

Memorization-Compression Cycles Improve Generalization

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Abstract

We prove theoretically that generalization improves not only through data scaling but also by compressing internal representations. To operationalize this insight, we introduce the Information Bottleneck Language Modeling (IBLM) objective, which reframes language modeling as a constrained optimization problem: minimizing representation entropy subject to optimal prediction performance. Empirically, we observe an emergent memorization–compression cycle during LLM pretraining, evidenced by oscillating positive/negative gradient alignment between cross-entropy and Matrix-Based Entropy (MBE), a measure for representation entropy. This pattern closely mirrors the predictive–compressive trade-off prescribed by IBLM and also parallels the biological alternation between active learning and sleep consolidation. Motivated by this observation, we propose Gated Phase Transition (GAPT), a training algorithm that adaptively switches between memorization and compression phases. When applied to GPT-2 pretraining on FineWeb dataset, GAPT reduces MBE by 50% and improves cross-entropy by 4.8%. GAPT improves OOD generalization by 35% in a pretraining task on arithmetic multiplication. In a setting designed to simulate catastrophic forgetting, GAPT reduces interference by compressing and separating representations, achieving a 97% improvement in separation — paralleling the functional role of sleep consolidation in biological learning process.

1 Introduction

Generalization occurs when learning from one task improves performance on another. The pursuit of generalization in pre-training LLM [11] has historically focused on scaling up data and parameter size [4], in post-training, Reinforcement Learning with Verifiable Reward (RLVR) [7] has gained attention. However, high quality data has been exhausted after years of tireless scraping, and RLVR is shown to only trim knowledge from baseline model [10] instead of ‘incentivize new reasoning pattern’.

We establish a theoretical upper bound on generalization error for deep learning models, indicating representation entropy as another dimension for improving generalization besides scaling data.

Theorem 1. Upper Bound on Generalization Error. *Let $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$ be random variables with an unknown joint distribution $P(X, Y)$, and suppose X is discrete with finite cardinality. Let f be a neural network with L intermediate representations forming a Markov chain:*

$$X \rightarrow R_1 \rightarrow \dots \rightarrow R_L \rightarrow \hat{Y}$$

Where R_l is internal representations, \hat{Y} is the prediction of the network. Then, for any dataset $\mathcal{D}_N = \{(x_i, y_i)\}_{i=1}^N$ sampled i.i.d. from $P(X, Y)$, the generalization error satisfies for $\alpha \in [1, +\infty)$:

$$\underbrace{\mathcal{L}_{P(X,Y)}(f, \ell)}_{\text{Generalization Error}} \leq \underbrace{\hat{\mathcal{L}}_{P(X,Y)}(f, \ell, \mathcal{D}_N)}_{\text{Empirical Error}} + \underbrace{\mathcal{O}\left(\log N \cdot \min_{1 \leq l \leq L} 2^{\alpha \cdot H(R_l)} / \sqrt{N}\right)}_{\text{Upper Bound on Generalization Gap}} \quad (1)$$

Intuitively, generalization performance of neural network can be improved by either increasing training data size, or by reducing entropy of its internal representations. We refer minimization of $H(R_l)$ as compression, and minimizing empirical loss as memorization.

The Information Bottleneck (IB) framework [8] characterizes the optimal representation as one that discards as much information from the input as possible, while preserving all information relevant to the output. Motivated by this, we introduce the Information Bottleneck Language Modeling (IBLM) objective, along with a theorem showing equivalence of IBLM with IB framework under language modeling.

Definition 1. Information Bottleneck Language Modeling (IBLM). Given a language model with internal representations $R_{1:L}$ and output token variable Y , the IBLM objective is:

$$\begin{aligned} \min \quad & \sum_{l=1}^L H(R_l) \\ \text{s.t.} \quad & \hat{Y} = \arg \min_{\hat{Y}} H(Y|\hat{Y}) \end{aligned} \quad (2)$$

Theorem 2. The IBLM objective defined in Equation 2 is equivalent to the classical Information Bottleneck formulation under the language modeling setting.

Note that $H(Y|\hat{Y})$ is the cross-entropy (CE) loss in conventional language modeling task [11]. In short, IBLM requires maximal compressing internal representation under optimal cross-entropy. To explicitly calculate $H(R)$, we adopt Matrix-based entropy (MBE), first proposed in [5], given a matrix $R \in \mathbb{R}^{s \times d}$ concatenating s token representations, MBE is given by

$$S_\alpha(T) = \frac{1}{1-\alpha} \log \left(\sum_i \left(\frac{\lambda_i(K)}{\text{Tr}(K)} \right)^\alpha \right)$$

where $K = RR^T$ is the Gram matrix. MBE essentially provide a continuous measure of matrix rank, by calculating entropy on distribution of singular values. It has been shown to exhibit a strong correlation with embedding quality [14], where it's also observed that later checkpoints in pre-trained base models has lower entropy, suggesting pre-training with cross-entropy loss alone leads to decrease in MBE.

