Subjective Questions on liner regression

1. Explain the linear regression algorithm in detail.
2. What are the assumptions of linear regression regarding residuals?
3. What is the coefficient of correlation and the coefficient of determination?
4. Explain the Anscombe's quartet in detail.
5. What is Pearson's R?
6. What is scaling? Why is scaling performed? What is the difference between normalized scaling and standardized scaling?
7. You might have observed that sometimes the value of VIF is infinite. Why does this happen?
8. What is the Gauss-Markov theorem?
9. Explain the gradient descent algorithm in detail.
10. What is a Q-Q plot? Explain the use and importance of a Q-Q plot in linear regression.

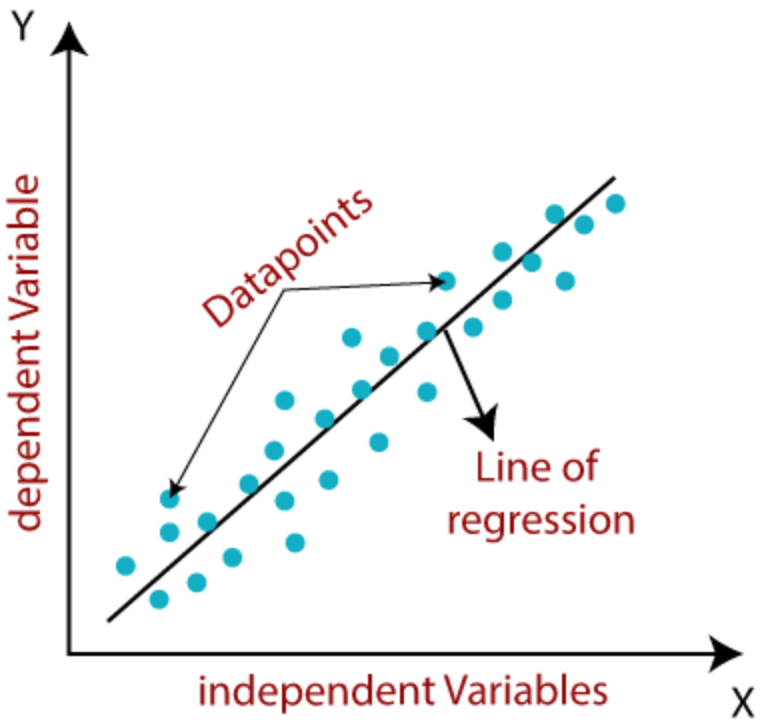
1. Explain the linear regression algorithm in detail.

Linear Regression is a machine learning algorithm based on **supervised learning**. It performs a **regression task**. Regression models a target prediction value based on independent variables. It is mostly used for finding out the relationship between variables and forecasting. Different regression models differ based on – the kind of relationship between dependent and independent variables, they are considering and the number of independent variables being used

Linear regression is one of the easiest and most popular Machine Learning algorithms. It is a statistical method that is used for predictive analysis. Linear regression makes predictions for continuous/real or numeric variables such as **sales, salary, age, product price,** etc.

Linear regression algorithm shows a linear relationship between a dependent (y) and one or more independent (y) variables, hence called as linear regression. Since linear regression shows the linear relationship, which means it finds how the value of the dependent variable is changing according to the value of the independent variable.

The linear regression model provides a sloped straight line representing the relationship between the variables



Mathematically, we can represent a linear regression as:

$$y= a_0+a_1x$$

Y= Dependent Variable (Target Variable)
X= Independent Variable (predictor Variable)
a0= intercept of the line (Gives an additional degree of freedom)
a1 = Linear regression coefficient (scale factor to each input value).

