PART A: OPTION THEORY

YASSEN TITROUQ & ISAK KRONEKVIST HJELM

MID SWEDEN UNIVERSITY

1. Introduction

Nvidia, a cornerstone of the ongoing AI revolution, has emerged as a $3 trillion dollar company that is not only fueling innovation but also cementing its position at the forefront of the Fourth Industrial Revolution. Its remarkable financial performance has solidified its status as one of the "Magnificent Seven" a select group of tech giants that powered 2023's stock market gains. Nvidia is among the top seven constituents of the S&P 500, demonstrating impressive and consistent financial growth over the years, with a strong outlook for future sustainability and expansion. The company's success is driven by its relentless commitment to innovation, leveraging strategic collaborations and partnerships to enhance its product offerings. (Yahoo News, 2023)

But what is Nvidia, and why does it matter? Founded in 1993, Nvidia operates in the semiconductors and semiconductor equipment industry. The company is publicly traded under the ticker symbol NVDA on the Nasdaq Global Select Market. Nvidia specializes in providing graphics, compute, and networking solutions on a global scale, with key operations in the United States, Taiwan, and China. Despite its market dominance, Nvidia finds itself navigating a tense geopolitical landscape, particularly due to the rising tensions between China and Taiwan. This situation poses a potential risk to Nvidia's supply chain, as the company relies heavily on Taiwan Semiconductor Manufacturing Company (TSMC) for chip production. This dependence exposes Nvidia to vulnerabilities, especially when compared to competitors that manufacture their own chips. (OneSafe, 2023)

While geopolitical uncertainty could disrupt Nvidia's supply chain, the company's growth trajectory has remained seemingly unstoppable. Its ability to outperform competitors and maintain investor confidence is notable. However, this has sparked debates among financial analysts about whether Nvidia's stock is overvalued. An overvalued stock, even with strong fundamentals and operating in a promising industry, carries the risk of future price corrections. Nvidia's rapid rise has led some to question whether its valuation reflects external market exuberance rather than intrinsic value. This perception of overvaluation could create trading risks, particularly for options traders. (Yahoo Finance, 2023)

2. Investment Decision

To make an accurate investment decision and build a well rounded understanding of Nvidia, it's essential to approach the analysis through a framework, illust in ***Figure 1***, of interconnected areas: business model, market and industry dynamics, valuation and fundamental metrics, and technical analysis. Together, these areas create a comprehensive picture of the company's potential, its current standing, and the risks and opportunities it faces.

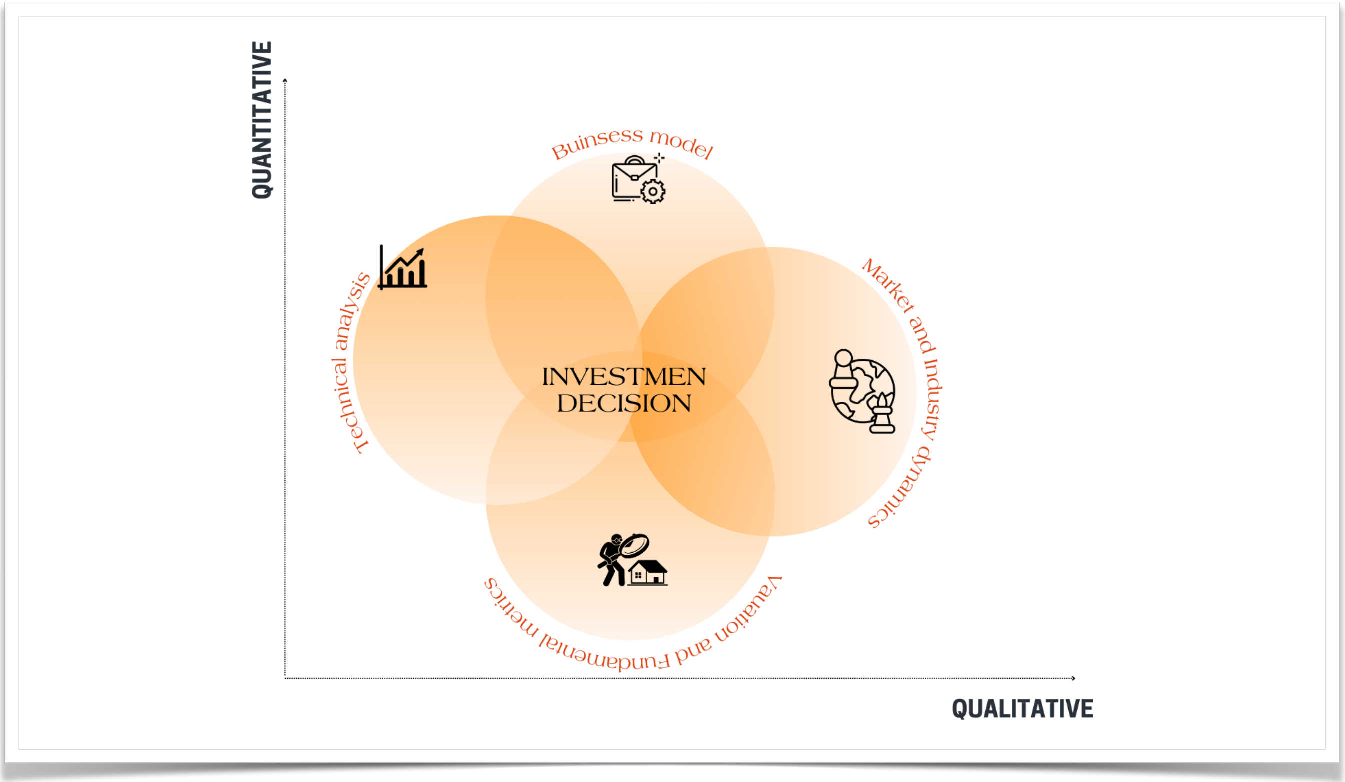


Figure 1 : Represents investment decision

The first step in this process involves examining Nvidia's business model, starting with an analysis of its financial health. This means evaluating key metrics such as revenue growth, profit margins, debt levels, and cash flow. By looking at how Nvidia has improved its performance from one reporting period to the next, we can better understand its capacity to sustain growth and adapt to changing market conditions. Beyond financials, it's important to consider the company's innovation processes, organizational structure, and leadership. Visionary investors like Warren Buffett often emphasize the importance of corporate governance, warning that frequent leadership changes or shifting ethics can undermine trust in a company's long-term direction. Nvidia's environmental and social commitments, as well as its collaborations with other companies to generate shared value, are also critical to understanding its strategic positioning. This foundational analysis offers insight into Nvidia's intrinsic value. Companies with strong fundamentals inspire confidence in their ability to weather challenges and grow, whereas weaker fundamentals can signal a riskier investment.

After analyzing Nvidia's internal strengths, the focus shifts outward to the dynamics of the market and industry in which it operates. Here, the lens broadens to examine Nvidia's impact on industry trends and its competitive positioning. How does Nvidia stack up against its rivals? Do its competitors pose significant risks? What role do broader economic conditions in its key regions, such as the United States, China, and Taiwan, play in its operations? Perhaps most crucially, what impact do geopolitical risks, such as rising tensions between China and Taiwan, have on Nvidia's supply chain and growth prospects? This external analysis highlights the opportunities and threats the company faces. Even the most fundamentally sound company can encounter challenges in an unfavorable industry or market. On the other hand, a thriving industry amplifies a company's growth potential and supports its innovation strategies, offering significant opportunities for expansion and value creation.

With the business model and market dynamics understood, the next focus is valuation and financial metrics. This involves assessing whether Nvidia's stock is attractively priced relative to its potential and the broader market. Valuation indicators such as the price-to-earnings (P/E) ratio, enterprise value to EBITDA (EV/EBITDA), and equity value help investors determine if Nvidia's stock is overvalued, undervalued, or fairly priced. A company may possess great fundamentals and operate in a promising industry, yet still be overvalued relative to its peers. Such a scenario could introduce risks, as inflated valuations often lead to market corrections. Conversely, undervaluation in a strong industry can present lucrative opportunities for value-driven investors. Valuation metrics are closely tied to industry dynamics, as favorable trends tend to elevate valuations, while unfavorable conditions can suppress them, creating room for strategic investments.

Finally, technical analysis plays a vital role in forming a complete picture of Nvidia. This involves analyzing stock price movements, trading volume, and market reactions to major events like quarterly earnings reports. Technical analysis helps identify price trends and optimal entry and exit points for trades. This area connects seamlessly to the previous stages of analysis; overvalued stocks can carry higher trading risks, while undervalued stocks may offer significant upside potential. Moreover, technical risks can eventually impact a company's fundamentals, reinforcing the need for a holistic and cyclical approach to evaluation.

Ultimately, these elements business model, market and industry dynamics, valuation and financial metrics, and technical analysis are deeply interconnected. A strong business model helps a company thrive in favorable market conditions, while healthy industry dynamics often elevate valuation. In turn, a well grounded valuation framework provides context for technical analysis, which can inform trading strategies and mitigate risks. This cyclical approach ensures that no aspect is analyzed in isolation and that new insights continually refine the overall investment thesis. By embracing this interconnected framework, investors can develop a nuanced understanding of Nvidia and make informed decisions in the dynamic landscape of trading and finance.

2.1 Business model and Historical Financial performance

NVIDIA has emerged as a global leader in accelerated computing, transitioning from a niche GPU manufacturer into a titan reshaping industries through artificial intelligence (AI) and high performance computing (HPC). With groundbreaking innovations, the company has demonstrated the unparalleled efficiency of GPUs in handling complex computational tasks, outpacing traditional CPUs and driving advancements in fields like AI and cryptocurrency. Beyond its technological impact, NVIDIA's financial performance has been extraordinary, with exponential revenue growth and enhanced profit margins reflecting its dominance in the tech sector. Under the visionary leadership of CEO Jensen Huang, NVIDIA's innovation strategies and organizational agility have cemented its position at the forefront of the computing revolution. Deciding whether to invest in NVIDIA requires careful consideration of several factors, even with the company's strong reputation and impressive track record. This analysis aims to provide investors with a comprehensive understanding of NVIDIA's transformation, offering a detailed examination of both qualitative aspects, such as leadership and innovation strategies, and quantitative metrics like revenue growth, profit margins, and market performance. By exploring these elements, we seek to equip potential investors with the insights needed to evaluate NVIDIA's financial health.(Securities.io, 2023)

2.1.2 Quantitative

To determine whether NVIDIA is an attractive investment, it is essential to analyze its performance relative to the S&P 500 index. As a member of the "Magnificent Seven" a group of highly influential companies in the market NVIDIA has gained significant attention. However, it is important to conduct a thorough analysis of key financial metrics to substantiate its potential as an investment opportunity.

2.1.2.1 Earning per Share(EPS) Growth and Revenue Growth

One of the critical parameters to assess is NVIDIA's Earnings Per Share (EPS) growth rate compared to the S&P 500. The EPS is a fundamental indicator of a company's profitability, reflecting the net income attributed to each share of common stock. A higher EPS growth rate often signals robust financial health and strong market demand. For context, referring to **Figure 2** the S&P 500 has recorded an EPS growth rate of 17.2% year-over-year in Q1 2024. This represents a slight decline from 18.9% in Q4 2023 but remains above Q3 2023's 13.9%. Moreover, these figures are significantly higher than the EPS growth rates in Q2 and Q1 2023, which stood at 4.3% and 2.6%, respectively. This trend highlights the overall positive trajectory of the broader market's profitability. To evaluate NVIDIA's potential, we must determine whether its EPS growth rate surpasses the S&P 500's benchmark of 17.2%. A higher growth rate would indicate that NVIDIA is outperforming the broader market in terms of profitability expansion, further solidifying its position as a compelling investment candidate. (S&P Global Market Intelligence, 2023)

