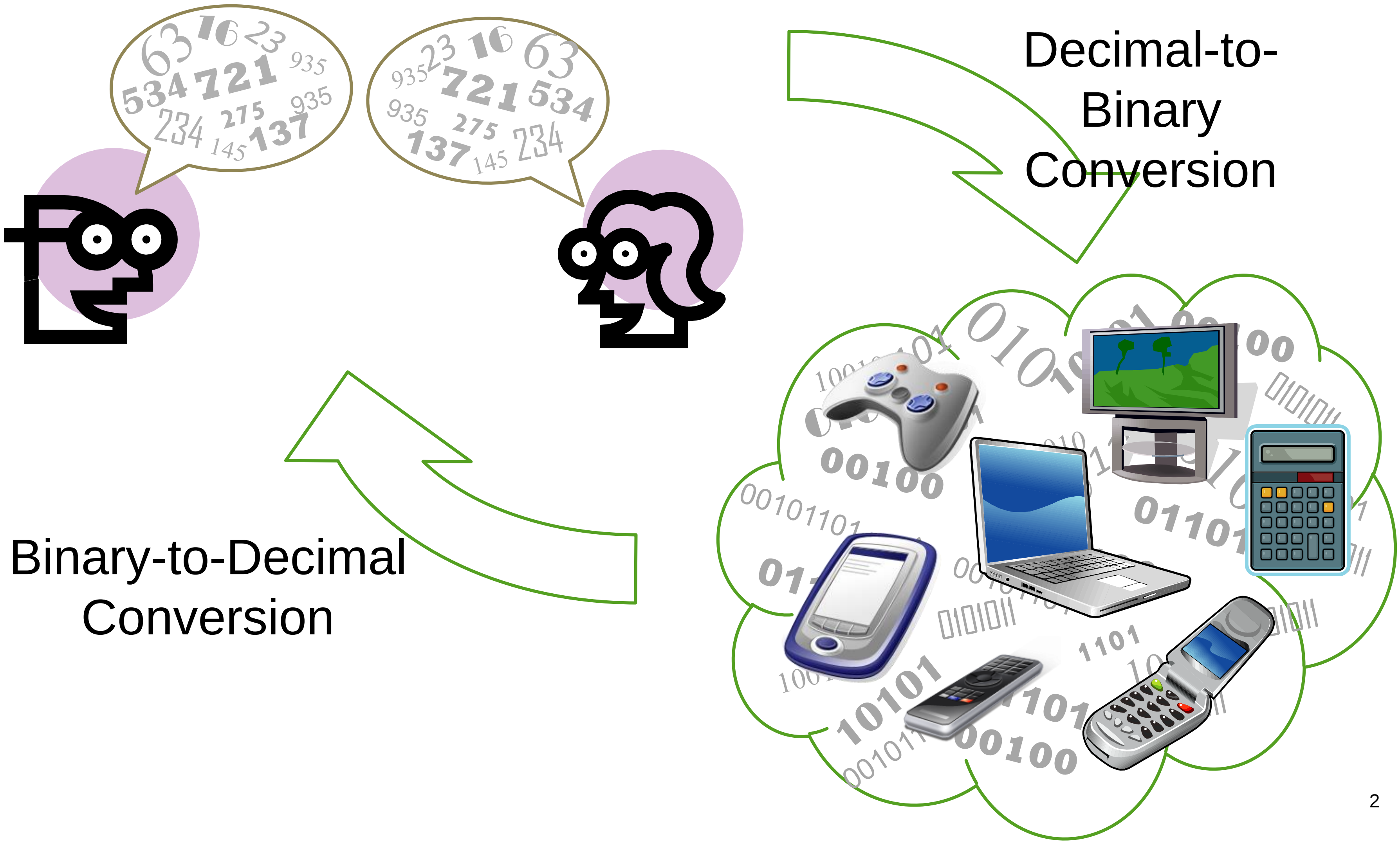
Mathematics for Computer Science I

CHAPTER 01 Binary Number System

1.2 : NUMBER CONVERSIONS

Bridging the Digital Divide



Why use Binary?

- At the lowest level, computers are based on billions of electrical elements that have only two states, (usually low and high voltage). By interpreting these as 0 and 1, it's very easy to build circuits for storing binary numbers and doing calculations with them. While it's possible to simulate the behavior of decimal numbers with binary circuits as well, it's less efficient. If computers used decimal numbers internally, they'd have less memory and be slower at the same level of technology.

Common Number Systems

System	Base	Symbols	Used by humans?	Used in computers?
Decimal	10	0, 1, … 9	Yes	No
Binary	2	0, 1	No	Yes
Octal	8	0, 1, … 7	No	No
Hexa-decimal	16	0, 1, … 9, A, B, … F	No	No

Quantities/Counting (1 of 3)

Decimal	Binary	Octal	Hexa-decimal
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	5	5
6	110	6	6
7	111	7	7

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Quantities/Counting (2 of 3)

Decimal	Binary	Octal	Hexa-decimal
8	1000	10	8
9	1001	11	9
10	1010	12	A
11	1011	13	B
12	1100	14	C
13	1101	15	D
14	1110	16	E
15	1111	17	F

Quantities/Counting (3 of 3)

Decimal	Binary	Octal	Hexa-decimal
16	10000	20	10
17	10001	21	11
18	10010	22	12
19	10011	23	13
20	10100	24	14
21	10101	25	15
22	10110	26	16
23	10111	27	17

Etc.

Decimal Number System

The number system that we use in our day-to-day life is the decimal number system. Decimal number system has base 10 as it uses 10 digits from 0 to 9. In decimal number system, the successive positions to the left of the decimal point represents units, tens, hundreds, thousands and so on.

Each position represents a specific power of the base (10). For example, the decimal number 1234 consists of the digit 4 in the units position, 3 in the tens position, 2 in the hundreds position, and 1 in the thousands position, and its value can be written as

$$(1\times1000) + (2\times100) + (3\times10) + (4\times1)$$
$$(1\times10^3) + (2\times10^2) + (3\times10^1) + (4\times10^0)$$
$$1000 + 200 + 30 + 1$$
$$1234$$

Binary Number System

Characteristics

- Uses two digits, 0 and 1.
- Also called base 2 number system
- Each position in a binary number represents a 0 power of the base (2). Example: 2^0
- Last position in a binary number represents an x power of the base (2). Example: 2^x where x represents the last position - 1.