Previous work [15] observed that in deterministic models (where $I(R; X) = H(R)$), training with a single loss target leads to a two-phase trend: a short initial memorization phase with rapid empirical loss reduction, followed by a monotonic decrease in $H(R)$, interpreted as a compression phase. This observation suggests a clean separation between learning and compression in deep networks.

However, our empirical analysis of GPT pretraining reveals a richer structure. By tracking the cosine similarity between gradients of CE and MBE, we observe that their alignment oscillates between positive and negative throughout training. This suggests that instead of proceeding through distinct phases, learning unfolds as a cyclic process, continuously alternating between expanding representations to capture data (memorization) and reorganizing them into more compact forms (compression). Importantly, this local oscillation occurs alongside a global trend of decreasing MBE, indicating that compression accumulates over time even as the network periodically re-enters memorization-like states. We refer to this phenomenon as the memorization–compression cycle.

In biological neural systems, learning is inherently cyclic, alternating between awake learning and sleep consolidation. In a seminal study [18], sleep was shown to resolve interference between conflicting memories by encouraging competition between them. When two conflicting tasks were presented sequentially, awake learning alone led to catastrophic forgetting, as new synaptic updates overwrote previously learned associations. Sleep consolidation overcame this by reorganizing synaptic weights—allocating distinct subsets to store each memory—achieving better retention than even interleaved learning.

Inspired by biological learning cycle, we introduce Gated Phase Transition (GAPT), a training algorithm that explicitly alternates between memorization (minimizing CE) and compression (minimizing CE and MBE) phases. GAPT is designed to approximate the optimal solution to the IBLM objective by dynamically switching phases based on learning signals, mirroring the role of consolidation in resolving representational conflict.

GAPT delivers consistent improvements across domains:

First, in LLM pre-training on FineWeb dataset, GAPT reduces Matrix-Based Entropy (MBE) by an average of 70.5% across layers while improving cross-entropy by 4.8%, outperforming standard language modeling and approaching the IBLM trade-off.

Second, in arithmetic generalization, GAPT reduces test cross-entropy by 35% and MBE by 47% when trained on 1–3 digit multiplication and tested on 4–6 digits, supporting Theorem 1.

Third, in a synthetic task with gradient interference, GAPT improves representational separation by 97% and reduces MBE by 91% relative to a mixed baseline—closely mirroring the conflict resolution behavior observed in biological sleep.

Our work makes several key contributions:

- **Theoretical Foundation:** We derive an upper-bound on generalization error showing that reducing representation entropy can improve generalization, alongside scaling data.
- **IBLM Objective:** We formulate Information Bottleneck Language Modeling (IBLM) as a constrained optimization problem, unifying representation entropy and cross entropy targets.
- **GAPT Algorithm:** We propose Gated Phase Transition (GAPT), a algorithm to solve IBLM alternates between memorization and compression phases based on dynamic training signals.
- **Empirical Results:** We show that GAPT improves LLM pre-training, arithmetic generalization, and conflict resolution.
- **Biological Analogy:** We relate the memorization–compression cycle in LLMs to the awake–sleep consolidation cycle in biological systems and validate compression’s similarity to consolidation.

The remainder of the paper expands on these contributions: Section 2 presents our theoretical framework and generalization bound; Section 3 details the GAPT algorithm; Section 4 presents empirical results, including the memorization–compression cycle and GAPT’s effectiveness; Section 5 discusses relevant work; Section 6 discusses broader implications.

2 Theory

Corollary 1 (Entropy Lower Bound for Finite Discrete Random Variables). *Let X be a discrete random variable with finite support Ω , where $|\Omega| = n$, and assume that $P(X = x) > 0$ for all $x \in \Omega$. Then there exists a constant $\beta \in (0, 1]$ such that:*

$$H(X) \geq \beta \cdot \log_2 |\Omega|.$$

Proof of Corollary 1 can be found in Appendix A.

Theorem 1. Upper Bound on Generalization Error. *Let $X \in \mathcal{X}$ and $Y \in \mathcal{Y}$ be random variables with unknown joint distribution $P(X, Y)$, and suppose X is discrete with finite cardinality. Let f be a neural network with L intermediate representations forming a Markov chain:*

$$X \rightarrow R_1 \rightarrow \dots \rightarrow R_L \rightarrow \hat{Y}$$

Then, for any dataset $\mathcal{D}_N = \{(x_i, y_i)\}_{i=1}^N$, there exists $\alpha \in [1, +\infty)$ s.t. the generalization error satisfies

$$\mathcal{L}_{P(X,Y)}(f, \ell) \leq \hat{\mathcal{L}}_{P(X,Y)}(f, \ell, \mathcal{D}_N) + \mathcal{O}\left(\frac{\log N \cdot \min_{1 \leq l \leq L} 2^{\alpha \cdot H(R_l)}}{\sqrt{N}}\right)$$

Proof. We begin by recalling the standard formulation of the generalization gap:

$$\text{Gen}_{P(X,Y)}(f, \ell, \mathcal{D}_N) := \mathcal{L}_{P(X,Y)}(f, \ell) - \hat{\mathcal{L}}_{P(X,Y)}(f, \ell, \mathcal{D}_N)$$