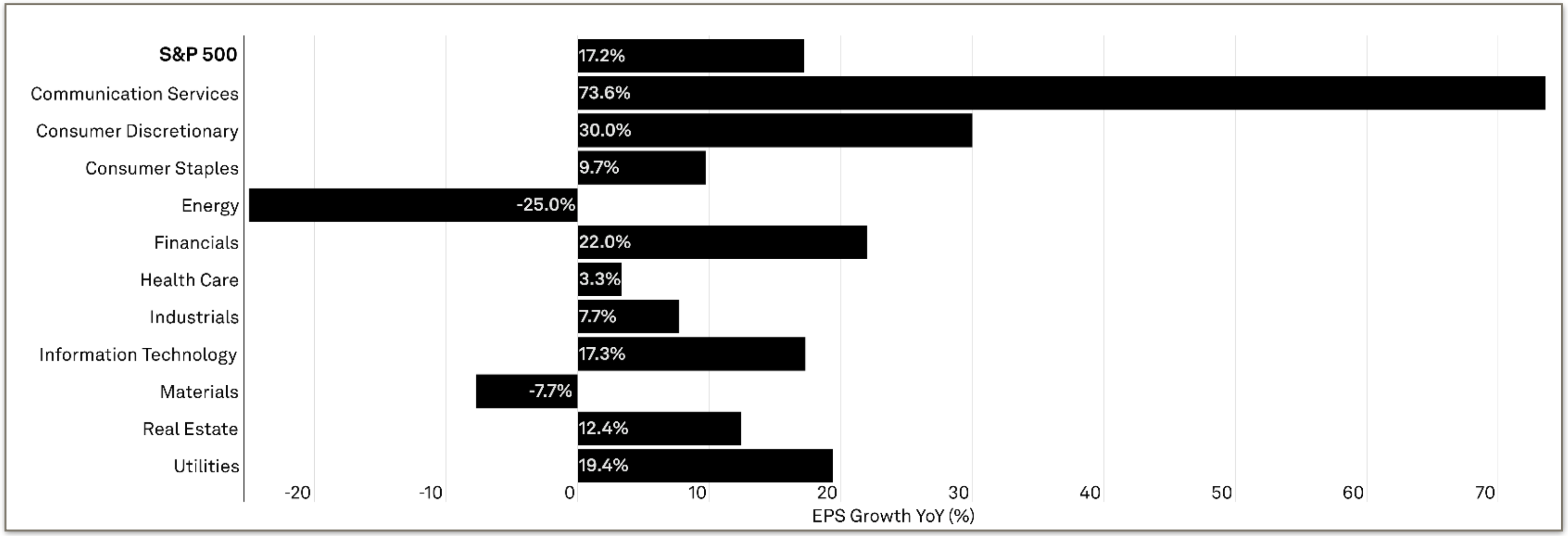


Figure 2 : Represent S&P 500 Q4 YoY EPS Growth (%). (S&P Global Market Intelligence, 2024)

Referring to **Table 1** and **Figure 3**, the mean EPS growth rate for NVIDIA is 290.92%. This figure significantly exceeds the S&P 500's Q1 2024 benchmark of 17.2%. NVIDIA's impressive average EPS growth indicates robust financial performance and operational excellence. Even during periods of market volatility, as observed in fiscal 2023, the company rebounded strongly, with fiscal 2024 displaying extraordinary earnings growth (e.g., 851.74% in Fiscal year 2024 Q2 and Fiscal year 2024 1267.53% in Q3). This exceptional mean growth of 290.92%, as shown in **Table 1**, highlights NVIDIA's ability to outperform the broader market consistently. It underscores the company's strong market demand, effective cost management, and innovation, making it a standout investment relative to the S&P 500, where the EPS growth rate was only 17.2%.

Table 1 : Represent Nvidia Revenue, EPS, Revenue Growth and EPS Growth per Fiscal Quarter

Fiscal Year	Fiscal Quarter	Revenue (in billions)	EPS (in USD)	Revenue Growth (%)	EPS Growth (%)
2022	Q1	$5.661B	0.08	83.86%	106.15 %
2022	Q2	$6.507B	0.1	68.31%	277.24 %
2022	Q3	$7.103B	0.1	50.3%	82.44 %
2022	Q4	$7.643B	0.12	52.77%	103.81 %
2023	Q1	$8.288B	0.06	46.41%	-16.12 %
2023	Q2	$6.704B	0.03	3.03%	-72.39 %
2023	Q3	$5.931B	0.03	-16.5%	-72.22 %
2023	Q4	$6.051B	0.06	-20.83%	-52.15 %
2024	Q1	$7.192B	0.08	-13.22%	28.11 %
2024	Q2	$13.51B	0.25	101.48%	851.74 %
2024	Q3	$18.12B	0.37	205.51%	1267.53 %
2024	Q4	$22.1B	0.5	265.28%	768.11 %
2025	Q1	$26.04B	0.6	262.12%	630.76 %
2025	Q2	$30.04B	0.68	122.4%	169.89 %
			Mean	***86.49 %***	***290.92%***

Note : Near the columns labeled "*Revenue Growth (%)*" and "*EPS Growth (%)*", green boxes indicate positive growth, signifying favorable performance, while red boxes indicate negative growth, representing unfavorable performance

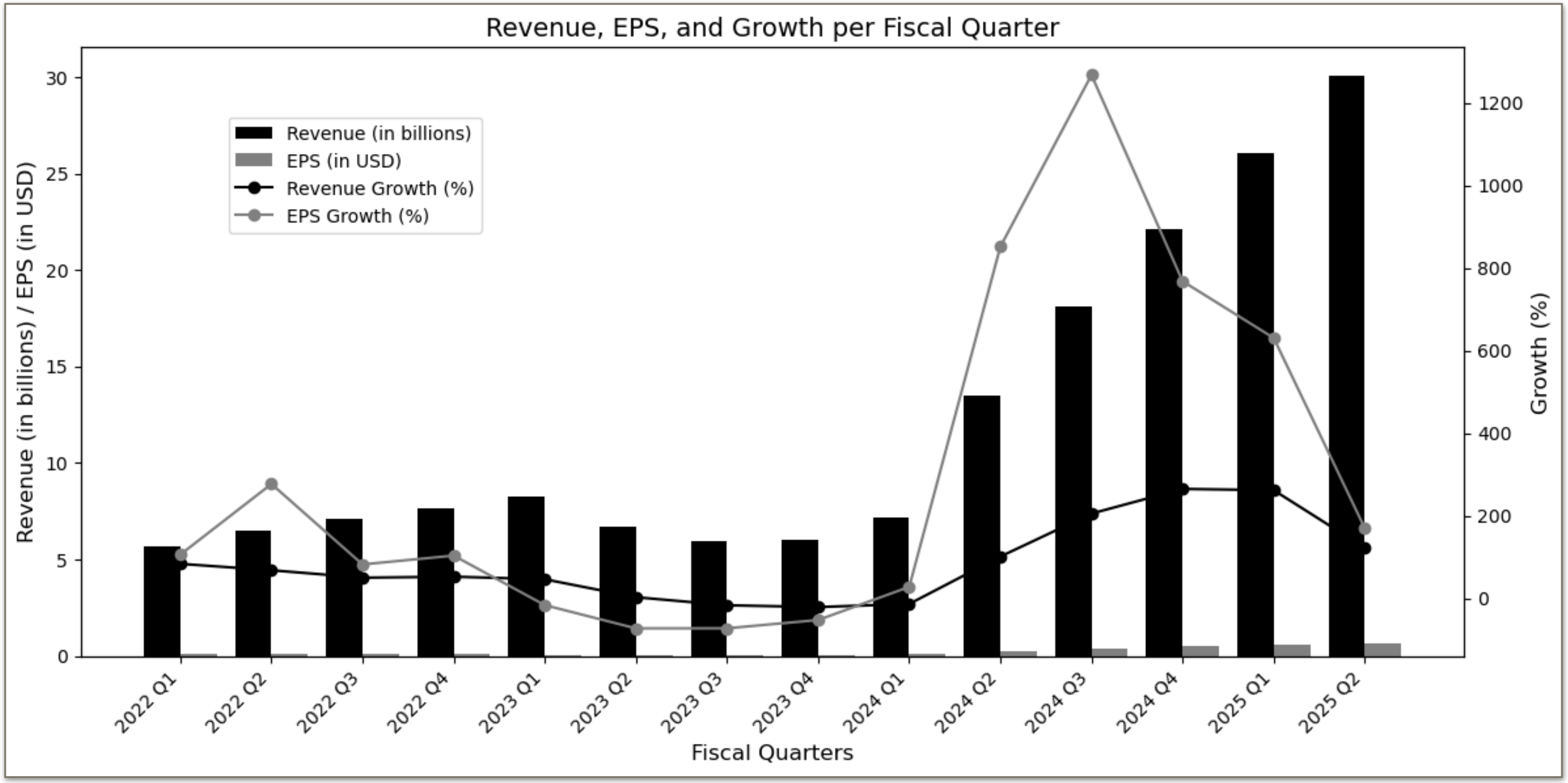


Figure 3 : Represent Nvidia Revenue, EPS, and Growth per Fiscal Quarter

Note : The graphs illustrate the data from **Table 1**, and the data has been plotted using Python.

Another key metric to evaluate NVIDIA's investment potential is its revenue growth rate compared to the S&P 500. Revenue growth provides insights into a company's ability to expand its operations and generate higher sales over time, which is essential for sustained profitability and market competitiveness. Referring to **Figure 4** In Q1 2024, the S&P 500 recorded a year-over-year revenue growth rate of 10.9%, showing a steady increase from 9.3% in Q4 2023 and 5.9% in Q3 2023. Earlier in the year, the growth rate stood at 3.9% for both Q2 and Q1 2023, signaling an overall positive trend in the revenue trajectory of the broader market. To assess NVIDIA's market performance, we must determine whether its revenue growth rate exceeds the S&P 500 benchmark of 10.9%. A higher revenue growth percentage would indicate that NVIDIA is not only outperforming the broader market but also demonstrating strong operational growth, which is crucial for long-term value creation. Such a comparison allows us as investors to gauge NVIDIA's relative position within the market and its ability to capitalize on evolving industry trends. (S&P Global Market Intelligence, 2024) (CSIMarket, n.d.)

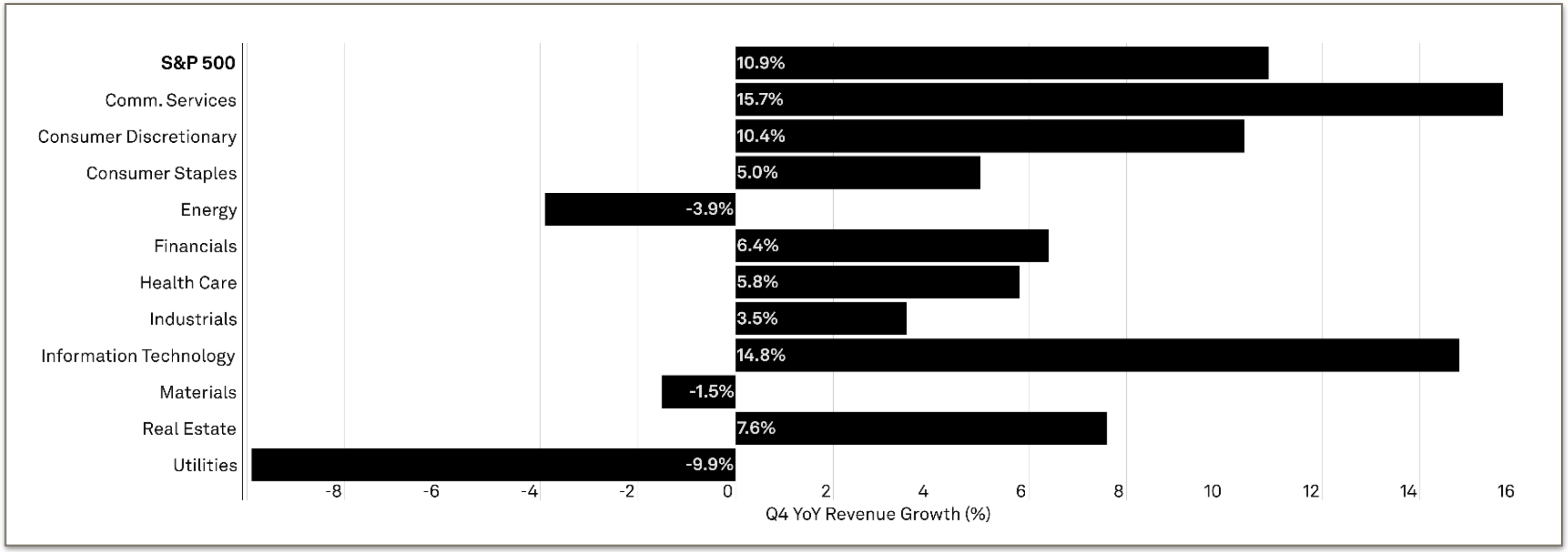


Figure 4 : Represent S&P 500 Q4 YoY Revenue Growth (%) (S&P Global Market Intelligence, 2024)

Referring to **Table 1**, the mean revenue growth rate for NVIDIA is 86.49%, a figure that substantially surpasses the S&P 500's Q1 2024 benchmark of 10.9%. NVIDIA's revenue growth performance is exceptional, far outpacing the broader market. With consistent increases observed in fiscal 2024 and 2025, such as 101.48% in Q2 2024 and over 265% in Q4 2024, NVIDIA demonstrates remarkable operational expansion and strong market demand.