Example

Binary Number: 10101_2

Calculating Decimal Equivalent −

Step	Binary Number	Decimal Number
Step 1	10101_2	$((1 \times 2^4) + (0 \times 2^3) + (1 \times 2^2) + (0 \times 2^1) + (1 \times 2^0))_{10}$
Step 2	10101_2	$(16 + 0 + 4 + 0 + 1)_{10}$
Step 3	10101_2	21_{10}

Note: 10101_2 is normally written as 10101.

Octal Number System

Characteristics

- Uses eight digits, 0,1,2,3,4,5,6,7.
- Also called base 8 number system
- Each position in an octal number represents a 0 power of the base (8). Example: 8^0
- Last position in an octal number represents an x power of the base (8). Example: 8^x where x represents the last position - 1.

Example

Octal Number − 12570_8

Calculating Decimal Equivalent −

Step	Octal Number	Decimal Number
Step 1	12570_8	$((1 \times 8^4) + (2 \times 8^3) + (5 \times 8^2) + (7 \times 8^1) + (0 \times 8^0))_{10}$
Step 2	12570_8	$(4096 + 1024 + 320 + 56 + 0)_{10}$
Step 3	12570_8	5496_{10}

Note: 12570_8 is normally written as 12570.

Hexadecimal Number System

Characteristics

- Uses 10 digits and 6 letters, 0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F.
- Letters represents numbers starting from 10. A = 10, B = 11, C = 12, D = 13, E = 14, F = 15.
- Also called base 16 number system.
- Each position in a hexadecimal number represents a 0 power of the base (16). Example 16^0.
- Last position in a hexadecimal number represents an x power of the base (16). Example 16^x where x represents the last position - 1.

Example −

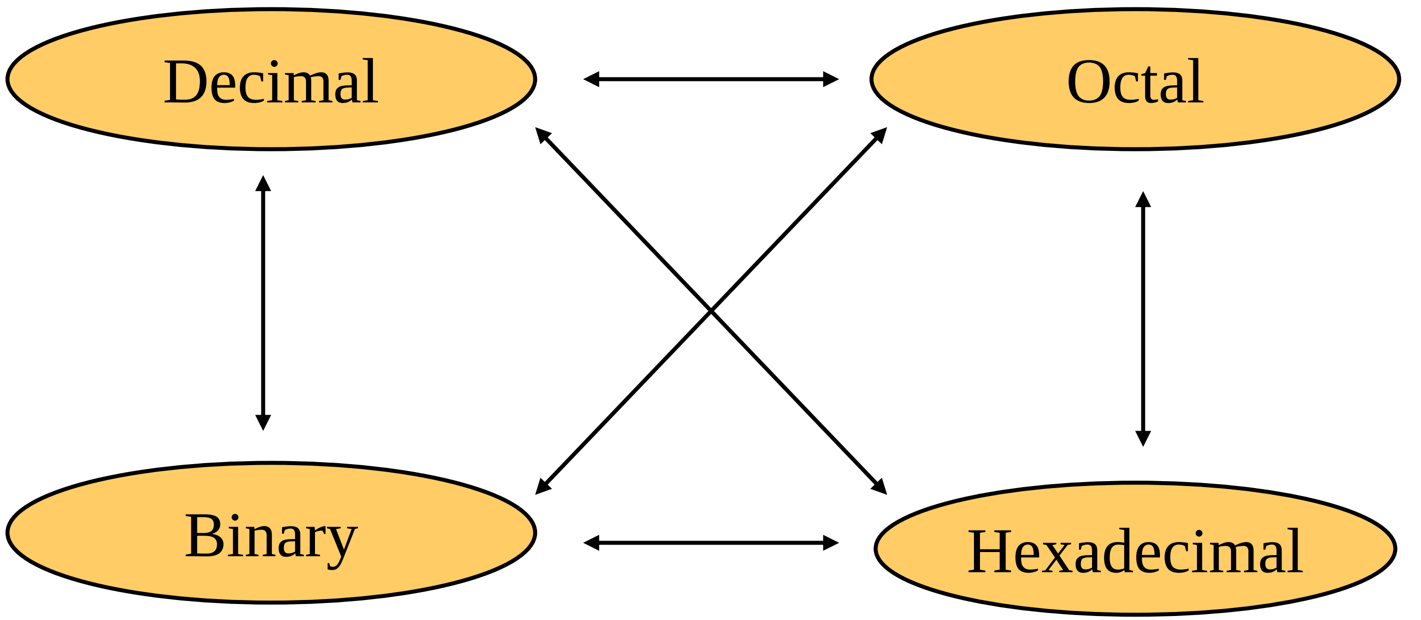
Hexadecimal Number: $19FDE_{16}$

Calculating Decimal Equivalent −

Step	Hexadecimal Number	Decimal Number
Step 1	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (F \times 16^2) + (D \times 16^1) + (E \times 16^0))_{10}$
Step 2	$19FDE_{16}$	$((1 \times 16^4) + (9 \times 16^3) + (15 \times 16^2) + (13 \times 16^1) + (14 \times 16^0))_{10}$
Step 3	$19FDE_{16}$	$(65536 + 36864 + 3840 + 208 + 14)_{10}$
Step 4	$19FDE_{16}$	106462_{10}

Conversion Among Bases

- The possibilities:

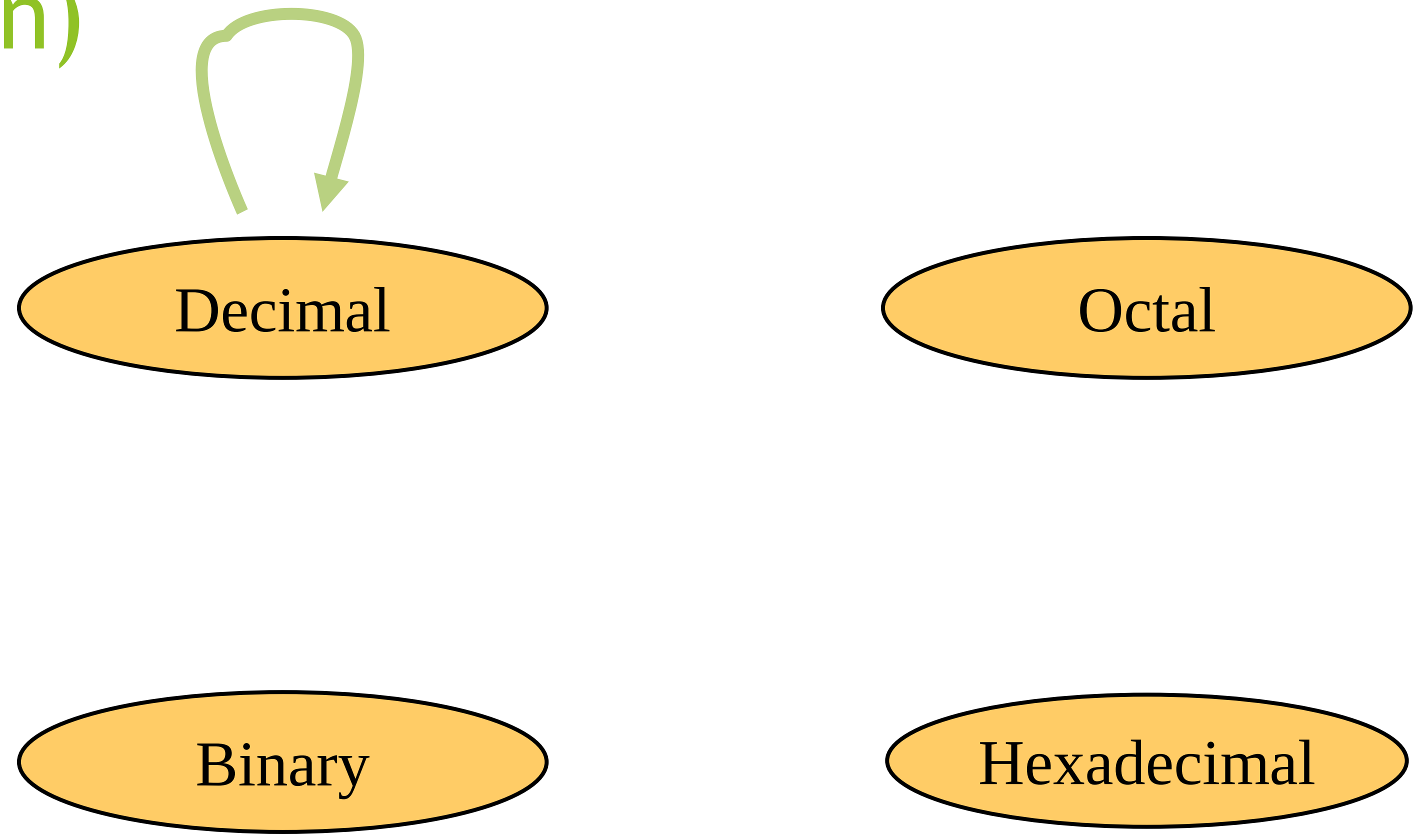


Quick Example

$$25_{10} = 11001_2 = 31_8 = 19_{16}$$

Base

Decimal to Decimal (just for fun)



