

DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING IMPERIAL  
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EE3-19  
REAL-TIME DIGITAL SIGNAL PROCESSING  
LAB 4 REPORT

YASMIN BABA - 01196634  
ZHI WEN SI - 01243716

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# 1. FILTER DESIGN

## 1.1 FILTER SPECIFICATION

The purpose of Lab 4 is to design and implement a FIR filter in addition to investigating methods that will allow it to run as fast as possible. The filter must satisfy the following specifications shown in *Figure 1*.

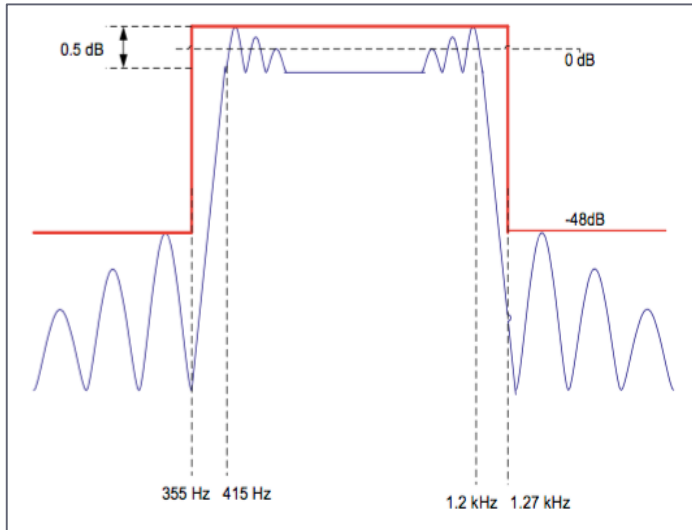


Figure 1: FIR Filter Specifications

In words, this is a bandpass filter with the following parameters:

High Pass Transition Band	355 Hz – 415 Hz
Low Pass Transition Band	1200 Hz – 1270 Hz
Passband Ripple	$\leq 0.5 \text{ dB}$
Stopband Gain	$\leq -48 \text{ dB}$

Table 1: FIR Filter Specifications

## 1.2 FILTER DESIGN USING MATLAB

Using Matlab, the FIR filter coefficients were designed via the Parks-McClellan algorithm using the following code shown in *Listing 1*.

```
%Filter Coefficient Calculator Via Parks McClellan Algorithm

rp = 0.5; %the passband ripple
sa = 48; %minimum stop band attenuation
fc = [355 415 1200 1270]; %cut off frequencies
fs = 8000; %sampling frequency
a = [0 1 0]; %band gains
%max deviations/ripples for each band
dev = [10^(-sa/20) (10^(rp/20)-1)/(10^(rp/20)+1) 10^(-sa/20)];

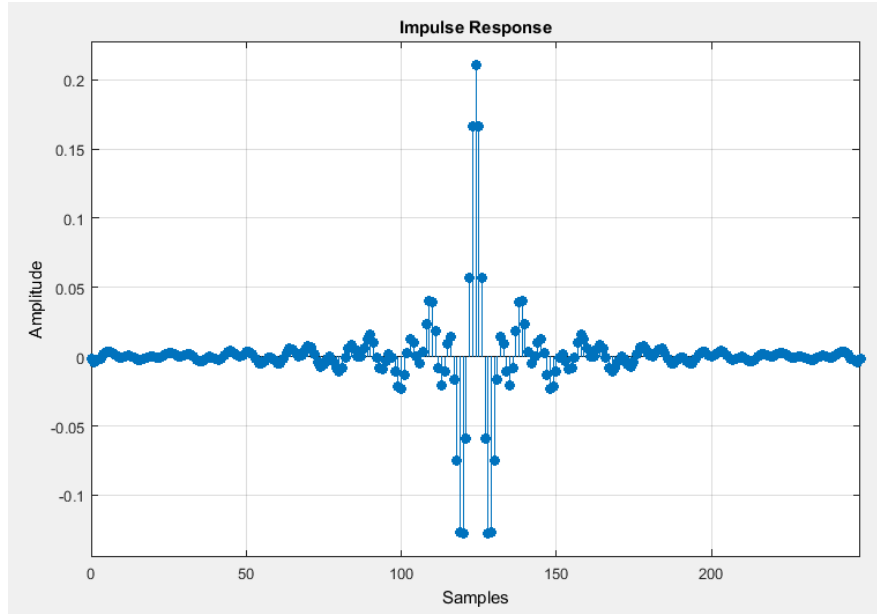
%calculates parameters used by the firpmord function
[N, fo, Ao, W] = firpmord(fc, a, dev, fs);
%calculates FIR filter coefficients
coefs = firpm(N, fo, Ao, W);

fvtool(coefs)
save fir_coefs3.txt coefs -ascii -double -tabs
```

Listing 1: Matlab Code for FIR Filter Design

A FIR filter with 249 real coefficients was produced, creating a 248<sup>th</sup> order FIR filter. Before proceeding to the frequency response of the filter, a sufficient understanding of a FIR filter is required. Note that all following plots have been acquired using the **fvtool** function with respect to the 249 coefficients designed in Matlab.

A FIR filter is a digitally designed filter, whose impulse response eventually falls to zero after a finite amount of time, as shown below in *Figure 2*. It therefore has limited memory represented by the order of the filter,  $M$ . A very important property of the FIR filter is that its impulse response is symmetrical. The filter coefficients are the values in the impulse response, hence are also symmetrical.



*Figure 2: Impulse Response*

The filter can be represented by the following time-domain difference equations, where  $M + 1$  represents the number of coefficients.

$$y(n) = b_0x(n) + b_1x(n - 1) + \dots + b_Mx(n - M)$$

Taking  $Z$ -transforms, the transfer function of the  $M^{th}$  order FIR filter is:

$$H(z) = b_0 + \frac{b_1}{z} + \dots + \frac{b_M}{z^M} = \sum_{k=0}^M b(k)z^{-k}$$

Given this transfer function, it is clear that the filter can have zeros anywhere in the  $z$  plane, in whose positions depend on the filter's coefficients. The filter, however, will only have its poles at the origin, where  $z = 0$ , as shown below in *Figure 3*. This is the difference compared to an IIR filter, which can have poles anywhere on the  $z$ -plane.

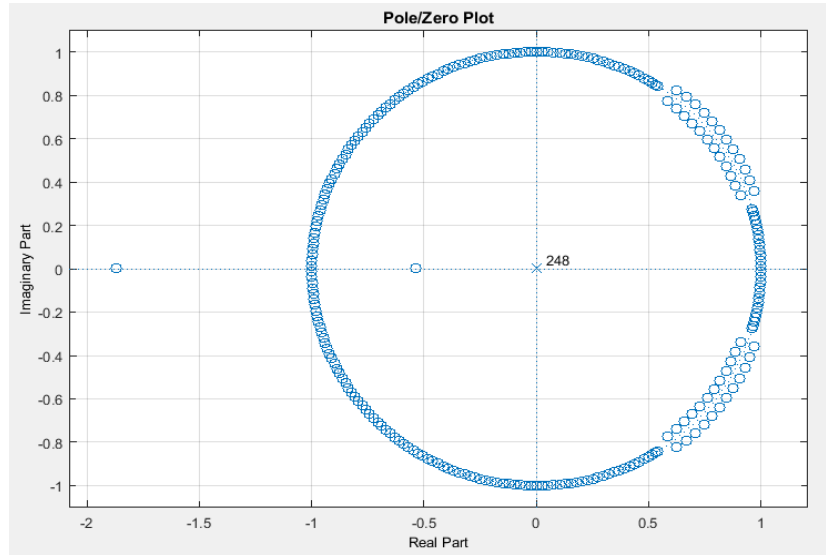


Figure 3: Z-Plane Plot

Only poles are considered when analysing the stability of the system. This is because if a pole exists outside the unit circle on the z-plane, the system will have an exponentially growing impulse response and thus will be unstable. It can be concluded that since the poles of a FIR filter are at the origin, the system is naturally stable.

Zeros, on the other hand, affect the magnitude response. The closer the zero to the unit circle, the larger the attenuation will be – hence zeros play no role in stability. Notice that there is a concentrated number of zeros around the stopband of the designed 248<sup>th</sup> ordered filter – this is desirable as it will attenuate the gain in the stopband, blocking unwanted signal frequencies. However, since there are only a finite number of zeros, the gain will never be able to attenuate to negative infinity, in which an ideal box shaped filter would. Having zeros so close together, and so close to the unit circle, produces the ripple in the stop-band observed in *Figure 4*. The gain can be viewed to have peaks and troughs, where each trough is the gain attenuation of one zero.

