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Project1 Phase3

P3.1(a):

Two noisy images are generated using the following formula with variance .2 and mean 0.

NoisyImage=OriginalImage + sqrt(Variance)*randn(256,256) + Mean

first image

second image

P3.1(b):

The noise can be reduced if we take average of above images. The derivation is as follows

Let we produce two noisy images $N_1(x,y), N_2(x,y)$
Adding noise to the image.

$$Img_1(x,y) = f(x,y) + N_1(x,y)$$

$$Img_2(x,y) = f(x,y) + N_2(x,y)$$

As the images are not related so their covariance is equal to zero

$$covar = E((Img_1 - \overline{Img_1})(Img_2 - \overline{Img_2})) = 0 \text{ —— } (A)$$

The avg of two images is given by

$$Img_{avg}(x,y) = \frac{1}{2}(Img_1avg(x,y) + Img_2avg(x,y))$$

The avg variance is given by

$$\sigma^2_{avg} = E(Img_{avg} - \overline{Img_{avg}})^2$$

$$= E\left[\left(\frac{Img_1 + Img_2}{2}\right) - \left(\frac{\overline{Img_1} + \overline{Img_2}}{2}\right)\right]^2$$

$$= E\left[\frac{1}{4}\left((Img_1 - \overline{Img_1}) + (Img_2 - \overline{Img_2})\right)^2\right]$$

$$= E\left[\frac{1}{4}(Img_1 - \overline{Img_1})^2 + (Img_2 - \overline{Img_2})^2 + 2(Img_1 - \overline{Img_1})(Img_2 - \overline{Img_2})\right]$$

$$= \frac{1}{4}\left[E(Img_1 - \overline{Img_1})^2 + E(Img_2 - \overline{Img_2})^2 + \underbrace{2E(Img_1 - \overline{Img_1})(Img_2 - \overline{Img_2})}_{\text{Covar}=0 \text{ From } (A)}\right]$$

$$= \frac{1}{4}(\sigma_1^2 + \sigma_2^2 + 0)$$

As both images have variance nearly equal to 0.2

$$= \frac{1}{4}(\sigma^2 + \sigma^2) = \frac{1}{4}(2\sigma^2) = \frac{1}{2}(\sigma^2)$$

P3.1(c):

After taking the average of above two images, the obtained image has less noise.

P3.2(a,b):

The equation to stretch the histogram of noisy image and the relation between variance of noisy image and image after stretching is given below.

Let we have a noisy image f_n

After stretching f_n we get new image g_n given by

(1)— $g_n = \frac{f_n - T_2}{T_1 - T_2} \times 255$ (As the image range is T_1 and T_2)

The variance of g_n is given by

$$\sigma_g^2 = E(g_n - \overline{g_n})$$

Putting g_n from (1)

$$\sigma_g^2 = E\left(\frac{f_n - T_2}{T_1 - T_2} \times 255 - \frac{\overline{f_n} - T_2}{T_1 - T_2}^{255}\right)$$

$$= 255\, E\left(\frac{f_n - \cancel{T_2} - \overline{f_n} + \cancel{T_2}}{T_1 - T_2}\right)$$

$$= \frac{255}{T_1 - T_2} E(f_n - \overline{f_n})$$

$$\boxed{\sigma_g^2 = \frac{255}{T_1 - T_2} \sigma_f^2}$$

P3.3:

The median filter is applied and variance is calculated which is less than the noisy image. After applying filter, the image is as follows.