These results highlight its ability to generate higher sales amid competitive and evolving industry conditions almost creating a monopoly market. The mean revenue growth rate of 86.49% solidifies NVIDIA's position as a leader in its sector. This growth rate reflects not only the company's ability to capture market share but also its effective strategies in addressing demand for its products and services, particularly in high growth segments like AI and advanced computing. Compared to the S&P 500's 10.9% revenue growth benchmark, NVIDIA's significant outperformance makes it an attractive investment candidate, showcasing its superior growth trajectory and competitive edge in the market.

2.1.2.2 Profit Margins

Profit margin is another crucial metric to analyze when evaluating NVIDIA's potential as an investment. Profit margins indicate the proportion of revenue that a company retains as profit after covering all expenses. This measure provides insight into a company's operational efficiency and its ability to manage costs relative to revenues. According to Tipper Alpha and **Figure 5** for Q1 2024 of the fiscal year 2025, the S&P 500 reported a net profit margin of 11.3%. Meanwhile a Forbes article estimates the profit margin at 11.7%, while other sources suggest a slightly lower figure of 10.1%. For consistency and to ensure a robust comparison, we will use the highest reported figure of 11.7% as a benchmark for evaluating NVIDIA's performance. Comparing NVIDIA's profit margin to this benchmark offers valuable insights into its profitability relative to the broader market. Historically, profit margins serve as a blunt but effective measure of a company's pricing power, cost management, and ability to generate earnings relative to its inputs, particularly labor and operational costs. Sustained high profit margins could signal a competitive advantage, but they also carry risks, such as increased scrutiny from regulators or changes in laws and accounting standards that could impact future earnings. (Lipper Alpha Insight, 2024) (Friesen, 2024) (DQYDJ, n.d.)

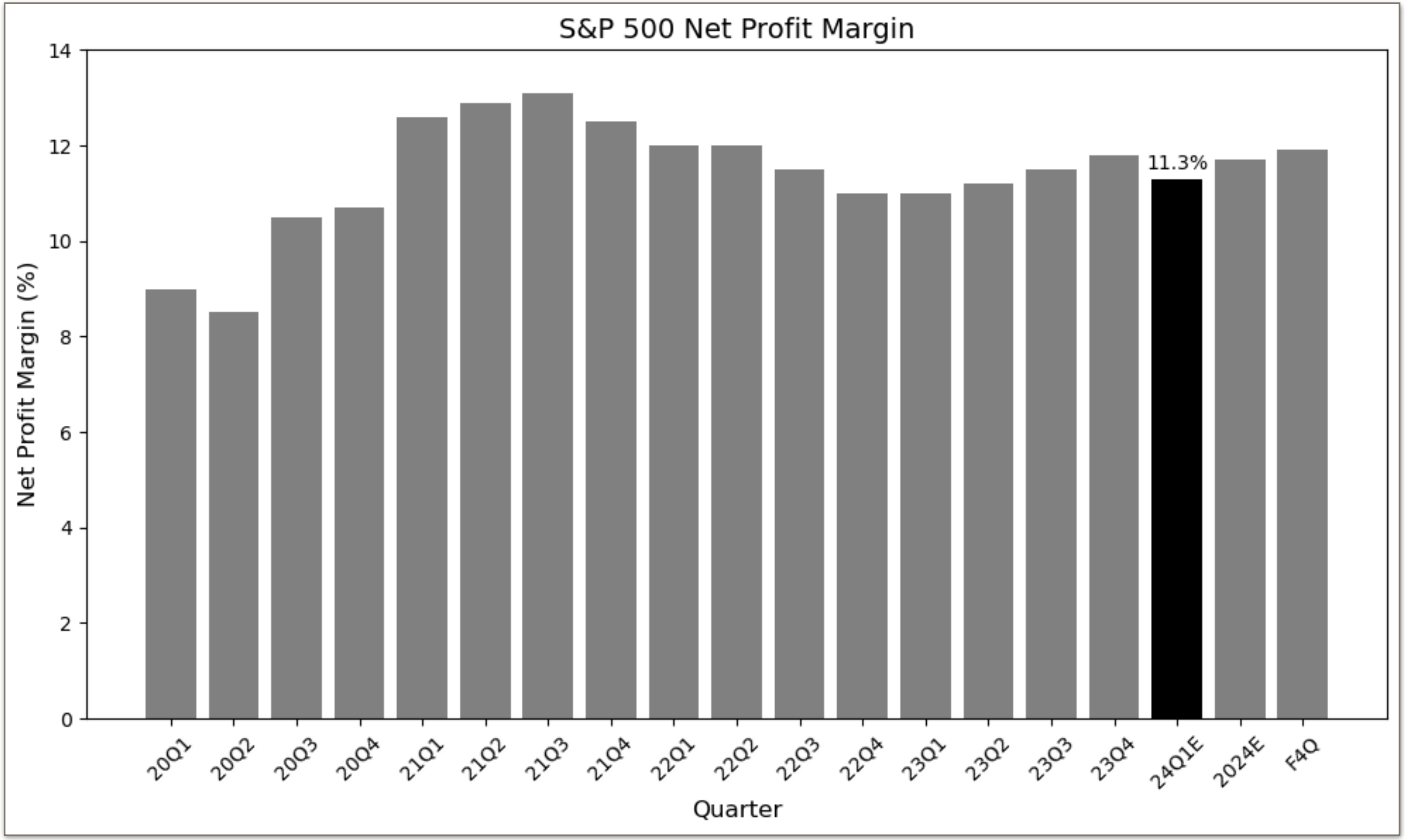


Figure 5: Represent S&P 500 Q4 YoY Revenue Growth (%)

Note : The graphs illustrate the data from taken from Lipper Alpha, and the data has been plotted using Python.

Referring to **Table 2**, the mean profit margin for NVIDIA across the reported quarters is 37.72%. This figure stands far above the S&P 500's Q1 2024 benchmark of 11.7%, demonstrating NVIDIA's superior profitability. NVIDIA's consistent ability to generate profit margins in the range of 30%–60% highlights its exceptional operational efficiency, pricing power, and cost management capabilities. Notably, in fiscal year 2025, NVIDIA achieved profit margins of 64.92% in Q1 and 62.06% in Q2, reinforcing its ability to retain a substantial portion of its revenue as profit. These margins reflect strong demand for NVIDIA's products, particularly in sectors like AI, GPUs, and advanced computing, and also indicate the company's competitive edge in managing input costs and maximizing profitability. Compared to the S&P 500 benchmark of 11.7%, NVIDIA's mean profit margin of 37.72% is more than three times higher. This significant outperformance positions NVIDIA as a highly efficient and profitable enterprise, appealing us to invest, seeking strong financial health and operational excellence. Such sustained high profit margins underscore NVIDIA's ability to maintain its market leadership while creating substantial shareholder value, further validating its position as a standout investment in comparison to the broader market.

Table 2 : Represent Nvidia Net Income, Revenue and Profit Margin per Fiscal Quarter

Fiscal Year	Fiscal Quarter	Net Income (USD)	Revenue (USD)	Profit Margin (%)
2022	Q1	$1.912B	$5.661B	34.55%
2022	Q2	$2.374B	$6.507B	37.56 %
2022	Q3	$2.464B	$7.103B	37.6 %
2022	Q4	$3.003B	$7.643B	38.86 %
2023	Q1	$1.618B	$8.288B	38.86 %
2023	Q2	$656M	$6.704B	7.44 %
2023	Q3	$680M	$5.931B	10.13 %
2023	Q4	$1.414B	$6.051B	20.76 %
2024	Q1	$2.043B	$7.192B	29.76 %
2024	Q2	$6.188B	$13.51B	50.34 %
2024	Q3	$9.243B	$18.12B	57.49 %

2025	Q1	$14.88B	$26.04B	64.92 %
2025	Q2	$16.6B	$30.04B	62.06 %
			Mean	***37.2%***

Note : Near the columns labeled "*Profit Margin (%)*", green boxes indicate positive growth, signifying favorable performance, while red boxes indicate negative growth, representing unfavorable performance

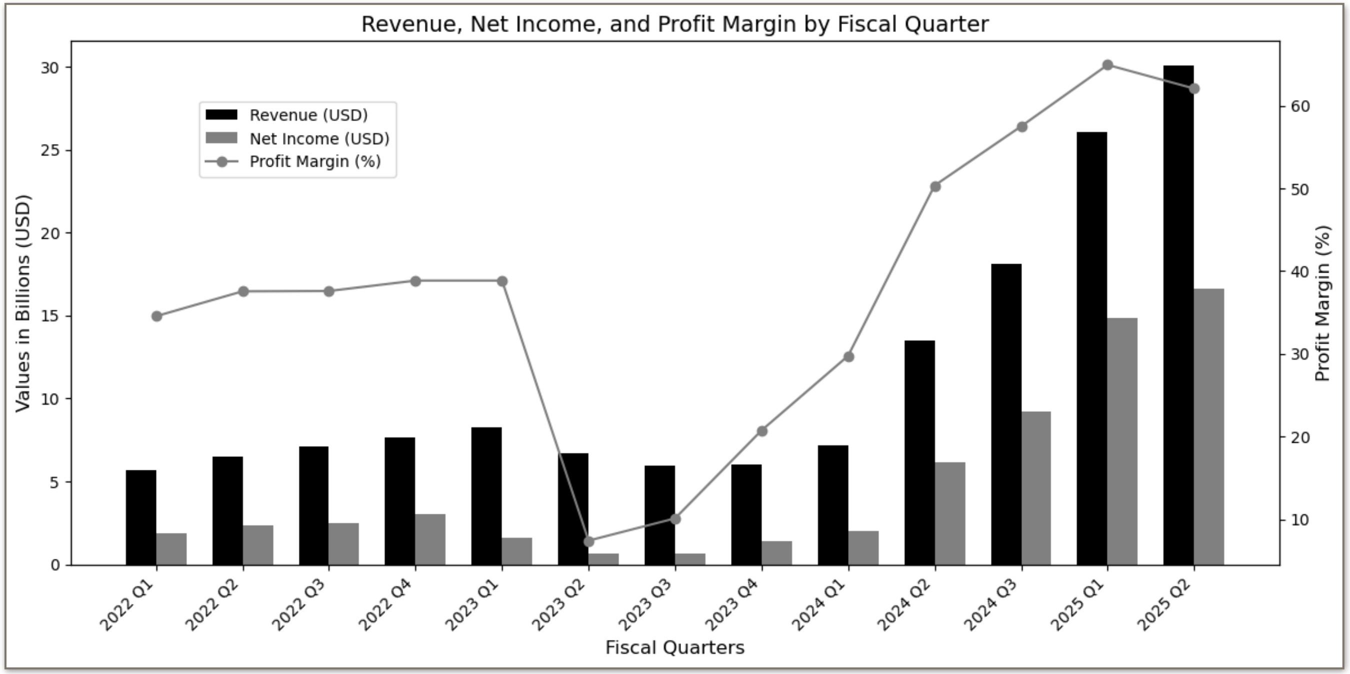


Figure 6 : Represent Nvidia Net Income, Revenue and Profit Margin per Fiscal Quarter

Note : The graphs illustrate the data from **Table 2**, and the data has been plotted using Python.

2.1.2.4 Cash flow Analysis

Capital expenditures (CapEx) and research and development (R&D) investments are critical indicators of a company's commitment to innovation and long-term growth. For NVIDIA, these metrics can be contextualized within the broader framework of the "Magnificent Seven," a group of highly influential companies in the market instead of the S&P 500. According to research conducted by Goldman Sachs, the total CapEx and R&D spending by the Magnificent Seven is projected to reach approximately $350 billion in 2024. This figure represents an extraordinary 61% of their collective operating free cash flow, highlighting the strategic focus of these companies on reinvestment and innovation. To put this into perspective, the Magnificent Seven's spending is three times greater than the combined CapEx and R&D expenditures of the remaining 493 companies in the S&P 500.As NVIDIA is a part of this elite group, we can use the 61% figure as a benchmark to evaluate its own spending levels. If NVIDIA's Cash flow growth exceed this benchmark, it would underscore its commitment to driving future growth through sustained innovation and reinvestment. Conversely, a lower percentage could prompt questions about whether NVIDIA is maintaining a competitive edge in these critical areas. This comparison not only reflects NVIDIA's standing within the Magnificent Seven but also offers us as investors a deeper understanding of its strategic priorities and alignment with broader industry trends in the tech market. (Friesen, 2024) (Forbes, n.d.)