Next slide…

Weight

$$125_{10} \Rightarrow 5 \times 10^{0} = 5$$

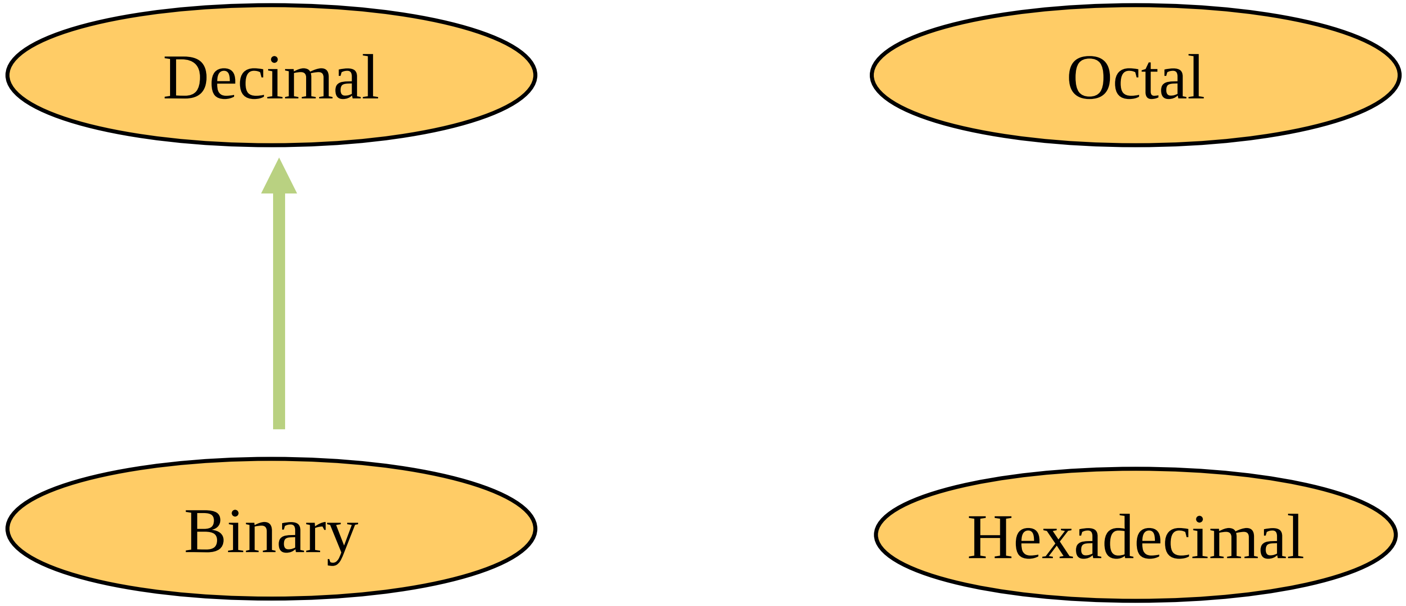
$$2 \times 10^{1} = 20$$

$$1 \times 10^{2} = \underline{100}$$

$$125$$

Base

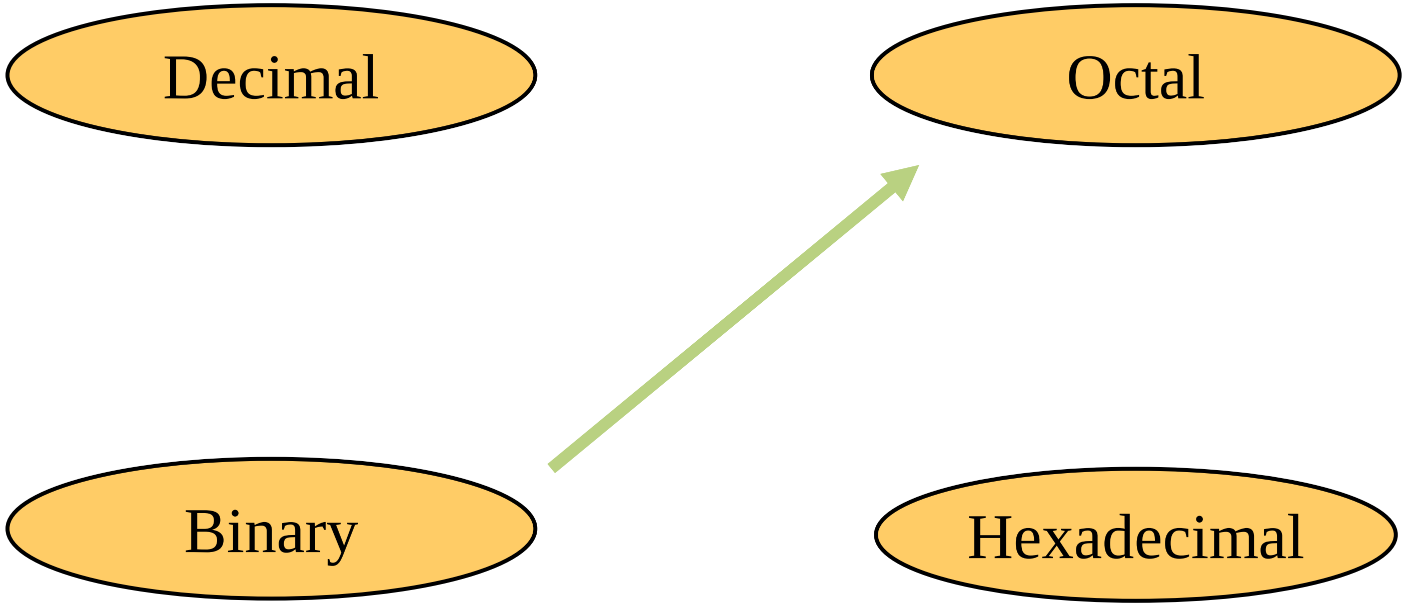
Binary to Decimal



Example

$$
\begin{array}{lrcrcr}
101011_2 \Rightarrow & 1 \times 2^0 & = & 1 \\
& 1 \times 2^1 & = & 2 \\
& 0 \times 2^2 & = & 0 \\
& 1 \times 2^3 & = & 8 \\
& 0 \times 2^4 & = & 0 \\
& 1 \times 2^5 & = & \underline{32} \\
& & & 43_{10}
\end{array}
$$

Binary to Octal



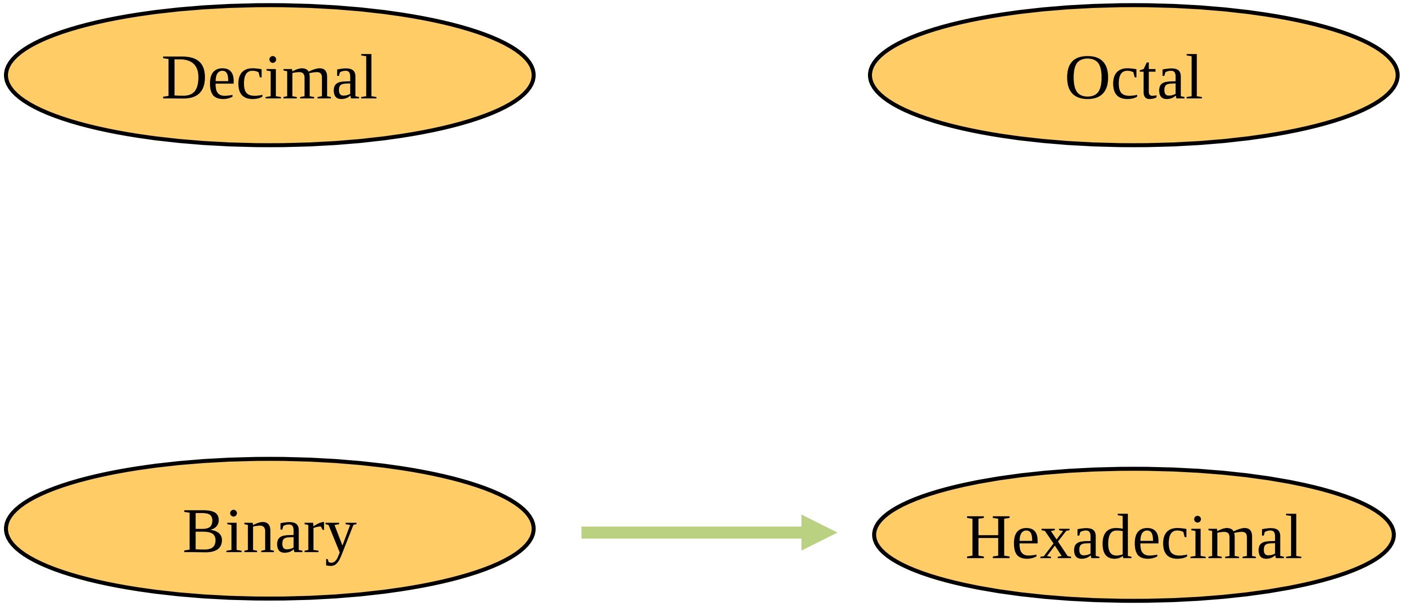
Example

$1011010111_2 = ?_8$

$$
\begin{array}{cccc}
1 & 011 & 010 & 111 \\
\downarrow & \downarrow & \downarrow & \downarrow \\
1 & 3 & 2 & 7
\end{array}
$$

