Convert non-linear equation to linear.
using Numerical method.

Exponential model

Sometimes in least square method, we will use exponential curve to achieve *minimum error* with the given data.

Let the required curve has the form:

$$y = ae^{bx}$$

To use the least square method of a straight line, first we must make a transformation to the required equation, this transformation called ***linearization*** by taking the inverse of the required equation of the curve as the following:

$$\ln y = \ln a + bx$$

This equation can be written as:

$$\boxed{Y = A + Bx} \qquad (2.4)$$

With the changes:

$$\boxed{Y = \ln y,\ \ A = \ln a,\ \ B = b,\ \ x = x.}$$

Then we can apply least square equations to equation (2.4) as:

$$\boxed{\begin{gathered} \sum Y_i = A(N) + B\sum x_i, \\ \sum x_i Y_i = A\sum x_i + B\sum x_i^2. \end{gathered}}$$

After solving the previous equations and getting the constants (A, B) we make a reverse operation to get (a, b) again, to express the equation $y = ae^{bx}$.

No.	Non-linear equation	Linear form	Relationship to $\hat{y} = a + b\hat{x}$	Values for least squares regression
1.	$y = cx^m$	$\ell n(y) = m\ell n(x) + \ell n(c)$	$\hat{y} = \ell n(y)$, $\hat{x} = \ell n(x)$ $b = m$, $a = \ell n(c)$	$\ell n(x_i)$ and $\ell n(y_i)$
2.	$y = c\,e^{mx}$	$\ell n(y) = mx + \ell n(c)$	$\hat{y} = \ell n(y)$, $\hat{x} = x$ $b = m$, $a = \ell n(c)$	x_i and $\ell n(y_i)$
3.	$y = c\,10^{mx}$	$\log(y) = mx + \log c$	$\hat{y} = \log(y)$, $\hat{x} = x$ $b = m$, $a = \log(c)$	x_i and $\ell n(y_i)$
4.	$y = \frac{1}{mx + c}$	$\frac{1}{y} = mx + c$	$\hat{y} = \frac{1}{y}$, $\hat{x} = x$ $b = m$, $a = c$	x_i and $\frac{1}{y_i}$
5.	$y = \frac{mx}{c + x}$	$\frac{1}{y} = \frac{c}{mx} + \frac{1}{m}$	$\hat{y} = \frac{1}{y}$, $\hat{x} = \frac{1}{x}$ $b = \frac{c}{m}$, $a = \frac{1}{m}$	$\frac{1}{x_i}$ and $\frac{1}{y_i}$
6.	$xy^c = d$ Gas equation	$\log y = \frac{1}{c}\log d - \frac{1}{c}\log x$	$\hat{y} = \log y$, $\hat{x} = \log x$ $a = \frac{1}{c}\log d$, $b = -\frac{1}{c}$	$\log x_i$ and $\log y_i$
7.	$y = cd^x$	$\log y = \log c + x \log d$	$\hat{y} - \log y$, $\hat{x} = x$ $a = \log c$, $b = \log d$	x_i and $\log y_i$
8.	$y = c + d\sqrt{x}$	$y = c + d\hat{x}$ where $\hat{x} = \sqrt{x}$	$\hat{y} = y$ and $\hat{x} = \sqrt{x}$ $a = c$ and $b = d$	$\sqrt{x_i}$ and y_i

Nonlinear machine learning algorithms:

Classification and Regression Trees, Naive Bayes, K-Nearest Neighbors, Learning Vector Quantization and Support Vector Machines.