Quantitative Trading Module

Mathematical Structure and Limitations of Financial Time Series

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Outline

- **Foundations of Stationarity**
  - Strict vs. Weak Stationarity
  - Financial Reality: Volatility Clustering and Regime Changes
- **Long Memory and Rough Volatility**
  - Power-law Kernels and Decay Rates
  - Fractional Brownian Motion and Hurst Parameters
- **Jump Dynamics and Path Regularity**
  - Modelling Jumps in Jump-Diffusion Frameworks
  - Hölder Regularity and Path Roughness
- **Heavy Tails and Statistical Limitations**
  - Power-law Exponents and Distributional Tails
  - Failure of Classical Limit Theorems and Gaussian Assumptions

Course Reference:
Futuretesting Quantitative Strategies
`http://ssrn.com/abstract=4647103`

Mathematical Properties of Asset Prices: Why Classical Statistics Fail in Markets

Statistical Properties and Probability

In the context of stochastic processes, **statistical properties** are defined entirely by the underlying **probability laws**.

- **Identity via Distribution:** All statistical measures (mean, variance, higher moments, etc.) are simply functional summaries of the **Joint Cumulative Distribution Function (CDF)**.
- **The Core Link:** If the joint distributions are identical, then any property derived from them—no matter how complex—must also be identical.
- **Conclusion:** For a strictly stationary process, 'statistical invariance' is a direct and total consequence of 'probabilistic invariance'.

Characterising a Stochastic Process

A collection of moments or statistical summaries is generally **not sufficient** to fully characterise a process due to the following constraints:

- **The Moment Problem:** A distribution is only uniquely determined by its moments if the distribution is 'well-behaved' (e.g., satisfying Carleman's condition). Some distinct distributions share identical moments.
- **Joint Distributions:** To fully characterize a process, one must know the **complete joint probability distribution** (Joint CDFs) for all possible finite collections of times $\{t_1, \ldots, t_n\}$.
- **The Gaussian Exception:** Statistical summaries like mean and covariance describe the process but do not 'define' it, **unless** the process is Gaussian. In the Gaussian case, the first two moments uniquely determine all joint distributions.

Expected Signature and Characterisation

The **expected signature** acts as a sequence of generalised moments for a stochastic process X.

- **Theorem (Chevyrev & Oberhauser):** Under certain regularity conditions (e.g., boundedness or decay conditions on the moments), the expected signature $\Phi(X) = E[S(X)_{0,T}]$ uniquely characterises the **Law** of the process.
- **Inclusion of Order:** This characterization requires *all* terms of the expected signature, including both the symmetric and anti-symmetric (Lévy area) parts.
- **Significance:** This allows us to represent a complex probability measure on path space as a single element in the tensor algebra.

Strict Stationarity and Moments

A stochastic process $(X_t)_{t \in \mathbb{R}}$ is strictly stationary if for any $n \in \mathbb{N}$ and $h \in \mathbb{R}$:

$$P(X_{t_1} \leq x_1, \ldots, X_{t_n} \leq x_n) = P(X_{t_1+h} \leq x_1, \ldots, X_{t_n+h} \leq x_n)$$

Implications for Moments:

- Since the joint distributions are identical, **all existing moments** and joint moments are invariant under time shifts.
- **First Moment:** $E[X_t] = \mu$ (constant for all t).
- **Second Moment:** $Var(X_t) = \sigma^2$ and $Cov(X_t, X_{t+\tau}) = \gamma(\tau)$ (depends only on the lag τ).
- This extends to all higher-order moments (e.g., skewness, kurtosis) provided they are mathematically finite.

Stationarity: Mathematical Foundations

Strict and Weak Stationarity

- A stochastic process $(X_t)_{t\in\mathbb{R}}$ is **strictly stationary** if all its finite-dimensional distributions are invariant under time shifts.
- Formally, for any $n \in \mathbb{N}$, any collection of times $\{t_1, \ldots, t_n\}$, and any $h \in \mathbb{R}$:

$$(X_{t_1}, \ldots, X_{t_n}) \stackrel{d}{=} (X_{t_1+h}, \ldots, X_{t_n+h})$$

- **Weak (second-order) stationarity** requires only:
  - Constant mean: $\mathbb{E}[X_t] = \mu$
  - Lag-dependent autocovariance:

$$\mathrm{Cov}(X_t, X_{t+h}) = \gamma(h)$$

Weak stationarity underpins most classical time-series models (ARMA, regression, correlation), yet is already a strong assumption in financial markets.