For discrete input variables X , it has been shown in [6] that the generalization gap is upper-bounded by:

$$\mathcal{L}_{P(X,Y)}(f, \ell) - \hat{\mathcal{L}}_{P(X,Y)}(f, \ell, \mathcal{D}_N) \leq \mathcal{O}\left(\frac{|\mathcal{X}| \log N}{\sqrt{N}}\right)$$

where $|\mathcal{X}|$ denotes the cardinality of the input space. By Corollary 1, we have $|\mathcal{X}| = \mathcal{O}(2^{\alpha \cdot H(X)})$ for $\alpha \in [1, +\infty)$, this suggests the upper bound could be reformulated via entropy

$$\mathcal{L}_{P(X,Y)}(f, \ell) - \hat{\mathcal{L}}_{P(X,Y)}(f, \ell, \mathcal{D}_N) \leq \mathcal{O}\left(\frac{2^{\alpha \cdot H(X)} \log N}{\sqrt{N}}\right)$$

Let f be a neural network with L intermediate representations forming a Markov chain:

$$X \rightarrow R_1 \rightarrow \dots \rightarrow R_L \rightarrow \hat{Y}$$

Fix any intermediate representation R_l , and decompose the network into $f = d \circ e$, where $e : \mathcal{X} \rightarrow \mathcal{R}_l$ maps inputs to $R_l = e(X)$, and $d : \mathcal{R}_l \rightarrow \mathcal{Y}$ predicts the output.

Applying the generalization bound to the representation R_l , we obtain:

$$\mathcal{L}_{P(X,Y)}(f, \ell) = \mathcal{L}_{P(R_l,Y)}(d, \ell_e) \quad (3)$$

$$\leq \hat{\mathcal{L}}_{P(R_l,Y)}(d, \ell_e, \mathcal{D}_N) + \mathcal{O}\left(\frac{2^{\alpha \cdot H(R_l)} \log N}{\sqrt{N}}\right) \quad (4)$$

$$\leq \hat{\mathcal{L}}_{P(X,Y)}(f, \ell, \mathcal{D}_N) + \mathcal{O}\left(\frac{2^{\alpha \cdot H(R_l)} \log N}{\sqrt{N}}\right) \quad (5)$$

Since the Markov structure ensures that each R_l is a valid bottleneck in the information flow, we can take the tightest such bound across all layers, yielding:

$$\mathcal{L}_{P(X,Y)}(f, \ell) \leq \hat{\mathcal{L}}_{P(X,Y)}(f, \ell, \mathcal{D}_N) + \mathcal{O}\left(\frac{\log N \cdot \min_{1 \leq l \leq L} 2^{\alpha \cdot H(R_l)}}{\sqrt{N}}\right)$$

This concludes the proof. \square

Theorem 2. *The Information Bottleneck Language Modeling (IBLM) objective defined in Equation 2 is equivalent to the classical Information Bottleneck formulation under the language modeling setting.*

Proof. The Information Bottleneck (IB) framework [8] defines the optimal representation R as the solution to:

$$\min_{p(r|x)} I(R; X) \quad (6)$$

$$\text{subject to } I(Y; R) = I(Y; X) \quad (7)$$

Intuitively, the optimal R is one that discards as much information from the input X as possible, while retaining all information necessary to predict the output Y . This representation is referred to as the *minimal sufficient statistic*.

In the case of large language models (LLMs), the network forms a deterministic Markov chain:

$$X \rightarrow R \rightarrow \hat{Y}$$

where R denotes an intermediate hidden representation and \hat{Y} is the predicted output.

Because R is deterministically computed from X , we have:

$$I(X; R) = H(R) - H(R|X) = H(R) \quad (8)$$

$$I(Y; X) \geq I(Y; R) \geq I(Y; \hat{Y}) \quad (9)$$

$$I(Y; \hat{Y}) = H(Y) - H(\hat{Y}|Y) \quad (10)$$

The first equality follows from the fact that $H(R|X) = 0$ for deterministic functions. The second line follows from the Data Processing Inequality (DPI) [3], which ensures that information cannot increase along a Markov chain.

Rewriting the loss of predictive information:

$$I(Y; X) - I(Y; R) \leq I(Y; X) - I(Y; \hat{Y}) \quad (11)$$

$$= H(Y|\hat{Y}) - H(Y|X) \quad (12)$$

Since $H(Y|X)$ is fixed (defined by the true distribution), minimizing $H(Y|\hat{Y})$ increases $I(Y; \hat{Y})$ and pushes it closer to $I(Y; X)$. In practice, minimizing $H(Y|\hat{Y})$ corresponds to minimizing the cross-entropy loss used in language modeling.

On the compression side, we know $H(R|X) = 0$ since neural network propagation is deterministic, therefore we have $I(X; R) = H(R)$. Thus, minimizing $H(R)$ directly minimizes the IB compression term $I(X; R)$, giving us a tractable surrogate for representation compression.