The values for x and y variables are training datasets for Linear Regression model representation

Types of Linear Regression

Linear regression can be further divided into two types of the algorithm:

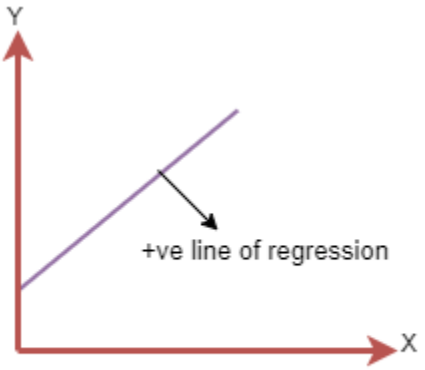
- **Simple Linear Regression:**
  If a single independent variable is used to predict the value of a numerical dependent variable, then such a Linear Regression algorithm is called Simple Linear Regression.
- **Multiple Linear regression:**
  If more than one independent variable is used to predict the value of a numerical

dependent variable, then such a Linear Regression algorithm is called Multiple Linear Regression.

Linear Regression Line

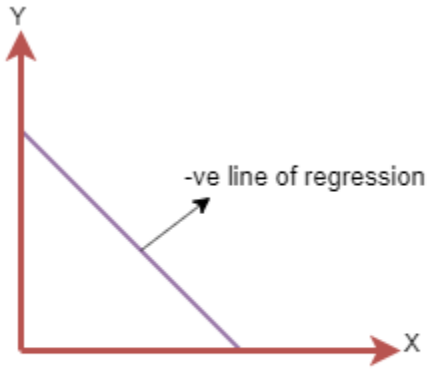
A linear line showing the relationship between the dependent and independent variables is called a **regression line**. A regression line can show two types of relationship:

- **Positive Linear Relationship:**
  If the dependent variable increases on the Y-axis and independent variable increases on X-axis, then such a relationship is termed as a Positive linear relationship.



The line equation will be: $Y= a_0+a_1x$

- **Negative Linear Relationship:**
  If the dependent variable decreases on the Y-axis and independent variable increases on the X-axis, then such a relationship is called a negative linear relationship.



The line of equation will be: $Y= -a_0+a_1x$

Finding the best fit line:

When working with linear regression, our main goal is to find the best fit line that means the error between predicted values and actual values should be minimized. The best fit line will have the least error.

The different values for weights or the coefficient of lines (a_0, a_1) gives a different line of regression, so we need to calculate the best values for a_0 and a_1 to find the best fit line, so to calculate this we use cost function.

Cost function-

- The different values for weights or coefficient of lines (a_0, a_1) gives the different line of regression, and the cost function is used to estimate the values of the coefficient for the best fit line.
- Cost function optimizes the regression coefficients or weights. It measures how a linear regression model is performing.
- We can use the cost function to find the accuracy of the **mapping function**, which maps the input variable to the output variable. This mapping function is also known as **Hypothesis function**.

For Linear Regression, we use the **Mean Squared Error (MSE)** cost function, which is the average of squared error occurred between the predicted values and actual values. It can be written as:

For the above linear equation, MSE can be calculated as:

$$MSE = 1\frac{1}{N}\sum_{i=1}^{n}(y_i - (a_1x_i + a_0))^2$$

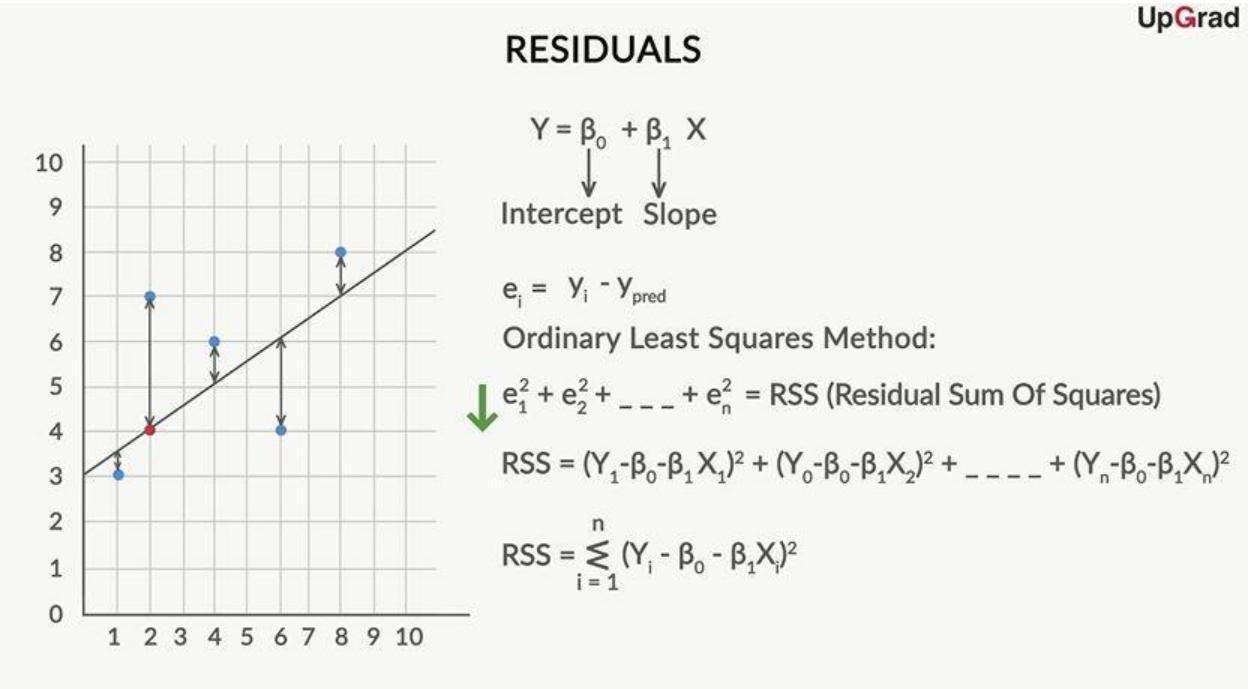
Where,

N=Total number of observation
Yi = Actual value
$(a1x_i+a_0)$= Predicted value.

Residuals: The distance between the actual value and predicted values is called residual. If the observed points are far from the regression line, then the residual will be high, and so cost function will high. If the scatter points are close to the regression line, then the residual will be small and hence the cost function.

The best-fit line is found by minimising the expression of RSS (Residual Sum of Squares) which is equal to the sum of squares of the residual for each data point in the

plot. Residuals for any data point is found by subtracting predicted value of dependent variable from actual value of dependent variable:



Gradient Descent:

- Gradient descent is used to minimize the MSE by calculating the gradient of the cost function.
- A regression model uses gradient descent to update the coefficients of the line by reducing the cost function.
- It is done by a random selection of values of coefficient and then iteratively update the values to reach the minimum cost function.

Y= Dependent Variable (Target Variable)
X= Independent Variable (predictor Variable)
a0= intercept of the line (Gives an additional degree of freedom)
a1 = Linear regression coefficient (scale factor to each input value).
ε = random error

The values for x and y variables are training datasets for Linear Regression model representation

Assumptions of linear regression

The following are some assumptions about dataset that is made by Linear Regression model −

Multi-collinearity − Linear regression model assumes that there is very little or no multi-collinearity in the data. Basically, multi-collinearity occurs when the independent variables or features have dependency in them.

Auto-correlation − Another assumption Linear regression model assumes is that there is very little or no auto-correlation in the data. Basically, auto-correlation occurs when there is dependency between residual errors.

Relationship between variables − Linear regression model assumes that the relationship between response and feature variables must be linear.

2. What are the assumptions of linear regression regarding residuals?

Assumption 1 : The Regression model is linear in its parameters (which are Coefficients and the error term).