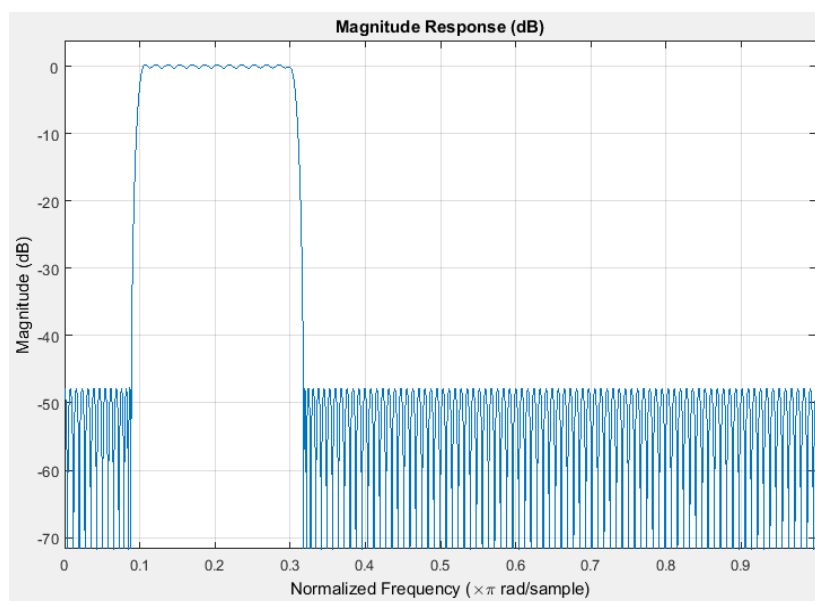
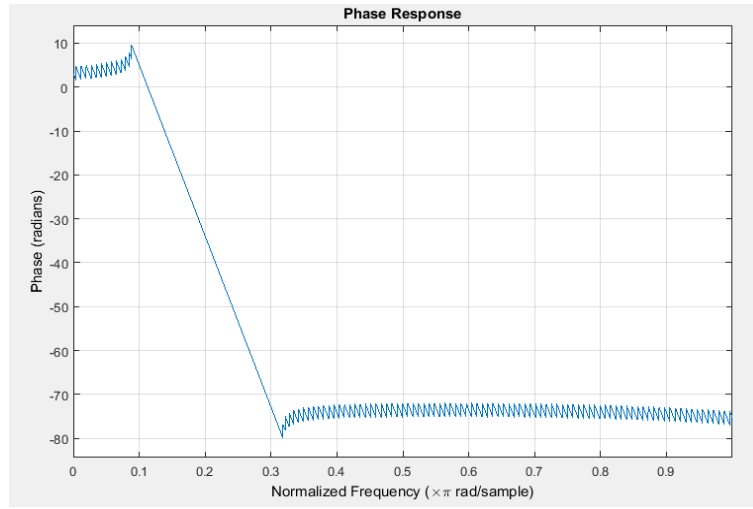


Figure 4: Magnitude Response

The phase response of the filter is shown in *Figure 5*.



*Figure 5: Phase Response*

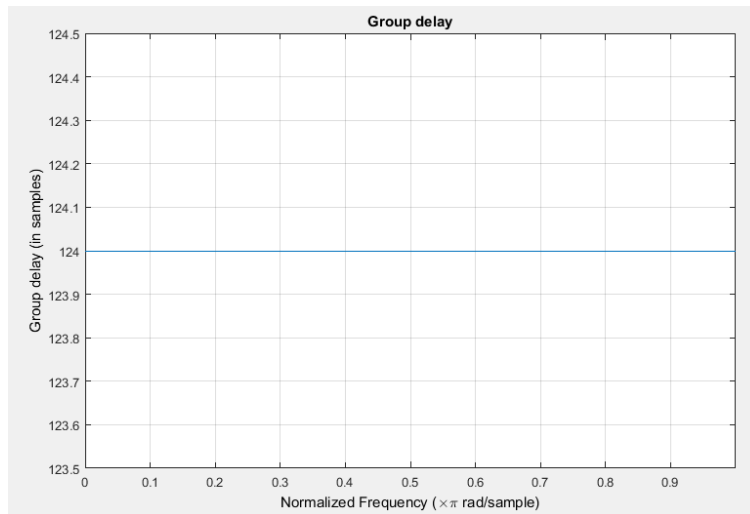
The FIR filter has an advantage of a linear phase in the passband. This is only possible when the filters coefficients are symmetrical. Since the group delay is calculated as,

$$\tau_g = -\frac{d\phi}{d\omega} = \frac{M-1}{2}$$

the delay through the filter is constant, preserving the shape of the waveform when it passes through the filter. This is especially important in audio signal processing where phase distortion is easily recognisable by the human ear.

In this case, the group delay has been measured to be 124, corresponding to the above equation.

$$\tau_g = \frac{M-1}{2} = \frac{249-1}{2} = 124$$



*Figure 6: Group Delay*

The 248<sup>th</sup> order filter does not meet the specification of remaining below a stopband gain of  $-48\text{ dB}$ . *Figures 7 and 8* illustrate the gain at the edges of the transition band whilst *Figure 9* compares the stopband gain to the desired threshold.

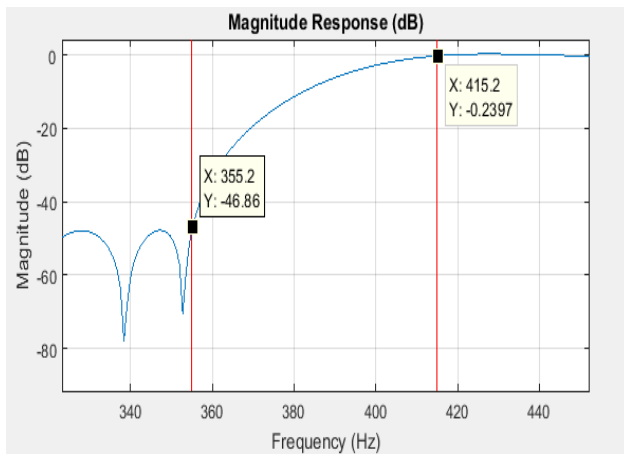


Figure 7: High Pass Transition Band

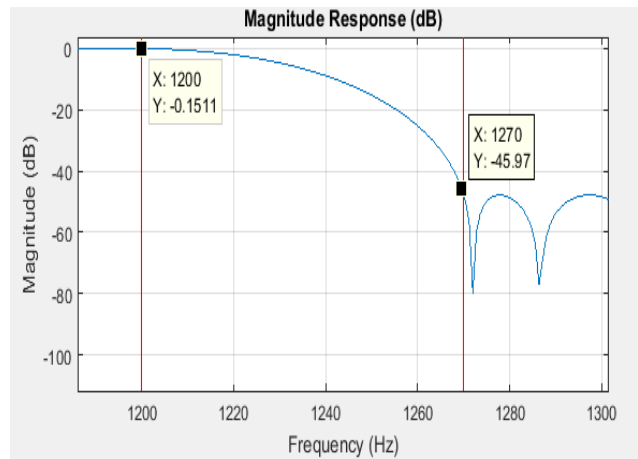


Figure 8: Low Pass Transition Band

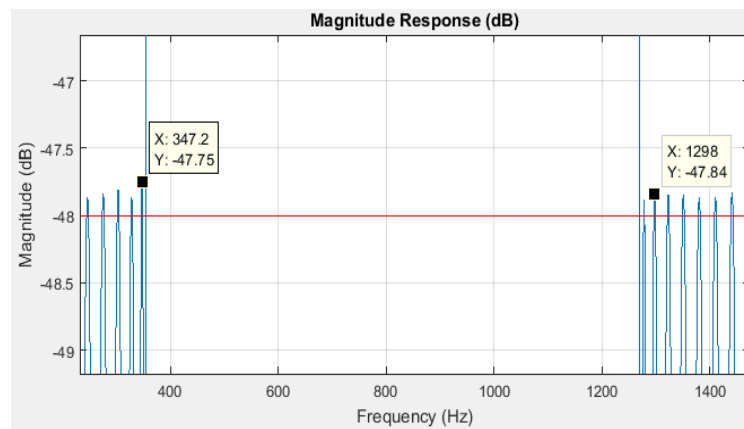


Figure 9: Minimum Stopband Gain Level

Furthermore, the passband ripple exceeds the maximum  $0.5\text{ dB}$  deviation, as measured in *Figure 10*.