$1011010111_2 = 1327_8$

Binary to Hexadecimal



Example

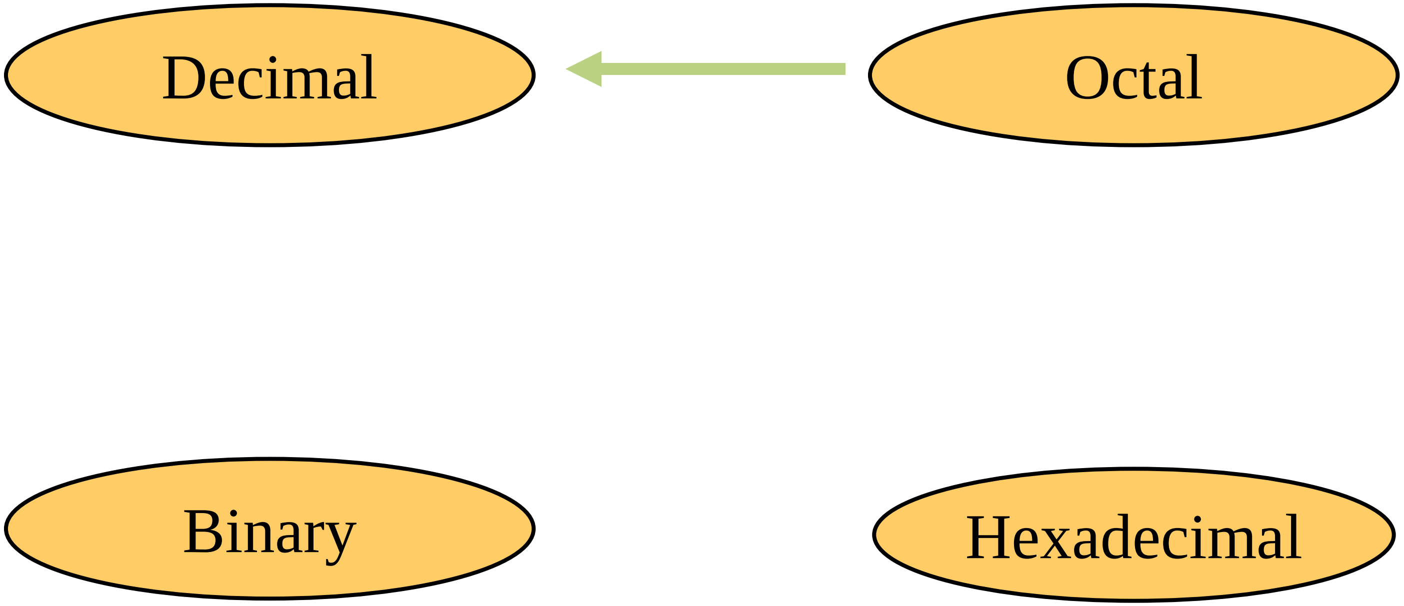
$1010111011_2 = ?_{16}$

10 1011 1011

2 B B

$1010111011_2 = 2BB_{16}$

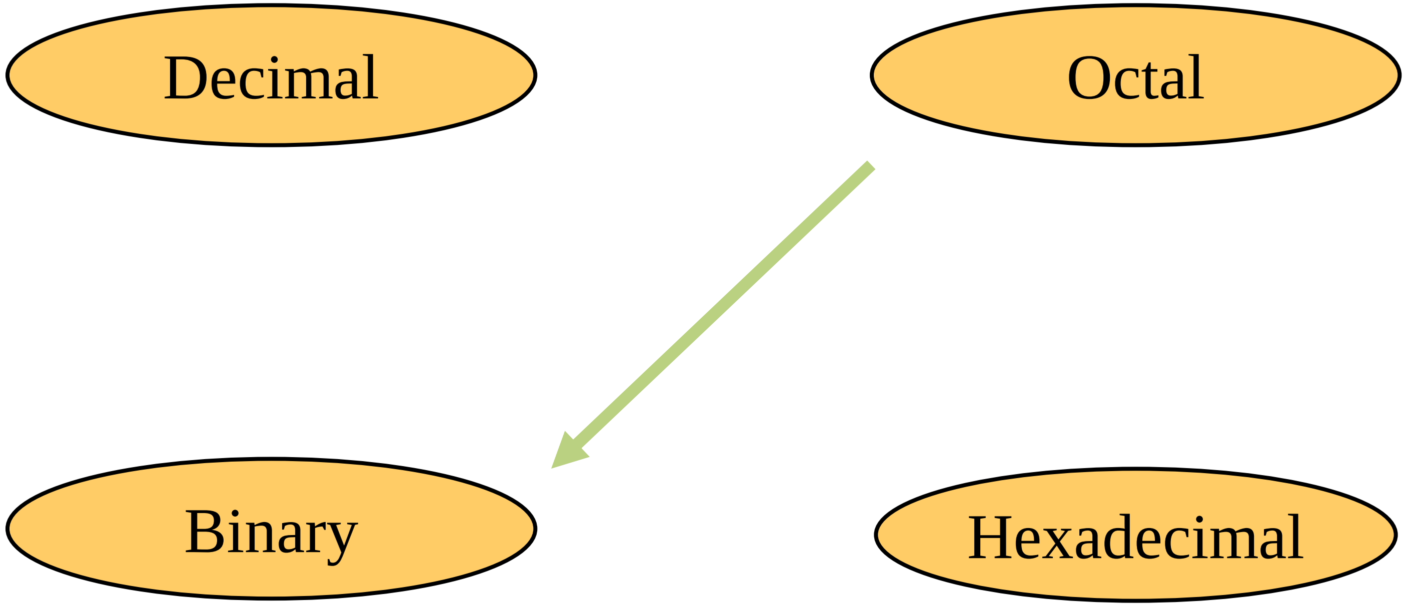
Octal to Decimal



Example

$$
\begin{array}{lrcrl}
724_8 \Rightarrow & 4 \times 8^0 & = & & 4 \\
& 2 \times 8^1 & = & 16 & \\
& 7 \times 8^2 & = & 448 & \\
\hline
& & & 468_{10} &
\end{array}
$$