Linearity: The change in the response variable due to one unit change in the predictor variable(Xk) is always constant irrespective of the current value of Xk

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k + \epsilon$$

The defining characteristic of linear regression is its functional form and to satisfy this assumption, the model should be correctly defined

Assumption 2 : Independent variables should not be perfectly correlated with each other (No Multicollinearity)

Checking for Multicollinearity:

Multicollinearity can be checked using *Variance Inflation factor(VIF).* VIF is calculated using the below formula.

$$VIF(\hat{\beta}_j) = \frac{1}{1 - R^2_{X_j|X_{-j}}}$$

VIF > 5 is generally considered problematic and VIF > 10 suggests a definite presence of collinearity

Assumption 3 : Mean of the residuals should be Zero

Assumption 4 : Residuals should not be correlated with the independent variables

As mentioned above, error term represents the unexplained variance in the response variable. Now if residuals are correlated with the independent variable, we can use the independent variables to predict the error which is fundamentally wrong. This correlation between error terms and independent variables is known as *endogeneity*.

Assumption 5 : Standard Deviation of the residuals should be constant (Homoscedasticity)

Assumption 6 : Residuals should not be correlated with each other.

Assumption 7 : Residuals should be normally distributed.

Assumption 8 : Independent variables should have positive variance.

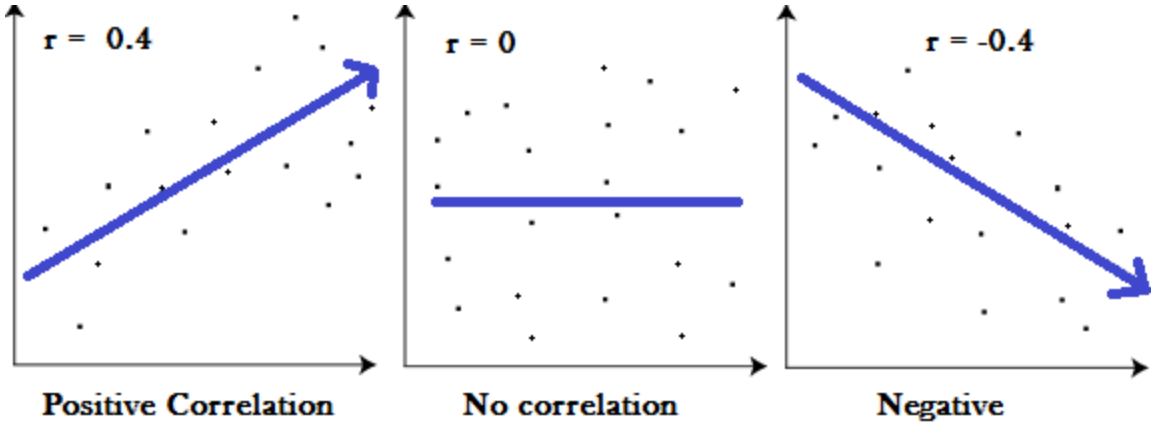
Assumption 9: Number of observations should be more than the number of features.

3. What is the coefficient of correlation and the coefficient of determination?

The correlation coefficient is a measure of linear association between two variables. Values of the correlation coefficient are always between -1 and +1. A correlation coefficient of +1 indicates that two variables are perfectly related in a positive linear sense, a correlation coefficient of -1 indicates that two variables are perfectly related in a negative linear sense, and a correlation coefficient of 0 indicates that there is no linear relationship between the two variables

Correlation coefficient formulas are used to find how strong a relationship is between data. The formulas return a value between -1 and 1, where:

- 1 indicates a strong positive relationship.
- -1 indicates a strong negative relationship.
- A result of zero indicates no relationship at all



The **coefficient of determination** (denoted by R^2). The coefficient of determination, R^2, is used to analyze how differences in one variable can be explained by a difference in a second variable Values can range from 0.00 to 1.00, or 0 to 100%.

- In terms of regression analysis, the coefficient of determination is an overall measure of the accuracy of the regression model.
- In simple linear regression analysis, the calculation of this coefficient is to square the *r* value between the two values, where *r* is the correlation coefficient.
- In a multiple linear regression analysis, R^2 is known as the multiple correlation coefficient of determination.
- It helps to describe how well a regression line fits (a.k.a., goodness of fit). An R^2 value of 0 indicates that the regression line does not fit the set of data points and a value of 1 indicates that the regression line perfectly fits the set of data points.
- By definition, R^2 is calculated by one minus the Sum of Squares of Residuals (SS_{error}) divided by the Total Sum of Squares (SS_{total}): $R^2 = 1 - (SS_{error} / SS_{total})$.

