £)	no log		0.77	0.66		-0.03	
population unemployment rate	log	0.86	$0.79 \\ 0.61$	0.77	$0.65 \\ 0.39$	0.37	0.89
percentage of the population that live in in-	no	$\begin{array}{c} 0.75 \\ 0.87 \end{array}$	0.84	$0.61 \\ 0.85$	0.39 0.76	$0.05 \\ 0.44$	$0.7 \\ 0.85$
come deprivation	no	0.07	0.04	0.00	0.70	0.44	0.65
US counties							
GDP per capita (in \$)	log	0.61	0.46	0.54	0.43	0.42	0.46
life expectancy at birth	no	0.01	0.40	0.54 0.71	0.43 0.64	0.42 0.55	0.40 0.71
median monthly two-bedroom rent (in \$)	log	0.75	0.70	0.71	0.52	0.50	0.60
- meater monenty two sections tent (in ϕ)	105	0.10	0.10		tinued		

Continued on next page

Variable	Transfor	No PCA	PCA components			its	Text
	mation		5	25	100	200	
personal income per capita (in \$)	no	0.73	0.66	0.68	0.54	0.40	0.61
population	log	0.87	0.81	0.82	0.83	0.82	0.90
unemployment rate	no	0.63	0.49	0.48	0.37	0.19	0.65
number of business establishments per 100,000	log	0.68	0.61	0.66	0.66	0.63	0.03
people							
percentage change in population compared to	no	0.52	0.44	0.45	0.38	0.27	0.41
the previous year							
proportion of mortgages being 90 or more days	no	0.71	0.65	0.65	0.51	0.34	0.41
delinquent							
US firms							
cost-to-revenue ratio	log	0.61	0.07	0.46	0.56	0.55	-0.03
debt-to-equity ratio	cubic	0.37	0.36	0.32	0.29	0.23	0.32
market capitalization	\log	0.91	0.89	0.90	0.91	0.90	0.83
price-earnings ratio	cubic	0.5	0.46	0.49	0.46	0.45	0.37
price-to-book ratio	cubic	0.28	0.14	0.25	0.19	0.11	0.39
return on assets	cubic	0.69	0.64	0.68	0.68	0.68	0.45
return on equity	cubic	0.67	0.62	0.66	0.65	0.65	0.44
return on revenue	cubic	0.61	0.60	0.59	0.58	0.57	0.48
total assets	log	0.93	0.88	0.92	0.93	0.93	0.81

Table AIII: Cross-validation performance of baseline model. The best performing approach in each row is highlighted in bold.

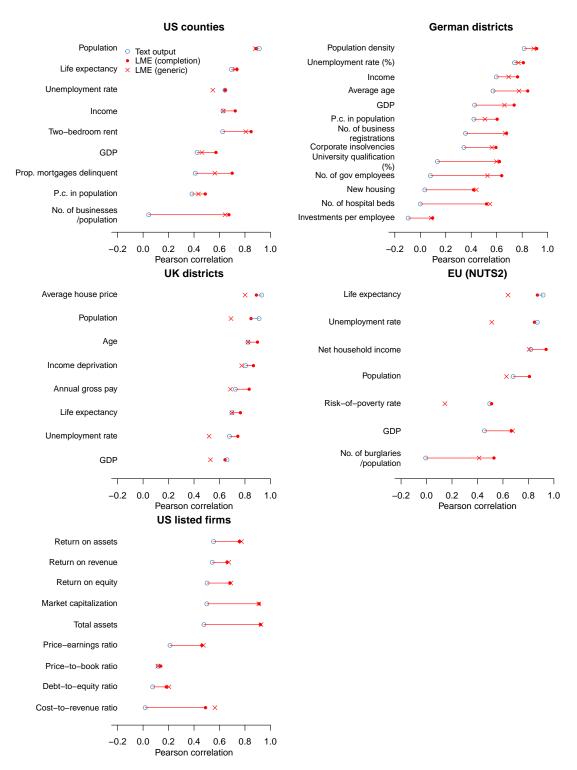


Figure AII: Cross-validation performance using Pearson correlation as the performance metric.

