# INFO-F-404 : Operating Systems II

We will use following notations :

- $O_i$ - Release time of Task $\tau_i$
- $C_i$ - Worst case execution time of Task $\tau_i$
- $D_i$ - Deadline of Task $\tau_i$
- $T_i$ - Period of Task $\tau_i$

## 1 Rate Monotonic

### System 1

Let's consider the system represented by Table 1. These are all *periodic*, *synchronous* tasks with *implicit deadline*. We will consider the case where these tasks (and jobs) are *independent* and *preemptible*.

| **Task index** | **Release time** | **WCET** | **Deadline** | **Period** |
|---|---|---|---|---|
| Task $\tau_1$ | 0 | 10 | 50 | 50 |
| Task $\tau_2$ | 0 | 20 | 80 | 80 |
| Task $\tau_3$ | 0 | 10 | 100 | 100 |
| Task $\tau_4$ | 0 | 50 | 200 | 200 |

Table 1: System of 4 periodic, synchronous tasks with implicit deadline.

**a)** Verify that this system could be scheduled using RM algorithm.

**Correction**

We can choose between two tests, let's start with the most simple (utilization of the processor):

$$U_{tot}(\tau) = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

$$U_{tot}(\tau) = \frac{10}{50} + \frac{20}{80} + \frac{10}{100} + \frac{50}{200} = 0.2 + 0.25 + 0.1 + 0.25 = 0.8$$

$$U_{tot} \leq ln(2), (ln(2) \approx 0.69)$$

We have $69\% < 80\% \leq 100\%$, so we can not claim anything about feasibility of this system. Note that we can also use $U_{tot} \leq n(2^{1/n} - 1)$, in this case we have $n = 4$, so $n(2^{1/n} - 1) = 0.7568$, but we still have $80\% > 75.68\%$.

We have to use the second method (based on the worst response time of the first job):

1. First of all, sort tasks by priority (assigned by RM, $p_i = \frac{1}{T_i}$) in decreasing order: $\tau_1 > \tau_2 > \tau_3 > \tau_4$.
2. After that we have to find the worst response time of the first job of each task. We can do so by finding the fixed point:
   - $W_0 = C_i$
   - $W_{k+1} = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{W_k}{T_j} \right\rceil \cdot C_j$

   till $W_{k+1} = W_k$ or $W_k > D_i$ (system is not feasible).

   For our system, we have:

   (a) $\tau_1$:
      i. $W_0 = 10$
      ii. $W_1 = 10 + 0 = 10$
      iii. Fixed point found, $W_1 \leq D_1 \rightarrow$ OK

   (b) $\tau_2$:
      i. $W_0 = 20$
      ii. $W_1 = 20 + \lceil \frac{20}{50} \rceil \cdot 10 = 20 + 10 = 30$
      iii. $W_2 = 20 + \lceil \frac{30}{50} \rceil \cdot 10 = 20 + 10 = 30$
      iv. Fixed point found, $W_2 \leq D_2 \rightarrow$ OK

   (c) $\tau_3$:
      i. $W_0 = 10$
      ii. $W_1 = 10 + \lceil \frac{10}{50} \rceil \cdot 10 + \lceil \frac{10}{80} \rceil \cdot 20 = 10 + 10 + 20 = 40$
      iii. $W_2 = 10 + \lceil \frac{40}{50} \rceil \cdot 10 + \lceil \frac{40}{80} \rceil \cdot 20 = 10 + 10 + 20 = 40$
      iv. Fixed point found, $W_2 \leq D_3 \rightarrow$ OK

   (d) $\tau_4$:
      i. $W_0 = 50$
      ii. $W_1 = 50 + \lceil \frac{50}{50} \rceil \cdot 10 + \lceil \frac{50}{80} \rceil \cdot 20 + \lceil \frac{50}{100} \rceil \cdot 10 = 50 + 10 + 20 + 10 = 90$
      iii. $W_2 = 50 + \lceil \frac{90}{50} \rceil \cdot 10 + \lceil \frac{90}{80} \rceil \cdot 20 + \lceil \frac{90}{100} \rceil \cdot 10 = 50 + 20 + 40 + 10 = 120$
      iv. $W_3 = 50 + \lceil \frac{120}{50} \rceil \cdot 10 + \lceil \frac{120}{80} \rceil \cdot 20 + \lceil \frac{120}{100} \rceil \cdot 10 = 50 + 30 + 40 + 20 = 140$
      v. $W_4 = 50 + \lceil \frac{140}{50} \rceil \cdot 10 + \lceil \frac{140}{80} \rceil \cdot 20 + \lceil \frac{140}{100} \rceil \cdot 10 = 50 + 30 + 40 + 20 = 140$
      vi. Fixed point found, $W_4 \leq D_4 \rightarrow$ OK