4. Explain the Anscombe's quartet in detail

Anscombe's Quartet

The statistician France Anscombe constructed the Anscombe dataset in 1973.

Anscombe created the dataset to demonstrate the importance of visualizing data and also to highlight the effect that outliers can have on a statistical findings of a dataset.

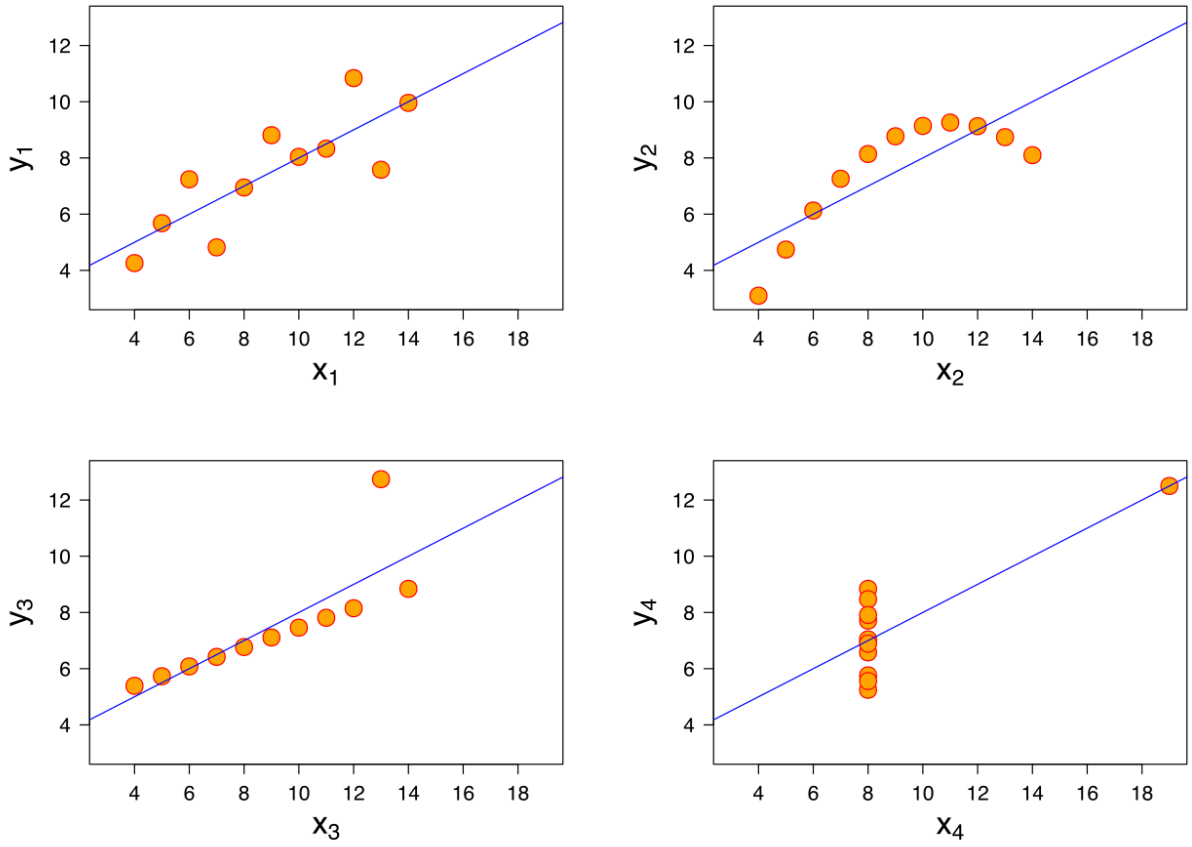
Anscombe's Quartet consists of four data sets, that when examined have nearly the identical statistical properties, yet when graphed the datasets tell a very different story.

