Project 0: Inaugural Project

Vision: The inaugural project teaches you to solve a simple economic model and present the results.

- **Objectives:** In your inaugural project, you should show that you can:
  1. Apply simple numerical solution and simulation methods
  2. Structure a code project
  3. Document code
  4. Present results in text form and in figures
- **Content:** In your inaugural project, you should:
  1. Solve and simulate a pre-specified economic model (see next page)
  2. Visualize results

  Example of structure: See this repository.
- **Structure:** Your inaugural project should consist of:
  1. A README.md with a short introduction to your project
  2. A single self-contained notebook (.ipynb) presenting the analysis
  3. Fully documented Python files (.py)
- **Hand-in:** On GitHub by uploading it to the subfolder *inaugralproject*, which is located in:

  github.com/NumEconCopenhagen/projects-YEAR-YOURGROUPNAME
- **Deadline:** See Calendar.
  - **Peer feedback:** After handing in, you will be asked to give peer feedback on the projects of two other groups.
    The solution will be made available after the peer feedback round has ended.
- **Exam:** Your inaugural project will be a part of your exam portfolio.
  You can incorporate feedback before handing in the final version.

Time Use of Couples

We consider a household with a *male* and a *female* member. They jointly maximize utility by choosing their time use:

1. Hours working *in the market*, L_M and L_F, at wages w_M and w_F.
2. Hours working *at home*, H_M and H_F.

Their choice of time use implies:

1. Consumption of market goods, $C = w_M L_M + w_F L_F$
2. Consumption of home production,

$$H = \begin{cases} \min\{H_M, H_F\} & \text{if } \sigma = 0 \\ H_M^{1-\alpha} H_F^{\alpha} & \text{if } \sigma = 1 \\ \left((1-\alpha) H_M^{\frac{\sigma-1}{\sigma}} + \alpha H_F^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} & \text{else} \end{cases}$$

   where σ is the elasticity of substitution, and $\frac{\alpha}{1-\alpha}$ is the productivity in home production for females relative to males.

3. Total consumption, $Q = C^{\omega} H^{1-\omega}$, where ω is the weight on market goods.

The household gets disutility from time spend working. The full maximization problem of the household is

$$\max_{L_M, H_M, L_F, H_F} \frac{Q^{1-\rho}}{1-\rho} - \nu \left(\frac{T_M^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}} + \frac{T_F^{1+\frac{1}{\epsilon}}}{1+\frac{1}{\epsilon}} \right), \ \rho > 1, \nu > 0, \epsilon > 0$$

s.t.

$$\begin{aligned} C &= w_M L_M + w_F L_F \\ H &= \begin{cases} \min\{H_M, H_F\} & \text{if } \sigma = 0 \\ H_M^{1-\alpha} H_F^{\alpha} & \text{if } \sigma = 1 \\ \left((1-\alpha) H_M^{\frac{\sigma-1}{\sigma}} + \alpha H_F^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} & \text{else} \end{cases} \\ Q &= C^{\omega} H^{1-\omega} \\ T_M &= L_M + H_M \\ T_F &= L_F + H_F \\ L_M, H_M, L_F, H_F &\geq 0 \\ T_M, T_F &\leq 24 \end{aligned}$$

The baseline parameters are:

1. **Preferences:** $\rho = 2$, $\nu = 0.001$, $\epsilon = 1$, $\omega = 0.5$
2. **Household production:** $\alpha = 0.5$, $\sigma = 1$
3. **Wages:** $w_M = w_F = 1$

Questions

We first assume that the choice set is *discrete* in half hours, specifically

$$L_M, L_F, H_M, H_F \in \left[\frac{0 \cdot 24}{48}, \frac{1 \cdot 24}{48}, \frac{2 \cdot 24}{48}, \ldots, \frac{48 \cdot 24}{48}\right]$$

Code is provided for solving the model in this case when $\sigma = 1$.

1. Illustrate how $\frac{H_F}{H_M}$ changes when varying $\alpha \in \{0.25, 0.50, 0.75\}$ and $\sigma = \{0.5, 1.0, 1.5\}$.
2. Plot $\log \frac{H_F}{H_M}$ against $\log \frac{w_F}{w_M}$ for $w_F \in [0.8, 0.9, 1.0, 1.1, 1.2]$.

We now assume that the choice set is *continuous*, i.e.

$$L_M, L_F, H_M, H_F \in [0, 24]$$

3. Plot $\log \frac{H_F}{H_M}$ against $\log \frac{w_F}{w_M}$ for $w_F \in [0.8, 0.9, 1.0, 1.1, 1.2]$

In Siminski and Yetsenga (2022), *Specialization, Comparative Advantage, and the Sexual Division of Labor*, the following regression is run on time use data of couples,

$$\log \frac{H_F}{H_M} = \beta_0 + \beta_1 \log \frac{w_F}{w_M}$$

They find that $\beta_0 \approx 0.4$ and $\beta_1 \approx -0.1$. Let $\hat{\beta}_0$ and $\hat{\beta}_1$ denote the same coefficients when estimated on data from the model for fixed w_M and $w_F \in [0.8, 0.9, 1.0, 1.1, 1.2]$.

4. Choose α and σ such it minimizes $\left(\beta_0 - \hat{\beta}_0\right)^2 + \left(\beta_1 - \hat{\beta}_1\right)^2$. Illustrate how the model fit the data and discuss the economic interpretation of the results.

Assume that additional empirical evidence shows that men and women are equally productive in home production, i.e. $\alpha = 0.5$.

5. Suggest and implement an extension of the model, and analyze whether or not it can help match the data when $\alpha = 0.5$.