Octal to Binary



Example

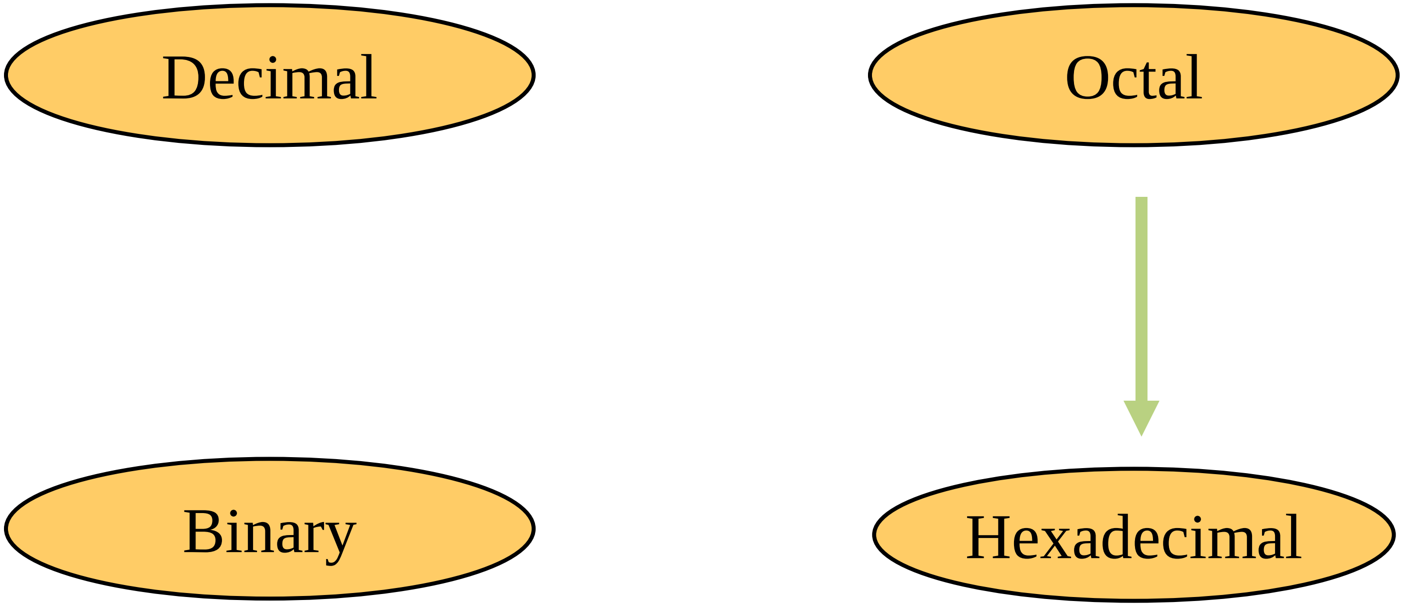
$705_8 = ?_2$

7 0 5

111 000 101

$705_8 = 111000101_2$

Octal to Hexadecimal



Example

$1076_8 = ?_{16}$

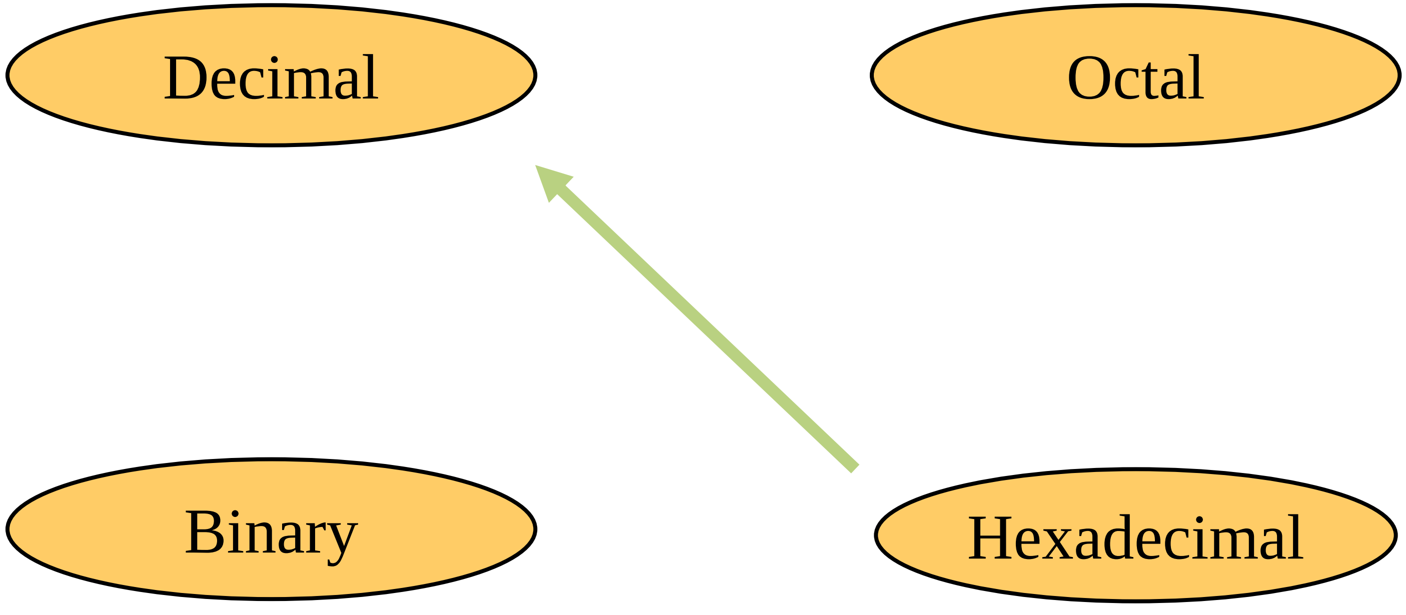
$$\begin{array}{cccc} 1 & 0 & 7 & 6 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 001 & 000 & 111 & 110 \end{array}$$

$$0010 \mid 0011 \mid 1110$$

$$2 \quad 3 \quad E$$

$1076_8 = 23E_{16}$

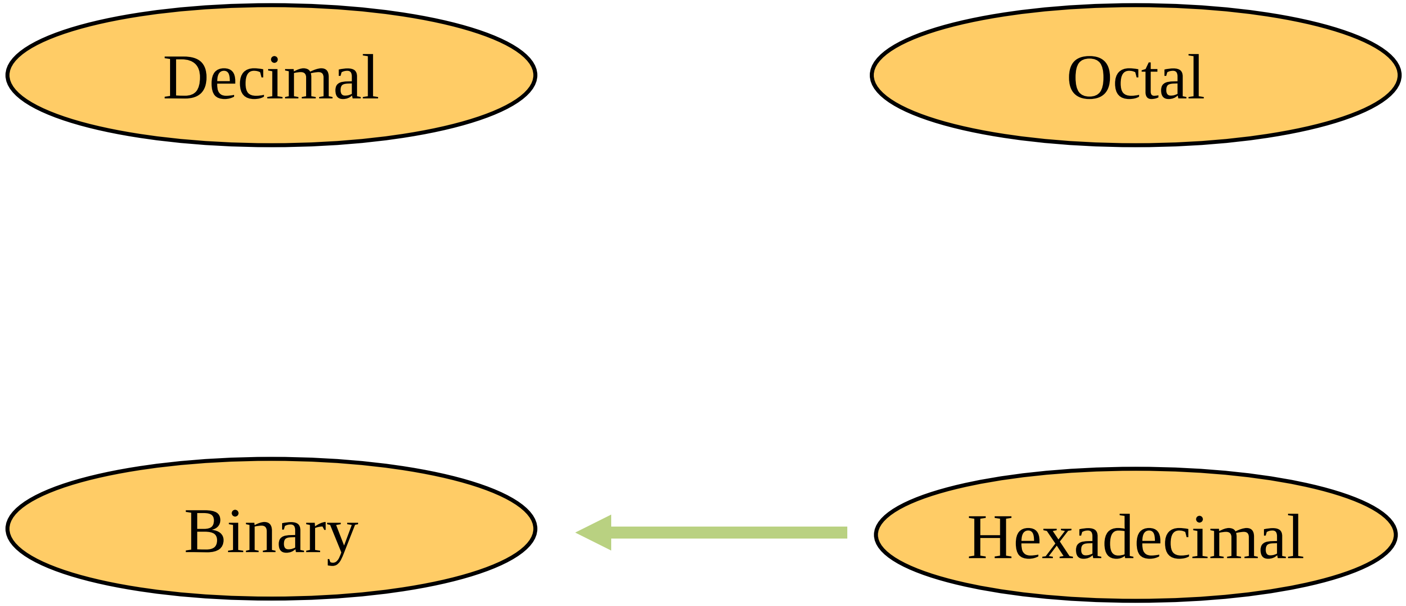
Hexadecimal to Decimal



Example

$$
\begin{array}{llllllll}
ABC_{16} \Rightarrow & C \times 16^0 & = & 12 \times 1 & = & 12 \\
& B \times 16^1 & = & 11 \times 16 & = & 176 \\
& A \times 16^2 & = & 10 \times 256 & = & 2560 \\
\hline
& & & & & 2748_{10}
\end{array}
$$