The mean cash flow growth for NVIDIA, as shown in **Table 3** and is 228.55%. This figure is remarkably strong compared to the S&P 500, where cash flow growth trends are significantly lower on average. NVIDIA's ability to generate exceptional cash flow growth underscores its financial strength and operational efficiency. While the S&P 500's cash flow growth tends to be much more modest, NVIDIA has achieved extraordinary results, particularly in fiscal year 2024 and 2025. For instance, growth rates of 399.84% in Q2 2024, 1770.41% in Q3 2024, and sustained growth above 400% in the following quarters highlight its ability to capitalize on market demand and manage cash flow effectively. This growth not only supports NVIDIA's ability to fund significant investments in CapEx and R&D but also provides flexibility to drive innovation, pursue acquisitions, and strengthen its competitive position within the "Magnificent Seven." By maintaining an average cash flow growth rate of 228.55%, NVIDIA far exceeds industry benchmarks, solidifying its role as a leader in the tech sector and an attractive candidate for investors seeking sustainable returns.Which makes us believe that NVIDIA's strong cash flow likely aligns with its strategic goals and helps it maintain a competitive edge in an innovation-driven market.

Table 3 : Represent Nvidia Cash from different operations and Cash Flow growth per Fiscal Quarter

Date	Fiscal Year	Fiscal Quarter	Cash Flow from Operations (in millions)	Growth (%)
2021-05-02	2022	Q1	1874	106.16%
2021-08-01	2022	Q2	2682	71.16%
2021-10-31	2022	Q3	1519	18.76%
2022-01-30	2022	Q4	3033	46.73%
2022-05-01	2023	Q1	1731	-7.63%
2022-07-31	2023	Q2	1270	-52.65%
2022-10-30	2023	Q3	392	-74.19%
2023-01-29	2023	Q4	2248	-25.88%
2023-04-30	2024	Q1	2911	68.17%
2023-07-30	2024	Q2	6348	399.84%
2023-10-29	2024	Q3	7332	1770.41%
2024-01-28	2024	Q4	11500	411.52%
2024-04-28	2025	Q1	15350	427.14%
2024-07-28	2025	Q2	14490	128.23%
2024-10-27	2025	Q3	17630	140.41%
			Mean	***228,55 %***

Note : Near the columns labeled "Growth (%)", green boxes indicate positive growth, signifying favorable performance, while red boxes indicate negative growth, representing unfavorable performance

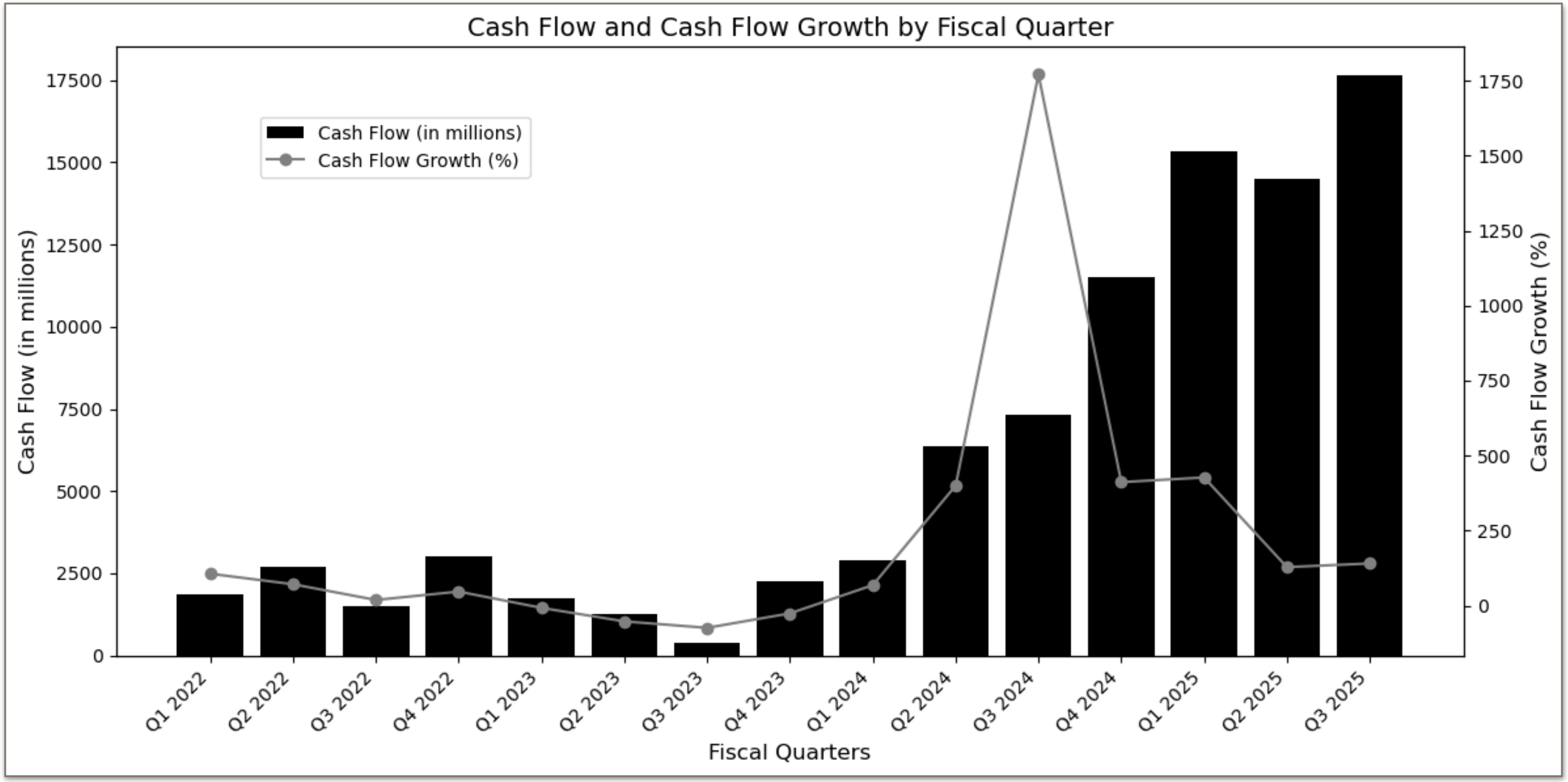


Figure 7 : Represent Nvidia Cash from different operations and Cash Flow growth per Fiscal Quarter

Note : The graphs illustrate the data from **Table 3**, and the data has been plotted using Python.

2.1.2.3 Debt levels

Debt is a critical component of corporate finance, one a crucial metric to analyzing which offering opportunities for growth NVIDIA has, while introducing potential risks. However, comparing one company's debt to another can be challenging due to several factors. Industries differ significantly in their capital requirements; for example, technology companies like NVIDIA often rely heavily on research and development (R&D) and capital expenditures (CapEx) to maintain competitiveness, while other sectors may accumulate debt for infrastructure or fixed costs. Additionally, the purpose of debt varies some companies strategically leverage it to fund acquisitions or high return projects, while others might be burdened with liabilities that inhibit growth. Moreover, the structure of debt matters; long term, low interest obligations are far less risky than short term liabilities exposed to fluctuating interest rates.(Damodaran, 2020) (Damodaran, 2012)

When assessing NVIDIA's debt, as shown in **Table 4** and **Figure 8** a clear trend of improvement emerges. During fiscal 2022, NVIDIA's total debt saw a sharp increase, growing 249.72% in Q1 and 71.6% in Q3. This spike likely reflects NVIDIA's strategic investments to capitalize on emerging market opportunities, such as its advancements in artificial intelligence and high performance computing. By fiscal 2023, however, NVIDIA's debt levels stabilized, hovering around $10.9 billion with minimal quarterly fluctuations. Importantly, from fiscal 2024 onward, NVIDIA began reducing its total debt consistently, achieving a 11.36% decrease across consecutive quarters. By Q2 2025, total debt stood at $9.71 billion, down from its peak levels in prior years. This reduction is a strong indicator of improved financial health. NVIDIA's ability to lower debt while simultaneously delivering impressive cash flow growth, as seen previously, highlights its operational strength and prudent financial management. Generating substantial cash flow allows NVIDIA to reduce its leverage without compromising growth initiatives, ensuring a healthier balance sheet and greater financial flexibility for the future. This flexibility is crucial, as it positions NVIDIA to invest further in innovation, weather economic downturns, and respond to opportunities with minimal constraints. Monitoring a company's debt is essential for investors because it provides a window into its financial resilience and risk profile. Excessive debt can erode profitability through high interest costs, reduce creditworthiness, and ultimately jeopardize the company's stability during economic challenges. On the other hand, well managed debt can fuel growth by funding strategic projects with high return potential. For NVIDIA, its declining debt levels and concurrent operational success demonstrate a balanced approach, where leverage is used as a tool for growth rather than becoming a liability. NVIDIA's effective debt management underscores its commitment to financial stability while maintaining the resources necessary to drive innovation and market leadership. By reducing total debt consistently and aligning it with strong cash flow generation, as mentioned in **2.1.2.4 Cash flow Analysis** NVIDIA enhances our confidence as investors and reinforces its position as a leader in the technology and semiconductor sector. This disciplined approach to debt not only reduces risk but also ensures the company is well prepared to sustain long term growth and profitability.

Table 4 : Represent Nvidia Total Debt and Debt Growth per Fiscal Quarter

Date	Fiscal Year	Fiscal Quarter	Total Debt (In Billions)	Growth (%)
2021-05-02	**2022**	Q1	6.963	249.72%
2021-08-01	**2022**	Q2	6.963	0.06%
2021-10-31	**2022**	Q3	11.943	71.60%
2022-01-30	**2022**	Q4	10.994	57.22%
2022-05-01	**2023**	Q1	10.946	57.20%
2022-07-31	**2023**	Q2	10.947	57.22%
2022-10-30	**2023**	Q3	10.949	-8.32%
2023-01-29	**2023**	Q4	10.95	0.06%
2023-04-30	**2024**	Q1	10.953	0.06%
2023-07-30	**2024**	Q2	10.954	0.06%
2023-10-29	**2024**	Q3	9.705	-11.36%
2024-01-28	**2024**	Q4	9.706	-11.36%
2024-04-28	**2025**	Q1	9.709	-11.36%
2024-07-28	**2025**	Q2	9.71	-11.36%

Note : Near the columns labeled "*Growth (%)*" green boxes indicate positive growth, signifying favorable performance, while red boxes indicate negative growth, representing unfavorable performance

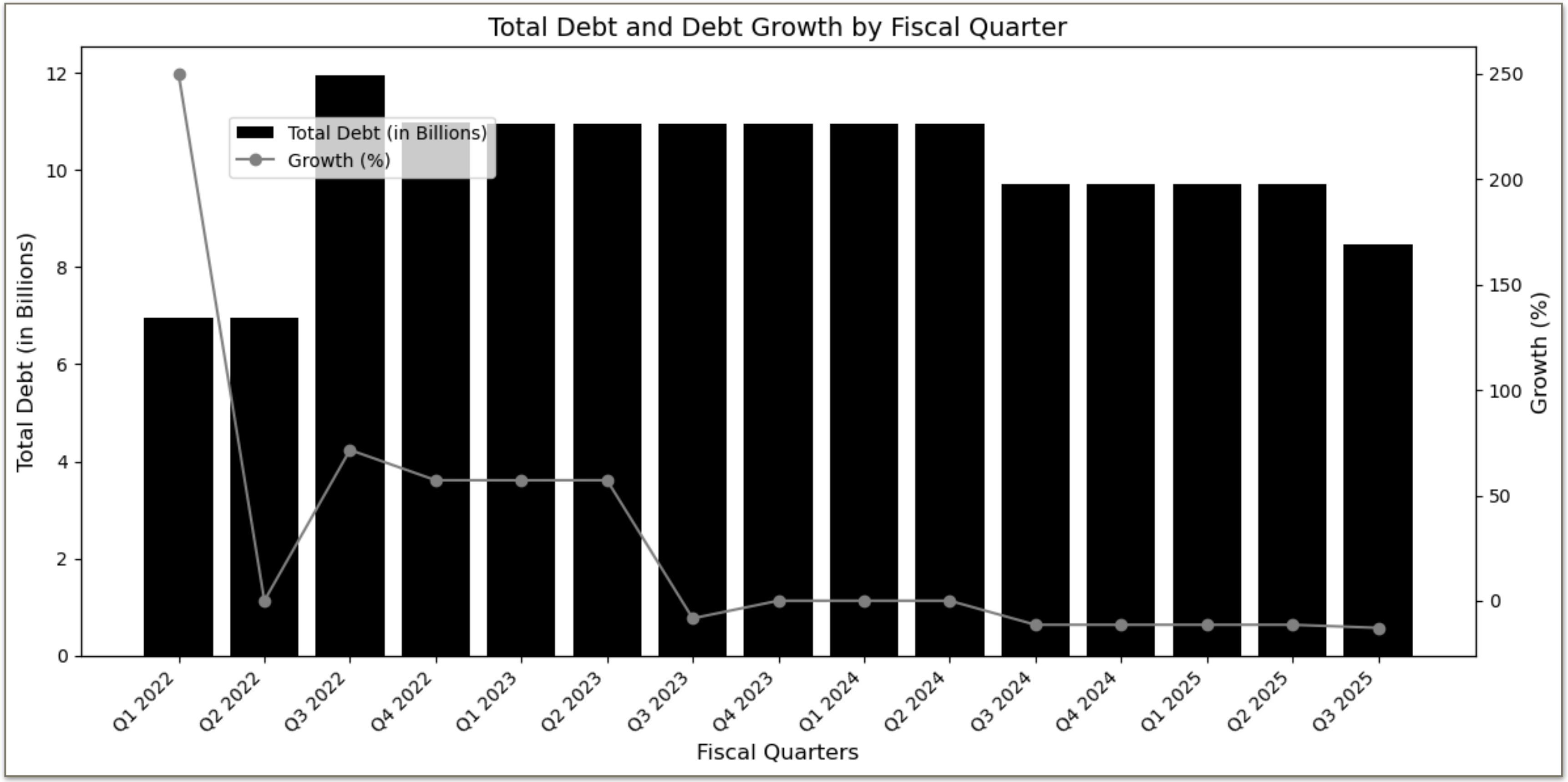


Figure 8 : Represent Nvidia Total Debt and Debt Growth per Fiscal Quarter

Note : The graphs illustrate the data from **Table 4**, and the data has been plotted using Python.