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**b)** Plot the scheduling of these 4 tasks using RM. Each job takes its worst case execution time (WCET) to end.

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**Correction**

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see Figure 1. All deadlines were met between $0$ and $200$, thus we know that this system is feasible (thanks to the feasibility interval $[0, \max\{D_i | i = 1, \ldots, n\})$)

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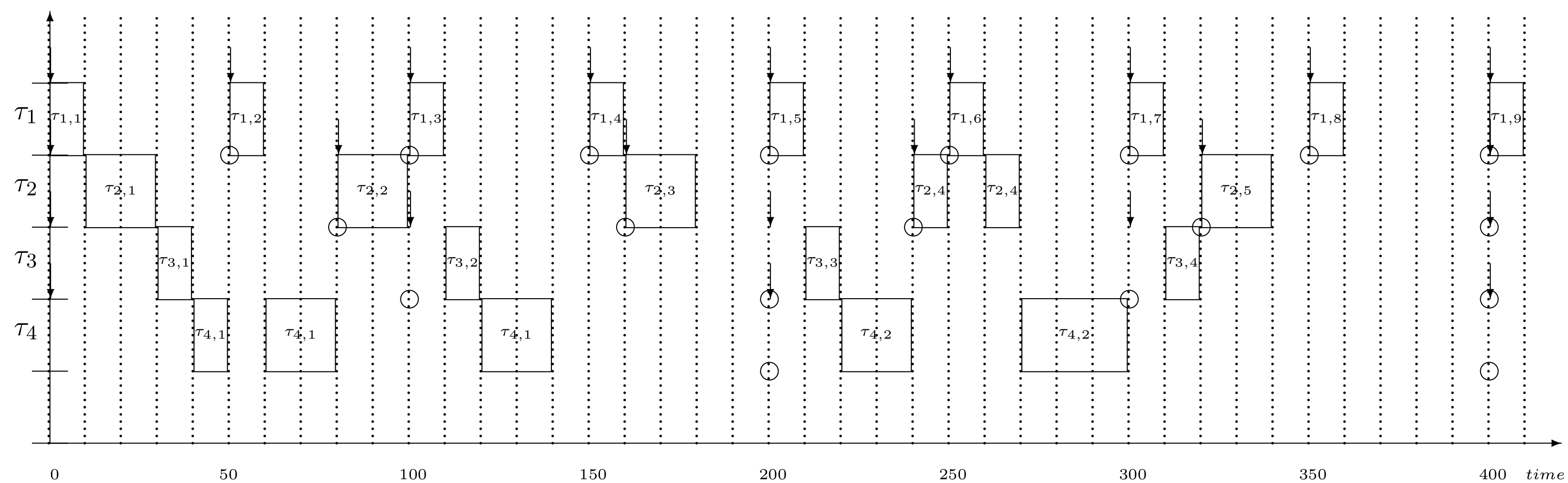


Figure 1: *RM Scheduling.*

**c)** Try to find a periodic system such as its utilization ($U_{total}$) is in range $[0.7; 1]$ and which could not be scheduled using RM algorithm.

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**Correction**

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see Table 2.

| Task index | Release time | WCET | Deadline | Period |
|---|---|---|---|---|
| Tâche $\tau_1$ | 0 | 20 | 40 | 40 |
| Tâche $\tau_2$ | 0 | 30 | 60 | 60 |

Table 2: RM can not schedule this system without missing a deadline.

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### System 2

Let's consider the system represented by Table 3. These are all *periodic*, *synchronous* tasks with *implicit deadline*. We will consider the case where these tasks (and jobs) are *independent* and *preemptible*.

| Task index | Release time | WCET | Deadline | Period |
|---|---|---|---|---|
| Task $\tau_1$ | 0 | 4 | 10 | 10 |
| Task $\tau_2$ | 0 | 3 | 15 | 15 |
| Task $\tau_3$ | 0 | 4 | 20 | 20 |

Table 3: System of 3 periodic, synchronous tasks with implicit deadline.

