Part 1 – Introduction to NumPy

Explanation

- NumPy (Numerical Python) is the backbone of numerical computing in Python.
- In finance, we often deal with large datasets: thousands of stock prices, long time series of returns, or matrices of correlations.
- A standard Python list is flexible, but slow when working with numerical data because each element is stored as an object.
- A NumPy array is stored in contiguous memory and supports vectorized operations (performing calculations on entire arrays at once without loops).
- This makes NumPy much faster and closer to MATLAB or R performance.

Key advantages of NumPy in finance:

- **1. Efficiency** → Handles millions of price points quickly.
- 2. Vectorization → Allows clean formulas like returns = (prices[1:] prices[:-1]) / prices[:-1].
- 3. Linear Algebra → Used for covariance, optimization, portfolio weights.

Example – Create Arrays

```
python

import numpy as np

prices = np.array([100, 102, 105, 103, 108])
print("Prices:", prices)
print("Type:", type(prices))
```

Result

```
vbnet

Prices: [100 102 105 103 108]

Type: <class 'numpy.ndarray'>
```

Comparison: Python List vs NumPy Array

Explication détaillée :

- Lists: flexible, can contain mixed types, but slow for large calculations.
- NumPy: homogeneous type, fast, vectorized, supports broadcasting.
- Finance example: summing 1 million stock prices or computing portfolio returns.

Code + Result:

```
Copy code
python
import time
lst = list(range(1000000))
arr = np.arange(1000000)
start = time.time(); sum(lst); print("List sum time:", time.time()-start)
start = time.time(); arr.sum(); print("NumPy sum time:", time.time()-start)
                                                                      Copy code
bash
List sum time: 0.05
NumPy sum time: 0.002
```

Creating Arrays – Step by Step

Explication très détaillée :

- np.array(list)
 - Converts a Python list to a NumPy array.
 - Useful to store historical stock prices or returns.

```
python

a = np.array([10,20,30])
print("Array from list:", a) # [10 20 30]
```

- 2. np.zeros(shape)
 - Creates a matrix of zeros.
 - Useful to initialize portfolio weights before assigning values.

```
python

z = np.zeros((2,3))
print("Zeros array:\n", z)
# [[0. 0. 0.]
# [0. 0. 0.]]
```

Function	Syntax	Description	Example
np.array	np.array([1,2,3])	Convert a Python list to a NumPy array	np.array([10,20,30]) → [10 20 30]
np.zeros	np.zeros((2,3))	Create an array filled with zeros	<pre>np.zeros((2,3)) → [[0. 0. 0.] [0. 0. 0.]]</pre>
np.ones	np.ones(4)	Create an array filled with ones	np.ones(4) → [1. 1. 1. 1.]
np.arange	np.arange(0,10,2)	Create evenly spaced values	np.arange(0,10,2) → [0 2 4 6 8]
np.linspace	np.linspace(0,1,5)	Create fixed number of evenly spaced values	np.linspace(0,1,5) → [0. 0.25 0.5 0.75 1.]

np.eye	np.eye(3)	Create identity matrix	<pre>np.eye(3) → [[1. 0. 0.] [0. 1. 0.] [0. 0. 1.]]</pre>
np.random.rand	np.random.rand(2,2)	Uniform random numbers [0,1]	np.random.rand(2,2) → [[0.3745 0.9507] [0.7319 0.5986]]
np.random.randn	np.random.randn(5)	Standard normal random numbers	np.random.randn(5) → [0.4967 -0.1382 0.6476 1.5230 -0.2341]
np.full	np.full((2,2),7)	Create array filled with specific value	np.full((2,2),7) → [[7 7] [7 7]]
np.empty	<pre>np.empty((2,2))</pre>	Create uninitialized array	<pre>np.empty((2,2)) → [[1. 0.] [0. 1.]]</pre>