2.1.1 Qualitative

The qualitative information is further utilized to determine whether Nvidia is a strong company by evaluating it against the following criteria: Is the organizational structure clear and understandable? Does the company appear attractive? Is it led by competent and visionary managers? Does it have a sustainable competitive advantage, such as a clear future development plan?

2.1.1.1 Organizational structure

NVIDIA's organizational structure is strategically designed to capitalize on two transformative technological trends: AI adoption and accelerated computing. As a world renowned technology company, NVIDIA has redefined modern computing through its groundbreaking innovations. Founded in 1993 and headquartered in Santa Clara, California, NVIDIA operates globally with a significant presence across North America, Europe, Latin America, and the Asia Pacific regions. The company serves diverse markets, including data centers, gaming, professional visualization, automotive, robotics, and AI solutions. At the core of NVIDIA's dominance is its pioneering work in GPU technology. NVIDIA's GPUs power the development of deep learning models, generative AI, and inference systems. These GPUs are essential for training large scale AI models, such as OpenAI's GPT, which rely on billions of data points to simulate human like understanding. NVIDIA offers full stack computing solutions that enable AI adoption and accelerate computational performance across industries. Its flagship AI GPUs, including the H100 and A100, are considered the backbone of AI development a crucial advantage in the Fourth Industrial Revolution (Industry 4.0), where AI-driven optimization of compute heavy tasks is the primary focus across diverse industry sectors. Unlike traditional CPUs, NVIDIA's GPUs specialize in parallel processing, dramatically increasing computational efficiency. This shift from CPU centric to GPU centric computing is groundbreaking and addresses applications requiring enormous computing power, such as scientific simulations, robotics, autonomous vehicles, and drug discovery. The NVIDIA CUDA programming model, combined with domain specific libraries, enables developers to unlock the full potential of NVIDIA hardware for compute intensive tasks. Together, these factors ensure NVIDIA remains indispensable in driving digital transformation across industries.(CNBC, 2024)(Untaylored, 2024)(Research-Methodology.net, n.d.)

NVIDIA's market leadership in AI chips is unparalleled, with the company holding between 70-95% of the market share for AI training processors. Its early investment in GPU acceleration and the CUDA software ecosystem has created a formidable technological moat, making it difficult for competitors to catch up. With the AI chip market projected to reach $400 billion annually within the next five years, NVIDIA is uniquely positioned to capture a substantial portion of this exponential growth. Beyond AI chips, NVIDIA has expanded its influence across multiple sectors. For instance, in the automotive industry, NVIDIA's DRIVE platform powers autonomous driving systems for companies like Tesla. The platform enables vehicles to perceive, map, and make decisions in real time, advancing safer and smarter automotive solutions. In healthcare and science, NVIDIA's GPUs accelerate drug discovery, genomics, and medical imaging. AI models trained on NVIDIA hardware enable scientists to analyze genome data, predict molecular interactions, and design potential treatments for diseases at unprecedented speeds. In the field of robotics and automation, companies like Amazon Robotics leverage NVIDIA's solutions to power autonomous robots that optimize warehousing and delivery processes. These AI powered systems rely on GPU acceleration to navigate complex environments efficiently. NVIDIA also drives industrial digitalization with its Omniverse platform, which allows industries like manufacturing and logistics to create digital twins virtual simulations of real world environments. For example, BMW uses Omniverse to optimize factory workflows, reducing costs and improving efficiency. NVIDIA's contributions extend further into energy optimization and climate solutions; its GPUs are used for climate simulations, renewable energy optimization, and infrastructure development to address global challenges like energy efficiency and climate change. These examples highlight the vast applicability of NVIDIA's hardware and software solutions across industries, proving its critical role in technological advancement. The impact is reflected in sectors such as healthcare, industry, and information technology, which collectively demonstrate a YoY EPS growth of approximately 30%, as represented in **Figure 2**, showcasing NVIDIA's strategic alignment with high growth markets. By fostering strategic partnerships with industry leaders such as Tesla and Amazon Robotics, NVIDIA not only validates its technological edge but also creates a stable operational foundation, ensuring scalability and long term success. NVIDIA's focus on energy efficiency further strengthens its position in both private and public sectors. For instance, NVIDIA's Blackwell GPUs are up to 20x more energy efficient than traditional CPUs for specific AI and high performance computing (HPC) workloads. Similarly, NVIDIA DPUs reduce power consumption by 25% by offloading essential data center functions from less efficient CPUs. These innovations enhance NVIDIA's attractiveness for government contracts and climate conscious initiatives, reinforcing its status as a future oriented leader.(Vestinda, 2024) (NVIDIA, 2024a) (NVIDIA, 2024b) (NVIDIA Investor Relations, n.d.)

However, NVIDIA's dominance in the AI market comes with challenges. As a pioneer, the company faces immense pressure to sustain its technological leadership while mitigating risks. Key competitors such as AMD, Intel, and emerging startups like D Matrix are aggressively entering the AI chip market. Hyperscalers like Google and Amazon are also developing in house AI chips to reduce their reliance on NVIDIA, posing a threat to its market share. Another challenge is the high cost of NVIDIA's flagship GPUs, such as the H100, which retail for over $30,000 per unit. This pricing pushes customers to seek cheaper alternatives, particularly for AI inference tasks. NVIDIA's monopolistic market position, while advantageous now, could become a vulnerability if competitors gain ground. The company's innovation momentum resembles a wave NVIDIA continuously generates energy and invests heavily to stay ahead, while competitors "surf" the wave, benefiting from NVIDIA's groundwork at a lower cost. Once the wave dissipates, competitors may glide forward, posing further challenges. NVIDIA is also heavily reliant on the AI boom. A slowdown in AI adoption or a disruptive technological breakthrough could undermine its growth trajectory. Additionally, as a hardware company with a global supply chain, NVIDIA faces geopolitical tensions, export restrictions, and supply chain disruptions that could impact operations. These risks will be further explored in Chapter **2.1.1.3: Innovation Strategies and Processes**. (FourWeekMBA, 2023) (FourWeekMBA, 2023)

2.1.1.2 Leadership & Ownership

NVIDIA stands out for its leadership and ownership dynamics. A key strength is that NVIDIA remains foundered under CEO and co founder Jensen Huang. Founder led companies consistently outperform the market because founders bring unparalleled market understanding, a long-term vision, and the ability to drive innovation. Huang has led NVIDIA for over 30 years, evolving it from a graphics chip manufacturer into a leader in gaming, artificial intelligence, and data centers. His tenure provides stability and strategic continuity, reinforcing our confidence in investing. However, a notable concern, as shown in **Table 5** and **Figure 9,** is NVIDIA's insider ownership, which stands at approximately 4%, far below the ideal threshold of 10% that aligns leadership interests with shareholders. While Huang's 3.5% , refer to **Table 6**, stake is significant in monetary value, the percentage raises questions about leadership's financial commitment. Furthermore, recent insider selling could signal caution about the company's valuation or growth trajectory, a potential red flag for us as investors. NVIDIA's ownership is dominated by institutional investors, holding 51% of shares, with The Vanguard Group as the largest stakeholder at 8.75%. High institutional ownership adds credibility and reflects confidence in the company's fundamentals but introduces risks. Institutional investors can trigger volatility if they collectively adjust their positions based on market sentiment. Such concentration means NVIDIA's stock price remains sensitive to institutional actions. The general public holds around 30% of shares, offering some diversification but limited influence compared to institutional giants. While retail investors benefit from NVIDIA's growth, they remain vulnerable to shifts driven by larger stakeholders.(MarketScreener, n.d.)(The Information, n.d.) (Yahoo Finance, 2024)

Table 5 : Represent Nvidia Shareholder type and percentage

Shareholder Type	Percentage
Institutional	51.1
Unknown	31.7
Other	7.8
Individuals Insiders	3.93
State Street Corp.	3.86

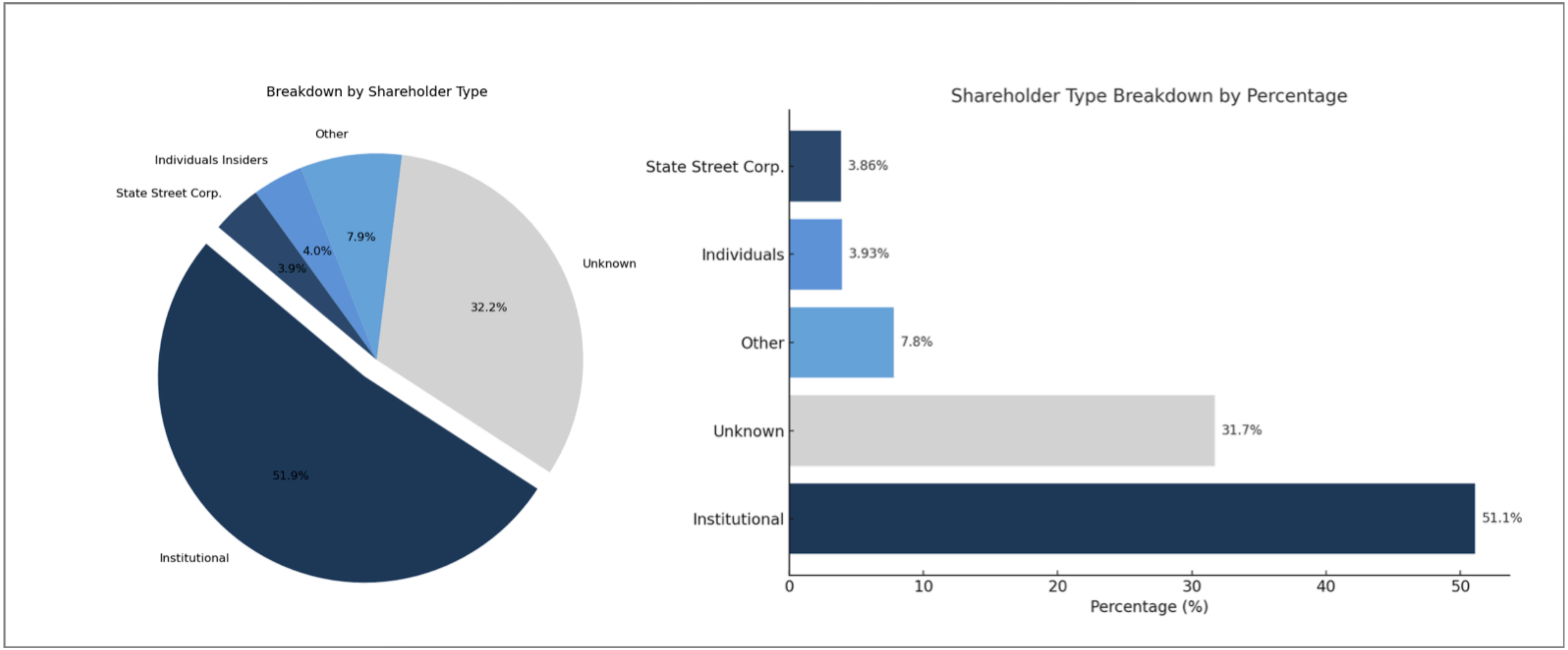


Figure 9 : Represent Nvidia Shareholder Type by Percentage

Note : The graphs illustrate the data from **Table 5**, and the data has been plotted using Python.