	I		II		III		IV	
	x	y	x	y	x	y	x	y
	10	8,04	10	9,14	10	7,46	8	6,58
	8	6,95	8	8,14	8	6,77	8	5,76
	13	7,58	13	8,74	13	12,74	8	7,71
	9	8,81	9	8,77	9	7,11	8	8,84
	11	8,33	11	9,26	11	7,81	8	8,47
	14	9,96	14	8,1	14	8,84	8	7,04
	6	7,24	6	6,13	6	6,08	8	5,25
	4	4,26	4	3,1	4	5,39	19	12,5
	12	10,84	12	9,13	12	8,15	8	5,56
	7	4,82	7	7,26	7	6,42	8	7,91
	5	5,68	5	4,74	5	5,73	8	6,89
SUM	99,00	82,51	99,00	82,51	99,00	82,50	99,00	82,51
AVG	9,00	7,50	9,00	7,50	9,00	7,50	9,00	7,50
STDEV	3,32	2,03	3,32	2,03	3,32	2,03	3,32	2,03

The summary statistics show that the means and the variances were identical for x and y across the groups :

- Mean of x is 9 and mean of y is 7.50 for each dataset.
- Similarly, the variance of x is 11 and variance of y is 4.13 for each dataset
- The correlation coefficient (how strong a relationship is between two variables) between x and y is 0.816 for each dataset

When we plot these four datasets on an x/y coordinate plane, we can observe that they show the same regression lines as well but each dataset is telling a different story :



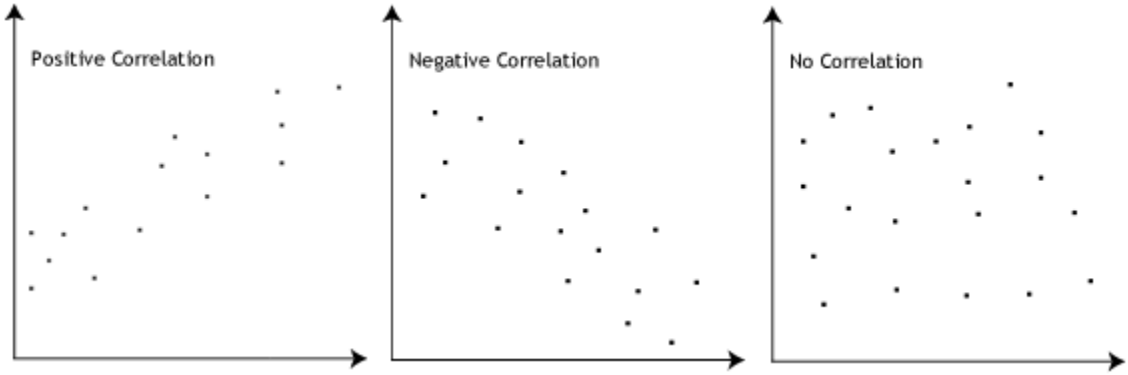
- Dataset I appears to have clean and well-fitting linear models.
- Dataset II is not distributed normally.
- In Dataset III the distribution is linear, but the calculated regression is thrown off by an outlier.
- Dataset IV shows that one outlier is enough to produce a high correlation coefficient.

This quartet emphasizes the importance of visualization in Data Analysis. Looking at the data reveals a lot of the structure and a clear picture of the dataset.

5. What is Pearson's R?

The Pearson product-moment correlation coefficient (or Pearson correlation coefficient, for short) is a measure of the strength of a linear association between two variables and is denoted by *r*. Basically, a Pearson product-moment correlation attempts to draw a line of best fit through the data of two variables, and the Pearson correlation coefficient, *r*, indicates how far away all these data points are to this line of best fit (i.e., how well the data points fit this new model/line of best fit).