**a)** Verify that this system could be scheduled using RM algorithm.

1. Try to use the technique based on the processor *utilization*.
2. Try to use the technique based on the *worst response time*.

## 2 Deadline Monotonic

### System 1

Let's consider the system represented by Table 4. These are all *periodic*, *synchronous* tasks with *constrained deadline*. We will consider the case where these tasks (and jobs) are *independent* and *preemptible*.

**a)** Verify that this system could be scheduled using DM algorithm.

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**Correction**

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Since we have a system with constrained deadline, we can not use the method based on the utilization factor. We have to use the second method:

| Task index | Release time | WCET | Deadline | Period |
|---|---|---|---|---|
| Task $\tau_1$ | 0 | 10 | 50 | 50 |
| Task $\tau_2$ | 0 | 20 | 40 | 80 |
| Task $\tau_3$ | 0 | 10 | 30 | 100 |
| Task $\tau_4$ | 0 | 50 | 150 | 200 |

Table 4: System of 4 periodic, synchronous tasks with constrained deadline.

1. Sort tasks by priority (assigned by DM, $p_i = \frac{1}{D_i}$) in decreasing order $\tau_3 > \tau_2 > \tau_1 > \tau_4$.
2. Find the worst response time of the first job of each task :
   (a) $\tau_3$:
      i. $W_0 = 10$
      ii. $W_1 = 10 + 0 = 10$
      iii. Fixed point found, $W_1 \leq D_3 \rightarrow$ OK
   (b) $\tau_2$:
      i. $W_0 = 20$
      ii. $W_1 = 20 + \lceil \frac{20}{100} \rceil \cdot 10 = 20 + 10 = 30$
      iii. $W_2 = 20 + \lceil \frac{30}{100} \rceil \cdot 10 = 20 + 10 = 30$
      iv. Fixed point found, $W_2 \leq D_2 \rightarrow$ OK
   (c) $\tau_1$:
      i. $W_0 = 10$
      ii. $W_1 = 10 + \lceil \frac{10}{100} \rceil \cdot 10 + \lceil \frac{10}{80} \rceil \cdot 20 = 10 + 10 + 20 = 40$
      iii. $W_2 = 10 + \lceil \frac{40}{100} \rceil \cdot 10 + \lceil \frac{40}{80} \rceil \cdot 20 = 10 + 10 + 20 = 40$
      iv. Fixed point found, $W_2 \leq D_1 \rightarrow$ OK
   (d) $\tau_4$:
      i. $W_0 = 50$
      ii. $W_1 = 50 + \lceil \frac{50}{100} \rceil \cdot 10 + \lceil \frac{50}{80} \rceil \cdot 20 + \lceil \frac{50}{50} \rceil \cdot 10 = 50 + 10 + 20 + 10 = 90$
      iii. $W_2 = 50 + \lceil \frac{90}{100} \rceil \cdot 10 + \lceil \frac{90}{80} \rceil \cdot 20 + \lceil \frac{90}{50} \rceil \cdot 10 = 50 + 20 + 40 + 10 = 120$
      iv. $W_3 = 50 + \lceil \frac{120}{100} \rceil \cdot 10 + \lceil \frac{120}{80} \rceil \cdot 20 + \lceil \frac{120}{50} \rceil \cdot 10 = 50 + 30 + 40 + 20 = 140$
      v. $W_3 = 50 + \lceil \frac{140}{100} \rceil \cdot 10 + \lceil \frac{140}{80} \rceil \cdot 20 + \lceil \frac{140}{50} \rceil \cdot 10 = 50 + 30 + 40 + 20 = 140$
      vi. Fixed point found, $W_4 \leq D_4 \rightarrow$ OK

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**b)** Plot the scheduling of these 4 tasks using DM. Each job takes its worst case execution time (WCET) to end.

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**Correction**

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see Figure 2. Once again, all deadlines were met between $0$ and $150$, thus we know that this system is feasible (thanks to the feasibility interval $[0, \max\{D_i | i = 1, \ldots, n\}]$).