Hexadecimal to Binary



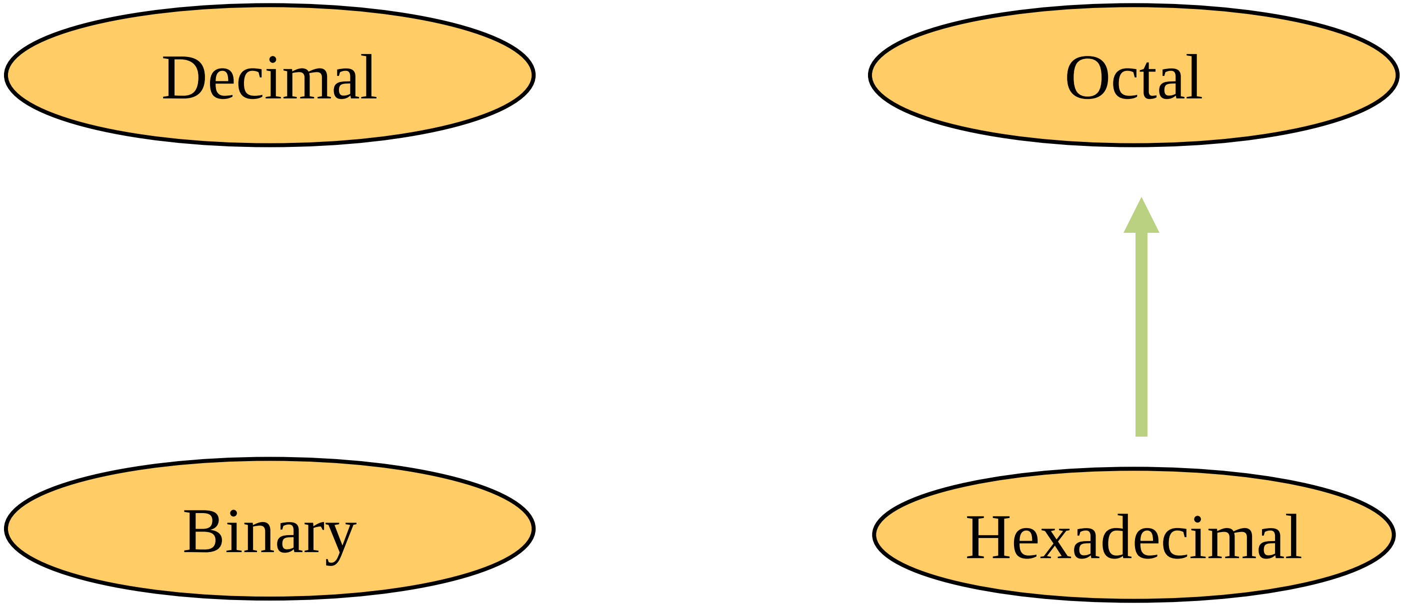
Example

$10AF_{16} = ?_2$

$$\begin{array}{cccc} 1 & 0 & A & F \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 0001 & 0000 & 1010 & 1111 \end{array}$$