Together, this shows that minimizing cross-entropy corresponds to satisfying the predictive constraint in IB, while minimizing representation entropy implements the compression objective—justifying the IBLM formulation. \square

3 Approach

While applying a Lagrangian objective (CE + λ ·MBE) is a natural approach to solving the constrained optimization in IBLM, we find it often leads to representation collapse: MBE converges to near-zero, but CE worsens as the model loses structure in its internal representations.

Inspired by the alternation between learning and consolidation in biological systems, we divide training into two phases: memorization, where the model minimizes cross-entropy (CE) loss, and compression, where it minimizes a weighted sum of CE and Matrix-Based Entropy (MBE). We propose Gated Phase Transition (GAPT), a training algorithm that dynamically alternates between these phases. GAPT tracks a global minimum CE loss and per-layer MBE histories, and uses patience-based gating to switch phases. Compression is exited early if CE degrades.

GAPT encourages localized compression—reorganizing existing knowledge without interfering with the acquisition of new information—and ensures that entropy reduction occurs only when it does not hinder memorization.

Algorithm 1 Gated Phase Transition (GAPT)

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1: Input: losses  $\mathcal{L}$ , thresholds  $\delta, \tau$ , patience  $p_m, p_c$ 
2: State:  $\phi \in \{1, 2\}$  (1 = mem, 2 = comp), counters  $s_m, s_c, E_{\min}, \text{MBE}_{\min}[i]$ 
3: Extract  $\mathcal{L}_{\text{ce}}, \{\text{MBE}_i\}$ ; update  $\Delta E \leftarrow E_{\min} - \mathcal{L}_{\text{ce}}, E_{\min} \leftarrow \min(E_{\min}, \mathcal{L}_{\text{ce}})$ 
4: if  $\phi = 1$  then ▷ Memorization
5:    $s_m \leftarrow 0$  if  $\Delta E > \delta$  else  $s_m += 1$ 
6:   if  $s_m \geq p_m$  then
7:      $\phi \leftarrow 2, s_c \leftarrow 0, E_{\min} \leftarrow \infty, \text{MBE}_{\min}[i] \leftarrow \infty$ 
8:   end if
9: else ▷ Compression
10:  if  $\mathcal{L}_{\text{ce}} > E_{\min} \cdot (1 + \tau)$  then
11:     $\phi \leftarrow 1, s_m \leftarrow 0$ 
12:  else
13:     $\Delta M \leftarrow \max_i(\text{MBE}_{\min}[i] - \text{MBE}_i)$ 
14:    for each  $i$  do
15:       $\text{MBE}_{\min}[i] \leftarrow \min(\text{MBE}_{\min}[i], \text{MBE}_i)$ 
16:    end for
17:     $s_c \leftarrow 0$  if  $\Delta M > \delta$  else  $s_c += 1$ 
18:    if  $s_c \geq p_c$  then
19:       $\phi \leftarrow 1, s_m \leftarrow 0$ 
20:    end if
21:  end if
22: end if

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4 Experiments

4.1 Natural Compression-Memorization Cycle

Our experimental setup follows the Modded-NanoGPT framework [23]. We remove FP8 matmul (due to hardware incompatibility with Hopper GPUs) and use a simplified block causal attention mask.

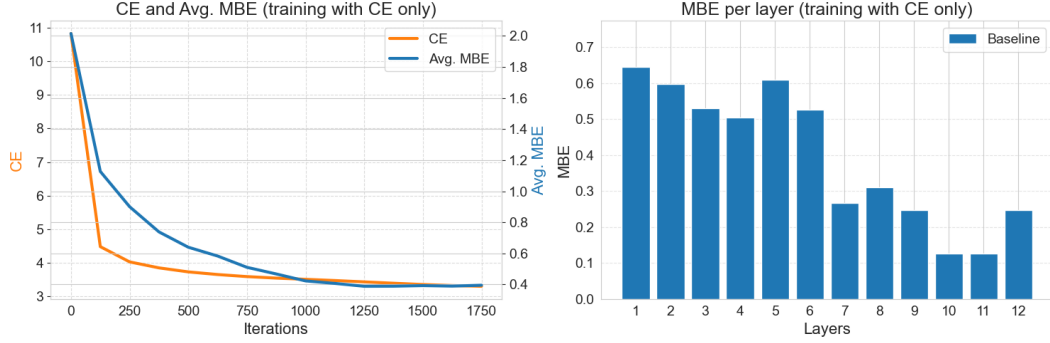


Figure 1: Left: CE and MBE loss curves during pretraining with CE loss only, showing implicit momentum for representation compression. Right: final per-layer MBE values. Later layers show lower MBE, indicating representation compression.