Stationarity: Financial Reality

Why Markets Are Not Stationary

- Financial markets violate the assumptions required for both strict and weak stationarity.
- Even if returns appear weakly stationary over short horizons, key empirical features break time-invariance:
  - Volatility clustering
  - Regime changes
  - Market microstructure evolution
  - Macroeconomic and policy interventions

Stationarity: Financial Reality

Why Markets Are Not Stationary

- Let log-returns be defined as:

$$r_t = \log\left(\frac{P_t}{P_{t-1}}\right)$$

While $\mathbb{E}[r_t] \approx 0$ locally, the conditional variance satisfies:

$$\mathbb{E}[r_t^2 \mid \mathscr{F}_{t-1}] = \sigma_t^2$$

which is time-varying.

This conditional heteroskedasticity directly contradicts second-order stationarity and motivates models such as GARCH, stochastic volatility, and regime-switching frameworks.

Stationarity: Market regimes

Structural non-stationarity:

- Financial markets transition between distinct regimes, such as low-volatility bull markets and high-volatility crisis periods. These regime changes induce *structural breaks*, violating stationarity.
- Mathematically, a financial time series X_t can be represented as:

$$X_t = \mu_t + \epsilon_t,$$

  where both the mean μ_t and the distribution of the innovations ϵ_t evolve over time.

Long-Memory Processes

A stochastic process (X_t) exhibits **long memory** if its autocorrelations $\rho(k)$ decay hyperbolically rather than exponentially, such that $\sum_{k=0}^{\infty} |\rho(k)| = \infty$.

- **Fractional Integration:** This behaviour is formally captured by the **ARFIMA**$(0, d, 0)$ model:

$$(1 - L)^d X_t = \epsilon_t$$

  L : lag operator, d : fractional differencing parameter.
- **The Operator:** For non-integer d, the operator $(1 - L)^d$ is defined via the binomial expansion:

$$(1 - L)^d = \sum_{k=0}^{\infty} \binom{d}{k} (-L)^k = 1 - dL + \frac{d(d-1)}{2!} L^2 - \dots$$

- **Significance:** This expansion shows that X_t depends on an infinite history of past innovations, creating 'long-range' persistence that traditional ARMA models cannot replicate.

Stationarity: Market regimes

Structural non-stationarity:

- Even models designed to capture persistence, such as long-memory or fractionally integrated processes,

$$(1 - L)^d X_t = \epsilon_t, \qquad d \in (0, 0.5),$$

  only model slowly decaying autocorrelations.

- These models still rely on an implicit stationarity assumption for the innovations ϵ_t, and therefore do not resolve the fundamental non-stationarity induced by regime shifts.

From Kernels to Power-Law Decay

The choice of the kernel $G(t-s) = \frac{1}{(c+t-s)^\alpha}$ is fundamental for modelling **long-range dependence** due to its asymptotic properties.

- **Algebraic vs. Exponential Decay:** Unlike standard ARMA or Markovian models where correlations decay exponentially ($e^{-\lambda|t-s|}$), this kernel decays **algebraically** (as a power of the lag).
- **Asymptotic Behaviour:** For large lags $\tau = t - s \gg c$:

  $$G(\tau) \approx \tau^{-\alpha}$$

  The constant $c > 0$ ensures the kernel remains finite (non-singular) as the lag approaches zero.
- **Long Memory Property:** When $0 < \alpha < 1$, the kernel is not integrable over $[0, \infty)$, i.e., $\int_0^\infty G(\tau)d\tau = \infty$. This lead to a 'long memory' effect where past shocks exert a persistent, non-negligible influence on the current state.

Long Memory Models: Power-law kernels

- Consider the kernel

$$G(t-s) = \frac{1}{(c+t-s)^\alpha}, \qquad c > 0,$$

  which exhibits **power-law decay** in the time lag $t-s$.
- The parameter $\alpha > 0$ controls the decay rate:
  - small $\alpha \Rightarrow$ slow decay, long memory,
  - large $\alpha \Rightarrow$ fast decay, short memory.
- In rough volatility models (e.g. the rough Bergomi model), a closely related kernel appears:

$$\int_0^t \frac{1}{(t-s)^\alpha}\, dW_s,$$

  where $\alpha = H + \frac{1}{2}$ and H denotes the Hurst parameter.