The Pearson correlation coefficient, *r*, can take a range of values from +1 to -1. A value of 0 indicates that there is no association between the two variables. A value greater than 0 indicates a positive association; that is, as the value of one variable increases, so does the value of the other variable. A value less than 0 indicates a negative association; that is, as the value of one variable increases, the value of the other variable decreases. This is shown in the diagram below:



For example, in the stock market, if we want to measure how two stocks are related to each other, Pearson *r* correlation is used to measure the degree of relationship between the two. The point-biserial correlation is conducted with the Pearson correlation formula except that one of the variables is dichotomous. The following formula is used to calculate the Pearson *r* correlation:

$$r_{xy} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{n\sum x_i^2 - (\sum x_i)^2}\sqrt{n\sum y_i^2 - (\sum y_i)^2}}$$

r_{xy} = Pearson r correlation coefficient between x and y
n = number of observations
x_i = value of x (for ith observation)
y_i = value of y (for ith observation)

6. What is scaling? Why is scaling performed? What is the difference between normalized scaling and standardized scaling?

Scaling is a technique to standardize the independent features present in the data in a fixed range. It is performed during the data pre-processing to handle highly varying magnitudes or values or units. If feature scaling is not done, then a machine learning algorithm tends to weigh greater values, higher and consider smaller values as the lower values, regardless of the unit of the values.

Example: If an algorithm is not using feature scaling method then it can consider the value 3000 meter to be greater than 5 km but that's actually not true and in this case, the algorithm will give wrong predictions. So, we use Feature Scaling to bring all values to same magnitudes and thus, tackle this issue.

How to Scale Features

There are four common methods to perform Feature Scaling.

1. **Standardisation:** Standardisation replaces the values by their Z scores.

$$x' = \frac{x - \bar{x}}{\sigma}$$

This redistributes the features with their mean **μ = 0** and standard deviation **σ =1** . sklearn.preprocessing.scale helps us implementing standardisation in python.

2. Mean Normalisation:

$$x' = \frac{x - \text{mean}(x)}{\max(x) - \min(x)}$$

This distribution will have values between **-1 and 1**with **μ=0**.

Standardisation and **Mean Normalization** can be used for algorithms that assumes zero centric data like **Principal Component Analysis(PCA).**

3. Min-Max Scaling:

$$x' = \frac{x - \min(x)}{\max(x) - \min(x)}$$

This technique re-scales a feature or observation value with distribution value between 0 and 1.

4. Unit Vector:

$$x' = \frac{x}{||x||}$$

Scaling is done considering the whole feature vecture to be of unit length.

Min-Max Scaling and **Unit Vector** techniques produces values of range [0,1]. When dealing with features with hard boundaries this is quite useful. For example, when dealing with image data, the colors can range from only 0 to 255.

Normalization vs. Standardization

The terms *normalization* and *standardization* are sometimes used interchangeably, but they usually refer to different things. *Normalization* usually means to scale a variable to have a values between 0 and 1, while *standardization* transforms data to have a mean of zero and a standard deviation of 1. This standardization is called a **z-score**, and data points can be standardized with the following formula:

$$z_i = \frac{x_i - \bar{x}}{s}$$

7. You might have observed that sometimes the value of VIF is infinite. Why does this happen ?

A large VIF implies that the variable is redundant with other variables in the data set. An infinite VIF value indicates that the corresponding variable may be expressed exactly by a linear combination of other variables ,which show an infinite VIF as well.

VIF is an index that provides a measure of how much the variance of an estimated regression coefficient increases due to collinearity. In order to determine VIF.

If all the independent variables are orthogonal to each other, then VIF = 1.0. If there is perfect correlation, then VIF = infinity. A large value of VIF indicates that there is a correlation between the variables.

If the VIF is 4, this means that the variance of the model coefficient is inflated by a factor of 4 due to the presence of multicollinearity. This would mean that that standard error of this coefficient is inflated by a factor of 2 (square root of variance is the standard deviation).

8. What is the Gauss-Markov theorem?

The Gauss Markov theorem says that, under certain conditions, the ordinary least squares (OLS) estimator of the coefficients of a linear regression model is the best linear unbiased estimator (BLUE), that is, the estimator that has the smallest variance among those that are unbiased and linear in the observed output variables.