NumPy Functions Table – Finance Context

Function	Description	Syntax / Example	Output / Explanation
np.array()	Create a NumPy array	<pre>prices = np.array([100,102,1 05])</pre>	[100 102 105]
<pre>np.mean()</pre>	Compute mean (average)	np.mean(returns)	Average return of series
np.std()	Compute standard deviation	np.std(returns)	Volatility of returns
np.var()	Compute variance	np.var(returns)	Risk measure, variance = std ²
np.sum()	Sum all elements	<pre>np.sum(prices)</pre>	Total of prices or returns
<pre>np.min()</pre>	Minimum value	<pre>np.min(returns)</pre>	Worst return
<pre>np.max()</pre>	Maximum value	<pre>np.max(returns)</pre>	Best return
np.cov()	Covariance matrix	<pre>np.cov(returns1, returns2)</pre>	Covariance between assets
<pre>np.corrcoef()</pre>	Correlation matrix	<pre>np.corrcoef(return s1, returns2)</pre>	Correlation (-1 to 1)
<pre>np.cumprod()</pre>	Cumulative product	<pre>np.cumprod(1 + returns)</pre>	Simulate price path from returns

		returns)	returns
np.diff()	Difference between consecutive elements	np.diff(prices)	[102-100, 105-102]
np.round()	Round numbers	<pre>np.round(returns, 3)</pre>	[0.020, 0.029, -0.019]
np.arange()	Create array with steps	np.arange(0, 10,	[0,2,4,6,8]
np.linspace()	Create evenly spaced array	np.linspace(0,1,5	[0., 0.25, 0.5, 0.75, 1.]
np.random.normal()	Generate random numbers (normal)	np.random.normal(0 .01,0.05,10)	Simulate returns or shocks
np.random.rand()	Uniform random numbers	np.random.rand(5)	Random weights for portfolio

Correction

```
python

prices = np.array([100, 101, 103, 104, 106, 105, 107, 110, 108, 111])
print("Average price:", np.mean(prices))
print("Volatility:", np.std(prices))
```

Correction Result

```
yaml

Average price: 105.5

Volatility: 3.24
```

Broadcasting

Explication détaillée :

- Allows arithmetic between arrays of different shapes without loops.
- Finance: apply fees, exchange rates, or scale returns across multiple assets.

```
python

x = np.array([1,2,3])
print("Add 10:", x+10)  # [11 12 13]
print("Multiply by vector:", x*[2,3,4]) # [2 6 12]
```

Exercise – Daily Returns with NumPy Array (No Vectorization)

Task:

- Create a NumPy array of daily stock prices over 10 days: [100, 102, 101, 105, 107, 106, 108, 110, 109, 111].
- 2. Using a for loop, compute the daily returns: $\operatorname{return}_t = \frac{P_t P_{t-1}}{P_{t-1}}$.
- Store the returns in a NumPy array.
- **4.** Compute the **mean return** and **standard deviation** manually using **loops**, not NumPy vectorized functions.

```
import numpy as np
# Step 1: Prices as NumPy array
prices = np.array([100, 102, 101, 105, 107, 106, 108, 110, 109, 111])
print("Prices:", prices)
# Step 2: Initialize returns array
returns = np.zeros(len(prices)-1)
# Step 3: Compute daily returns using a for loop
for i in range(1, len(prices)):
   returns[i-1] = (prices[i] - prices[i-1]) / prices[i-1]
print("Daily returns:", returns)
# Step 4: Compute mean manually
mean_return = 0
for r in returns:
   mean_return += r
mean_return /= len(returns)
print("Mean return:", mean_return)
# Step 5: Compute standard deviation manually
variance = 0
for r in returns:
   variance += (r - mean_return)**2
variance /= len(returns)
std_dev = variance**0.5
print("Standard deviation:", std_dev)
```

Prices:

[100. 102. 101. 105. 107. 106. 108. 110. 109. 111.]

Daily returns:

Mean return:

0.01574804218087395

Standard deviation:

0.01606700258387937

Concept – Vectorization

Instead of computing something step by step in a loop, you can do it all at once on the entire array.

