Yasaman Mostafavi
CS 411
F21
A 20439542

A. Let: $a = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}$ $c = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$ find

1. $2a - b$ $\quad 2a = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$ $\quad 2a - b = \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$

2. $\hat{a}$ a unit vector in the direction of a

1. find the magnitude $\|\vec{a}\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

2. divide vector by its magnitude $\hat{a} = \begin{bmatrix} 1/\sqrt{14} \\ 2/\sqrt{14} \\ 3/\sqrt{14} \end{bmatrix}$

3. $\|a\|$ and the angle of a realitive to the positive x axis

$\|a\| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

4. the direction cosines of a

$\cos\alpha = \frac{a_x}{\|a\|} = \frac{1}{\sqrt{14}}$ $\quad \cos\beta = \frac{a_y}{\|a\|} = \frac{2}{\sqrt{14}}$ $\quad \cos\gamma = \frac{a_z}{\|a\|} = \frac{3}{\sqrt{14}}$

5. the angle between a and b

$a \cdot b = |a||b|\cos(\alpha)$

$a \cdot b = 1\times4 + 2\times5 + 3\times6 = 32$

$\cos^{-1}\left(\frac{32}{\sqrt{14}\cdot\sqrt{77}}\right) = 12{,}9°$

* accidentally did #10. 6 on next page.

10. $a \times b$ and $b \times a$

$c_x = a_y b_z - a_z b_y = 2\cdot6 - 3\cdot5 = -3$

$c_y = a_z b_x - a_x b_z = 3\cdot4 - 1\cdot6 = 6$ $\quad a \times b = \langle -3, 6, -3 \rangle$

$c_z = a_x b_y - a_y b_x = 1\cdot5 - 2\cdot4 = -3$

$d_x = b_y a_z - b_z a_y = 5\cdot3 - 6\cdot2 = 3$

$d_y = b_z a_x - b_x a_z = 6\cdot1 - 4\cdot3 = -6$ $\quad b \times a = \langle 3, -6, 3 \rangle$

$d_z = b_x a_y - b_y a_x = 4\cdot2 - 5\cdot1 = 3$

6. a·b and b·a

$1\cdot4+2\cdot5+3\cdot6 = 32 \quad a\cdot b = b\cdot a = 32$

7. a·b by using the angle between a and b

$a\cdot b = |a||b|\cos\theta = \sqrt{14}\sqrt{77}\cos(12.9) = 32.0042$

8. the scalar projection of b onto a

Scalar projection $\vec{b}$ to $\vec{a}$: $comp_a b = \frac{a\cdot b}{|a|} = \frac{32}{\sqrt{14}}$

9. a vector perpendicular to a

a vector is perp/orthogonal to a if $\theta = \frac{\pi}{2}$ or $a\cdot b = 0$

$1\cdot c_x + 2\cdot c_y + 3\cdot c_z = 0 \quad \langle 1, 1, -1\rangle$ is perp. b/c $a\cdot c = 0$

10. on prev. page

11. a vector perpendicular to both a and b.

a vector that would be perpendicular to a and b, is the normal vector of their cross product i.e.

$\langle -3, 6, -3\rangle$ or $\langle 3, -6, 3\rangle$

12. the linear dependency between a, b, and c

1. set up a homeogeneous system of equations

$$\left[\begin{array}{ccc|c} 1 & 4 & 1 & 0 \\ 2 & 5 & -1 & 0 \\ 3 & 6 & 3 & 0 \end{array}\right] \xrightarrow{2.\ rref} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$c_1 + 3c_3 = 0$

$1c_2 - 1c_3 = 0$

$0 = 0$

3. the system has values that are non-zero therefore, is lineairly dependent

13. a^Tb and ab^T

$a^Tb = [1\ 2\ 3]\begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} = (1\cdot4 + 2\cdot5 + 3\cdot6) = 32$