**Note**: notice that if a FTP algorithm RM (defined only for systems with implicit deadline), it will assign priorities as follows : $\tau_1 > \tau_2 > \tau_3 > \tau_4$. Once $\tau_1$ and $\tau_2$ are executed (instant $t = 30$) $\tau_3$ misses its deadline.

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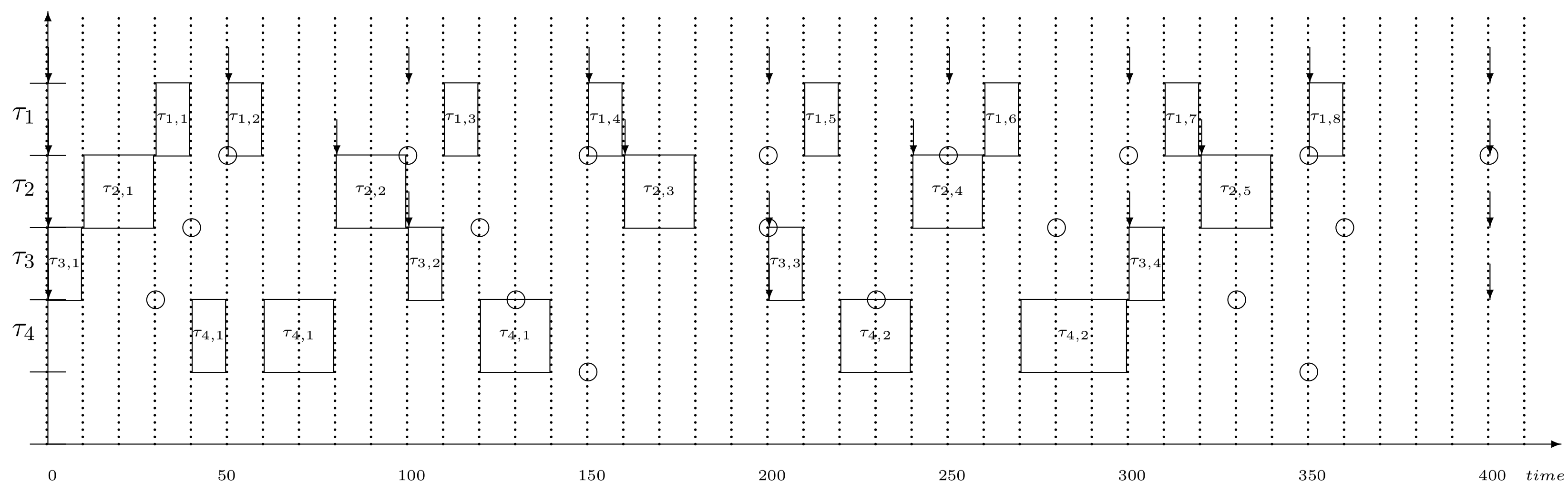


Figure 2: *DM Scheduling.*

### System 2

Let's consider the system represented by Table 5. These are all *periodic*, *synchronous* tasks with *constrained deadline*. We will consider the case where these tasks (and jobs) are *independent* and *preemptible*.

| Task index | Release time | WCET | Deadline | Period |
|---|---|---|---|---|
| Task $\tau_1$ | 0 | 3 | 5 | 5 |
| Task $\tau_2$ | 0 | 2 | 8 | 9 |
| Task $\tau_3$ | 0 | 1 | 4 | 11 |

Table 5: System of 3 periodic, synchronous tasks with constrained deadline.

**a)** Verify that this system could be scheduled using DM algorithm.

**b)** Is there any other algorithm that can successfully schedule $\tau$ by assigning fix priorities to each task?

## 3 Systems with arbitrary deadline

Let's consider the system represented by Table 6. These are all *periodic*, *synchronous* tasks with *arbitrary deadline*. We will consider the case where these tasks (and jobs) are *independent* and *preemptible*.

| Task index | Release time | WCET | Deadline | Period |
|---|---|---|---|---|
| Taske $\tau_1$ | 0 | 10 | 50 | 50 |
| Taske $\tau_2$ | 0 | 20 | 40 | 80 |
| Taske $\tau_3$ | 0 | 10 | 150 | 100 |
| Taske $\tau_4$ | 0 | 50 | 220 | 200 |

Table 6: System of 3 periodic, synchronous tasks with arbitrary deadline.