Interpretation and implications of long memory

- Power-law kernels induce **long-range dependence**: shocks to volatility decay slowly and remain influential far into the future.
- This behaviour contrasts sharply with exponential kernels (e.g. Ornstein–Uhlenbeck dynamics), where dependence vanishes rapidly.
- Empirically, long memory provides a compelling explanation for:
  - persistent volatility clustering,
  - slow decay of autocorrelations in squared returns,
  - the observed roughness of volatility paths.
- From a modelling perspective, long-memory processes:
  - break the Markov property,
  - invalidate many classical filtering and calibration techniques,
  - require fractional calculus or non-Markovian simulation methods.
- These features align closely with empirical findings across equities, FX, and rates, particularly at high frequencies.

Fractional Brownian motion and roughness

- A central object in long-memory modelling is **fractional Brownian motion** (fBm), denoted $(B_t^H)_{t\geq 0}$, indexed by the **Hurst parameter** $H \in (0,1)$.
- fBm is a zero-mean Gaussian process with covariance

$$\mathbb{E}[B_t^H B_s^H] = \frac{1}{2}\left(t^{2H} + s^{2H} - |t-s|^{2H}\right).$$

- The value of H governs memory and path regularity:
  - $H = \frac{1}{2}$: standard Brownian motion (no memory),
  - $H > \frac{1}{2}$: persistent, long-range dependence,
  - $H < \frac{1}{2}$: anti-persistent behaviour and **rough paths**.
- Empirical studies consistently find $H \approx 0.1$–0.2 for volatility, motivating the class of **rough volatility models**.

Rough volatility: empirical motivation

- High-frequency data reveal that volatility paths are significantly **rougher** than those generated by classical stochastic volatility models.
- Empirically, the log-volatility process exhibits:
  - very low Hurst exponents ($H \ll \frac{1}{2}$),
  - rapidly oscillating paths,
  - strong short-term dependence with long-lasting impact.
- Classical models (e.g. Heston or SABR) impose excessive smoothness and fail to reproduce these features.
- Rough volatility models replace Markovian dynamics with fractional or Volterra-type processes, achieving:
  - better fit to volatility surfaces,
  - improved short-maturity smile dynamics,
  - realistic forward variance behaviour.
- This paradigm shift has reshaped modern volatility modelling in both academia and practice.

Example: Power-law decay

When $\alpha = 0.6$, the decay of the kernel $G(t-s)$ is slow.

1. **Memory effects**
   - The kernel G assigns substantial weight to distant past values when computing the effect at time t.
   - Since $\alpha < 1$, the decay of past increments is slow, implying long-range dependence.
   - This behaviour is consistent with empirical observations such as volatility clustering and persistent autocorrelations in absolute returns.

2. **Relation to fractional integration**
   - For $\alpha \in (0,1)$, the kernel mimics that of a fractional integral of order $1-\alpha$.
   - The choice $\alpha = 0.6$ corresponds to a fractional integration parameter $d = 0.4$, indicating strong persistence while remaining stationary (since $d < \frac{1}{2}$).

Example: Power-law decay

When $\alpha = 0.6$, the decay of the kernel $G(t - s)$ is slow.

1. **Connection to rough volatility**
   - With $\alpha = 0.6$, we have $H = 0.1$, placing the process firmly in the *rough* regime.
   - Empirical studies typically find volatility roughness in the range $H \in [0.05, 0.2]$, consistent with this specification.

Modelling jumps

Definition

- Let X_t be a càdlàg process (right-continuous with left limits). A *jump* at time t is defined by:

$$\Delta X_t = X_t - X_{t^-},$$

where $X_{t^-} = \lim_{s \uparrow t} X_s$ denotes the left-hand limit.

- The process X_t is said to exhibit jumps if there exists at least one $t \in [0, T]$ such that $\Delta X_t \neq 0$.

Modelling jumps

The jump–diffusion framework

- A standard jump–diffusion model takes the form:

$$dX_t = \mu\, dt + \sigma\, dW_t + J_t\, dN_t,$$

  where:
  - W_t is a Brownian motion,
  - N_t is a Poisson process,
  - J_t represents the (random) jump size.

Apparent continuity in discretely sampled financial data

The mechanism explained

- Conceptually, when drawing the path of a Lévy process with jumps, one must *lift the pen* at each jump time to represent the discontinuity.

- In modern electronic markets, price updates occur at extremely high frequencies (e.g., microseconds).