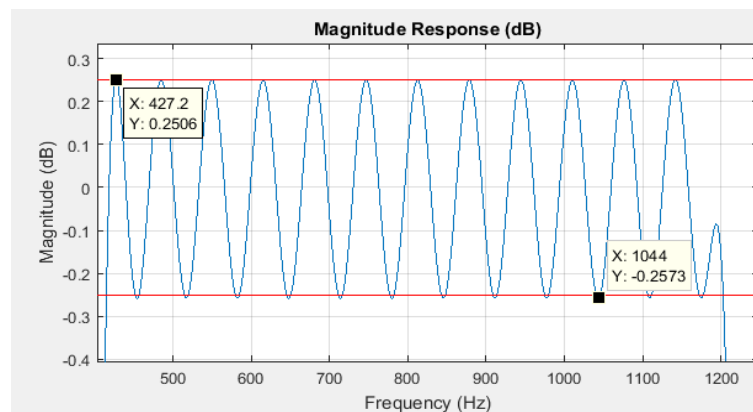


Figure 10: Gain Deviation in the Passband

In order to meet the specifications, the filter coefficients have to be increased. As the Parks-McClellan algorithm only produces filters of even orders, an even number of coefficients must be added before the **firmp** function. After some trial and error, an addition of 10 coefficients was decided to be sufficient.

The transition band has been measured as follows, illustrating the gain lies below  $-48\text{ dB}$  at the band edges.

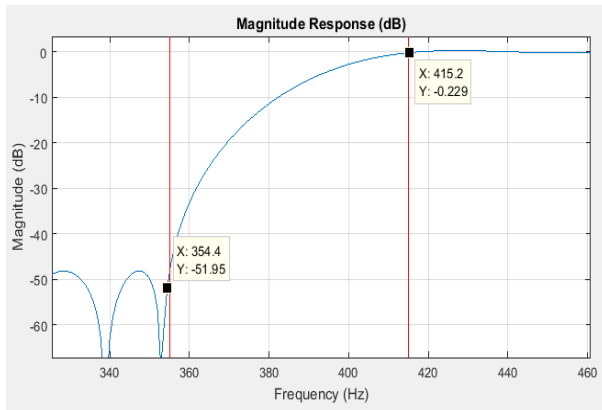


Figure 11: High Pass Filter Transition Band

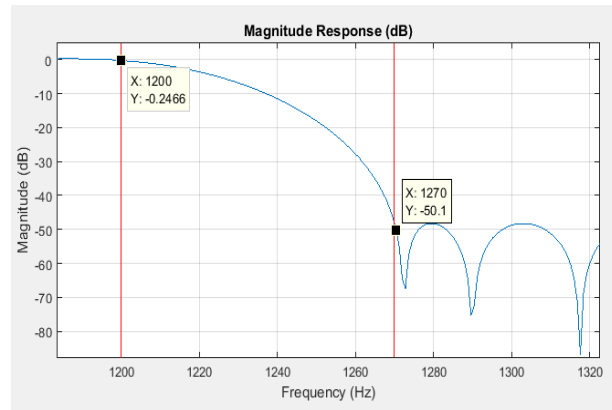


Figure 12: Low Pass Filter Transition Band

The stopband ripple is also lies below  $-48\text{ dB}$ , as desired.

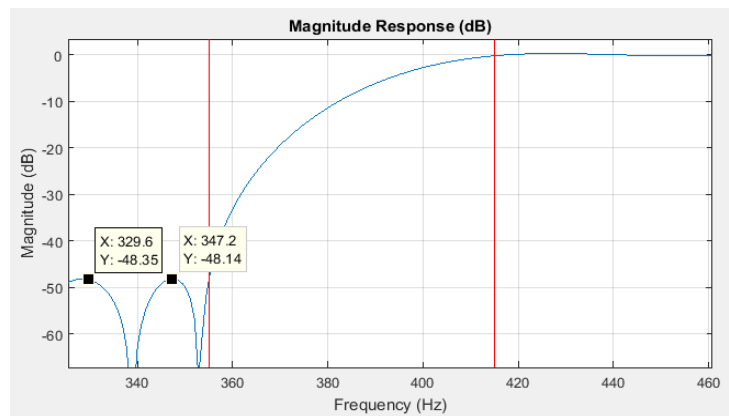


Figure 13: Stopband Gain

Finally, the passband ripple lies within the desired  $0.5\text{ dB}$  deviation.

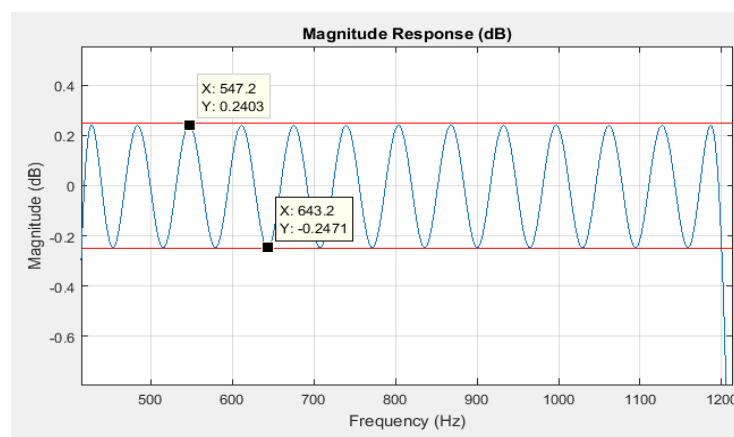


Figure 14: Passband Gain Deviation



The following plots have been repeated for the new 258<sup>th</sup> order filter for completion.