**a)** Find the feasibility interval for this system.

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**Correction**

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Since we deal with a system with arbitrary deadline we can not use test methods based on the utilization nor the worst response time. It means that we have to simulate the execution of this system to find out if the is a way to assign fix priorities to tasks in order to schedule the system. Thus we have to find the feasibility interval.

For systems with arbitrary deadline the feasibility interval is : $[0, \lambda_n)$, where $\lambda$ is the fixed point of the following equation:

$$\lambda_n = \sum_{i=1}^{n} \left\lceil \frac{\lambda_n}{T_i} \right\rceil \cdot C_i$$

We can calculate $\lambda$ using the following iterative method:

- $W_0 = \sum_{i=1}^{n} C_i$
- $W_{k+1} = \sum_{i=1}^{n} \left\lceil \frac{W_k}{T_i} \right\rceil \cdot C_i$

So for our system described by Table 6 we have:

1. $W_0 = 10 + 20 + 10 + 50 = 90$
2. $W_1 = \lceil \frac{90}{50} \rceil \cdot 10 + \lceil \frac{90}{80} \rceil \cdot 20 + \lceil \frac{90}{100} \rceil \cdot 10 + \lceil \frac{90}{200} \rceil \cdot 50 = 20 + 40 + 10 + 50 = 120$
3. $W_2 = \lceil \frac{120}{50} \rceil \cdot 10 + \lceil \frac{120}{80} \rceil \cdot 20 + \lceil \frac{120}{100} \rceil \cdot 10 + \lceil \frac{120}{200} \rceil \cdot 50 = 30 + 40 + 20 + 50 = 140$
4. $W_3 = \lceil \frac{140}{50} \rceil \cdot 10 + \lceil \frac{140}{80} \rceil \cdot 20 + \lceil \frac{140}{100} \rceil \cdot 10 + \lceil \frac{140}{200} \rceil \cdot 50 = 30 + 40 + 20 + 50 = 140$
5. Fixed-point is found, the feasibility interval is: $[0, W_3) = [0, 140)$

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# 4 Audsley

Let's consider the system represented by Table 7. These are all *periodic*, *asynchronous* tasks with *constrained deadline*. We will consider the case where these tasks (and jobs) are *independent* and *preemptible*.

| Task index | Release time | WCET | Deadline | Period |
|---|---|---|---|---|
| Task $\tau_1$ | 100 | 10 | 20 | 30 |
| Task $\tau_2$ | 50 | 20 | 50 | 50 |
| Task $\tau_3$ | 0 | 30 | 100 | 150 |

Table 7: System of 3 periodic, asynchronous tasks with constrained deadline.

**a)** Find the study interval, use the expression $[O_{\max}, O_{\max} + 2 \cdot P]$.

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**Correction**

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First we have to find $P$ and $O_{\max}$, they are given by formulas:

$$P = lcm\,\{T_i \mid i = 1, \ldots, n\}$$

$$O_{\max} = \max\{O_i \mid i = 1, \ldots, n\}$$

So, we have : $P = lcm(30, 50, 150) = 150$ and $O_{\max} = \max(100, 50, 0) = 100$

Now we can find the study interval : $[100, 100 + 2 \cdot 150] = [100, 400]$

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**b)** Plot the scheduling of these 3 tasks in the interval $[0, 400]$ using Audsley. Each job takes its worst case execution time (WCET) to end.

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**Correction**

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$\tau_1 > \tau_2 > \tau_3$.

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**c)** Find the study interval, use the expression $[0, S_n + P]$.

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**Correction**

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In order to find the interval $[0, S_n + P)$ we have to find $S_n$ (from previous exercise we already know that $P = 150$). $S_n$ can be calculated using the following expression (Tasks are sorted by priority):

$S_1 = O_1$
$S_i = \left\{ O_i + \left\lceil \frac{(S_{i-1} - O_i)^+}{T_i} \right\rceil \cdot T_i \right\}$ for $i = 2, \ldots, n$
$(x)^+ = max(x, 0)$

In our case we have $\tau_1 > \tau_2 > \tau_3$, so :

1. $S_1 = O_1 = 100.$
2. $S_2 = O_2 + \lceil \frac{S_1 - O_2}{T_2} \rceil \cdot T_2 = 50 + 1 \cdot 50 = 100.$
3. $S_3 = O_3 + \lceil \frac{S_2 - O_3}{T_3} \rceil \cdot T_3 = 0 + 1 \cdot 150 = 150.$

Now we can find the interval : $[0, 150 + 150]$ = $[0, 300]$.