$ab^T = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}[4\ 5\ 6] =$

B. Let $A=\begin{bmatrix}1&2&3\\4&-2&3\\0&5&-1\end{bmatrix}$, $B=\begin{bmatrix}1&2&1\\2&1&-4\\3&-2&1\end{bmatrix}$ $C=\begin{bmatrix}1&2&3\\4&5&6\\-1&1&3\end{bmatrix}$ $d=\begin{bmatrix}1\\2\\3\end{bmatrix}$

$2A=\begin{bmatrix}2&4&6\\8&-4&6\\0&10&-2\end{bmatrix}$ $2A-B=\begin{bmatrix}1&2&5\\6&-5&10\\-3&12&-1\end{bmatrix}$

2. AB and BA

$AB=\begin{bmatrix}14&-2&-4\\9&0&15\\7&7&-21\end{bmatrix}$ $BA=\begin{bmatrix}9&3&8\\6&-18&13\\-5&15&2\end{bmatrix}$

3. $(AB)^T$ and B^TA^T

$(AB)^T=\begin{bmatrix}14&9&7\\-2&0&7\\-4&15&-21\end{bmatrix}=B^TA^T$

4. $|A|$ and $|C|$

$|A|=1(-2\cdot-1-3\cdot5)-2(4\cdot-1-0)+3(4\cdot5)=55$

$|C|=1(15-6)-2(12+6)+3(4+5)=0$

5. the matrix (A, B or C) in which the row vectors form an orthogonal set.

set $A=\{[1\ 2\ 3],[4\ -2\ 3],[0\ 5\ -1]\}$

$[1\ 2\ 3]\cdot[4\ -2\ 3]=4+(-4)+9=9$ X not orthagonal

set B is orthogonal, as all rows are parallel w/eachother

6. A^{-1} and B^{-1}

$A^{-1}=\left[\begin{array}{ccc|ccc}1&2&3&1&0&0\\4&-2&3&0&1&0\\0&5&-1&0&0&1\end{array}\right]$ $R_2\to-4R_1+R_2$ $\left[\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&-10&-9&-4&1&0\\0&5&-1&0&0&1\end{array}\right]$

$\frac{R_2}{-10}\left[\begin{array}{ccc|ccc}1&2&3&1&0&0\\0&1&9/10&4/10&1/10&0\\0&5&-1&0&0&1\end{array}\right]$ $R_2\to(-2)R_2+R_1$; $\left[\begin{array}{ccc|ccc}1&0&6/5&1/5&1/5&0\\0&1&9/10&4/10&-1/10&0\\0&5&-1&0&0&1\end{array}\right]$

$R_3\to-5R_2+R_3\left[\begin{array}{ccc|ccc}1&0&6/5&1/5&1/5&0\\0&1&9/10&4/10&-1/10&0\\0&0&-11/2&-2&1/2&1\end{array}\right]\to\left[\begin{array}{ccc|ccc}1&2&6/5&1/5&1/5&0\\0&1&9/10&4/10&-1/10&0\\0&0&1&4/11&-1/11&-2/11\end{array}\right]$

$$\left[\begin{array}{ccc|ccc}1&0&0&-13/55&17/55&12/55\\0&1&9/10&2/5&-1/10&0\\0&0&1&4/11&+1/11&-2/11\end{array}\right] R_2 \to -\frac{9}{10}R_3+R_2 \left[\begin{array}{ccc|ccc}1&0&0&-13/55&17/55&12/55\\0&1&0&4/55&-1/55&9/55\\0&0&1&4/11&-1/11&-2/11\end{array}\right]$$

$$B^{-1}=\left[\begin{array}{ccc|ccc}1&2&1&1&0&0\\2&1&-4&0&1&0\\3&-2&1&0&0&1\end{array}\right] \to \left[\begin{array}{ccc|ccc}1&2&1&1&0&0\\0&-3&-6&-2&1&0\\0&-8&-2&-3&0&1\end{array}\right]$$

