Quantitative Trading

Empirical Properties of Financial Time Series

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8th January 2026

Outline

- **Distributional Properties**
  - Heavy Tails, Non-Gaussianity, and Power-Laws
  - Statistical and Economic Consequences of Tail Risk
- **Temporal Dynamics**
  - Volatility Clustering and Persistence
  - Non-Stationarity and Structural Breaks
- **Asymmetry and Discontinuities**
  - Jumps, Leverage Effects, and Nonlinear Dynamics
  - Path Dependence and Memory Effects
- **Market Microstructure and Interconnectivity**
  - Microstructure Noise and Frictions
  - Cross-Asset Spillovers and Contagion
- **Modelling Implications**
  - Transitioning from Classical to Robust Frameworks

Course Reference:
Futuretesting Quantitative Strategies
`http://ssrn.com/abstract=4647103`

Characteristics of Financial Time Series

- Financial time series exhibit a rich set of empirical regularities that sharply contradict the assumptions of classical statistical models such as independence, normality, and stationarity.
- These recurring empirical patterns, known as *stylised facts*, are remarkably robust: they appear across asset classes, markets, geographical regions, and time horizons.
- Stylised facts are not modelling artefacts; they are structural features of financial markets that any realistic model must confront.
- Understanding these characteristics is essential for:
  - building credible forecasting and trading models,
  - designing robust risk and stress-testing frameworks,
  - interpreting econometric results in a meaningful way.

Goal of this course: to connect empirical market behaviour with appropriate statistical, econometric, and mathematical modelling choices.

Heavy Tails and Non-Gaussianity

Empirical departure from normality

- Financial returns exhibit systematic deviations from the Gaussian distribution.
- Extreme events occur far more frequently than predicted by normal models — a phenomenon known as *heavy* or *fat tails*.
- Both large negative returns (crashes) and large positive returns (rallies) are observed with unexpectedly high probability.
- Empirical return distributions display pronounced excess kurtosis and often skewness.

Key implication: Gaussian models severely underestimate the probability and impact of extreme market movements, leading to systematic underestimation of financial risk.

Empirical Evidence and Power-Law Tails

Empirical studies across asset classes (equities, FX, commodities, rates) and markets show that the tails of return distributions decay much more slowly than in the Gaussian case. In particular, tail probabilities follow a power-law behaviour:

$$\mathbb{P}(|X_t| > x) \sim C\,x^{-\alpha}, \qquad \alpha \in (2, 4),$$

where α is known as the *tail index* and $C > 0$ is a constant.

- Power-law decay implies a significantly higher probability of extreme returns than exponential (Gaussian) tails.
- Empirically, $\alpha \approx 3$ are frequently observed for daily returns.
- For $2 < \alpha < 4$, variance is finite, but higher moments such as kurtosis are theoretically infinite or highly unstable.
- Tail behaviour is remarkably stable across markets, even when central distributions differ.

Interpretation: extreme events are not anomalies - they are an intrinsic feature of financial markets.

Statistical Implications of Heavy Tails

The presence of heavy tails has profound consequences for statistical modelling and inference in finance:

- **Unstable moments:** If $\alpha \leq 4$, the fourth moment (kurtosis) is theoretically infinite; if $\alpha \leq 2$, even the variance does not exist. This undermines estimators and hypothesis tests that rely on finite moments.
- **Sensitivity to outliers:** Extreme observations occur frequently and exert disproportionate influence on classical estimators such as the sample mean, variance, and covariance.
- **Breakdown of classical limit theorems:** For $\alpha < 2$, sums of returns converge to a stable (Lévy α-stable) distribution rather than a normal distribution, invalidating standard Central Limit Theorem arguments.

Statistical Implications of Heavy Tails

- **Slow convergence:** Even when variance exists, convergence to asymptotic distributions can be extremely slow, making large-sample approximations unreliable in practice.

Consequence: robust and tail-aware statistical tools are essential for estimation, inference, and risk measurement in financial applications.

Economic and Practical Consequences

Assuming normality when modelling financial returns leads to a systematic underestimation of extreme risk, with significant economic consequences:

- **Risk management failures:** Gaussian-based Value-at-Risk (VaR) severely underestimates the frequency and magnitude of large losses, particularly during market stress.
- **Option mispricing:** Models such as Black–Scholes assign negligible probability to large price moves, leading to underpriced deep out-of-the-money options and inadequate hedging strategies.
- **Misestimated risk premia:** Investors demand compensation for tail risk; models that ignore heavy tails fail to capture this premium, distorting expected return estimates.