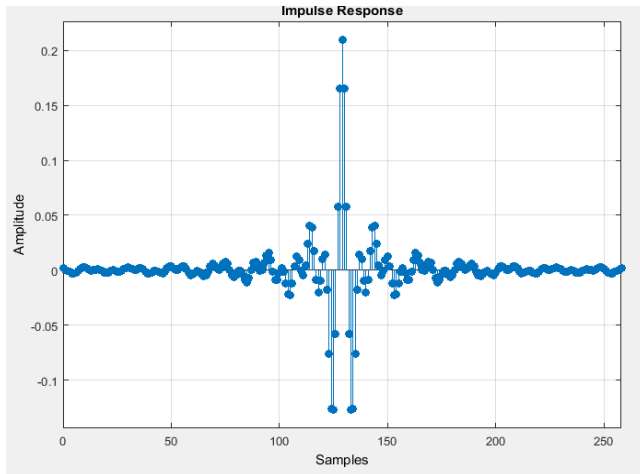


Figure 15: Impulse Response

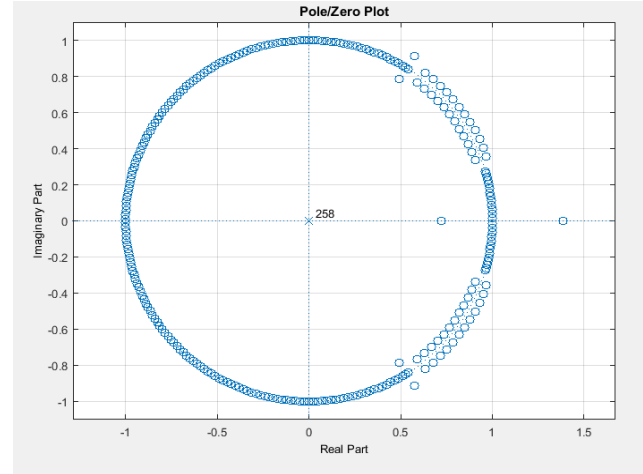


Figure 16: Pole-Zero Plot

The new group delay is,

$$\frac{M - 1}{2} = \frac{259 - 1}{2} = 129$$

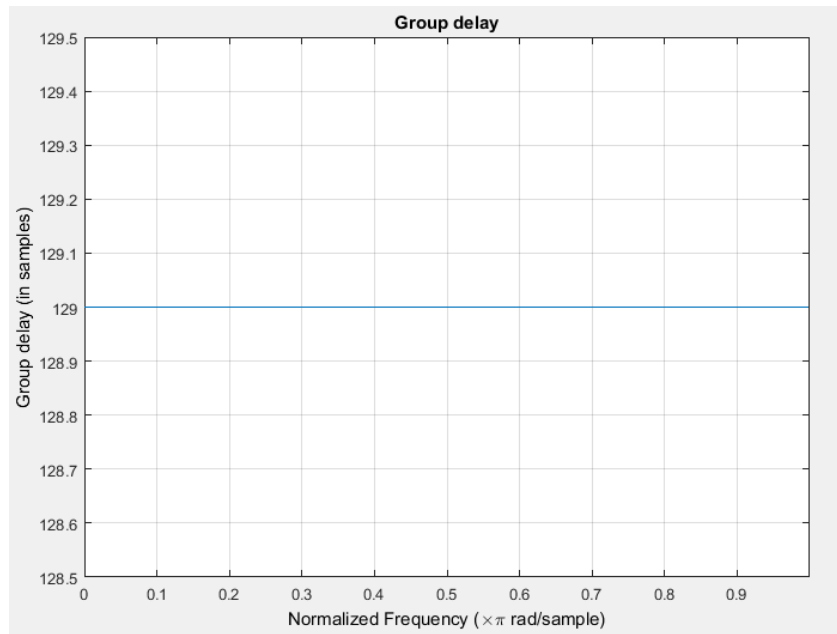


Figure 17: Group Delay

## 2. FILTER IMPLEMENTATION

Using the new Matlab designed 259 filter coefficients, the filter has been implemented via non-circular and circular buffer methods. Five different functions have been created and will be discussed in terms of their execution times. Before explaining how each function is implemented, the code outside the filter functions will be discussed briefly.

The global variables have been initialised and the filter coefficients from Matlab included as follows.

```
39 // define a 259 element delay buffer
40 #define N 259
41 // global variables
42 short x[N] = {0};
43 short X[2*N] = {0};
44 int index = N-1;
45 //include filter coefficients
46 #include "fir_coefs259.txt"
```

*Listing 2*

Each time the interrupt service routine is called, a sample is read from the ADC of the audio chip via **mono\_read\_16Bit**. This sample is then filtered using either non-circular or circular buffer methods and is written to the DAC via **mono\_write\_16Bits**. As these samples must be 16 bits long, all buffer arrays used throughout the code have been declared as **short**. Moreover, the arrays are declared global such that they do not re-initialise each time the ISR function is called.

```
171 /***** WRITE YOUR INTERRUPT SERVICE ROUTINE HERE*****/
172 void ISR_AIC(void)
173 {
174     short sample_in;
175     double output;
176
177     sample_in = mono_read_16Bit();
178
179     output = non_circ_FIR(sample_in);
180
181     mono_write_16Bit((Int16)output); // converts the output to 16 bits long
182
183 }
```

*Listing 3*

Finally, the above breakpoints have been added in order to measure the number of clock cycles each filter implementation function takes.

## 2.1 A NON-CIRCULAR BUFFER

The first filter design was realised via a non-circular buffer. The filter is perceived as a ‘delay line’, where all samples are stored in an array and are shuffled down each time a new sample arrives. In other words, the array is buffered via a FIFO method (first element in, first element out). The following **non\_circ\_FIR** function implements the buffer system in addition to convoluting the samples with the appropriate filter coefficients.

```
150 /***** Non-Circ Convolution Function *****/
151 double non_circ_FIR(short sample)
152 {
153     int i;
154     int j;
155     double sum = 0;
156
157     for (i = N-1; i>0; i--)
158     {
159         //shift elements down the buffer
160         x[i] = x[i-1];
161     }
162     //store new sample at the beginning
163     x[0] = sample;
164
165     for (j = 0; j<N; j++)
166     {
167         //convolution
168         sum += h[j]*x[j];
169     }
170
171     return sum;
172 }
```

Listing 4

Each time a new sample arrives, the elements in the array  $x$  are shifted to the right using a *for* loop. Once shifted, the new sample is then stored in the first element of the array,  $x[0]$ . Convolution occurs in the second *for* loop, in which convolution is defined as the following MAC (multiply accumulate operation) equation.

$$\sum_{k=0}^{N-1} h[k]x[n-k]$$

The result is stored in the variable *sum*, which is then returned to the **ISR\_AIC** function to be written to the DAC.

## 2.2 IMPROVING THE PERFORMANCE USING THE COMPILER

In order to evaluate the performance of the **non\_circ\_FIR** function, the number of clock cycles were measured, firstly with no optimisation and then with optimisation levels `-o0` and `-o2`.

Before looking at the results of the different optimisation levels, a better understanding of each level is conducted <sup>[1]</sup>.

- **-o0:**
  - Performs control-flow-graph simplification
  - Allocates variables to registers
  - Performs loop rotation
  - Eliminates unused code
  - Simplifies expressions and statements
  - Expands calls to functions declared inline
- **-o2:**
  - Performs software pipelining
  - Performs loop optimizations
  - Eliminates global common subexpressions
  - Eliminates global unused assignments
  - Converts array references in loops to incremented pointer form
  - Performs loop unrolling

At optimisation level `-o0`, the expressions and statements in the loop are simplified to reduce the complexity. Whereas at optimisation level `-o2`, loop unrolling and software pipelining further reduces the execution time by decreasing the number of times the loop condition is tested and carrying out multiple iterations in parallel.

Based on the results measured, optimisation level `-o2` had significantly increased the speed of the function.

Optimisation Level	Number of Clock Cycles
None	16,644
-o0	13,511
-o2	4,220

*Table 2: Execution Time using Non Circular Buffering*

## 2.3 SCOPE TRACES

In order to verify that the FIR filter has been correctly generated, output signals were observed and compared their subsequent input signals using the oscilloscope.

