MA301.3 – Advanced Mathematics for Computing

Sample Paper

Answer all questions

Question-1 (Vectors)

Mark the correct response with a pen on the answer sheet provided. **[20 Marks]**

(i) Let the vectors u and v be $(-1, 1,3)$ and $(-1, -2, 3)$. Then the vector $3u - 2v$ is given by:

(a) $(-1, 7, -3)$
(b) $(-1, 7, \; 3)$
(c) $(1, 7, -3)$
(d) $(1, \; 7, \; 3)$
(e) $(-1, -1, -3)$

(ii) Find x, y, z such that $(x - y - z, \; x + 2y - z, \; 2x - y + z) = (6, 0, 1)$

(a) $x = 1, \; y = -2, \quad z = 3$
(b) $x = 1, \; y = -2, \quad z = -3$
(c) $x = -1, \; y = -2, \quad z = 3$
(d) $x = 1, \; y = 2, \quad z = 3$
(e) $x = -2, \; y = 1, \quad z = 3$

(iii) For any vectors u, v, w in $\mathbb{R}^n$ and any scalars k, k' in $\mathbb{R}$, which of the following statements are correct?

I. $(u + v) + w \neq u + (v + w)$
II. $u + v \neq v + u$
III. $u.v = v.u$
IV. $k(u + v) = ku + kv$ and $(k + k')u = ku + k'u$

(a) III only (b) IV only (c) I and II (d) III and IV (e) All of the above

(iv) Suppose u and v are two vectors where $u = (k, 17, k, -5)$ and $v = (-5, -k, 3k, -7)$ Find value/s of k such that these vectors are orthogonal?

(a) 5 only (b) $\frac{7}{3}$ only (c) 5 and $\frac{-7}{3}$ (d) 5 and $\frac{7}{3}$ (e) None of the above

(v) Suppose u and v are two vectors where $u = (1, 2, -1)$ and $v = (3, -1, 2)$. The projection of the vector u onto v is given by:

(a) $\frac{1}{14}(3, -1, 2)$ (b) $\frac{-1}{\sqrt{14}}(3, -1, 2)$ (c) $\frac{-1}{14}(3, -1, 2)$ (d) $\frac{-1}{14}(1, 2, -1)$ (e) $\frac{-1}{6}(1,2, -1)$

(vi) Which of the following are unit vectors?

I. $(\frac{5}{\sqrt{61}}, \frac{6}{\sqrt{61}})$

II. $\frac{1}{\sqrt{2}}(1,0,0,1)$

III. $(\frac{4}{\sqrt{50}}, \frac{5}{\sqrt{50}}, \frac{9}{\sqrt{50}})$

IV. $(0,1,0,1)$

(a) I and II only (b) I and IV only (c) I, II and III (d) I, III and IV (e) All of the above

(vii) Which of the following are true statements for the vectors $u = (2,1,1)$ and $v = (1,1,2)$

I. For the above vectors u, v in R^3 $|u.v| \geq \|u\| \, \|v\|$

II. Dot product between the two vectors u and v is 6.

III. Cross product between the vectors u, v in R^3 is $i - 3j - k$

(a) I and II only (b) II and III only (c) I, II and III (d) I and III only (e) None of the above

(viii) The equation of the hyperplane H in $\mathbb{R}^4$ that passes through the point P(1,-2,-3,5) is $2x_1 + 3x_2 - x_3 + 2x_4 - 9 = 0$. The normal to H is given by

(a) $[2, 3, -1,2]$
(b) $[2,3,-1,2,9]$
(c) $[-9,2,-1,3,2]$
(d) $[2,-1,3,2]$
(e) $[-2,-3,1,-2]$

(ix) Consider the curve $F(t) = [t^2 - 3, \; 2t^2, \; t + 1]$ in $\mathbb{R}^3$. The unit tangent vector to the curve when $t = 1$ is given by

(a) $(2, 4,1)$
(b) $(\frac{4}{\sqrt{21}}, \frac{2}{\sqrt{21}}, \frac{1}{\sqrt{21}})$
(c) $(\frac{2}{\sqrt{21}}, \frac{4}{\sqrt{21}}, \frac{1}{\sqrt{21}})$
(d) $(2t, 4t ,1)$
(e) $\frac{1}{21}(2,4,1)$

(x) Find the parametric equation of the line L, through the points P(3, -2, 1) and Q(2, 1, 3) in the direction of QP.