Table 6 : Equities Percentage and Value of the company owned

Name	Equities	Percentage	Valuation (In Billions)
Vanguard Fiduciary Trust Co.	2143786860	8.754%	296 $
BlackRock Advisors LLC	1417824137	5.789%	196 $
Fidelity Management & Research Co. LLC	997153684	4.072%	138 $
State Street Corp.	945738025	3.862%	131 $
Jensen Huang	859323531	3.509%	119 $
Geode Capital Management LLC	546079492	2.23%	75,495 $
T. Rowe Price International Ltd.	407607620	1.664%	56,352$
JPMorgan Investment Management, Inc.	404892154	1.653%	55,976 $
BlackRock Life Ltd.	375552284	1.533%	51,920$
Eaton Vance Management	329700529	1.346%	45,581$

2.1.1.3 Products and Future Development

As mentioned in **2.1.1.1 Organizational structure** at the core of Nvidia's success is its complete technology ecosystem, combining hardware and software to support AI, data analysis, scientific research, and other demanding computing tasks. Central to this are Nvidia's GPUs, powered by CUDA a programming tool that allows GPUs to efficiently handle complex tasks, particularly those needed for AI. Alongside CUDA, Nvidia offers a wide range of software tools and applications that make it easier for businesses to use AI and get the best performance from its hardware, making Nvidia a key player across many industries. Adding to this foundation are Mellanox's Data Processing Units (DPUs), which help make data centers faster and more efficient by taking over heavy networking and storage tasks from CPUs. Combined with Nvidia's GPUs, these DPUs help build powerful, scalable systems that can handle the growing demand for AI. On top of this, Nvidia introduced the Grace CPU in 2023, a processor designed to link thousands of GPU accelerated servers into one system. This creates a unified platform capable of managing massive amounts of data, which is crucial for modern AI applications. (Reddit, 2013)(Tom's Hardware, 2024)

Looking ahead, Nvidia's production plans offer strong growth potential. Deliveries of the H200 GPU are set to ramp up in Q3 2024, with production of its next-generation Blackwell GPUs (GB200) expected to begin around the same time. What makes this particularly important for our decision in trading Nvidia's options is the timing Nvidia's fiscal Q3 2025 report will be released shortly. This report will provide a key opportunity for Nvidia to prove it can meet production targets and capitalize on demand. If the company delivers as expected, strong results could drive up the stock price, especially with the growing interest in Blackwell GPUs and AI hardware. The combination of Nvidia's clear product pipeline, its ability to scale up production, and the critical role its technology plays in AI makes it a strong investment opportunity. With upcoming fiscal results aligning closely with Blackwell production, Nvidia has the potential to outperform expectations. If the company meets its commitments and demand remains high, the stock price could see notable gains, making it attractive. (TrendForce, 2024) (TweakTown, 2024)

2.2 Market And Industry Dynamics

How will the current economic and geopolitical landscape impact Nvidia? The Democratic Progressive Party, which supports Taiwan's independence, secured a third consecutive term in 2024. In response, Beijing has intensified both political and military pressure on Taipei. Adding to this tension, the upcoming U.S. election could further escalate conflicts between China and the U.S., especially if Donald Trump wins. Trump's platform includes steep import tariffs, with proposed rates as high as 60% on Chinese made goods. If enacted, these tariffs would significantly challenge Xi Jinping's ambitions to transform China into a global technology powerhouse, straining relations between the world's two largest economies even further. Given these developments, how well can Nvidia navigate this geopolitical storm? Nvidia operates in critical and often contentious regions, including the U.S., China, Taiwan, and Latin America, as outlined **2.1.1.1 organizational structure**. Conflicts in these areas particularly between China and Taiwan or China and the U.S. pose a serious risk to Nvidia's market stability, potentially impacting revenue and profitability. Negative political shifts in any of these regions could hurt Nvidia's supply chain, sales, and overall business operations.

2.2.1 Geographical Region, Market and Geopolitics

Tensions in geopolitics between Taiwan and China, as well as the USA and China, are set to heavily impact Nvidia's stock. As a cornerstone of the AI and semiconductor industries, Nvidia's reliance on Taiwan Semiconductor Manufacturing Company (TSMC) for chip production makes it especially vulnerable to disruptions in these increasingly strained relationships. The Taiwan Strait has long been a geopolitical flashpoint, with China asserting its claim over Taiwan. Recent military exercises by China near Taiwan have escalated fears of conflict, threatening the stability of the global tech supply chain. TSMC, the world's largest contract chipmaker and a vital supplier for Nvidia, is at the heart of this tension. Any conflict or disruption in Taiwan would severely impact TSMC's ability to produce advanced chips, which are critical for Nvidia's GPU production. Taiwan's chipmakers have also reportedly developed a 'kill switch' mechanism, enabling them to shut down production remotely to safeguard trade secrets in the event of an invasion. While this protects intellectual property, it introduces a significant risk of production halts, leaving companies like Nvidia exposed to massive supply chain vulnerabilities.(OneSafe, 2024)

Compounding these risks is Nvidia's position in the ongoing US-China tech rivalry. American restrictions on the export of Nvidia's advanced A100 and H100 chips to China have already reduced its revenue from the Chinese market, which accounted for just 17% of Nvidia's revenue in 2023, down from 26% in 2021. These restrictions also disrupt Nvidia's dominance in the AI chip sector, as Chinese companies and research institutions seek alternatives. Adding to this complexity, China has launched an antitrust investigation into Nvidia related to its Mellanox acquisition. This scrutiny not only threatens Nvidia's reputation but also raises barriers to its partnerships in Asia, further complicating its global strategy. The consequences of these tensions ripple across the tech sector. Nvidia and its competitor AMD, both heavily dependent on TSMC, face the same vulnerability: any disruption in Taiwan's chip production could dramatically impact their operations and stock performance. For TSMC itself, geopolitical uncertainties have created heightened volatility as investors weigh the potential fallout of escalating tensions. The precarious balance of power between Taiwan, China, and the United States underscores the fragility of the tech industry's supply chain. For Nvidia, this means navigating a highly uncertain landscape where geopolitical events can have immediate and far-reaching impacts. The 'kill switch' mechanism, while designed to protect intellectual assets, further illustrates the delicate situation, where any escalation could abruptly disrupt the flow of advanced chips. (Nasdaq, 2024)

Ultimately, the intertwined challenges of geopolitical instability and trade restrictions put Nvidia at significant risk. The company's ability to maintain its technological lead while mitigating supply chain disruptions will be crucial. For investors, these geopolitical dynamics demand close attention, as the stakes for Nvidia and the broader tech industry continue to escalate.

2.3 Trading Comp Valuation and fundamental metrics

Note: While the mathematical calculations underlying this analysis are not detailed here due to space constraints, they are thoroughly documented in the accompanying Excel file "Trading Comp Valuation.excl".

The trading comparable valuation analysis has also been calculated to offer a detailed view of Nvidia's relative valuation within the semiconductor industry, highlighting how it measures against its key competitors. This approach evaluates Nvidia alongside peers like AMD, TSM (Taiwan Semiconductor Manufacturing), Intel, ARM, Qualcomm, Broadcom, and Micron Technology. By analyzing valuation multiples such as EV/Revenue, EV/EBITDA, and P/E ratios, we gain a clearer perspective on Nvidia's market positioning. The analysis reveals a striking disparity in valuation. Nvidia's EV/Revenue multiple for Q1'24 is a remarkable 163.8x, significantly higher than the industry mean of 73.5x and the median of 44.4x. Across subsequent quarters, Nvidia continues to trade at a substantial premium. For instance, by Q2'25, Nvidia's multiple stands at 144.0x compared to the sector mean of 50.1x. This trend is consistent with the EV/EBITDA multiples, where Nvidia reaches 292.7x in Q1'24, nearly double the sector mean of 156.3x and well above the median of 87.2x. The P/E ratios further underscore Nvidia's premium, with a staggering 333.3x for Q1'24 against the sector average of 135.3x. When compared with competitors, Nvidia's valuation stands out sharply. Companies like AMD and Broadcom show much more modest multiples, with AMD's EV/Revenue at 44.1x for Q1'24 and Broadcom at 44.6x. TSM, often considered a leader in semiconductor manufacturing, trades at 59.6x EV/Revenue, reflecting a more conservative valuation. ARM, though notable for its innovation, has unusually high multiples, such as 221.4x for EV/Revenue, but lacks Nvidia's dominance in growth segments like AI. Meanwhile, Intel trades at far lower multiples, with an EV/Revenue of just 8.1x for Q1'24, showcasing its struggles in maintaining competitive growth. Nvidia's consistently higher multiples can largely be attributed to its leadership in the AI sector. The company's dominance in AI accelerators and its robust growth in data center demand projected to reach $500 billion annually justify some of its premium valuation. Furthermore, Nvidia's innovative product pipeline, such as its next generation Blackwell chips, solidifies its position as a market leader. These factors drive investor confidence and contribute to Nvidia's elevated multiples relative to its peers.

However, the trading comp analysis also underscores potential risks. Nvidia's high valuation reflects optimistic growth expectations that may be tempered by external factors. Regulatory scrutiny, as seen with the DOJ's antitrust investigation, supply chain challenges, and geopolitical tensions, particularly involving Taiwan (a critical supplier of semiconductors), pose significant uncertainties. These risks highlight the possibility that Nvidia's premium valuation might not be sustainable if market conditions shift.

Trading comparable valuation is an essential tool for contextualizing a company's market valuation relative to its competitors. It provides a benchmark to determine whether a stock is overvalued or undervalued based on industry norms. In Nvidia's case, while its leadership position and growth trajectory justify a higher valuation, the magnitude of its multiples consistently double or even triple those of its peers raises questions about overvaluation. Yet, for investors like us with a higher risk tolerance, Nvidia's strategic advantages and dominance in AI might make its premium valuation more palatable. Ultimately, the decision to invest hinges on the following question: Can Nvidia continue to outperform its peers and maintain its market leadership amidst a rapidly evolving industry landscape.

2.4 Technical Analysis

*Note: Further data on the Nvidia technical analysis can be found in **Appendix A2: Technical Analysis**.*

The decision to invest in NVIDIA is guided by a combination of technical analysis and historical price performance data, which together provide a comprehensive picture of the stock's behavior following significant events. **Table 7** presented demonstrates the price changes of NVIDIA one day after key report dates, showing an average increase of +5.99%. This data highlights NVIDIA's strong market performance and positive investor sentiment during these periods, making it an attractive candidate for investment. The upward momentum seen after major announcements, such as the +24.38% increase on May 24, 2023, and the +16.41% increase on February 21, 2024, reflects the market's confidence in NVIDIA's ability to meet or exceed expectations. While there are occasional negative movements, such as the -6.39% decline on August 28, 2024, these are overshadowed by the broader trend of significant gains. This indicates that, even in instances of temporary dips, NVIDIA's stock has shown resilience and the ability to recover, reaffirming its strength as an investment. Technical analysis plays a critical role in evaluating NVIDIA's investment potential. By examining percentage retracements, I can identify key support and resistance levels, which are crucial in determining whether a price decline is a temporary pullback within an uptrend or a sign of a broader reversal. For example, NVIDIA's recovery following past retracements suggests that such dips have historically been opportunities to buy rather than warnings of a sustained downtrend. The stock's ability to bounce back and generate consistent returns further solidifies its reliability as an investment option. Additionally, understanding retracement levels helps establish optimal entry points for investment. If NVIDIA's stock price retraces to a key support level but maintains its upward trend, this indicates a strong buying opportunity. The historical data shows that even during periods of slight declines, NVIDIA has demonstrated the resilience to rebound, providing confidence in the stock's long-term growth prospects. In making this decision, it is also important to consider NVIDIA's position within the broader technology market. The company has consistently innovated and maintained a leading position in its industry, which aligns with the positive market reactions observed in the data. This strategic advantage, combined with strong historical performance and reliable retracement patterns, reinforces the rationale for investing in NVIDIA. (TipRanks, n.d.) (Murphy, 1999)

Table 7: Represent price ranges

Report Date	Price 1 Day Before	Price 1 Day After	Percentage Change
Nov 20, 2024	145.88	146.66	+0.53%
Aug 28, 2024	125.59	117.57	-6.39%
May 22, 2024	94.93	103.77	+9.31%
Feb 21, 2024	67.45	78.52	+16.41%
Nov 21, 2023	49.92	48.7	-2.44%
Aug 23, 2023	47.09	47.14	+0.11%
May 24, 2023	30.52	37.96	+24.38%
		Mean	**+ 5.99%**

Note : Near the columns labeled "*Growth (%)*" green boxes indicate positive growth, signifying favorable performance, while red boxes indicate negative growth, representing unfavorable performance. The data in the table has been derived from the various values obtained through analyzing the chart in ***Appendix A2: Technical Analysis***s.