$$\left[\begin{array}{ccc|ccc}1&2&1&1&0&0\\0&1&2&2/3&-1/3&0\\0&-8&-2&-3&0&1\end{array}\right] \to \left[\begin{array}{ccc|ccc}1&0&-3&-1/3&2/3&0\\0&1&2&2/3&-1/3&0\\0&0&14&7/3&-8/3&1\end{array}\right]$$

$$\left[\begin{array}{ccc|ccc}1&0&-3&-1/3&2/3&0\\0&1&2&2/3&-1/3&0\\0&0&1&1/6&-4/21&1/14\end{array}\right] \to \left[\begin{array}{ccc|ccc}1&0&0&1/6&2/21&3/14\\0&1&0&1/3&1/21&-1/7\\0&0&1&1/6&-4/21&1/14\end{array}\right]$$

7. C^{-1} $C^{-1} = \text{DNE} \to |C| = 0$

8. The product Ad

$$Ad=\begin{bmatrix}1&2&3\\4&-2&3\\0&5&-1\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}=\begin{bmatrix}1\cdot1+2\cdot2+3\cdot3\\4\cdot1+-2\cdot2+3\cdot3\\0\cdot1+5\cdot2+-1\cdot3\end{bmatrix}=\begin{bmatrix}14\\9\\7\end{bmatrix}$$

9. The projection of the rows of A onto the vector d w/o normalizin d.

$$\text{proj}_d[1\,2\,3]=\frac{da_1}{|d|^2}\cdot a=\frac{14}{\sqrt{14}^2}\cdot[1\,2\,3]=[1\,2\,3]$$

$$\text{proj}_d[4\,-2\,3]=\frac{da_2}{|d|^2}\cdot a=\frac{9}{\sqrt{14}^2}\cdot[1\,2\,3]=\left[\frac{9}{14},\frac{18}{14},\frac{27}{14}\right]$$

$$\text{proj}_d[0\,5\,-1]=\frac{da_3}{|d|^2}\cdot a=\frac{7}{(\sqrt{14})^2}[1\,2\,3]=\left[\frac{7}{14},\frac{14}{14},\frac{21}{14}\right]$$

10. the linear combination of the coulumns of A using elem of d

$$\left[\begin{array}{ccc|c}1&2&3&1\\4&-2&3&2\\0&5&-1&3\end{array}\right]\to\left[\begin{array}{ccc|c}1&2&3&1\\0&-10&-9&-2\\0&5&-1&3\end{array}\right]\to\left[\begin{array}{ccc|c}1&2&3&1\\0&1&9/10&1/5\\0&0&-11/2&2\end{array}\right]$$

$$\left[\begin{array}{ccc|c}1&0&0&57/55\\0&1&0&29/55\\0&0&1&-4/11\end{array}\right]\quad x_1=\frac{57}{55},\ x_2=\frac{29}{25},\ x_3=-4/11$$

11. the solution for the equation $Bx=d$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 2 & 1 & -4 & 2 \\ 3 & -2 & 1 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & -3 & -6 & 0 \\ 0 & -8 & -2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 1/4 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & -7/4 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \rightarrow \boxed{\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]} \quad x = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

12. the solution for the equation $Cx=d$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \\ -1 & 1 & 3 & 3 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & -3 & -6 & -2 \\ 0 & 3 & 6 & 4 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 2/3 \\ 0 & 3 & 6 & 4 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 1 \\ 0 & 1 & 2 & 2/3 \\ 0 & 0 & 0 & 2 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$ cannot reduce further

C. Let $f(x)=x^2+3$ $g(x)=x^2$, $g(x,y)=x^2+y^2$ find: 9

1. first and second derivitives of $f(x)$ w/ respect to x

$f'(x)=2x$; $f''(x)=2$

2. the partial derivatives

$\frac{dg}{dx}$ and $\frac{dg}{dy}$ $\quad \frac{dg}{dx}=2x$, $\frac{dg}{dy}=2y$