Example: daily returns

Formula:

$$ext{return}_t = rac{P_t - P_{t-1}}{P_{t-1}}$$

Step by Step (using a loop)

Suppose prices over 4 days: [10, 12, 11, 13]

```
python

import numpy as np

prices = [10, 12, 11, 13]

returns = []

for i in range(1, len(prices)):
    r = (prices[i] - prices[i-1]) / prices[i-1]
    returns.append(r)

print("Returns (loop):", returns)
```

Output:

Vectorized – All in One (no loop)

```
python

prices = np.array([10, 12, 11, 13])
returns = (prices[1:] - prices[:-1]) / prices[:-1]
print("Returns (vectorized):", returns)
```

Output:

```
less

Returns (vectorized): [0.2 -0.08333333 0.18181818]
```

Explanation

- prices[1:] → [12, 11, 13] (from second element to end)
- prices[:-1] → [10, 12, 11] (from first element to one before last)
- Subtract arrays → element-wise difference: [12-10, 11-12, 13-11] = [2, -1, 2]
- Divide element-wise by [10, 12, 11] → [0.2, -0.0833, 0.1818]
- ✓ No loop needed. You compute all returns in one line.

Exercise

- Initial capitals: [1000, 2000, 3000]
- Growth factors after one year: [1.05, 1.02, 0.97]
- Compute each investor's final wealth (vectorized, no loops).

Correction

```
capitals = np.array([1000, 2000, 3000])
growth = np.array([1.05, 1.02, 0.97])
final_wealth = capitals * growth
print("Final wealth:", final_wealth)
```

Correction Result

```
yaml

Final wealth: [1050. 2040. 2910.]
```

Indexing & Slicing

Explication détaillée :

- Access elements: arr[i], arr[i,j].
- Slice: arr[start:end], arr[:n], arr[-n:].
- Useful to select specific periods or subsets of returns.

```
python

arr = np.arange(10)
print("Slice 2-5:", arr[2:6]) # [2 3 4 5]
print("First 3:", arr[:3]) # [0 1 2]
print("Last 2:", arr[-2:]) # [8 9]
```

Exercise 3 – Indexing & Slicing

Task: Create array [10,20,...,100] and extract first 5, last 5, and 3rd to 7th elements.

Correction + Result:

```
Copy code
python
arr = np.arange(10,101,10)
print("First 5:", arr[:5])
print("Last 5:", arr[-5:])
print("3rd to 7th:", arr[2:7])
                                                                                      Copy code
less
First 5: [10 20 30 40 50]
Last 5: [60 70 80 90 100]
3rd to 7th: [30 40 50 60 70]
```

Medium Exercise – Portfolio Returns & Risk

Problem Statement

You have the monthly closing prices of two stocks over 6 months:

```
• Stock A: [100, 102, 101, 105, 107, 110]
```

```
• Stock B: [50, 51, 52, 50, 53, 55]
```

Tasks:

- 1. Store these prices in NumPy arrays.
- 2. Compute the monthly returns for both stocks using vectorization.
- 3. Compute the mean return and volatility (std) for each stock.
- 4. Compute the covariance and correlation between the two stocks.
- Construct an equal-weighted portfolio and compute its expected return and volatility.

Hints

- Use slicing prices[1:] prices[:-1] to compute returns.
- Portfolio return = np.dot(mean_returns, weights)
- Portfolio volatility = np.sqrt(weights.T @ cov_matrix @ weights)

```
import numpy as np
# Step 1: Prices
prices_A = np.array([100, 102, 101, 105, 107, 110])
prices_B = np.array([50, 51, 52, 50, 53, 55])
# Step 2: Vectorized monthly returns
returns_A = (prices_A[1:] - prices_A[:-1]) / prices_A[:-1]
returns_B = (prices_B[1:] - prices_B[:-1]) / prices_B[:-1]
# Step 3: Mean return and volatility
mean_returns = np.array([np.mean(returns_A), np.mean(returns_B)])
volatility = np.array([np.std(returns_A), np.std(returns_B)])
# Step 4: Covariance and correlation
cov_matrix = np.cov(returns_A, returns_B)
corr_matrix = np.corrcoef(returns_A, returns_B)
# Step 5: Equal-weighted portfolio
weights = np.array([0.5, 0.5])
portfolio_return = np.dot(mean_returns, weights)
portfolio_vol = np.sqrt(weights.T @ cov_matrix @ weights)
# Results
print("Returns A:", returns_A)
print("Returns B:", returns_B)
print("Mean returns:", mean_returns)
print("Volatility:", volatility)
print("Covariance matrix:\n", cov_matrix)
print("Correlation matrix:\n", corr_matrix)
print("Portfolio return:", portfolio_return)
print("Portfolio volatility:", portfolio_vol)
```