The **Gauss Markov theorem** tells us that if a certain set of assumptions are met, the ordinary least squares estimate for regression coefficients gives you the *best linear unbiased estimate (BLUE)* possible.

Gauss Markov Assumptions

There are five Gauss Markov assumptions (also called *conditions*):

1. **Linearity**: the parameters we are estimating using the OLS method must be themselves linear.
2. **Random**: our data must have been randomly sampled from the population.
3. **Non-Collinearity**: the regressors being calculated aren't perfectly correlated with each other.
4. **Exogeneity**: the regressors aren't correlated with the error term.
5. **Homoscedasticity**: no matter what the values of our regressors might be, the error of the variance is constant

We can summarize the Gauss-Markov Assumptions succinctly in algebra, by saying that a linear regression model represented by

$y_i = x_i' \beta + \varepsilon_i$

and generated by the ordinary least squares estimate is the best linear unbiased estimate (BLUE) possible if

- $E\{\varepsilon_i\} = 0$, $i = 1, \ldots, N$
- $\{\varepsilon_1 \ldots\ldots \varepsilon_n\}$ and $\{x_1 \ldots.., x_N\}$ are independent
- $cov\{\varepsilon_i, \varepsilon_j\} = 0$, $i, j = 1, \ldots., N$ $I \neq j$.

- $V\{\varepsilon_1 = \sigma^2, i= 1, \ldots.N$

The first of these assumptions can be read as "The expected value of the error term is zero.". The second assumption is collinearity, the third is exogeneity, and the fourth is homoscedasticity.

9. Explain the gradient descent algorithm in detail.

Gradient descent is an optimization algorithm used to minimize some function by iteratively moving in the direction of steepest descent as defined by the negative of the gradient. In machine learning, we use gradient descent to update the parameters of our model. Parameters refer to coefficients in Linear Regression and weights in neural networks

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Cost function

A Loss Functions tells us "how good" our model is at making predictions for a given set of parameters. The cost function has its own curve and its own gradients. The slope of this curve tells us how to update our parameters to make the model more accurate.

Step-by-step

Now let's run gradient descent using our new cost function. There are two parameters in our cost function we can control:
m(weight) and b(bias). Since we need to consider the impact each one has on the final prediction, we need to use partial derivatives. We calculate the partial derivatives of the cost function with respect to each parameter and store the results in a gradient.

Given the cost function:

$$f(m,b)=1N\sum i=1n(yi-(mxi+b))2$$

The gradient can be calculated as:

$$f'(m,b)=\left[\left[dfdmdfdb\right]\right]=[1N\sum -2xi(yi-(mxi+b))1N\sum -2(yi-(mxi+b))]$$

To solve for the gradient, we iterate through our data points using our new m
and b values and compute the partial derivatives. This new gradient tells us the slope of our cost function at our current position (current parameter values) and the direction we should move to update our parameters. The size of our update is controlled by the learning rate.

10. What is a Q-Q plot? Explain the use and importance of a Q-Q plot in linear regression.

Q–Q (quantile-quantile) plot is a probability plot, which is a graphical method for comparing two probability distributions by plotting their quantiles against each other.

A Q–Q plot is used to compare the shapes of distributions, providing a graphical view of how properties such as location, scale, and skewness are similar or different in the two distributions. Q–Q plots can be used to compare collections of data, or theoretical distributions.

The use of Q–Q plots to compare two samples of data can be viewed as a non-parametric approach to comparing their underlying distributions

A **Q–Q plot** is a plot of the quantiles of two distributions against each other, or a plot based on estimates of the quantiles. The pattern of points in the plot is used to compare the two distributions.

Q Q Plots (Quantile-Quantile plots) are plots of two quantiles against each other. A quantile is a fraction where certain values fall below that quantile. For example, the median is a quantile where 50% of the data fall below that point and 50% lie above it. The purpose of Q Q plots is to find out if two sets of data come from the same distribution. A 45 degree angle is plotted on the Q Q plot; if the two data sets come from a common distribution, the points will fall on that reference line.