Economic and Practical Consequences

- **Systemic risk amplification:** Underestimation of tail risk contributes to insufficient capital buffers, increasing the likelihood of contagion and financial instability during crises.

Lesson: incorrect distributional assumptions are not innocuous — they directly affect pricing, capital allocation, and financial stability.

Modelling Heavy Tails

To address the empirical failure of Gaussian models, several alternative modelling frameworks have been developed:

- **Stable and Lévy processes:** Generalisations of Brownian motion that allow for heavy tails and, in some cases, infinite variance. These models capture extreme behaviour but pose challenges for calibration and interpretation.
- **Flexible parametric distributions:** Student-t, Generalised Hyperbolic, and Normal Inverse Gaussian distributions capture excess kurtosis while often retaining finite variance.
- **Extreme Value Theory (EVT):** Focuses explicitly on the statistical behaviour of extremes and provides theoretically grounded tools for tail risk estimation (e.g., tail indices, extreme quantiles).

Modelling Heavy Tails

- **Jump-based models:** Jump-diffusion and pure-jump processes introduce discontinuities to capture rare but impactful events.
- **Data-driven approaches:** Machine learning models (e.g. GANs, normalising flows, variational autoencoders) can learn complex, non-Gaussian return distributions without strong parametric assumptions.

Trade-off: richer tail behaviour improves realism but often increases model complexity and estimation risk.

Volatility Clustering and Persistence

Financial markets exhibit strong temporal dependence in volatility: periods of high volatility tend to be followed by high volatility, and tranquil periods by tranquil periods. This phenomenon is known as *volatility clustering*.

- Returns r_t are often close to serially uncorrelated, consistent with weak-form market efficiency.
- In contrast, measures of volatility such as $|r_t|$ or r_t^2 exhibit significant positive autocorrelation.
- Formally, for small lags h:

$$\mathrm{Cov}(r_t^2, r_{t+h}^2) > 0.$$

- Volatility shocks decay slowly over time, indicating strong persistence and long memory effects.

Interpretation: volatility is predictable even when returns are not, motivating ARCH/GARCH and stochastic volatility models.

Non-Stationarity and Structural Breaks

Financial time series are inherently non-stationary: their statistical properties evolve over time in response to economic, institutional, and behavioural forces.

- Key quantities such as the mean return, volatility, correlations, and tail behaviour are time-varying.
- Regime changes may be driven by macroeconomic cycles, monetary policy shifts, liquidity conditions, or technological and regulatory changes.
- Standard econometric methods assuming time-invariant parameters can produce misleading inference and unstable forecasts.

Non-Stationarity and Structural Breaks

Structural breaks

Structural breaks correspond to abrupt and persistent changes in the data-generating process, for example:

- central bank interventions or policy regime shifts,
- financial crises and systemic events,
- changes in market microstructure or trading rules.

Implication: models must allow for regime dependence, parameter instability, or adaptive learning.

Jumps and Discontinuities

Empirical asset price dynamics exhibit sudden and discrete movements that cannot be explained by continuous diffusion models alone.

- Jumps arise from earnings announcements, macroeconomic releases, geopolitical events, regulatory actions, or liquidity shocks.
- Such events generate abrupt price changes that dominate short-horizon risk and contribute disproportionately to tail behaviour.
- Ignoring jumps leads to systematic underestimation of both volatility and tail risk.

Jumps and Discontinuities

A commonly used representation is the jump–diffusion model:

$$dX_t = \mu\, dt + \sigma\, dW_t + J_t\, dN_t,$$

where W_t is Brownian motion, N_t is a Poisson process, and J_t denotes the random jump size.

Key insight: in many markets, jump risk rather than diffusive risk drives extreme losses and option prices.

Leverage Effects and Asymmetric Volatility

Empirical evidence shows that negative returns tend to increase future volatility more than positive returns of the same magnitude. This phenomenon is known as the *leverage effect*.

- After a price decline, firms become more highly leveraged, increasing perceived risk and future volatility.
- Negative shocks trigger stronger behavioural and institutional responses, such as risk aversion, margin calls, and forced deleveraging.
- Volatility responses are therefore asymmetric with respect to the sign of returns.

Modelling implications:

- Symmetric volatility models fail to capture observed dynamics.
- Asymmetric GARCH models (EGARCH, GJR-GARCH) and stochastic volatility models with leverage terms are empirically superior.

Autocorrelation in Higher Moments

While raw asset returns typically exhibit little to no linear autocorrelation, higher moments display strong and persistent time dependence.