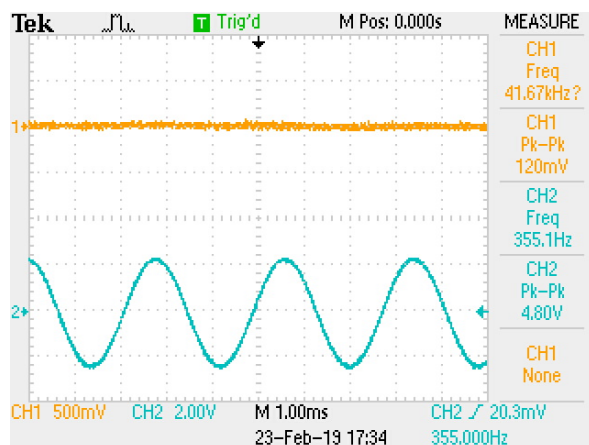


Figure 18: 355 Hz Input Frequency

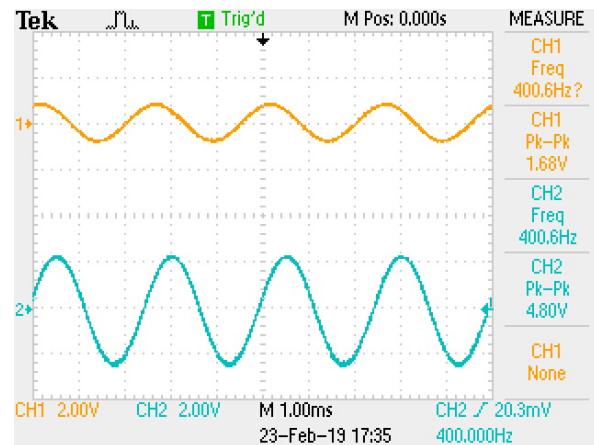


Figure 19: 400 Hz Input Frequency

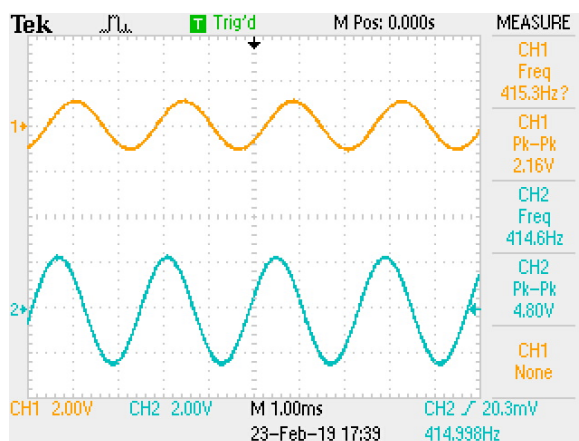


Figure 20: 415 Hz Input Frequency

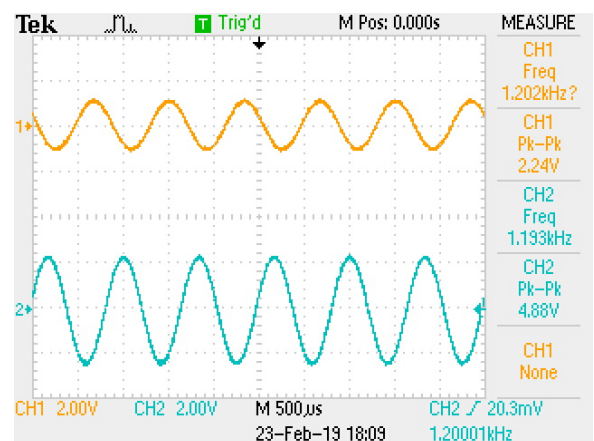


Figure 21: 1200 Hz Input Frequency

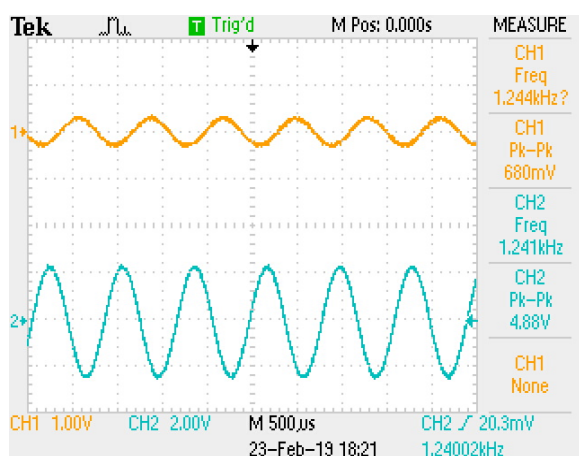


Figure 22: 1245 Hz Input Frequency

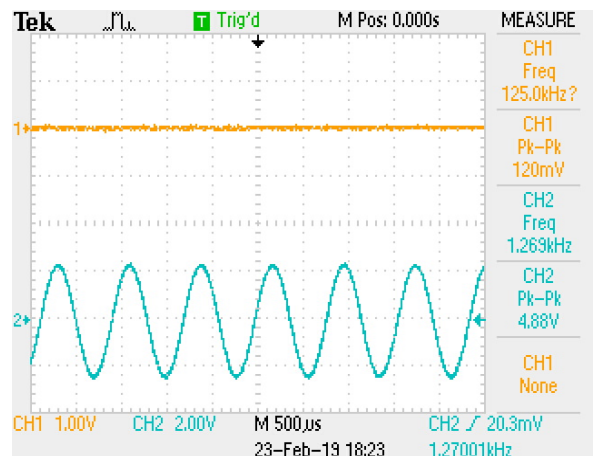
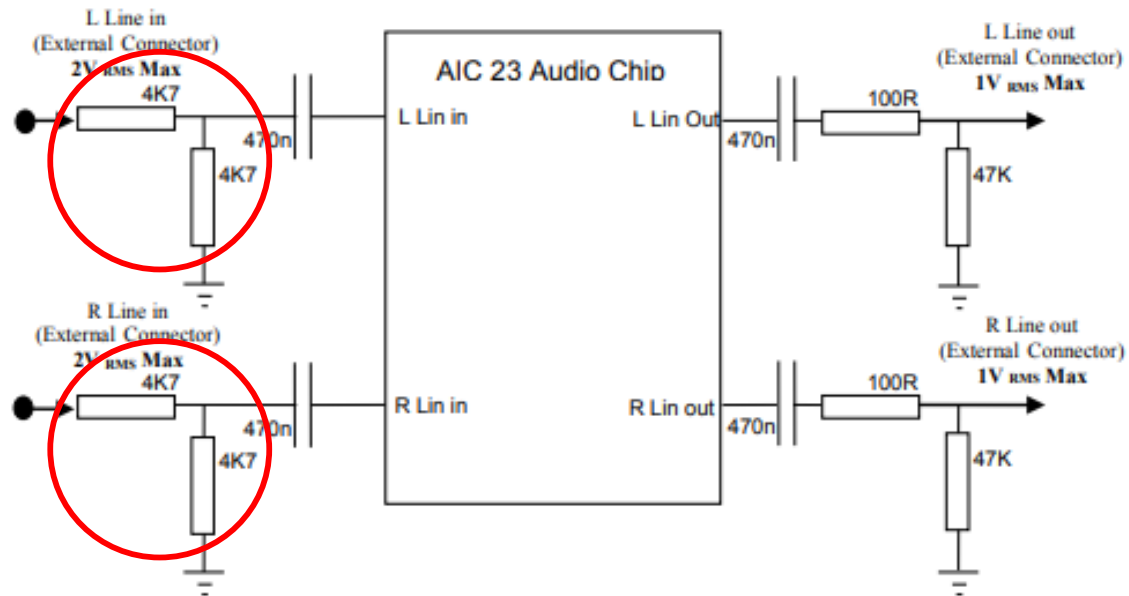


Figure 23: 1270 Hz Input Frequency

As desired, no signals were observed below 355 Hz and above 1270 Hz. Furthermore, signals in the middle of the transition band were measured with the correct frequencies, however were attenuated. One unexpected finding was that signals in the passband were attenuated by roughly 0.5 dB, as illustrated in *Figures 20 and 21*.

This unexpected attenuation is due to the potential divider found at the input of the audio chip. It halves the amplitude of the input signal before reaching the ADC.



*Figure 24: External Connections to the Audio Chip*

Another interesting finding observed was there is a phase shift between the input and output signals, i.e. the peaks of the two signals do not coincide. This is expected due to the group delay of the implemented filter. This observation is further discussed in Section 4 of the report.

## 3. DESIGNING A FASTER FILTER

### 3.1 METHOD 1 – CIRCULAR BUFFERING

As convolution is a critical operation in digital signal processing, the execution time is desired to be as low as possible, which can be achieved using efficient code. The previously used non-circular buffer is not the most efficient method due to the need to copy data from one element location to the next each time a new sample arrives.

Circular buffers can be used to eliminate this shifting process by storing the samples in an array with a size large enough to fit the new sample and all previous samples. In other words, when a new sample is inputted, it will be stored in a new location compared to the previous samples. Once the end of the array has been reached, a wrap-around effect occurs. The oldest sample will now be replaced with the newest sample as it will no longer be needed for following convolution calculations.

The first circular buffer method has been implemented via the following code.

```
174 /***** Circ Convolution Function 1 *****/
175 double circ_FIR_1(short sample)
176 {
177     int i;
178     double sum = 0;
179
180     //store new sample in array
181     x[index] = sample;
182
183     for(i = 0; i < N; i++){
184         //convolution
185         //incrementing the position in array after each iteration
186         //perform wrap around via modulus function
187         sum += h[i]*x[(index+i)%N];
188     }
189
190     //decrement index to prepare for next sample
191     //ie to store it in a new location in array
192     index--;
193     //if index is negative
194     if(index < 0){
195         //wrap around the array
196         index = N - 1;
197     }
198     return sum;
199 }
```

Listing 5

The global variable *index* can be perceived as a ‘pointer’, mapping the new sample to a new location in the array *x*. Once the new sample has been stored, a *for* loop implements convolution with the according FIR coefficients. Within the loop, the sample locations are based on *index*, which is incremented by the iterator *i* in order to access older samples.

However, as the first sample no longer begins at  $x[0]$ , there is a risk that the programme may access undefined locations in the buffer. To avoid this hazard, a modulus operation has been applied. This function returns the remainder of the division and thus the correct location to be accessed.

For example, consider the end case when  $index + i$  is equal to 259:

$$(index + i) \% N = 259 \% 259 = 0$$

Therefore 0 is returned, remaining within the bounds of the buffer array.

Once convolution has ended,  $index$  is decremented in order to prepare for the new sample. When  $index$  becomes a negative number, it is set back to  $N - 1$ , avoiding any accessing of undefined locations in  $x$ .

The function has the following clock cycle measurements, which have been found to be longer than the non-circular buffer implementation. This is believed to be due to the modulus operation, which comprises of three arithmetic operations: division, multiplication and subtraction.

$$A \% B = A - floor(A/B) * B$$

The amount of time needed to compute the result from a modulo is therefore longer than loading/storing data during shifting.