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# 5 Earliest Deadline First

Let's consider the system represented by Table 8. These are all *periodic*, *synchronous* tasks with *constrained deadline*. We will consider the case where these tasks (and jobs) are *independent* and *preemptible*.

| Task index | Release time | WCET | Deadline | Period |
|---|---|---|---|---|
| Task $\tau_1$ | 0 | 10 | 50 | 50 |
| Task $\tau_2$ | 0 | 20 | 40 | 80 |
| Task $\tau_3$ | 0 | 10 | 30 | 100 |
| Task $\tau_4$ | 0 | 50 | 150 | 200 |

Table 8: System of 4 periodic, synchronous tasks with constrained deadline.

**a)** Find the study interval for this system (for EDF algorithm).

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**Correction**

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The study interval for EDF is $[0, L)$, where $L$ is given by :

$$L = \sum_{i=1}^{n} \left\lceil \frac{L}{T_i} \right\rceil \cdot C_i$$

$L$ can be calculated using the following iterative approach:

$W_0 = \sum_{i=1}^{n} C_i$
$W_{k+1} = \sum_{i=1}^{n} \left\lceil \frac{W_k}{T_i} \right\rceil \cdot C_i$

In our case we have :

1. $W_0 = 10 + 20 + 10 + 50 = 90,$
2. $W_1 = \lceil \frac{90}{50} \rceil \cdot 10 + \lceil \frac{90}{80} \rceil \cdot 20 + \lceil \frac{90}{100} \rceil \cdot 10 + \lceil \frac{90}{200} \rceil \cdot 50 = 120,$

3. $W_2 = \lceil \frac{120}{50} \rceil \cdot 10 + \lceil \frac{120}{80} \rceil \cdot 20 + \lceil \frac{120}{100} \rceil \cdot 10 + \lceil \frac{120}{200} \rceil \cdot 50 = 140,$
4. $W_3 = \lceil \frac{140}{50} \rceil \cdot 10 + \lceil \frac{140}{80} \rceil \cdot 20 + \lceil \frac{140}{100} \rceil \cdot 10 + \lceil \frac{140}{200} \rceil \cdot 50 = 140$

The fixed point is found, now we have our interval: $[0, 140)$

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**b)** Plot the scheduling of these 3 tasks in the interval $[0, 400]$ using EDF. Each job takes its worst case execution time (WCET) to end.

**c)** Find a system of periodic tasks that could be scheduled using EDF, but not using DM.

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**Correction**

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see Table 9

| Task index | Release time | WCET | Deadline | Period |
|---|---|---|---|---|
| Task $\tau_1$ | 0 | 50 | 100 | 100 |
| Task $\tau_2$ | 0 | 50 | 130 | 200 |
| Task $\tau_3$ | 0 | from 1 to 50 | 150 | 200 |

Table 9: Systeme schedulable by EDF but not by DM.

If DM is used, the task $\tau_3$ misses its deadline at $t = 150$.

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# 6 Least Laxity First

Let's consider the system represented by Table 10. These are all *periodic*, *synchronous* tasks with *constrained deadline*. We will consider the case where these tasks (and jobs) are *independent* and *preemptible*.

| Task index | Release time | WCET | Deadline | Period |
|---|---|---|---|---|
| Task $\tau_1$ | 0 | 10 | 50 | 50 |
| Task $\tau_2$ | 0 | 20 | 40 | 80 |
| Task $\tau_3$ | 0 | 10 | 30 | 100 |
| Task $\tau_4$ | 0 | 50 | 150 | 200 |

Table 10: System of 4 periodic, synchronous tasks with constrained deadline.

**a)** Plot the scheduling of these 4 tasks in the interval $[0, 200]$ using LLF. Consider the case when all priorities are recalculated every 10 time units. Each job takes its worst case execution time (WCET) to end.