- **Low-frequency observation:** when prices are sampled every 1, 5, or 10 minutes:
  - Only discrete snapshots of the price process are observed.
  - The evolution between sampling times is unobserved.
  - Any jump occurring between two observations is visually replaced by a straight line segment.

- As a result, genuinely discontinuous price paths may appear continuous in discretely sampled data.

Example in the Black-Scholes model

Corporate events such as discrete dividends and earnings per share (EPS) announcements induce jumps in stock prices:

$$S(t_i) = S(t_i^-) + \mathrm{sgn}(d_i)\, d_i,$$

where d_i denotes the jump magnitude at event time t_i.

- Consider a discrete time grid $\mathscr{T} = \{t_0 = 0, t_1, \ldots, t_N = T\}$, with initial spot price $S_{t_0} = 50$.
- Assume a single proportional dividend at time t_i of the form:

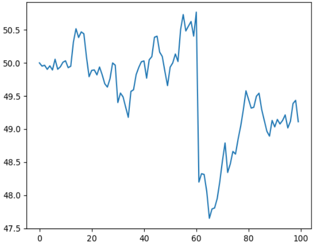
$$\tilde{d}_i = \alpha S_{t_i}, \qquad \alpha = 5\%.$$

In the Black-Scholes framework, the price dynamics are given by:

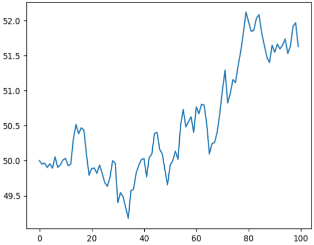
$$\frac{dS_t}{S_{t^-}} = \mu\, dt + \sigma\, dW_S^{\mathbb{P}}(t) + \frac{\overline{D}_t}{S_{t^-}}\, dt,$$

where $\mu = r - q - \lambda\sigma$, and $\overline{D}_t = \sum_{i=0}^{\infty} \delta(t - t_{d_i})$, with $\delta(\cdot)$ denoting the Dirac delta function.

Example in the Black-Scholes model



(a) With proportional dividend



(b) Without dividend

Figure 1: Stock price paths in the Black–Scholes model with and without a proportional dividend.

Jumps and their role in financial time series

Key implications

- **Risk modelling:** Jumps generate fat tails in return distributions, invalidating Gaussian assumptions and materially affecting risk measures such as Value-at-Risk (VaR) and Expected Shortfall.
- **Option pricing:** Jump risk alters implied volatility surfaces and necessitates models such as Merton's jump–diffusion or exponential Lévy models for accurate pricing.
- **Forecasting and control:** The presence of jumps undermines smoothness and Markovian assumptions commonly used in time-series forecasting, motivating non-local or regime-switching frameworks.

Characterising Local Behaviour

Beyond global statistical properties, the **fine-scale structure** of a path is determined by how it behaves over infinitesimal intervals. This leads us to the concept of **local smoothness**.

- **Path-wise Analysis:** While stationarity and long memory describe the relationship between distant points, we often need to quantify the 'jaggedness' of the path at any given moment.
- **Intuition of Smoothness:**
  - A differentiable function is 'smooth'.
  - A Brownian motion is continuous but nowhere differentiable, it is 'rough'.
- **Transition to Regularity:** To formalise these differences, we use a metric that measures the maximum rate of change relative to the time increment $|t - s|$.

This brings us to the formal definition of Hölder regularity.

Hölder regularity

Definition

- The Hölder exponent α_H quantifies the local smoothness of a function or stochastic process.
- A function X_t is said to be α_H-Hölder continuous if there exists a constant $C > 0$ such that:

$$|X_t - X_s| \leq C|t - s|^{\alpha_H}, \qquad \forall\, s, t.$$

- The exponent α_H characterises:
  - The roughness of sample paths,
  - How continuous, jagged, or irregular the signal is,
  - How rapidly values can change over small time intervals.

Hölder regularity

Interpretation

- $\alpha_H = 1$: smooth, differentiable paths; typical of deterministic functions
- $\alpha_H = \frac{1}{2}$: Brownian motion (almost surely); classical diffusion behaviour
- $\alpha_H < \frac{1}{2}$: rougher than Brownian motion; characteristic of fractional noise and rough volatility
- $\alpha_H \approx 0.3$: very rough, highly irregular paths; consistent with empirical volatility estimates
- $\alpha_H \to 0$: extremely jagged behaviour; near-discontinuous paths dominated by sharp variations or jumps

Empirical studies show that realised volatility paths in financial markets are well modelled with $\alpha_H \in [0.1, 0.4]$, indicating roughness well beyond that of Brownian motion.