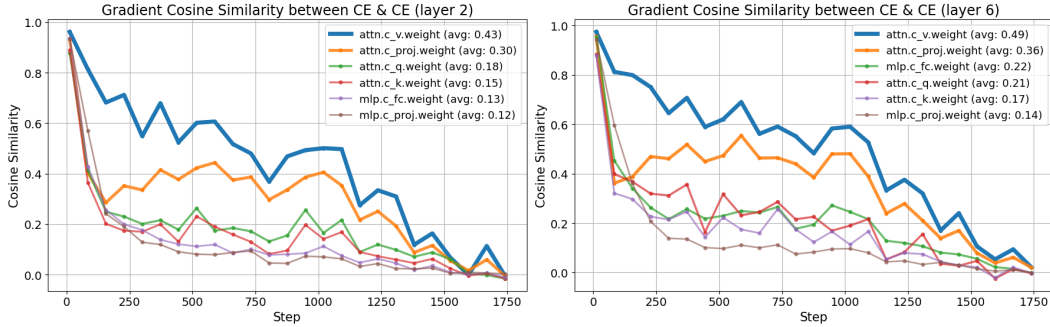


Figure 2: Cosine similarity between CE gradients across batches. CE gradients become increasingly decorrelated over time, reflecting diminishing shared signal.

All experiments are conducted on 8xL40 GPUs, training a 12-layer GPT model on a 0.73B-token FineWeb training set, evaluated on its corresponding validation split. We train on CE loss only. We log cross-entropy (CE) and Matrix-Based Entropy (MBE) gradients at every training step. For each iteration, we record both CE and MBE gradients across multiple batches and compute the average cosine similarity both (1) across batches (CE vs. CE), and (2) between CE and MBE gradients, within and across batches.

In Figure 1, we observe that training with CE loss alone leads to a consistent decrease in MBE across layers, confirming observations from [14]. However, unlike the "compression valley" phenomenon described in that work, our results show that layers 7–12 have **lower** MBE than layers 1–6. This difference is likely due to our architectural design: we adopt a U-Net-like skip connection pattern in which layer 1 is connected to layer 12, layer 2 to layer 11, and so on. As a result, the lower MBE observed in later layers likely reflects more compact representations in the decoding stages.

We further analyze CE gradient behavior. Figure 2. shows that gradient consistency (measured by cosine similarity across batches) declines over time across all layers. This indicates an increasing signal-to-noise ratio in CE gradients as training progresses.

We next inspect cosine similarity between CE and MBE gradients. As shown in Figure 3, we observe recurring sign flips that indicate an implicit alternation between **memorization** (negative similarity) and **compression** (positive similarity) phases, even without explicitly optimizing for entropy.

To characterize this oscillation, we analyze the gradient signal across training using three measures: (1) standard deviation (oscillation strength), (2) zero-crossing rate (oscillation frequency), and (3) peak-to-average power ratio from the power spectral density (periodicity strength). Figure 4 summarizes these metrics across layers and parameter types. We find that attention parameters exhibit

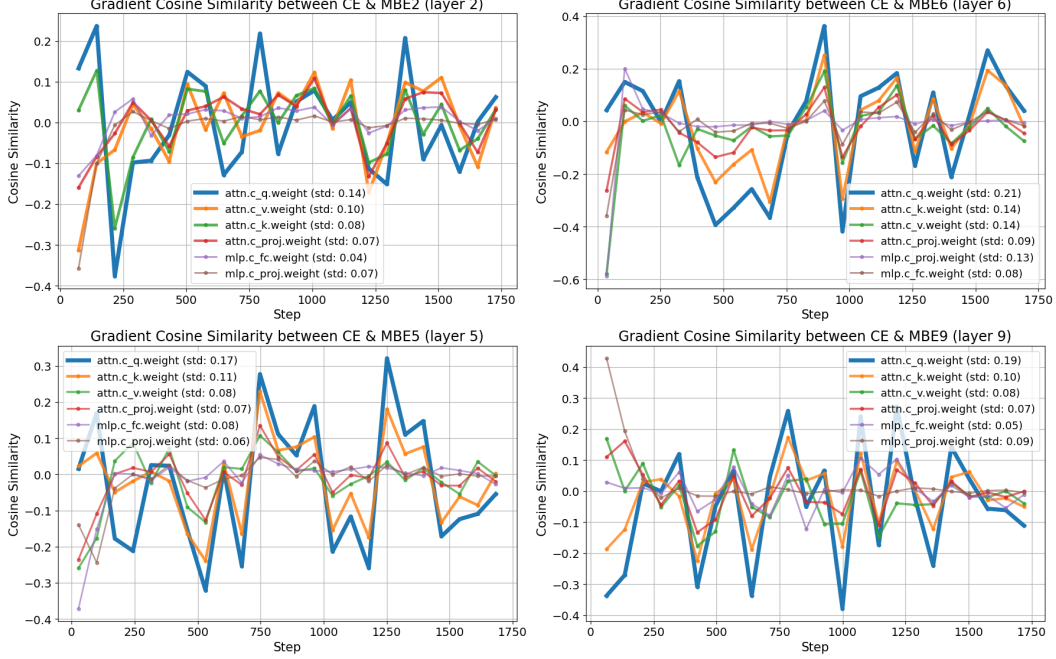


Figure 3: Cosine similarity between CE and MBE gradients over training. Alternating positive and negative phases indicate emergent memorization–compression cycles.

