Homework 7

Saturday, November 27, 2021 12:07 PM

(a) Explain the difference between forward and inverse texture mapping. Explain which is the preferred (forward or inverse mapping) method for performing texture mapping.

Forward mapping scans the texture and determines where it will lie on the object, which may cause problems with overlapping pixels, or holes. Inverse texture mapping looks at the target (the object) and figures out which pixel should be copied from the texture. Now, holes and overlapping pixels will not occur, because of this, inverse mapping is favorable.

(b) Let $v_1 = [1,2]$, $v_2 = [4,2]$, $v_3 = [4,6]$ be texture coordinates (i.e. vertices in a texture image) and $v'_1 = [1,2]$, $v'_2 = [1,4]$, $v'_3 = [4,2]$ be the corresponding coordinates in the rendered image that is being produced. Write the equation for computing the forward map between texture and image coordinates, then compute the forward map. Write the equation for computing the inverse map, then compute the inverse map.

Forward map

$$\underset{3\times3}{M} = \underset{3\times3}{[v'_1\ v'_2\ v'_3]}\ \underset{3\times3}{[v_1\ v_2\ v_3]^{-1}}$$

$$M = \begin{bmatrix} 1 & 1 & 4 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 4 \\ 2 & 2 & 6 \\ 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 3/4 & -1/2 \\ 2/3 & -1/2 & 7/3 \\ 0 & 0 & 1 \end{bmatrix}$$

Inverse map

$$\underset{3\times3}{M^{-1}} = \underset{3\times3}{[v_1\ v_2\ v_3]}\ \underset{3\times3}{[v'_1\ v'_2\ v'_3]^{-1}}$$

$$M = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 2 & 6 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 4 \\ 2 & 4 & 2 \\ 1 & 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 3/2 & -3 \\ 4/3 & 0 & 2/3 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Explain the advantage of bump-mapping over standard texture mapping in increasing the level of realism. Explain how bump-mapping creates the illusion of bumps without modifying the geometry of the surface.

Bump mapping is significantly cheaper computationally than standard texture mapping because instead of actually changing the object, instead we just change the normals upon the object to simulate texture. A major downside however is that bump mapping obviously will not change the shadows of a texture, and may create some uncanny visuals.

(d) Given a bump image $b(x, y)$ explain how the normal $n = (n_x, n_y, n_z)$ at location (x, y) is changed using the bump image.

The texture image will be used as a height map, and we can use the following equation to determine exactly how the normal is changed:

$$\begin{bmatrix} n^*_x \\ n^*_y \\ n^*_z \end{bmatrix} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} - b_u \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - b_u \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

where $(b_u\ b_v)$ are the height map

(e) Let the intensity of the image at location $(4, 5)$ be 20 and the intensity at location $(5, 5)$ be 25. Compute the displacement that will be added to the x coordinate of the normal using bump mapping.

The displacemen that will be added to the x coordinate of the normal is the intensity at $x=5$ minus the intensity at $x=4$, resuting in $25-20=5$ and thus the displacement is 5.

(f) Given that the R,G,B intensity you read from a normal map is $(0.2, 0.3, 0.9)$, what is the normal vector that will be decoded from this intensity.

decoding: $n = n^* \cdot 2 - 1$

$n_R = (0.2 \cdot 2) - 1 = -0.6$

$n_B = (0.3 \cdot 2) - 1 = -0.4$

$n_G = (0.9 \cdot 2) - 1 = 0.8$

(g) Given a triangle with vertices $p_1 = [1, 2, 1]$, $p_2 = [4, 2, 2]$, $p_3 = [4, 6, 2]$ and texture coordinates

(g) Given a triangle with vertices $p_0 = [1,2,1]$, $p_1 = [4,2,2]$, $p_2 = [4,6,3]$ and texture coordinates of $(u_0, v_0) = [1,2]$, $(u_1, v_1) = [1,4]$, $(u_2, v_2) = [4,2]$, compute the normal N, tangent T, and bi-tangent B.

$E_1 = P_1 - P_0$

$E_2 = P_2 - P_0$

$\Delta u_1 = u_1 - u_0$

$\Delta u_2 = u_2 - u_0$

$$[T\ B] = [E_1\ E_2] \begin{bmatrix} \Delta u_1 & \Delta u_2 \\ \Delta v_1 & \Delta v_2 \end{bmatrix}^{-1}$$

$N = T \times B$

$E_1 = [4,2,2] - [1,2,1] = [3,0,1]$

$E_2 = [4,6,3] - [1,2,1] = [3,4,2]$

$$[T\ B] = \begin{bmatrix} 3 & 3 \\ 0 & 4 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 3 \\ 2 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 3/2 \\ 4/3 & 0 \\ 2/3 & 1/2 \end{bmatrix}$$

$$\begin{array}{cc} T & B \end{array}$$

$$N = T \times B = \begin{bmatrix} 1 \\ 4/3 \\ 2/3 \end{bmatrix} \times \begin{bmatrix} 3/2 \\ 0 \\ 1/2 \end{bmatrix} = \begin{bmatrix} 3/2 \\ 1/2 \\ -2 \end{bmatrix}$$

(h) Given the vectors T, B, N from the previous question, write the transformation matrix that will transform the normals from tangent space (normal map coordinate system) to the world coordinate system.

$M^{-1} = \begin{bmatrix} | & | & | \\ T & B & N \\ | & | & | \end{bmatrix}$ describes the matrix for transforming from tangent $\rightarrow$ model space. so

$$M^{-1} = \begin{bmatrix} 1 & 3/2 & 3/2 \\ 4/3 & 0 & 1/2 \\ 2/3 & 1/2 & -2 \end{bmatrix} = \text{Transformation matrix for tangent} \rightarrow \text{model space.}$$

(i) Given the transformation matrix from the previous question, write the transformation matrix that will transform the light direction vector L to the normal map coordinate system.

it has been proven to us $M^{-T} = \begin{bmatrix} T \\ B \\ N \end{bmatrix}^{-T} = [T\ B\ N]^T = \begin{bmatrix} T \\ B \\ N \end{bmatrix} = M$, therefore the transformation matrix for the light direction L to the normal map coordinate system is simply M.

$$M = \begin{bmatrix} — T — \\ — B — \\ — N — \end{bmatrix} = \begin{bmatrix} 1 & 4/3 & 2/3 \\ 3/2 & 0 & 1/2 \\ 3/2 & 1/2 & -2 \end{bmatrix}$$