Heavy Tails and Asymptotic Decay

While Hölder regularity describes the local smoothness of a path, the **global distribution** of its values often exhibits deviations from the Gaussian bell curve, particularly in finance.

- **Tail Behaviour:** We are interested in the probability of observing extreme values far from the mean.
- **Asymptotic Scaling:** A distribution follows a **power-law** if the probability of an outcome exceeding a threshold x decays as a power of x:

  $$P(|X_t| > x) \sim L(x)x^{-\alpha}, \quad x \to \infty$$

  where $L(x)$ is a slowly varying function.
- **Significance:** This indicates that "outliers" are much more frequent than in a Normal distribution (where decay is exponential, e^{-x^2}).

This decay rate is quantified by the power-law exponent, which we define next.

The power-law exponent

Definition

- The power-law tail exponent α characterises the heaviness of distributional tails:

$$P(|X_t| > x) \sim x^{-\alpha} \quad \text{as } x \to \infty,$$

  with empirical estimates in financial data typically satisfying $\alpha \in (0, 4)$.

- The exponent α quantifies:
  - The likelihood of extreme events (large returns or jumps),
  - The rate at which tail probabilities decay,
  - Which statistical moments (mean, variance, kurtosis) are finite.

The power-law exponent: Interpretation

- $\alpha = \infty$: Gaussian behaviour; Brownian motion with exponentially decaying tails
- $\alpha > 4$: light-tailed distributions; all moments finite; extreme events are rare
- $\alpha = 4$: transition point; finite variance but borderline excess kurtosis
- $\alpha = 3$: heavy-tailed regime; finite variance but infinite fourth moment; large moves occur more frequently
- $\alpha \in (2, 3)$: very heavy tails; finite mean but infinite variance; risk dominated by extreme events
- $\alpha \in (1, 2)$: extremely heavy tails; infinite mean and variance; dynamics dominated by jumps
- $\alpha < 1$: ultra-heavy tails; no finite moments; behaviour entirely driven by extreme outliers

Power-law decay vs. power-law tails

Key implications include

1. Power-law decay in time (as in $G(t-s)$)
   - Controls temporal memory or dependence.
   - Relevant in volatility modelling, rough paths, and fractional processes.
   - Does not necessarily imply large jumps: a process with slowly decaying autocorrelations due to a kernel like G may still be continuous.
2. Power-law tails in distributions (as in $P(X > x) \sim x^{-\alpha}$)
   - Refers to heavy tails in the marginal distribution of returns or volatility.
   - Implies large jumps or extreme events.
   - When $\alpha < 2$ variance is infinite; when $\alpha < 1$, even the mean is infinite.

The effect of $\alpha < 1$

When $\alpha < 1$ both of the following are true in different contexts:

Key implications include

1. In time: Power-law decay kernels imply long-range dependence or memory in processes, not jumps.
2. In size/distribution: Power-law distribution tails with $\alpha < 1$ imply very heavy tails and extreme jumps.

Examples of a model that combines both memory and jumps include rough Levy model or fractional jump-diffusion.

Power-Law Tails: Empirical Evidence

Tail Behaviour of Financial Returns

Numerous empirical studies across asset classes and markets show that the tails of return distributions decay according to a power law:

$$\mathbb{P}(|r_t| > x) \sim Cx^{-\alpha}, \qquad \alpha \in (2, 4)$$

where α is the *tail index*.

- The decay is significantly slower than the exponential decay implied by the Gaussian distribution.
- Empirical estimates often find:

$$\alpha \approx 3 \quad \text{for daily returns}$$

- Implications:
  - Variance exists ($\alpha > 2$), but higher moments may be unstable
  - Sample kurtosis and skewness become unreliable
  - Extreme events dominate risk measures

Power-Law Tails: Empirical Evidence

Power-law tails are remarkably stable across markets and time periods, reinforcing their status as a universal stylised fact of financial returns.

Statistical Consequences of Heavy Tails

Implications for Estimation and Inference

- **Unstable moments**
  - If $\alpha \leq 4$, the theoretical kurtosis is infinite
  - If $\alpha \leq 2$, even the variance does not exist
- **Sensitivity to outliers**
  - Sample mean and variance are dominated by rare extreme observations
  - Standard errors become unreliable
- **Failure of classical limit theorems**
  - For $\alpha < 2$, the Central Limit Theorem no longer applies
  - Sums of returns converge to α-stable distributions, not Gaussian

As a result, statistical tools that rely on finite moments and Gaussian asymptotics can lead to misleading inference in financial applications.