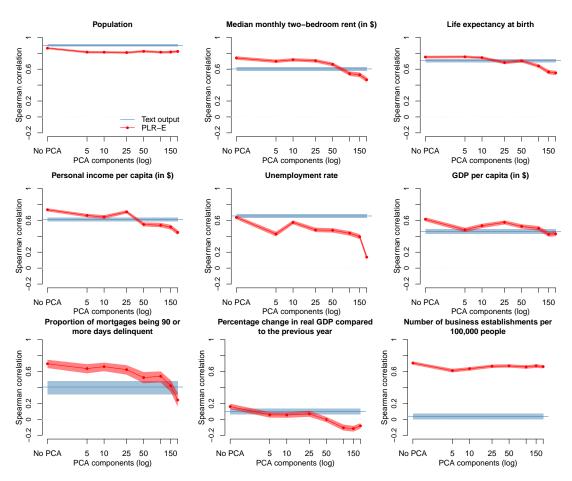


Figure AIII: Performance of base model in US firms dataset as a function of number of by PCA components.

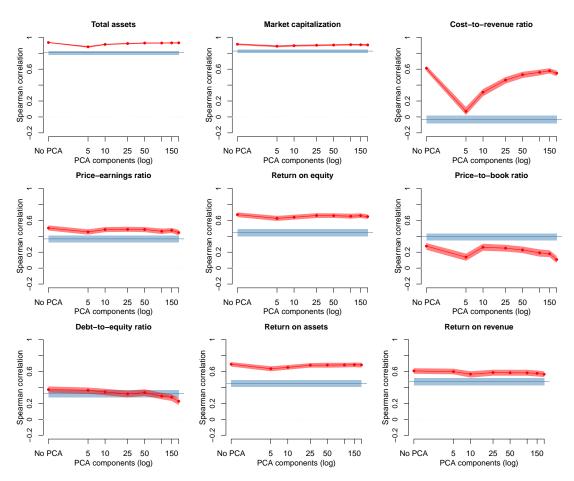


Figure AIV: Performance of base model in US firms dataset as a function of number of by PCA components.

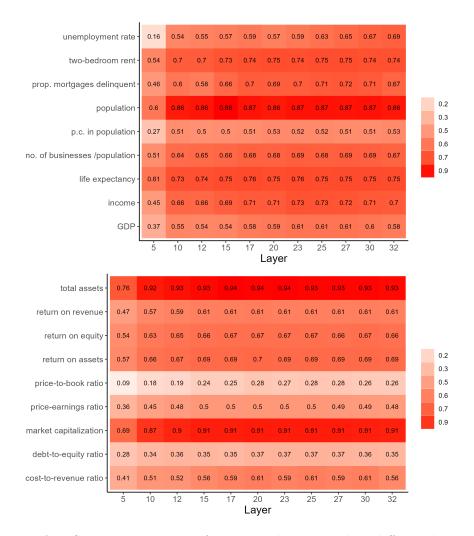


Figure AV: Comparing LME performance when trained on different layers.

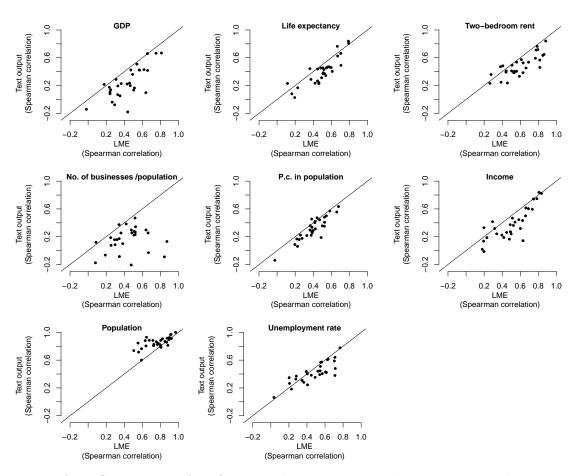


Figure AVI: Comparison of performance between LME and text output within states with at least 50 counties.

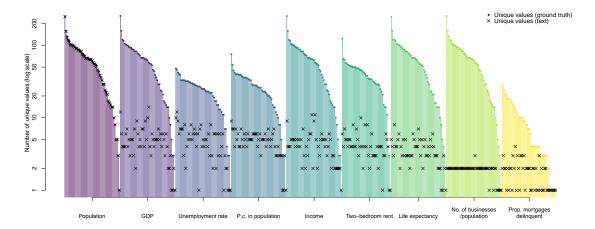


Figure AVII: Number of unique ground truth and text output values for each state. Each vertical lines represents one state; states are ordered by the decreasing number of unique values observed.

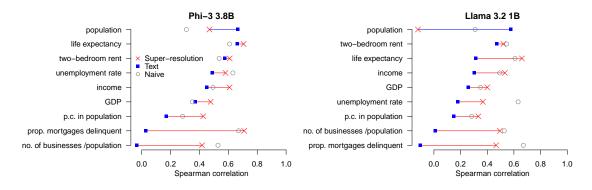


Figure AVIII: Super-resolution results for Phi3 3.8B and Llama 3.2. 1B.