$10AF_{16} = 0001000010101111_2$

Hexadecimal to Octal



Example

$1F0C_{16} = ?_8$

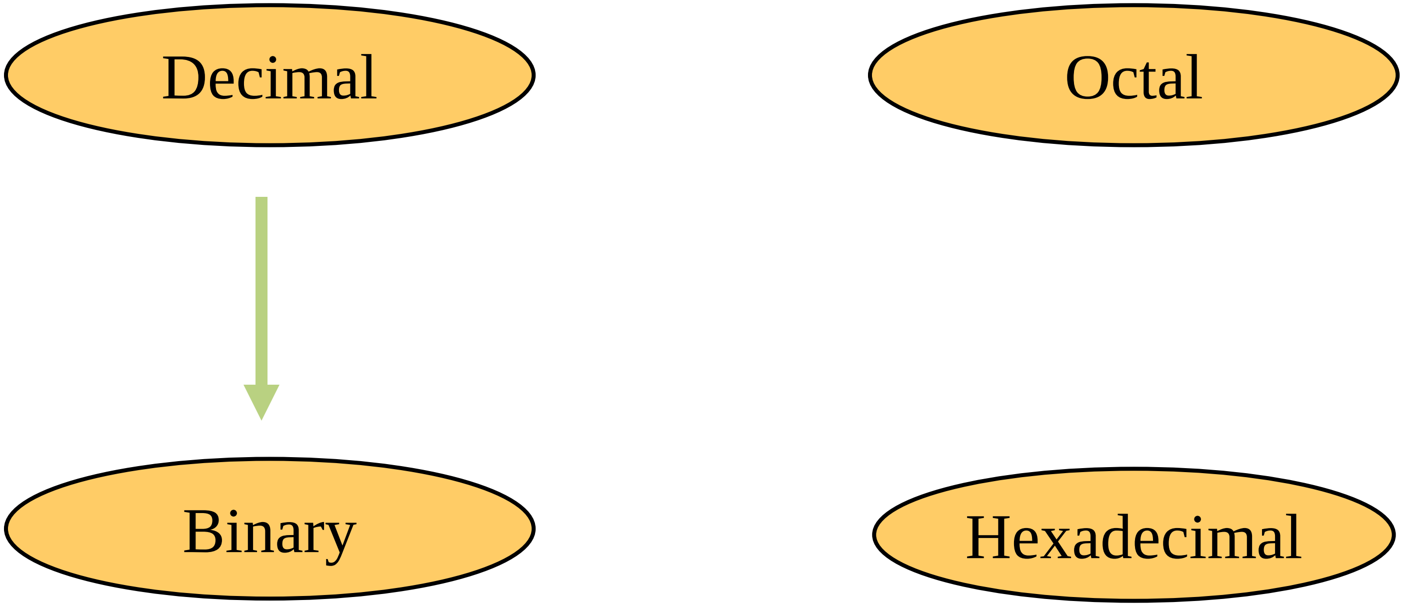
1	F	0	C
0001	1111	0000	1100

$0|001|111|1\,00|00\,1|100$

1	7	4	1	4

$1F0C_{16} = 17414_8$

Decimal to Binary



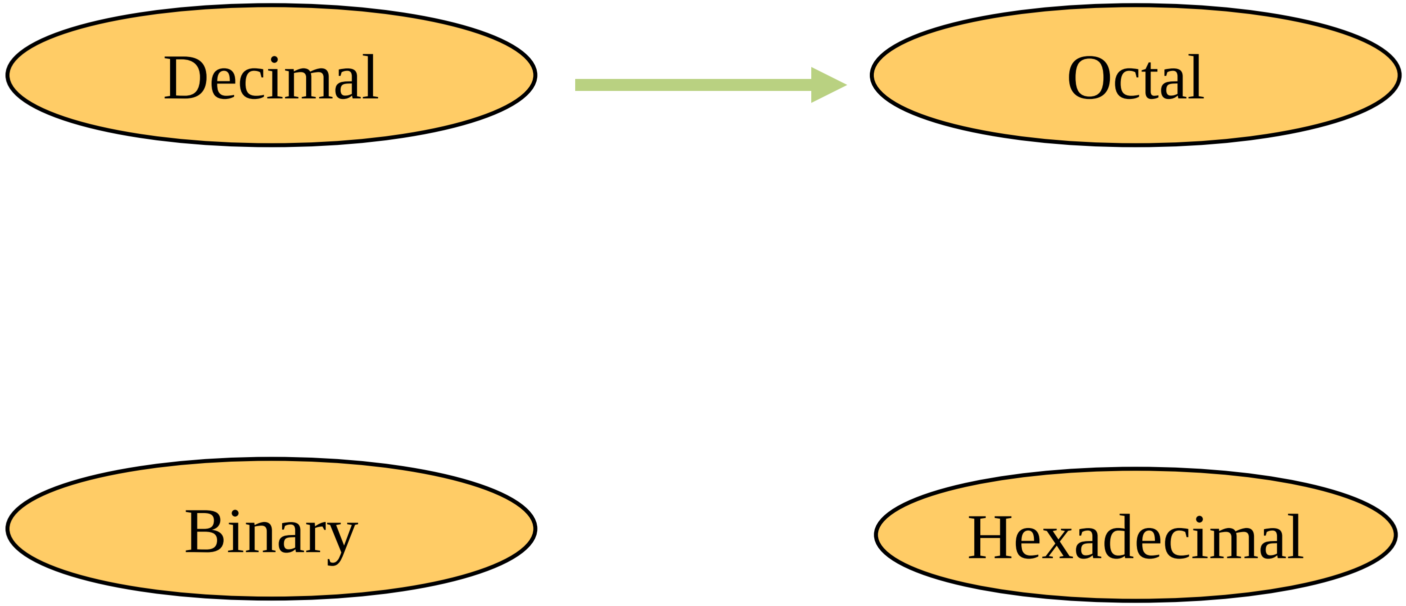
Example

$125_{10} = ?_2$

2	125	
2	62	1
2	31	0
2	15	1
2	7	1
2	3	1
2	1	1
	0	1

$125_{10} = 1111101_2$

Decimal to Octal



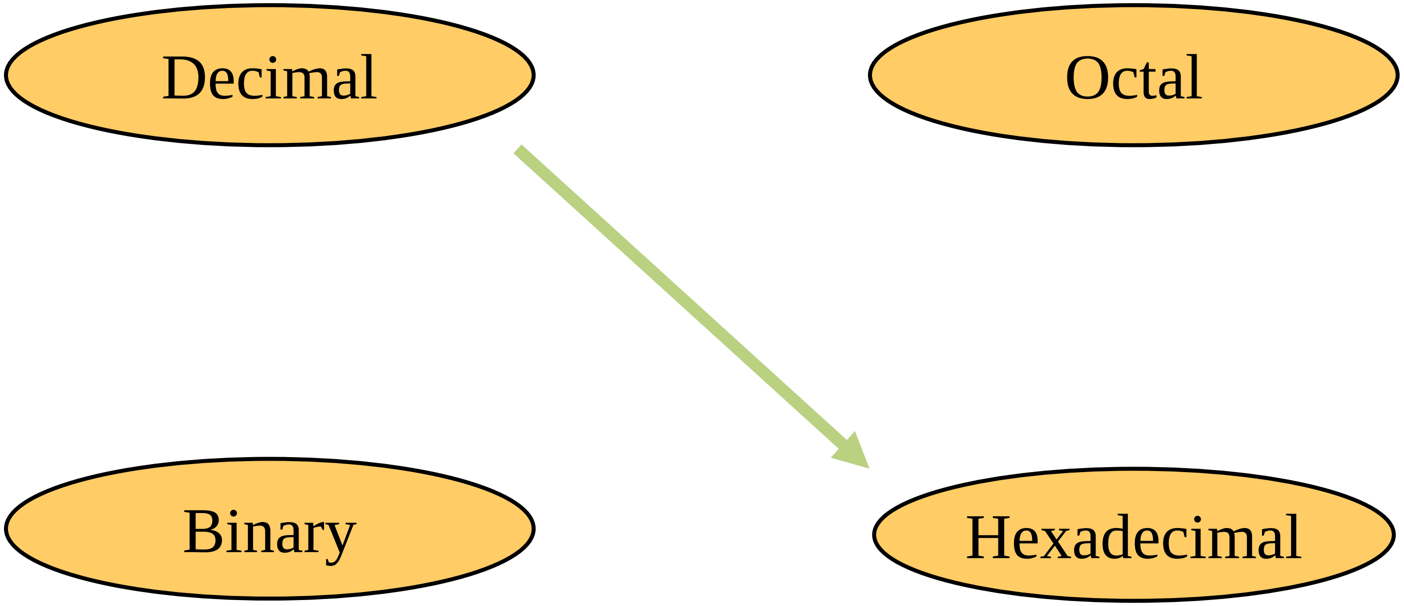
Example

$1234_{10} = ?_8$

8	1234	
8	154	2
8	19	2
8	2	3
	0	2

$1234_{10} = 2322_8$

Decimal to Hexadecimal



Example

$1234_{10} = ?_{16}$

16	1234	
16	77	2
16	4	13 = D
	0	4

$1234_{10} = 4D2_{16}$

Thank you