(a) $(3 + t, \; -2 - 3t, \; 1 - 2t)$
(b) $(3 + t, \; -2 + 3t, \; 1 - 2t)$
(c) $(3 + t, \; -2 - 3t, \; 1 + 2t)$
(d) $(3 - t, \; 2 - t, \; 1 - 2t)$
(e) $(3 - t, \; -2 + 3t, \; 1 + 2t)$

Question 2 (Differentiation) **[20 marks]**

1. Find the limits of the following expressions
   i. $\lim_{x \to 0} \frac{x}{\sqrt{1+x} - 1}$ (2 marks)
   ii. $\lim_{x \to \infty} \frac{2x^2 - 1}{3x^2 + 1}$ (2 marks)

2. Differentiate $\sqrt{x}$ using the first principles. (3 marks)

3. Differentiate the following with respect to x:
   i. $y = 4x^3 - 3x^2 - 1$ (2 marks)
   ii. $y = 3x \sin x + \cos^{-1} x$ (2 marks)
   iii. $y = \sin(\cos x)$ (2 marks)
   iv. $y = \frac{1 - x^2}{1 + x^2}$ (2 marks)

4. Show the following:
   If $y = 3e^{2x} \cos(2x - 3)$, verify that $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 8y = 0$ (5 marks)

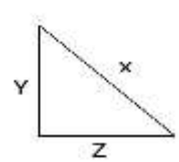
Question 3 (Partial Differentiation) **[25 marks]**

1. If $z = x \cos y - y \cos x$, Find $\frac{\partial z}{\partial x}, \frac{\partial z}{\partial y}, \frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial y^2}$ and $\frac{\partial^2 z}{\partial x \partial y}$ (5 marks)

2. If $z = yf(x^2 - y^2)$ show that $y\frac{\partial z}{\partial x} + x\frac{\partial z}{\partial y} = \frac{xz}{y}$ (5 marks)

3. If $y = \frac{ws^3}{d^4}$, find the percentage change in y when w increases by 3% of w, s decrease by 2% of s and d increases by 1% of d. (7 marks)

4. In the triangle with a 90 degree angle shown below, x is increasing at 2 cm/s while y is decreasing at 3 cm/s. Calculate the rate at which z is increasing when x= 5 cm & y=3 cm. (8 marks)



Question 4 (Integration & Multiple Integrals) **[20 marks]**

1. Integrate the following functions:
   i. $\int (8x^3 - 3x^2 + 1)\, dx$ (2 marks)
   ii. $\int \frac{x^2}{1+x^3} dx$ (2 marks)
2. Using integration by parts, find $\int e^x sin(x)\, dx$ (4 marks)
3. Integrate the following
   i. $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\, dydx$
   ii. $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dzdydx$ (6 marks)
4. Find the area of the triangle formed by the vertices (2,1), (5,4) & (8,2). Use the double integral to find the solution. (6 marks)

Question 5 (Proving Techniques) **[15 marks]**

1. Prove that if x & y are odd integers, then xy is odd using direct proof. (3 marks)

2. Briefly explain why you should select an indirect proof over a direct proof. Then, prove that if $n^2 > 16$ then $n > 4$ for any positive integer n using an appropriate proving technique. (4 marks)

3. Show that if n^3+3 is odd, then n is even using proof by contradiction. (4 marks)

4. Prove the following by Mathematical Induction: (4 marks)
   $1 + 2 + 3 + \cdots + n = \frac{n}{2}(n + 1)$

MCQ ANSWER SHEET

(1)	(a)	(b)	(c)	(d)	(e)
(2)	(a)	(b)	(c)	(d)	(e)
(3)	(a)	(b)	(c)	(d)	(e)
(4)	(a)	(b)	(c)	(d)	(e)
(5)	(a)	(b)	(c)	(d)	(e)
(6)	(a)	(b)	(c)	(d)	(e)
(7)	(a)	(b)	(c)	(d)	(e)
(8)	(a)	(b)	(c)	(d)	(e)
(9)	(a)	(b)	(c)	(d)	(e)
(10)	(a)	(b)	(c)	(d)	(e)

*** End of Question Paper ***