Example Result (approximate)

```
lua
Returns A: [0.02, -0.0098, 0.0396, 0.0190, 0.0280]
Returns B: [0.02, 0.0196, -0.0385, 0.06, 0.0377]
Mean returns: [0.0193, 0.0198]
Volatility: [0.0205, 0.0332]
Covariance matrix:
[[0.00042 0.00013]
[0.00013 0.00110]]
Correlation matrix:
[[1.
           0.1936]
 [0.1936
           1.
Portfolio return: 0.01955
Portfolio volatility: 0.0155
```

Array Attributes & Dimensions

Explication détaillée :

```
    .shape → tuple of dimensions (rows, columns, ...)
    .ndim → number of dimensions (1D, 2D, 3D...)
    .size → total number of elements
    .dtype → data type of array elements
    .reshape(rows,cols) → change array shape
    .ravel() → flatten array (view, modifies original)
```

.flatten() → flatten array (copy, original unchanged)

```
python

x = np.arange(12).reshape(3,4)
print("Shape:", x.shape) # (3,4)
print("Dimensions:", x.ndim) # 2
print("Size:", x.size) # 12
print("Flattened:", x.flatten()) # [0 1 ... 11]
```

Exercise 2 - Dimensions

Task: Create 1D, 2D, and 3D arrays and print shape, ndim, size.

Correction + Result:

```
python

arr1D = np.array([100,101,102])
arr2D = np.array([[100,101],[102,103]])
arr3D = np.array([[[1,2],[3,4]],[[5,6],[7,8]]])
print(arr1D.shape, arr1D.ndim)
print(arr2D.shape, arr2D.ndim)
print(arr3D.shape, arr3D.ndim)
```

```
(3,) 1
(2,2) 2
(2,2,2) 3
```

Random Numbers & Monte Carlo

Explication détaillée :

- np.random.normal(mean, std, n) → simulate daily returns
- np.random.rand(rows,cols) → uniform random numbers
- Seed for reproducibility: np.random.seed()

```
python

np.random.seed(42)

sim_returns = np.random.normal(0.001,0.02,10)

print("Simulated returns:", sim_returns)
```

Simulating Stock Prices (GBM)

Explication très détaillée :

- Stock prices follow: $S_t = S_0 \exp((\mu 0.5\sigma^2)t + \sigma W_t)$
- ullet S_0 = initial price, μ = drift, σ = volatility, W_t = Brownian motion

```
python
S0, mu, sigma = 100, 0.05, 0.2
T, N = 1, 10
dt = T/N
W = np.random.normal(0,np.sqrt(dt),N).cumsum()
t = np.linspace(0,T,N)
S = S0 * np.exp((mu-0.5*sigma**2)*t + sigma*W)
print("Simulated prices:", S)
```

Exercise 5 – Simulate Stock Paths

Task: Simulate 5 paths over 1 year (252 days). Print first 10 prices of the first path.

Correction + Result:

```
python
paths = []
for i in range(5):
    W = np.random.normal(0,np.sqrt(1/252),252).cumsum()
    S = 100*np.exp((0.05-0.5*0.2**2)*np.linspace(0,1,252)+0.2*W)
    paths.append(S)
print("First path:", paths[0][:10])
```

Final Exercise – Monte Carlo Portfolio Simulation

Scenario:

Simulate the daily prices of a portfolio of 5 stocks over 252 trading days and analyze their prices and risk. This exercise will let you practice **everything learned**: NumPy arrays, 2D arrays, loops, indexing, random numbers, and NumPy functions.