3. Option Strategy Plan

Table 8 : Represent Options in the Context of Market Instruments

	Debt	Equity	Derivatives
Money Market (short term)	• T-Bills • CD's • Eurodollars • Fed Funds		• **Options** • Futures • Forward Contracts
Capital Market (Long term	• T-Bonds • Agency bonds • Municipals • Corporate bonds	• Common Stock • Preferred Stock	• LEAPS • Swaps

Options are versatile financial instruments that grant the holder specific rights regarding the purchase or sale of an asset at a predetermined price, by a certain date. There are two main types of options: call options and put options. A call option gives the holder the right, but not the obligation, to buy the underlying asset for a specified price (the strike price) on or before the expiration date. Conversely, a put option gives the holder the right to sell the underlying asset for the strike price within the same time frame. The expiration date marks the last day the option can be exercised. Options are categorized further based on their exercise style. American options can be exercised at any time up to and including the expiration date, while European options can only be exercised on the expiration date itself. Although most exchange traded options are American due to their flexibility, European options are often easier to analyze and are widely used for theoretical and pricing models. When an investor takes a position in options, it can be a long position, where the option is purchased, or a short position, where the option is sold or written. A long call position hopes for an increase in the underlying asset's price, while a long put position anticipates a price decrease. On the other hand, the writer of a call or put option takes on the opposite exposure, earning an upfront premium while bearing potential liabilities later. Options can be classified based on the relationship between the stock price (S) and the strike price (K). A call option is considered in the money if $S > K$, at the money if $S = K$, and out of the money if $S < K$. For put options, the classifications are reversed. The intrinsic value of an option is the immediate value it would have if exercised, calculated as $\max(S - K, 0)$ for calls and $\max(K - S, 0)$ for puts. Any additional value an option holds due to the time left until expiration is referred to as its time value. Option contracts are standardized by exchanges to ensure transparency and liquidity. A typical contract in the United States represents 100 shares of the underlying stock. The expiration date is fixed, often set as the third Friday of the expiration month, with options trading available for multiple expiration cycles. Strike prices are determined by the exchange and are usually spaced at $2.50, $5, or $10 intervals, depending on the stock's price. Market makers play a crucial role in ensuring liquidity by quoting bid and offer prices for options, facilitating seamless transactions. However, writers of options must maintain margin accounts to cover potential liabilities. This system is overseen by organizations like the Options Clearing Corporation, which ensures the integrity and execution of trades. Beyond exchange-traded options, a significant portion of options is traded over the counter (OTC). These OTC options are customized to meet the specific needs of corporate treasurers or fund managers. Although they lack the standardization of exchange-traded options, OTC options offer greater flexibility in terms of strike prices, expiration dates, and other contract specifications. (Tastytrade, n.d.) (Hull, 2017)

3.1 Option strategy

Based on Nvidia's trading valuation, business model, and technical analysis, the company appears to be in a bullish phase. Key supporting factors include Nvidia's dominance in the AI market, with expectations to lead the AI accelerator market, and significant growth in data center demand projected at approximately $500 billion annually. Additionally, the anticipated impact of Nvidia's new product, the Blackwell chip, likened to the revolutionary iPhone launch, underscores its growth potential. However, Nvidia's geopolitical and industry dynamics warrant caution. Regulatory investigations by the U.S. Justice Department over potential antitrust violations and supply chain constraints, including production delays for the Blackwell chip, pose risks. The supply chain challenges and geopolitical tensions could catalyze a bearish shift in the market sentiment. Financial performance pressures, such as declining gross margins, add further uncertainty. Given these dynamics, we might lean toward bullish option strategies while maintaining the flexibility to shift toward bearish or neutral strategies if geopolitical or industry factors materialize negatively. The dual nature of Nvidia's market outlook highlights the importance of tailoring option strategies dynamically to align with evolving conditions.

We can use ***Figure 1 in Appendix.A4 Figure*** as a decision making tool, guiding through various strategies that align with the market outlook and volatility expectations. Nvidia, with an implied volatility of 55%, falls into a high volatility environment, making it particularly relevant to analyze strategies suitable for such conditions while considering both bullish and bearish scenarios. In high volatility markets, investors with a bullish outlook can utilize strategies such as selling out of the money put options to capitalize on rising prices. This approach, known as a short put, offers a high probability of profit at 80% and allows us to benefit from Nvidia's upward momentum. Alternatively, a covered call strategy, which involves combining a long stock position with a short out of the money call, generates income while providing exposure to

Nvidia's growth. Neutral sentiment in high volatility scenarios could lead to strategies like a wide iron condor or a strangle, both of which aim to profit from range bound price movements, with the latter offering a probability of profit of 70%. For bearish sentiment, strategies like short calls or call ratio credit spreads can be employed. The latter involves selling two out of the money calls while buying one call closer to the money, offering an 85% probability of profit and serving as an effective hedge against potential price declines.

Nvidia's current market positioning supports a bullish outlook, driven by its dominance in the AI market, its pivotal role in data center growth, and the anticipation surrounding its upcoming Blackwell chip. These factors underline Nvidia's long-term potential and make it an attractive asset for bullish strategies. However, risks such as regulatory investigations by the U.S. Justice Department, supply chain disruptions, and declining gross margins cannot be ignored. These challenges introduce the possibility of a bearish shift in sentiment, highlighting the need for adaptable strategies. Given Nvidia's strong growth potential, a short put strategy or a covered call aligns well with its current high volatility and bullish trajectory. These strategies not only leverage Nvidia's upward momentum but also could generate income, making them ideal for the current market environment. However, considering the potential for bearish shifts due to regulatory concerns or supply chain issues, a call ratio credit spread provides a balanced approach, limiting downside risk while allowing for moderate declines or stable prices. For us who foresee Nvidia maintaining its range, a strangle could also be a viable choice, taking advantage of its high implied volatility to earn premiums.

3.2 Risk personalities

Another important factor to take in consideration is risk personalities. When engaging in options trading, it is crucial to recognize and understand the role of risk personality in shaping investment decisions. Different investors have varying attitudes toward risk, which influence their strategies and expectations. Broadly, these personalities can be categorized into risk averse, risk neutral, and risk seeking profiles. Risk averse investors prioritize the preservation of their capital above all else. For these individuals, the goal is to minimize potential losses, even if it means accepting lower returns. They prefer strategies with limited downside risks, such as covered calls or calendar spreads, which offer modest but steady returns. These strategies align with their cautious nature, as they avoid significant exposure to volatility and focus on predictable outcomes. Risk neutral investors strike a balance between risk and reward, weighing the probabilities of profit and loss equally. For them, the decision making process revolves around expected value rather than an emotional aversion to losses or a preference for large gains. These investors often select strategies like strangles or iron condors, which take advantage of market conditions and implied volatility without leaning too heavily on either extreme of the risk spectrum. Their goal is to achieve consistent, incremental growth over time. Risk seeking investors, however, embrace risk as a necessary element in the pursuit of high returns. They are willing to tolerate significant fluctuations in their portfolio and accept the possibility of losses if it means capturing outsized gains. Strategies such as short puts or debit spreads appeal to these investors, particularly in bullish market environments with high volatility, such as Nvidia's current situation. This willingness to take on substantial risk sets them apart from their more conservative counterparts. Howard Marks, a renowned investor and author, also stated, "You can't take the same actions as everyone else and expect to outperform." Marks highlights that investors' willingness to accept varying levels of risk often determines their outcomes. He also points out that investors can create new risks through their strategies, particularly when venturing into uncharted or unconventional areas of the market. This perspective emphasizes that risk taking is not just about accepting pre-existing risks but also about innovating and identifying unique opportunities that align with one's risk tolerance. (Oaktree Capital, n.d.)

In our case, we align with a risk seeking profile but with a calculated approach. We are willing to take on higher risks in options trading, as we believe the potential for outsized returns justifies the associated uncertainty. For example, in Nvidia's bullish market scenario, strategies like short puts or call ratio credit spreads are particularly appealing because they offer significant reward potential. However, we also impose a constraint on our risk exposure: we are prepared to accept limited losses as long as there remains a reasonable probability of earning some return. This balance ensures that while we aim for substantial gains, we do not expose ourselves to untenable downside risks. Moreover, as Marks suggests, our willingness to embrace risk is not static it evolves with market conditions and our broader objectives. We recognize that taking calculated risks or even creating new risks through innovative strategies can open up opportunities that others might overlook. This mindset aligns with our goal of achieving high returns while maintaining a disciplined approach to managing losses. By understanding and leveraging our risk personality, we position ourselves to navigate the complexities of the market effectively and pursue our financial objectives with confidence.

3.1.1 Planned Bullish Market Strategy

Hoping on an increase in the Nvidia stock the long call strategy is a straightforward yet powerful options trading strategy. It is employed when an investor is bullish on the underlying stock. It involves purchasing a call option, which grants the right, but not the obligation, to buy the underlying asset at a specific strike price before the option's expiration date. This strategy benefits from upward price movements in the underlying stock and is most effective when there is a strong expectation that the stock price will rise significantly. In the given trade, illustrated in **image 10**, the strike price is set at $162.5, and the expiration date is December 29, 2024. The cost of the trade is $200, which represents the premium paid to acquire the call option. The breakeven price for this trade is $165.50, calculated by adding the cost of the trade to the strike price. For this strategy to be profitable, the price of NVIDIA stock must exceed the breakeven point at expiration. The risk in a long call strategy is limited to the premium paid, which, in this case, is $200. This is the maximum loss we can incur, even if NVIDIA stock trades significantly below the strike price or becomes worthless. On the other hand, the potential profit is theoretically unlimited because as the stock price rises beyond the breakeven point, the gains increase exponentially.

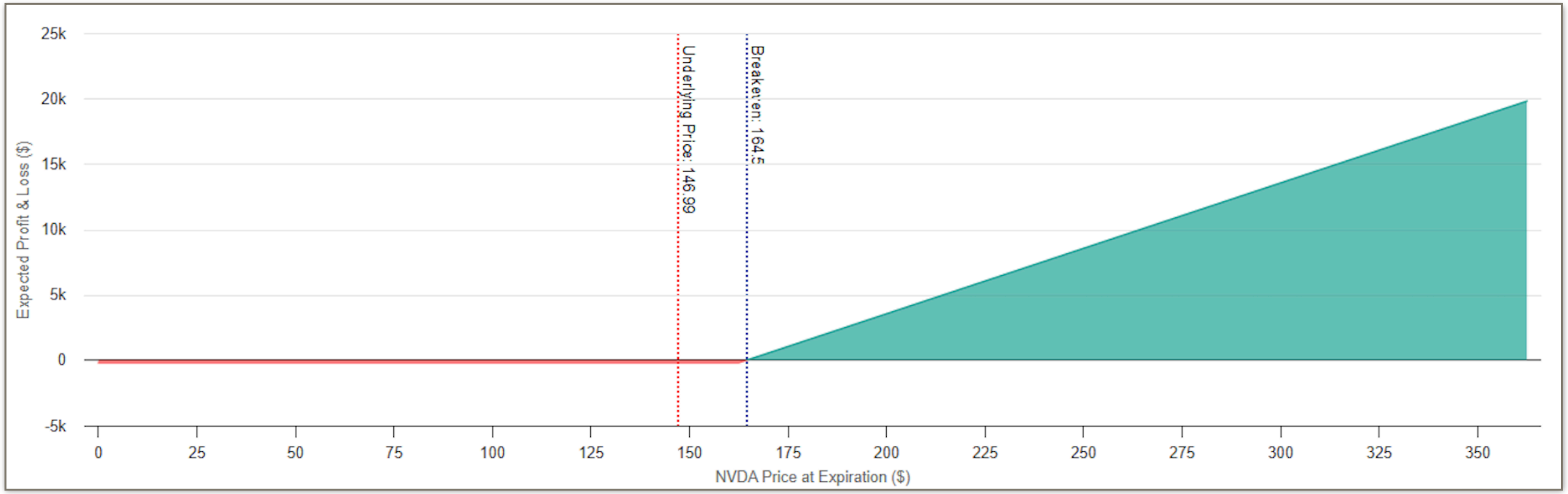


Figure 10 : Illustrate the Long call strategy

The profit and loss graph illustrates, **image 10**, this clearly. The flat section of the graph below the strike price represents the maximum loss, while the steep upward slope beyond the breakeven point signifies the unlimited profit potential. This makes the long call strategy appealing for us risk seeking traders who are confident about significant bullish movement in the stock price. The strategy hinges on a positive Q3 FY25 report, potentially including the announcement of an upcoming release for the Blackwell chip. Additionally, the expectation is that, even with the U.S. elections underway, any significant actions, such as Trump replacing Biden and introducing sanctions on China, would not take effect until January. This delay could help mitigate immediate negative impacts on NVIDIA's stock, allowing for short-term growth. This specific trade is particularly relevant in a bullish scenario where NVIDIA is expected to capitalize on growth opportunities, such as AI market dominance and increasing demand for data center infrastructure, as highlighted in prior analyses. However, like all trades, there are risks. If NVIDIA fails to meet growth expectations or faces macroeconomic challenges, the stock may not reach the breakeven point, resulting in a loss.