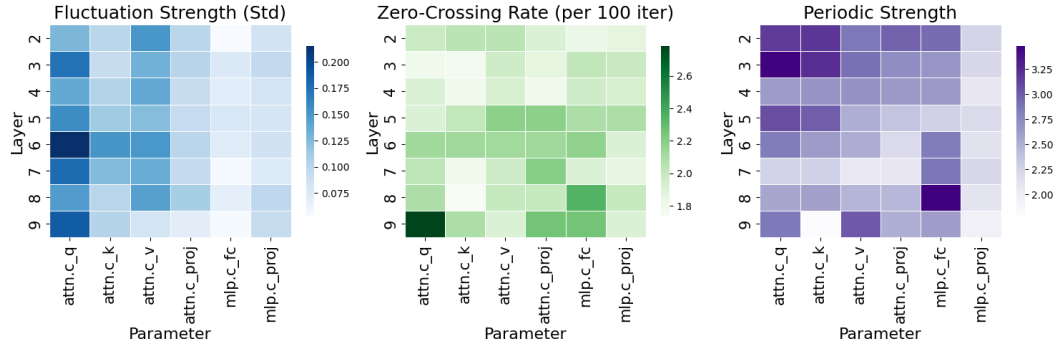


Figure 4: Oscillation metrics between CE and MBE gradients across layers and parameter groups. Left: standard deviation; center: zero-crossing rate; right: periodic strength (peak-to-mean PSD ratio).

stronger and more frequent oscillations than MLP parameters. Interestingly, earlier layers show higher oscillation frequency, while no layer demonstrates strong rhythmic periodicity—suggesting that oscillation is irregular and state-driven, rather than strictly periodic.

This suggests an intriguing perspective on learning: while training on a fixed dataset induces a global momentum toward representation compression, the local dynamics alternate between phases of memorization and compression.

4.2 Pre-training with GAPT

In our second experiment, we retain the same GPT pre-training setup but incorporate the GAPT algorithm to test whether it offers a better solution to IBLM objective defined in Equation 2 ,

compared to a baseline model trained solely on the cross-entropy (CE) loss. MBE regularization during compression phase in GAPT is done from layer 2 to 9.

Model	CE Loss
Baseline	3.31
GAPT (Ours)	3.15 (-4.8%)

Table 1: Cross-entropy loss on the FineWeb validation set.

As shown in Table 1, GAPT reduces cross-entropy on the FineWeb validation set by 4.8% compared to the CE-only baseline. In addition to reducing test CE loss, GAPT also significantly compresses internal representations. Figure 5 (left) shows the layer-wise MBE for both models. We observe consistent reductions across all regularized layers. The per-layer MBE values and their relative improvements are summarized in Figure 5 (right).

GAPT reduces MBE by an average of 70.5% across layers 2–9 while improving validation cross-entropy by 4.8%. This suggests that explicitly alternating between memorization and compression phases offers an effective solution to the constrained optimization objective in IBLM (Equation 2), and can exceed baseline training with cross-entropy target alone.

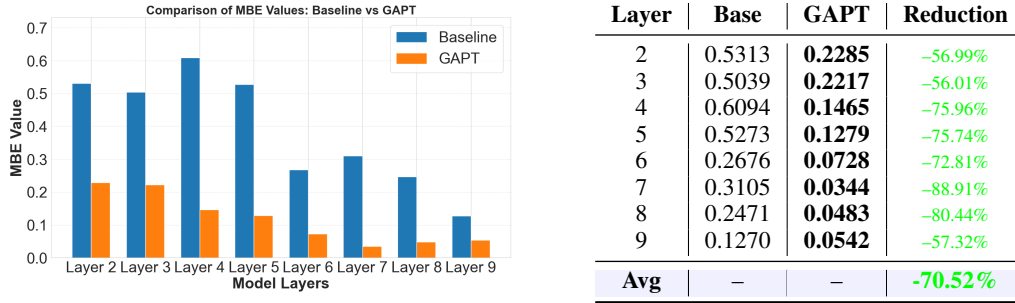


Figure 5: Left: Layer-wise MBE for baseline vs. GAPT. Right: Per-layer MBE reduction with GAPT.

4.3 Conflicting Memory Resolution

Inspired by [18], which showed that sleep consolidation helps resolve memory conflicts, we design a synthetic experiment to test whether GAPT can mitigate representation interference between conflicting experiences. We use a 2-layer MLP f_θ with randomly initialized parameters and define a symmetric shift $\Delta\theta$. Inputs $X_1, X_2 \in \mathbb{R}^{10}$ are sampled from Gaussians with shared variance but distinct means:

$$X_1 \sim \mathcal{N}([1, 1, 1, 1, 1, 0, 0, 0, 0, 0], \sigma^2 I),$$

$$X_2 \sim \mathcal{N}([0, 0, 0, 0, 0, 1, 1, 1, 1, 1], \sigma^2 I)$$

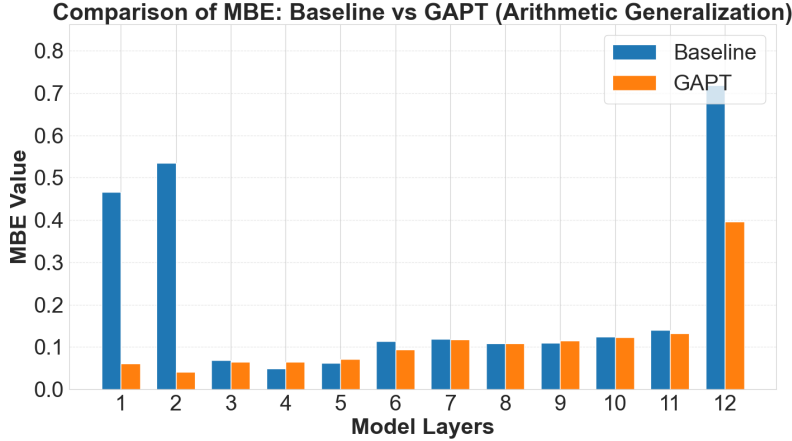
The targets are defined as $Y_1 = f_{\theta+\Delta\theta}(X_1)$ and $Y_2 = f_{\theta-\Delta\theta}(X_2)$, producing two tasks with negatively aligned gradients. We compare GAPT against four baselines: single-task training, ordered training, mixed training, and GAPT applied to mixed batches with MBE regularization.

Table 2 summarizes results. Ordered learning, where the model is trained on one experience and then on the other, suffers from catastrophic forgetting: performance on the first task degrades significantly after exposure to the second. Mixed training alleviates this issue, achieving low L1 loss on both tasks and moderate representation separation. However, GAPT improves over mixed training in both respects: it maintains the same L1 accuracy while achieving a 97% increase in separation ratio and a 91% reduction in MBE.

These results suggest that the compression encouraged by GAPT not only preserves generalization performance but also promotes the disentanglement of conflicting memories. Moreover, compression

Strategy	L1 (Pos)	L1 (Neg)	MBE (Pos)	MBE (Neg)	Dist.	Sep. Ratio
Pos-only	0.02	0.71	0.18	0.45	–	–
Neg-only	0.92	0.02	0.36	0.36	–	–
Pos \rightarrow Neg	0.43	0.04	0.19	0.15	2.43	2.84
Neg \rightarrow Pos	0.02	0.57	0.19	0.43	1.86	2.33
Mixed	0.03	0.03	0.10	0.22	3.66	4.11
GAPT + MBE (Ours)	0.03	0.03	0.02 (−80%)	0.02 (−91%)	6.64 (+81%)	8.08 (+97%)

Table 2: Performance on conflicting experience learning. Lower L1/MBE and higher separation indicate better generalization and disentanglement.



Metric	Baseline (1085)	GAPT (Ours, 898)	Change
Entropy (ID)	0.010	0.011	+10%
Entropy (OOD)	4.334	2.817	−35%
Avg MBE (0–11)	0.218	0.115	−47%

Figure 6: Up: Comparison of baseline vs. GAPT on arithmetic generalization. Bottom: Arithmetic generalization performance summary. GAPT improves OOD generalization and yields more compact representations.

and separation emerge in tandem during training, closely resembling the consolidation behavior observed in biological neural systems during sleep. This supports the view that compression serves not only as a generalization mechanism, but also as a functional tool for resolving interference in memory [18].

4.4 Arithmetic Generalization

To evaluate whether GAPT improves generalization in pre-training language models, we conduct a controlled experiment using a synthetic arithmetic dataset. We pre-train a GPT-2 model from scratch to perform integer multiplication. The training dataset contains 10 million multiplication equations between integers with 1–3 digits. For evaluation, we prepare two test sets: an in-domain (ID) set with 10,000 examples also from the 1–3 digit range, and an out-of-domain (OOD) set with 10,000 examples involving 4–6 digit multiplications. An additional OOD validation set with 1,000 examples is used for early stopping.

We tokenize the input and output sequences at the per-digit level and train the model for 1,750 iterations with a batch size of 16. Due to observed instability in OOD entropy, we adopt an early stopping strategy that halts training if validation loss increases by more than 20% after iteration 800. We expect future work to further stabilize GAPT in OOD settings.

As shown in Figure 6, GAPT improves generalization substantially. It reduces OOD entropy by 35% and average representation entropy (MBE) by 47%, while maintaining similar performance on the

in-domain set. This supports our theoretical prediction that minimizing representation entropy leads to stronger generalization under the Information Bottleneck Language Modeling (IBLM) framework.

Interestingly, while MBE regularization is only applied to a subset of layers, we observe that GAPT achieves lower MBE even in unregularized layers (e.g., layers 0, 1, and 11), suggesting a degree of entropy compression generalization across the network. Notably, MBE in layer 1 is reduced by 92%, and in layer 11 by 45%.

5 Relevant Work

The **Information Bottleneck (IB)** method was introduced in [2] to formalize the goal of retaining only task-relevant information in representations by minimizing entropy. A theoretical connection between generalization and the cardinality of discrete inputs was established in [6], and later applied to deep networks in [8]. However, the entropy–generalization link remained incomplete due to the gap between cardinality and entropy measures.