Optimization Level	Number of Cycles
None	18,232
-o0	16,133
-o2	13,533

Table 3: Execution Time using a Circular Buffer and a Modulus Function



### 3.2 METHOD 2 – REPLACING ARITHMETIC OPERATION WITH BRANCHING

In order to speed up the function's execution time, the modulus operation has been removed and replaced with an *if* statement.

```
201 /***** Circ Convolution Function 2 *****/
202 double circ_FIR_2(short sample)
203 {
204     int i;
205     double sum = 0;
206     //store new sample in array
207     x[index] = sample;
208
209     for(i = 0; i < N; i++){
210         //convolution
211         sum += h[i]*x[index];
212         index++; //increment index
213         //if index reaches the end of the array
214         if(index == N){
215             index = 0; //wrap around
216         }
217     }
218
219     index--;
220     if(index < 0){
221         index = N - 1; //wrap around
222     }
223     return sum;
```

Listing 6

Again, the pointer *index* is incremented each time the loop repeats. However, instead of adding *index* and *i* together, *index* itself is now incremented. There is no need to worry about losing the original value *index* holds as it is returned after exiting the *for* loop. An *if* condition has been placed to avoid *index* from overflowing.

This approach cuts down the number of clock cycles as comparisons can be done in parallel whereas previously taking the modulus with a number that is not a power of two can be a fairly expensive operation.

Optimization Level	Number of Cycles
None	15,116
-o0	13,784
-o2	1,026

Table 4: Execution Time using Circular Buffering and If Statements

### 3.3 METHOD 3 – EXPLOITING SYMMETRY

As discussed in Section 1.1, the impulse response of the FIR filter is symmetric, resulting in symmetrical coefficients. Considering this property, the *for* loop can be reduced by half, reducing the function's number of clock cycles.

In order to find a pattern between the array locations of the symmetric coefficients, convolution of an array of size 5 has been considered.

index	h[0]	h[1]	h[2]	h[3]	h[4]
4	x[4]	x[0]	x[1]	x[2]	x[3]
3	x[3]	x[4]	x[0]	x[1]	x[2]
2	x[2]	x[3]	x[4]	x[0]	x[1]
1	x[1]	x[2]	x[3]	x[4]	x[0]
0	x[0]	x[1]	x[2]	x[3]	x[4]

Due to the symmetry property of the coefficients,  $h[3] = h[1]$  and  $h[4] = h[0]$ . The common terms can be grouped together to produce the following table. As the filter has an odd number of coefficients, there will always be an odd one out which lies in the middle of the array.

index	h[0]	h[1]	h[2]
4	x[4] + x[3]	x[0] + x[2]	x[1]
3	x[3] + x[2]	x[4] + x[1]	x[0]
2	x[2] + x[1]	x[3] + x[0]	x[4]
1	x[1] + x[0]	x[2] + x[4]	x[3]
0	x[0] + x[4]	x[1] + x[3]	x[2]

The same method of convolution in a **for** loop will be implemented, however, since there are now two coefficients of equal values, two pointers will be used to access them simultaneously. The pointers motions is imagined as one travelling up the array, whilst the other travels down, retrieving the symmetrical coefficients.

As an example, refer to the values in the first column and row in the table: **ind\_1** will point to  $x[4]$  whilst **ind\_2** will point to  $x[3]$ . Throughout the convolution **for** loop, **ind\_1** will be incremented and **ind\_2** will be decremented.

Referring to the pattern found in the table, the code in *Listing 7* has been split into three different **if else** statements:

1. **Index lies in the upper half of the array:**  
Overflowing will only occur via  $x[ind\_1]$  hence only one **if** statement exists within the **for** loop.
2. **Index lies in the lower half of the array:**  
Overflowing will only occur via  $x[ind\_2]$  hence only one **if** statement exists within the **for** loop.
3. **Index lies in the middle of the array:**  
No overflow will occur hence no **if** statement exists.

If they had not been split in such a manner, both overflow scenarios would need to be checked in every loop of the **for** loop. Therefore, splitting it in such a way, and reducing the for loops by a half, has led to lower clock cycle measurements.

Optimization Level	Number of Cycles
None	9,944
-o0	6,415
-o2	669

Table 5: Execution Time Using Circular Buffering and Symmetry of Coefficients

```

226 /***** Circ Convolution Function 3 *****/
227 double circ_FIR_3(short sample){
228     int ind_1 = index; //incrementing pointer
229     int ind_2 = index-1; //decrementing pointer
230     int i;
231     double sum = 0;
232
233     x[index] = sample; //store sample
234
235     if(index > (N-1)/2){//if index belongs in upper half of array
236         for(i = 0; i < (N-1)/2; i++){
237             if(ind_1 > N-1){//check for overflow
238                 ind_1 = 0;
239             }
240             //perform convolution using symmetry property
241             sum += h[i]*(x[ind_1] + x[ind_2]);
242             ind_1++;
243             ind_2--;
244         }
245         //middle case
246         sum += h[(N-1)/2]*x[ind_1];
247     }
248     else if(index < (N-1)/2){//if index belongs in lower half of array
249         for(i = 0; i < (N-1)/2; i++){
250             if(ind_2 < 0){//check for overflow
251                 ind_2 = N-1;
252             }
253             sum += h[i]*(x[ind_1] + x[ind_2]);
254             ind_1++;
255             ind_2--;
256         }
257         //middle case
258         sum += h[(N-1)/2]*x[ind_1];
259     }
260     else{//if index belongs in center of array
261         for(i = 0; i < (N-1)/2; i++){
262             sum += h[i]*(x[ind_1] + x[ind_2]); //no overflow checks
263             ind_1++;
264             ind_2--;
265         }
266         //middle case
267         sum += h[(N-1)/2]*x[ind_1];
268     }
269
270     index--;
271     if(index < 0){
272         index = N - 1; //overflow check
273     }
274     return sum;
275 }

```

Listing 7

### 3.4 METHOD 4 – DOUBLING THE BUFFER

The execution time of the function can be further reduced by doubling the size of the array in which the samples are stored.

```
277 /***** Circ Convolution Function 4 *****/
278 double circ_FIR_4(short sample){
279     int ind_1 = index;
280     int ind_2 = index-1+N;
281     int i;
282     double sum = 0;
283
284     X[index] = sample; //store sample in 'first' array
285     X[index+N] = sample; //store sample in 'second' array
286
287     for(i = 0; i < (N-1)/2; i++){
288         //convolution using symmetry
289         sum += h[i]*(X[ind_1] + X[ind_2]);
290         ind_1++;
291         ind_2--;
292     }
293     //middle case
294     sum += h[(N-1)/2]*X[ind_1];
295
296     index--;
297     if(index < 0){
298         index = N - 1; //overflow prevention
299     }
300     return sum;
301 }
```

Listing 8

Two sets of samples will now be present in the array, allowing the pointer *ind\_1* to access locations previously described as ‘undefined’, i.e. in the case of a 4<sup>th</sup> order filter:  $x[5]$ ,  $x[6]$  etc. Moreover, by initialising *ind\_2* to begin at the end of the array, no overflow will occur as it decrements within the *for* loop.

In other words, all overflow scenarios have been avoided and hence has reduced the number of *if* statements and thus the number of branching calculations and clock cycles.

Optimization Level	Number of Cycles
None	8,504
-o0	5,375
-o2	585

Table 6: Execution Times using a Double Array

## 4. MEASUREMENT OF FILTER RESPONSE

The frequency response of the latter, fastest filter implementation has been found using the spectrum analyser.