Non-Gaussian Returns and Heavy Tails

Empirical Distribution of Returns

- Empirical financial returns deviate sharply from the Gaussian assumption.
- Return distributions exhibit:
  - **Heavy (fat) tails**: extreme events occur far more frequently than predicted by the normal distribution
  - **Excess kurtosis**: sharp peak around the mean and slow tail decay
  - **Frequent outliers**: large gains and losses are not rare
- As a consequence, Gaussian-based models systematically underestimate tail risk.
- These deviations are observed consistently across:
  - Asset classes (equities, FX, rates, commodities)
  - Markets and geographies
  - Time horizons (from intraday to monthly)

Non-Gaussian Returns and Heavy Tails

Heavy tails are one of the most robust *stylised facts* in financial time series and represent a fundamental challenge to classical statistical modelling.

Economic and Risk Management Implications

Consequences of Assuming Normality

- **Underestimation of tail risk**
  - Gaussian Value-at-Risk (VaR) severely underpredicts extreme losses
  - Historical drawdowns occur far more frequently than models suggest
- **Option mispricing**
  - Black–Scholes assumes log-normal returns with thin tails
  - Market-implied-vol embed crash risk absent from the model
- **Misestimated risk premia**
  - Investors demand compensation for tail risk
  - Gaussian models fail to capture downside asymmetry

Systematic underestimation of extreme events can lead to undercapitalisation and amplify systemic risk during periods of market stress.

Leverage effects and asymmetric volatility

- Empirical evidence shows that **negative returns increase future volatility more than positive returns of the same magnitude**.
- This phenomenon is known as the **leverage effect** and contradicts symmetric volatility models.
- Economically, price drops increase financial leverage, raise default risk, and amplify risk perceptions.
- Behaviourally, losses trigger stronger reactions than gains, generating asymmetric feedback loops in volatility.
- Asymmetry is captured by models such as:
  - EGARCH
  - GJR-GARCH
  - Stochastic volatility models with correlated shocks

Ignoring leverage effects leads to biased volatility forecasts and systematic option mispricing, particularly for downside risk.

Autocorrelation in higher moments

- Raw asset returns typically exhibit **little to no linear autocorrelation**, consistent with weak-form market efficiency.
- In contrast, **higher moments**, such as volatility, skewness, and kurtosis, display strong and persistent temporal dependence.
- Empirically:
  - Squared and absolute returns are highly autocorrelated.
  - Volatility shocks decay slowly over time.
- This behaviour underpins:
  - GARCH and stochastic volatility models
  - Time-varying risk premia
  - Volatility and tail-risk forecasting

Time dependence in higher moments implies that risk is predictable even when returns are not.

Path dependence and memory effects

- Financial time series often exhibit **path dependence**: the impact of a shock depends on recent market history.
- Identical returns can lead to different outcomes depending on:
  - preceding trends or drawdowns,
  - volatility regimes,
  - time elapsed since the last major shock.
- This behaviour violates the **Markov property**, a key assumption of many classical stochastic models.
- Memory effects arise from:
  - investor learning and adaptation,
  - liquidity cycles,
  - institutional constraints and risk limits.

Capturing memory requires models with latent states, long-range dependence, or recurrent structures.

Asymmetry and nonlinear dynamics

- Market responses to shocks are often **nonlinear and asymmetric**.
- A price decrease and a price increase of the same magnitude can generate very different future dynamics.
- Losses typically trigger:
  - stronger risk aversion,
  - forced deleveraging,
  - liquidity withdrawal.
- These mechanisms produce nonlinear feedback loops, volatility bursts, and regime shifts.
- Linear models fail to capture these effects, motivating:
  - threshold and regime-switching models,
  - nonlinear stochastic volatility,
  - behavioural and agent-based frameworks.

Ignoring nonlinearities leads to systematic underestimation of downside risk and tail dependence.

Key takeaways

Summary

- Financial time series exhibit **long memory**, **rough paths**, and **jumps**, violating classical Gaussian assumptions.
- Power-law kernels capture persistent dependence and link naturally to fractional and rough volatility models.
- Jumps arise from both market microstructure and economic events, but may appear continuous under low-frequency sampling.
- Hölder regularity provides a precise notion of path roughness, while power-law exponents quantify tail risk and extreme events.

The end

Thank You !