A) Compute the DH parameters and setup the parameters table. Derive the kinematic map A(d1, θ2) . What is the corresponding workspace? Write it mathematically.

Configuration space:

$$d_1 \in [-1{,}0], \theta_2 \in [0{,}2\pi]$$

$$C = C_1 \times C_2$$

DH Parameters:

link	θ_i	d_i	a_i	α_i
1	0	$l_1 + d_1$	0	0
2	θ_2	0	l_2	0

Transformation matrix:

$$A_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & l_1 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & l_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A(d_1, \theta_2\,) = A_1.A_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & l_2\sin(\theta_2) \\ 0 & 0 & 1 & l_1 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Workspace:

$$W = \left\{ o_0 + \begin{bmatrix} l_2\cos(\theta_2) \\ l_2\sin(\theta_2) \\ l_1 + d_1 \end{bmatrix}, \begin{bmatrix} d_1 \\ \theta_2 \end{bmatrix} \in C \right\}$$

B1) Consider two different sets of values of d∗1, θ∗2. For each, print the corresponding kinematic transformation matrix A and verify that it is identical to that computed manually.

$$l_1 = 2.5, l_2 = 1$$

1- For $\theta = 30°, d = -0.5$:

$$A(d_1, \theta_2) = A_1.A_2 = \begin{bmatrix} 0.866 & -0.5 & 0 & 0.866 \\ 0.5 & 0.866 & 0 & 0.5 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2- For $\theta = 180°, d = -1$:

$$A(d_1, \theta_2) = A_1.A_2 = \begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is same with the result of ROS except the coordinates x and y because ROS doesn't use DH convention to set up the axis so, x and y are changed at the results of ROS and there is a minus factor on x coordinate.

B2- Now consider o2- the origin of frame F2 associated with link 2. For each sets of values of d∗1, θ∗2, add code to compute and print the corresponding workspace point o0

$$O^0 = A.O^2 = A.\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} l_2\cos(\theta_2) \\ l_2\sin(\theta_2) \\ l_1 + d_1 \\ 1 \end{bmatrix}$$

1- For $\theta = 30°, d = -0.5$:

$$O^0 = \begin{bmatrix} 0.866 \\ 0.5 \\ 2 \\ 1 \end{bmatrix}$$

2- For $\theta = 180°, d = -1$:

$$O^0 = \begin{bmatrix} -1 \\ 0 \\ 1.5 \\ 1 \end{bmatrix}$$

B3- Now consider o0 as found in previous part. First, manually set up the inverse kinematics equations and solve and print d1, θ2. Compare them with the original pair d∗1, θ∗2. Now use moveit inverse kinematics api to solve and print d1, θ2 and again compare them with the original pair.

1-

$$O^0 = \begin{bmatrix} 0.866 \\ 0.5 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} l_2\cos(\theta_2) \\ l_2\sin(\theta_2) \\ l_1 + d_1 \\ 1 \end{bmatrix}$$

It is obvious that $\theta = 30°, d = -0.5$

2-

$$O^0 = \begin{bmatrix} -1 \\ 0 \\ 1.5 \\ 1 \end{bmatrix} = \begin{bmatrix} l_2\cos(\theta_2) \\ l_2\sin(\theta_2) \\ l_1 + d_1 \\ 1 \end{bmatrix}$$

It is obvious that $\theta = 180°, d = -1$

ROS output also gives the same results.

B4- Finally, write code so that your robot moves through the workspace in a continual manner. Inspecting this workspace, describe the kind of tasks where this type of robot could be used and could not be used.

Workspace is a cylindrical surface so we can do the work that needs to be done in cylindrical surfaces such as carrying objects and pick and place. However, we can’t do transportation since it is not mobile nor the assembly operations.