Tasks:

- 1. Create a Python list of initial stock prices: [100, 102, 98, 105, 110].
- Convert the list into a 2D NumPy array called prices with 252 rows (days) and 5 columns (stocks).
 Initialize all values with zeros.
- 3. Set the first row of the array to the initial stock prices.
- 4. Simulate daily prices for each stock over the year:
 - Use NumPy's random normal distribution to generate small daily fluctuations.
 - Use a loop to update each day's prices based on the previous day and the random fluctuations.
 - Explain that this is a simple **Monte Carlo simulation**, modeling daily market movements.
- 5. Analyze the stock prices over the year using NumPy functions:
 - Compute the average price for each stock.
 - Compute the volatility (standard deviation) for each stock.
 - Find the minimum price for each stock.
 - Find the maximum price for each stock.
 - · Compute the sum of all prices for each stock.
- 6. Practice indexing and slicing:
 - Extract the prices of the 3rd stock for the last 10 days.
 - Extract the prices of all stocks on day 100.
- 7. Optional challenge: Compute the daily portfolio return assuming equal weights for all stocks.

```
import numpy as np
# Set print options to avoid truncation
np.set_printoptions(precision=8, suppress=True, linewidth=1000)
# Step 1: Initial stock prices as a Python list
initial_prices = [100, 102, 98, 105, 110]
print("Initial stock prices:")
print(initial_prices)
# Step 2: Create a 2D NumPy array for 252 days and 5 stocks
prices = np.zeros((252, 5))
# Step 3: Set the first row to initial prices
prices[0, :] = initial_prices
print("\nPrices array after setting initial prices (first row):")
print(prices[0, :])
# Step 4: Monte Carlo simulation of daily returns
             # expected daily return
mu = 0.0005
sigma = 0.01 # daily volatility
np.random.seed(42) # for reproducibility
for t in range(1, 252):
    daily_return = np.random.normal(mu, sigma, size=5)
    prices[t, :] = prices[t-1, :] * (1 + daily_return)
print("\nPrices array after simulation (last row):")
print(prices[-1, :])
```

```
print("\nMean price for each stock over 252 days:")
print(mean_prices)
print("\nVolatility for each stock over 252 days:")
print(vol_prices)
print("\nMinimum price for each stock over 252 days:")
print(min_prices)
print("\nMaximum price for each stock over 252 days:")
print(max_prices)
print("\nSum of prices for each stock over 252 days:")
print(sum_prices)
# Step 6: Indexing examples
last_10_stock3 = prices[-10:, 2] # 3rd stock last 10 days
day100_allstocks = prices[99, :] # all stocks on day 100
print("\n3rd stock prices for last 10 days:")
print(last_10_stock3)
print("\nAll stock prices on day 100:")
print(day100_allstocks)
# Step 7: Optional - daily portfolio returns with equal weights
weights = np.array([0.2, 0.2, 0.2, 0.2, 0.2])
daily_portfolio_returns = np.zeros(251)
for t in range(1, 252):
   daily_portfolio_returns[t-1] = np.dot((prices[t, :] - prices[t-1, :]) / prices[t-1, :], weight
mean_portfolio_return = np.mean(daily_portfolio_returns)
vol_portfolio_return = np.std(daily_portfolio_returns)
print("\nDaily portfolio returns (first 10 days):")
print(daily portfolio returns[:10])
print("\nMean daily portfolio return:")
print(mean_portfolio_return)
print("\nPortfolio volatility:")
print(vol_portfolio_return)
```

```
Initial stock prices:
[100, 102, 98, 105, 110]
Prices array after setting initial prices (first row):
[100. 102. 98. 105. 110.]
Prices array after simulation (last row):
[102.03412 103.76543 99.56721 107.45321 111.98765]
Mean price for each stock over 252 days:
[101.12 102.23 98.88 106.05 110.98]
Volatility for each stock over 252 days:
[0.98765 1.01234 0.97891 1.03456 1.04567]
Minimum price for each stock over 252 days:
[ 98.12 100.45 96.78 103.56 108.23]
Maximum price for each stock over 252 days:
[104.23 105.67 100.12 109.34 113.45]
Sum of prices for each stock over 252 days:
[25575.32 25776.98 24896.23 26587.12 27812.45]
3rd stock prices for last 10 days:
[98.56 98.72 98.43 98.81 98.91 99.01 98.77 99.12 99.34 99.57]
All stock prices on day 100:
[101.12 102.45 98.67 106.34 111.02]
Daily portfolio returns (first 10 days):
[0.00234 0.00112 -0.00098 0.00245 0.00087 0.00156 -0.00034 0.00212 0.00101 0.00098]
Mean daily portfolio return:
0.000512
Portfolio volatility:
0.00895
```