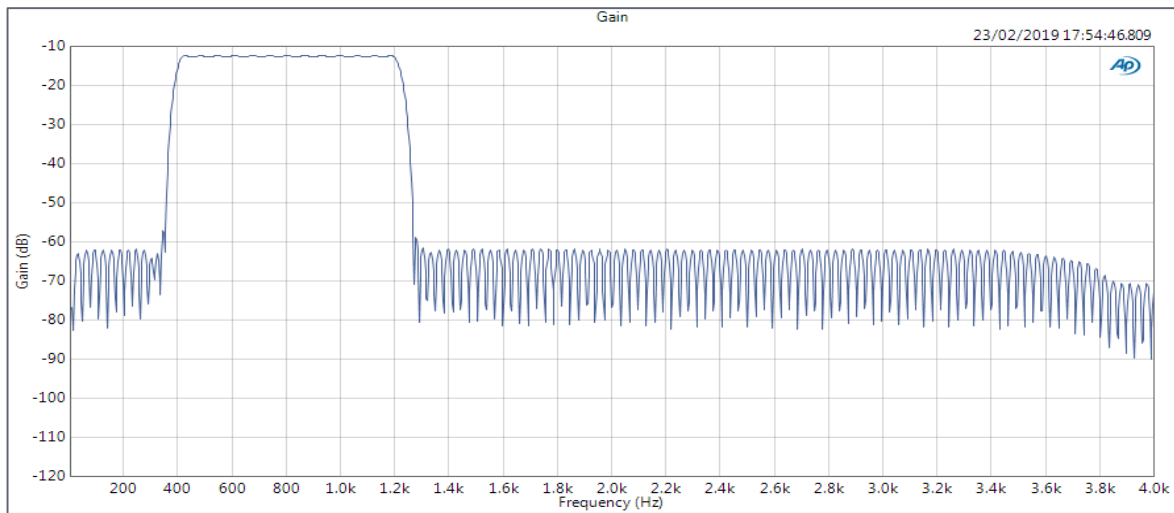


Figure 25: Magnitude Response

The gain in the passband is not the expected 0 dB but lies around  $-12\text{ dB}$ . This is due to a gain factor of  $\frac{1}{4}$  found between the spectrum analyser and the input of the codec, as shown below in Figure 26. This has been implemented as a safety precaution, preventing signals of significant amplitudes damaging the DSK unit.

$$20 \log \left( \frac{1}{4} \right) \approx -12.04$$

The magnitude response can thus be perceived to have been shifted down by 12 dB. If this ‘offset’ didn’t exist, the gain at the passband would be 0 dB and the stopband would have a maximum gain at  $-50\text{ dB}$ .

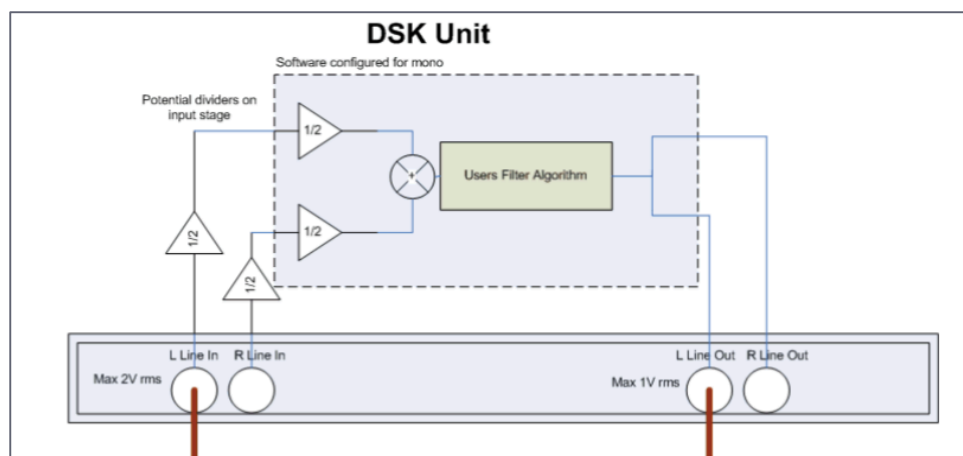


Figure 26: Connections Between the Spectrum Analyser and DSK

Furthermore, notice how the gain begins to roll as 4 kHz is approached. This is due to the low pass filter at the output of the audio chip that has a cut off frequency of 4 kHz.

To check if the filter has met the specifications and the predictions on Matlab, the gain difference between the transition band was measured, as shown below. It was found to be  $-43.456\text{ dB}$ , which does not match the specification of  $-48\text{ dB}$ .

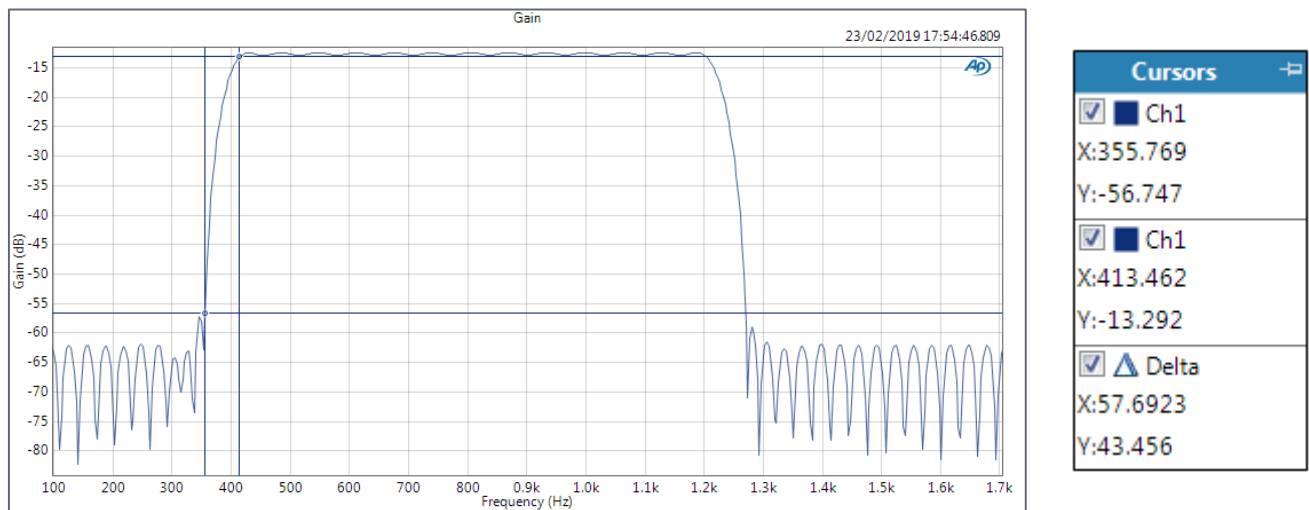


Figure 27: High Pass Transition Band

On the other hand, the passband ripple does satisfy the  $0.5\text{ dB}$  deviation limit. It was measured as  $0.468\text{ dB}$ .

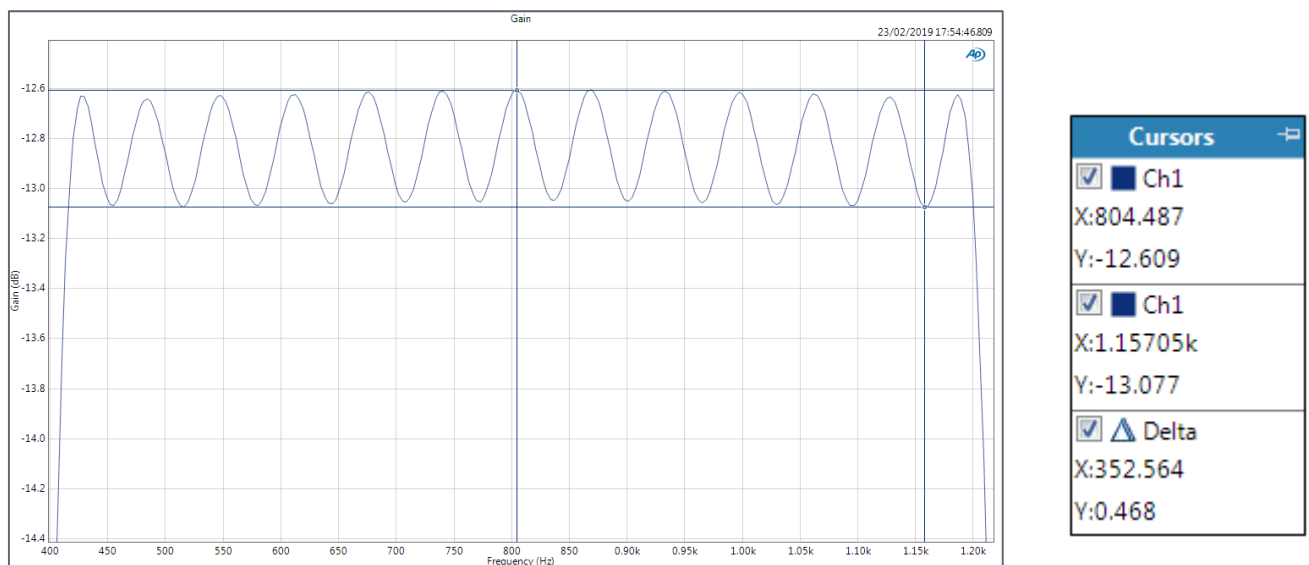


Figure 28: Passband Deviation

Figure 29 illustrates the measured phase response of the system. A linear phase response is found within the passband and is more clearly shown in Figure 30. However, unlike the Matlab plots, the phase does not flatten out in the stopband.

