Ant Colony Optimization (ACO)

Nature-Inspired Optimization Algorithm

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What is ACO?

- Metaheuristic inspired by the behavior of real ants.
- Real ants find shortest paths by laying down **pheromones**.
- **Positive feedback**: stronger pheromone trails attract more ants.
- **Distributed search**: many ants explore simultaneously.

How ACO Works (High Level)

1. Initialize pheromones on all paths.
2. Place each ant at a random starting point.
3. Each ant builds a solution step-by-step.
4. Update pheromones (evaporation + reinforcement).
5. Repeat for multiple iterations.

ACO Main Formulas

Probability to choose next node j:

$$P_{ij} = \frac{[\tau_{ij}]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{k \in \text{allowed}}[\tau_{ik}]^{\alpha}[\eta_{ik}]^{\beta}}$$

Pheromone Update Rule:

$$\tau_{ij} = (1-\rho)\tau_{ij} + \Delta\tau_{ij}$$

- α: importance of pheromone
- β: importance of heuristic
- ρ: evaporation rate

Example 1: Find Shortest Path

Cities: A, B, C, D
Distances:

- A-B: 2, A-C: 3, A-D: 4
- B-C: 1, B-D: 5, C-D: 2

Task:

- Start with equal pheromones.
- Run one iteration of ant movement.

Example 1: Step-by-Step Solution

- Step 1: Initialize pheromone $\tau = 1$ on all edges.
- Step 2: Each ant moves based on calculated probabilities.
- Step 3: After tours, update pheromone trails.
- Step 4: Display updated pheromone table.

Example 1: Solving Step-by-Step

Step 1: Calculate probabilities (example for moving from A):

$$P(A \to B) = \frac{1^{\alpha}(1/2)^{\beta}}{1^{\alpha}(1/2)^{\beta} + 1^{\alpha}(1/3)^{\beta} + 1^{\alpha}(1/4)^{\beta}}$$

Assume $\alpha = 1$, $\beta = 2$.

Step 2: Simulate ant movements (random based on probabilities).

Step 3: After tour complete:

- Shorter tours deposit more pheromone.
- Update pheromone trails according to performance.

Example 2: Maze Solving (Only Define)

Problem:

- A 5x5 grid maze.
- Ants must find shortest path from Start to End.
- Heuristic: move closer to goal; avoid walls.

Example 3: Job Scheduling (Only Define)

Problem:

- Assign 4 jobs to 3 machines.
- Objective: minimize total completion time.
- Each path: a possible job-machine assignment.

Parameters Tuning Tips

- α: pheromone importance
- β: heuristic importance
- ρ: evaporation rate
- Number of ants: related to problem size

Summary

- ACO is a powerful metaheuristic for combinatorial problems.
- Inspired by real-world ant behavior.
- Strong solutions reinforced over time.
- Careful parameter tuning is crucial.