- Autocorrelation of r_t is often close to zero, consistent with arbitrage and informational efficiency.
- In contrast, volatility-related quantities such as $|r_t|$, r_t^2, and implied volatility are highly autocorrelated.
- Time variation is also observed in skewness and kurtosis, especially during periods of market stress.

Implication:

- Volatility and higher moments are forecastable even when returns are not.
- This motivates stochastic volatility models, GARCH-type models, and frameworks with time-varying higher moments.

Path Dependence and Memory Effects

Financial time series often exhibit path dependence: the impact of a new shock depends on the recent history of the market.

- The same return shock may have different consequences depending on whether the market is trending, range-bound, or recovering from a drawdown.
- Volatility dynamics depend on the sequence and clustering of past shocks, not only on the current state.
- Empirical evidence suggests the presence of long memory in volatility and trading activity.

Modelling consequences:

- Purely Markovian models with short memory may fail to capture observed dynamics.
- Models with long memory, regime dependence, or state augmentation are often required.

Asymmetry and Nonlinear Dynamics

Financial markets respond to shocks in a nonlinear and asymmetric manner: identical shocks of opposite sign often generate very different dynamics.

- A 1% price drop typically has a stronger impact on volatility, liquidity, and risk perception than a 1% price increase.
- Investor behaviour is often convex in losses due to risk aversion, constraints, and behavioural biases.
- Feedback mechanisms can amplify shocks, producing volatility bursts, cascades, and regime shifts.

Implications for modelling:

- Linear models fail to capture asymmetric responses and feedback effects.
- Nonlinear models (threshold models, regime-switching models, nonlinear GARCH) are empirically more realistic.

Market Microstructure Noise and Frictions

At high frequencies, observed asset prices are contaminated by market microstructure effects that obscure the underlying economic signal.

- Bid–ask bounce, discrete pricing, order book dynamics, and latency effects introduce noise into high-frequency returns.
- Transaction prices may deviate from the latent efficient price due to liquidity and execution constraints.
- As sampling frequency increases, noise can dominate the true return signal.

Consequences:

- Naively computed high-frequency returns are biased and exhibit spurious autocorrelation.
- Specialised models and estimators are required (noise-robust volatility estimators, subsampling, state-space models).

Key lesson: more data does not necessarily mean more information.

Market Microstructure Noise and Frictions

At high sampling frequencies, observed asset prices are contaminated by market microstructure effects that distort true price dynamics.

- Bid–ask bounce, discreteness of prices, and order-processing delays introduce spurious volatility.
- Latency, asynchronous trading, and order book dynamics generate noise unrelated to fundamental value changes.
- At very high frequencies, noise often dominates the true price signal.

Consequences:

- Naively computed high-frequency returns can be misleading.
- Standard estimators of volatility and correlation become biased.
- Specialised models and estimators (e.g. realised volatility with noise correction) are required.

Cross-Asset Spillovers and Contagion

Financial markets are interconnected, and during periods of stress shocks often propagate across assets, sectors, and geographical regions.

- Dependencies across assets strengthen in crises, undermining diversification benefits.
- Correlations are time-varying and tend to increase sharply during market downturns.
- Extreme losses often occur simultaneously across markets, indicating strong tail dependence.

Cross-Asset Spillovers and Contagion

Modelling challenges:

- Constant-correlation models fail precisely when they matter most.
- Dynamic correlation models (e.g. DCC-GARCH) capture time variation but may miss tail dependence.
- Copula-based and multivariate extreme value models explicitly address joint tail risk.

Key risk insight: diversification breaks down in crises, not in normal times.

Implications for Financial Modelling

The empirical characteristics of financial time series pose fundamental challenges to classical econometric and statistical approaches.

- Assumptions of normality, linearity, independence, and stationarity are systematically violated in financial data.
- Standard performance metrics (e.g. MSE, correlation) often fail to capture economically relevant risks embedded in volatility clustering, jumps, and tail events.
- Models that fit average behaviour well may perform disastrously during periods of stress, precisely when risk management matters most.

Implications for Financial Modelling

Consequences:

- Financial models must be statistically coherent *and* economically meaningful.
- Tail risk, regime changes, and nonlinear dynamics must be treated as core features, not secondary corrections.
- Modern approaches combine classical stochastic models with regime-switching, jump components, and data-driven methods capable of capturing memory and structural change.

Central message: understanding the empirical structure of financial time series is a prerequisite for sound modelling, pricing, and risk management.

The end

Thank You !