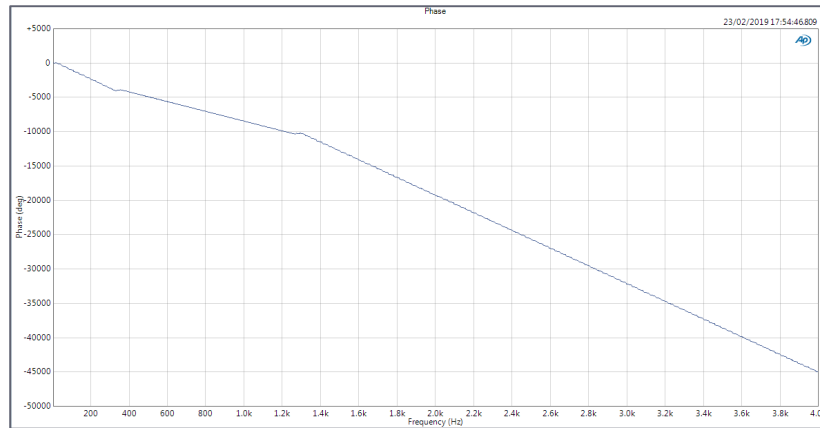


Figure 29: Phase Response

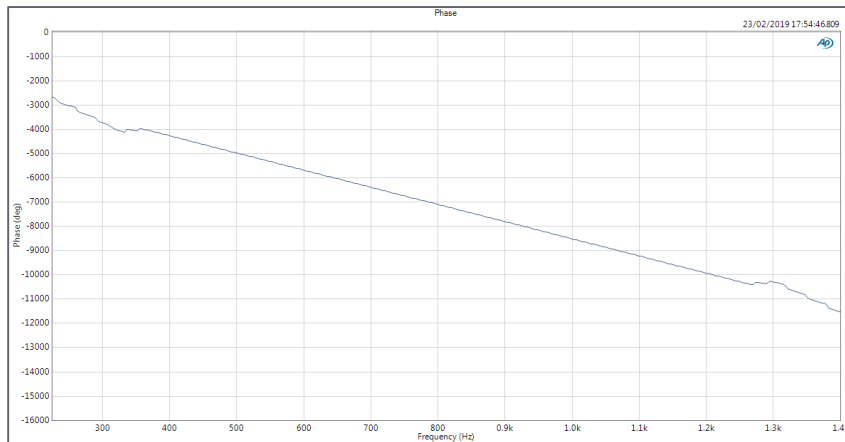


Figure 30: Zoomed in Phase Response

In order to understand the decreasing phase response in the stopband, the phase response excluding the FIR filtering has been additionally measured and is shown below in Figure 31.

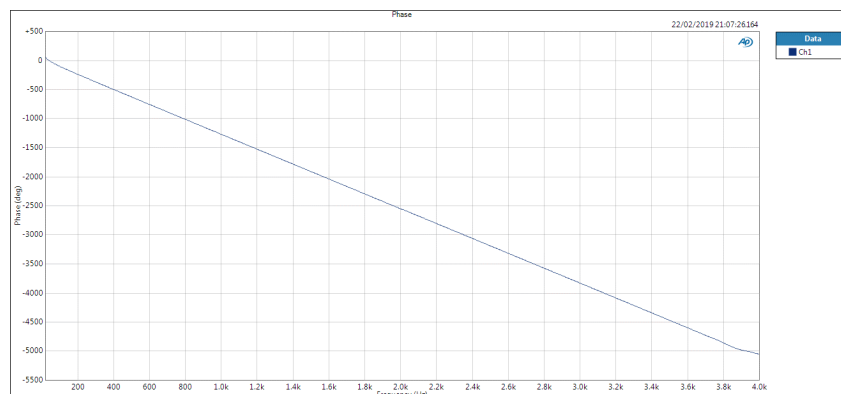
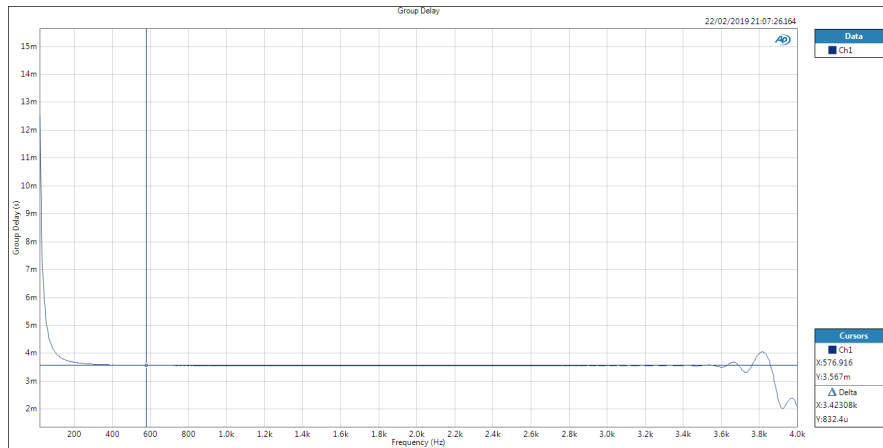


Figure 31: Unfiltered Phase Response



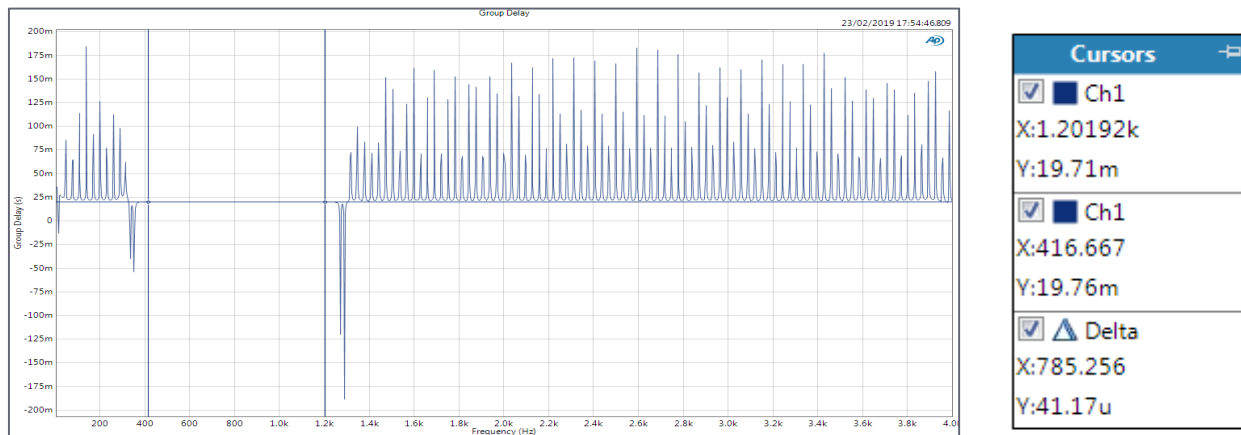
The group delay of the system is the reason for this linearly decreasing phase response. As discussed previously, the group delay is calculated as the derivative of the phase, hence a constant group delay results in a linear phase response. *Figure 32* shows the group delay of the unfiltered system.



*Figure 32: Group Delay of Unfiltered System*

The delay was measured to be around  $3.567 \times 10^{-3}$  and is believed to be caused by the circuitry within overall system. Matlab does not take these parasitic delays into account when measuring the phase response and hence assumed there was 0 delay in the stopband and thus a constant phase.

The group delay of the filtered system was then measured for comparison (*Figure 33*).



*Figure 33: Group Delay of Filtered System*

The expected group delay of the filter was previously measured in Section 1.2 to be 129. In the time domain, this corresponds to a value of,

$$\frac{129}{f_{\text{sampling}}} = \frac{129}{8000} = 16.125 \times 10^{-3}$$

However, due to the extra delay previously discussed, it does not correspond to the predicted value and has been measured to be  $19.76 \times 10^{-3}$ .

It can be concluded that even more filter coefficients would be needed to meet the desired specifications.

## 5. REFERENCES

- [1] <http://www.ti.com/lit/ug/spru187u/spru187u.pdf>