Empirical evidence of a two-phase learning dynamic—early memorization followed by entropy compression—was presented in [15]. To quantify entropy in neural networks, matrix-based entropy (MBE) was proposed in [5], and later applied to LLMs in [14], where it correlated with embedding quality and revealed compression trends across checkpoints.

LLMs such as GPT-2 [11] generalize across tasks by scaling training data and parameters [4], effectively compressing the training corpus [13, 17]. While post-training methods like instruction tuning [9] improve usability, LLMs still struggle on out-of-distribution tasks such as reverse reasoning [22] and multi-hop inference [20].

Recent advancements in RL with verifiable rewards (RLVR) have improved mathematical and coding performance [7], though often by narrowing base model behavior [10]. Finally, vocabulary design has seen progress through curriculum-based tokenization, where dynamic vocabulary growth based on modeling entropy leads to improved pretraining efficiency [21].

6 Implication

Generalization occurs when learning from one task improves performance on another. Our findings suggest that generalization can emerge from compressing internal representations under predictive constraints—without relying on massive amounts of demonstrative data. This ability may allow artificial systems to discover novel patterns and principles, reducing dependence on large-scale supervision. However, it also raises important safety concerns: a system that generalizes well but lacks transparency or interpretability may pose greater risks than one that mostly memorizes. As we move toward AGI, it is essential to understand and govern the mechanisms behind representation compression and generalization to ensure safe, aligned, and accountable behavior.

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A Appendix

Theorem 3 (Minimum Probability Entropy Bound). *Let X be a discrete random variable with sample space Ω , where $|\Omega| = n$. Suppose there exists a constant $\alpha \in (0, \frac{1}{n}]$ such that $P(X = x) \geq \alpha$ for all $x \in \Omega$. Then the entropy of X is bounded below by:*

$$H(X) \geq -(1 - \alpha(n - 1)) \log_2(1 - \alpha(n - 1)) - (n - 1)\alpha \log_2(\alpha).$$

Furthermore, for sufficiently large n and small α such that $\beta = \alpha n \ll 1$, this bound approximates to:

$$H(X) \geq \beta \log_2(n)$$

Proof. Under the constraint that $P(X = x) \geq \alpha$ for all $x \in \Omega$, the entropy

$$H(X) = - \sum_{x \in \Omega} P(x) \log_2 P(x)$$

is minimized when the distribution is as imbalanced as possible: one outcome has the highest allowed probability, and all others are assigned the minimum α . The worst-case distribution is:

$$\begin{aligned} P(x_1) &= 1 - \alpha(n - 1), \\ P(x_i) &= \alpha \quad \text{for } i = 2, \dots, n. \end{aligned}$$

The resulting entropy is:

$$H(X) = -(1 - \alpha(n - 1)) \log_2(1 - \alpha(n - 1)) - (n - 1)\alpha \log_2(\alpha).$$

For small α and large n such that $\alpha n \ll 1$, we approximate have:

$$1 - \alpha(n - 1) \approx 1 - \alpha n.$$

Substituting this into the expression:

$$\begin{aligned} H(X) &\approx -(1 - \alpha n) \log_2(1 - \alpha n) - \alpha n \log_2(\alpha) \\ &= -(1 - \alpha n) \log_2(1 - \alpha n) - \alpha n \log_2(\alpha n) + \alpha n \log_2(n) \\ &= h(\alpha n) + \alpha n \log_2(n) \end{aligned}$$

where $h(p) = -p \log_2 p - (1 - p) \log_2(1 - p)$ is the binary entropy function.

Since $h(\alpha n) \geq 0$ and $\log_2(1/\alpha) \geq \log_2(n)$ for $\alpha \leq 1/n$, we get:

$$H(X) \geq \alpha n \log_2(n) = \beta \log_2(n)$$

which completes the proof. \square

Corollary 1 (Entropy Lower Bound for Finite Discrete Random Variables). *Let X be a discrete random variable with finite support Ω , where $|\Omega| = n$, and assume that $P(X = x) > 0$ for all $x \in \Omega$. Then there exists a constant $\beta \in (0, 1]$ such that:*

$$H(X) \geq \beta \cdot \log_2 |\Omega|.$$

Proof. Let $\varepsilon := \min_{x \in \Omega} P(X = x)$, which exists and is strictly positive since X is discrete with finite support. By the Minimum Probability Entropy Bound (Theorem 3), we have:

$$H(X) \geq -(1 - \varepsilon(n - 1)) \log_2(1 - \varepsilon(n - 1)) - (n - 1)\varepsilon \log_2(\varepsilon).$$

For small ε and large n , this approximates to:

$$H(X) \geq \varepsilon n \log_2\left(\frac{1}{\varepsilon}\right),$$

which is linear in $n = |\Omega|$. Setting $\beta := \varepsilon n$, the result follows:

$$H(X) \geq \beta \cdot \log_2 |\Omega|.$$

\square