3.1.2 Scenario 2, Neutral Market Strategy

Supposing that the market will stay neutral the strategy we will invest in is is a Long Call Butterfly Spread, a multi leg options strategy that combines the purchase and sale of call options with different strike prices but the same expiration date. This strategy is designed for scenarios where the stock price remain relatively stable within a defined range. It is a popular choice in markets with low implied volatility or when the investor has a neutral to slightly bullish outlook for the underlying stock.

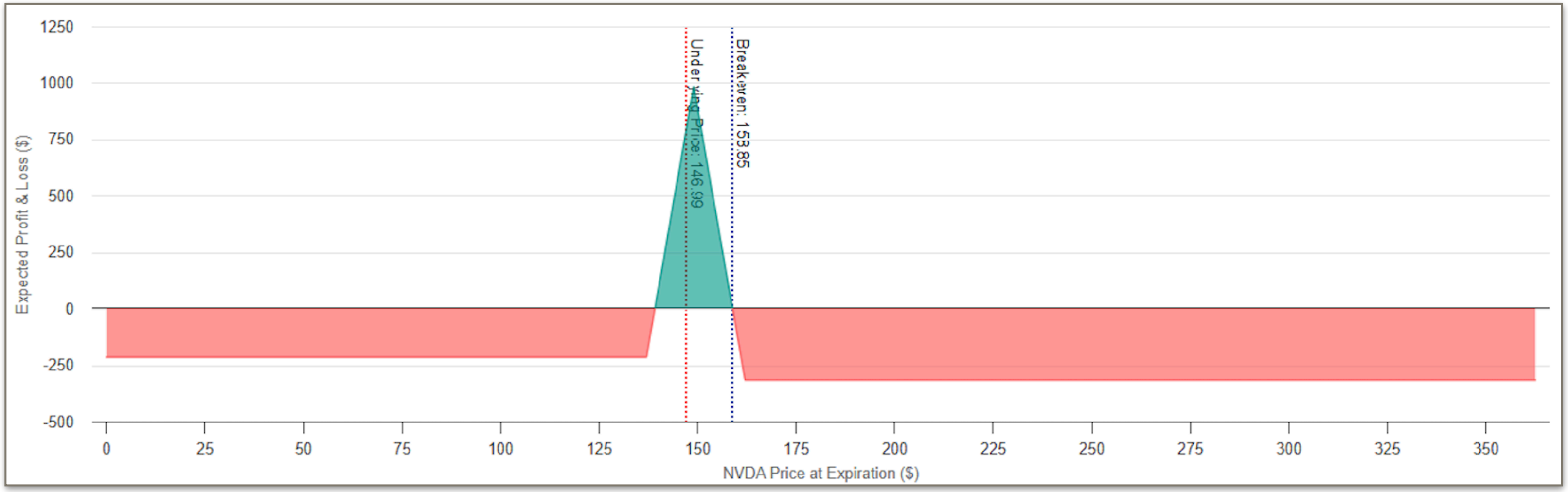


Figure 11 : Represent the Long Call Butterfly spread strategy

In this particular setup, illustrated in **image 11**, the strategy involves buying one call option at a lower strike price of $137.00, selling two call options at a middle strike price of $149.00, and buying another call option at a higher strike price of $162.00. The cost of the trade amounts to $215.00, which is the total premium paid for entering the position. The trade offers a defined risk reward profile, the maximum potential profit is $983.75, achievable if the stock price closes precisely at the middle strike price of $149.00 at expiration. The maximum loss is capped at $315.00, which occurs if the stock price falls below $137.00 or rises above $162.00 at expiration. The breakeven points for this strategy are $138.85 on the lower end and $158.85 on the upper end. Profitability is achieved as long as the stock price remains within this range. The Long Call Butterfly Spread is particularly effective in scenarios where the underlying stock's price is expected to remain range bound, as it allows the investor to benefit from minimal price movement. At the same time, the strategy accommodates a slightly bullish expectation, as the upper breakeven point extends into higher price territory. The appeal of this approach lies in its low cost of entry, clearly defined risk, and favorable reward to risk ratio. In this case, we are risking $315.00 for a potential profit of $983.75, representing a high potential return relative to the risk.

Given that NVIDIA's current price is $146.99, this setup aligns well with a neutral or slightly bullish outlook hoping the geopolitical factors affecting NVIDIA are likely to drive the market down after an initial surge following the release of the Q3 FY25 report, which typically leads to an increase in the stock price. The expectation is that the market will rise around November 21, driven by positive sentiment from the earnings report. However, geopolitical factors and the overvaluation identified in the trading comp analysis are expected to bring the price back down, resulting in an overall net zero change in the stock price by the expiration date on November 29. The breakeven range between $138.85 and $158.85 suggests that we expects NVIDIA's stock to remain stable or experience mild upward movement until the Nov 29 2024 expiration date. The strategy takes advantage of this anticipated price stability while ensuring that the risk is capped, providing a level of security even if the stock price moves unexpectedly.

3.3.3 Planned Bearish Market Strategy

Assuming the market will decline, the strategy to be applied would be a Bear Put Spread, which is a bearish options trading strategy designed to profit from a moderate decline in the price of Nvidia stock. This strategy involves buying a put option at a higher strike price and simultaneously selling another put option at a lower strike price. Both options share the same expiration date. The primary objective of the Bear Put Spread is to limit the downside risk of the trade while also capping the maximum potential profit.

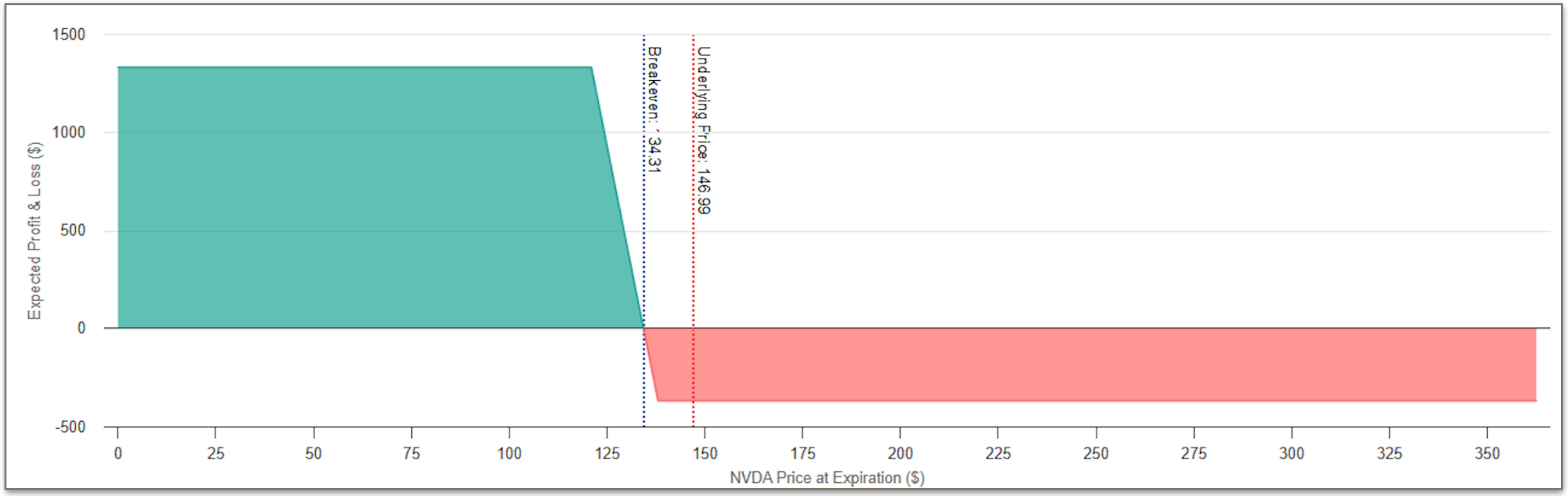


Figure 12 : Represent the Bear Put Spread strategy

From the setup, illustrated in **image 12**, in the trade information, we have to purchase one put option with a strike price of $138 for $4.90 and sell one put option with a strike price of $121 for $1.21. The net premium paid for the strategy is $369, which represents the total cost of entering the trade. The breakeven price for the strategy is calculated at $134.31, meaning that Nvidia's stock price needs to fall below this level for the strategy to generate a profit at expiration. **Image 12** visually represents the profit and loss potential of the Bear Put Spread. The green area highlights the profit range, which begins when the stock price drops below $134.31. The maximum profit, amounting to $1,331, is achieved when Nvidia's stock price is at or below the lower strike price of $121 at expiration. On the other hand, the maximum loss is limited to the premium paid for the trade, $369, which occurs if Nvidia's stock price remains at or above the higher strike price of $138. This strategy is particularly suitable if NVIDIA fails to meet expectations in the release of the Q3 FY25 report the 21 November. For instance, delays in the Blackwell chip or a decline in revenue could significantly impact the market. Additionally, the new U.S. elections may escalate geopolitical tensions between China and the U.S., which would likely have a negative effect on NVIDIA. These factors could foreseeably lead to a significant drop in their stock price, aligning with the strategy's anticipated outcomes. By selling the lower strike put, we reduce the overall cost of the trade compared to buying a standalone put option. However, this also caps the maximum profit at the difference between the two strike prices minus the net premium paid.

Another strategy that has to be taken in consideration if the prices fall is the long put strategy involves the purchase of a put option, granting the holder the right to sell the underlying asset at a specified strike price before the expiration date. In this particular example, a long put is purchased for NVIDIA with a strike price of $138.00, expiring on December 29, 2024, at a cost of $4.90 per option. The graphical representation of the profit and loss chart for this strategy, illustrated in **image 13**, shows a bearish expectation, where we benefits if the underlying stock price falls significantly below the breakeven point.

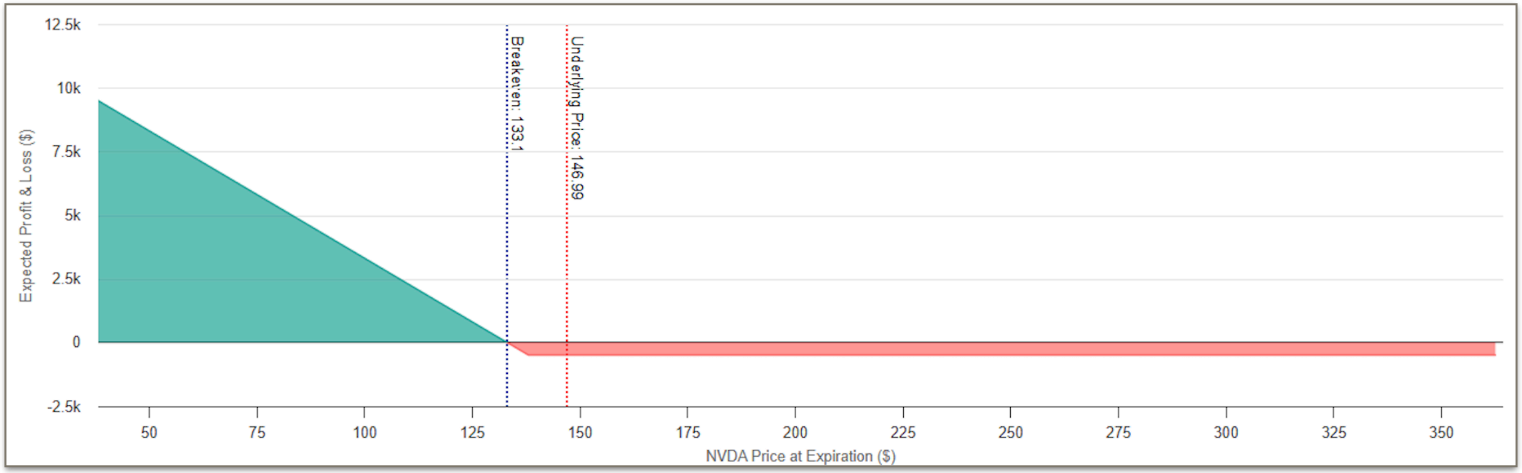


Figure 13 : Represent the Long put

Analyzing the P&L chart, the breakeven price is calculated as the strike price minus the premium paid for the option, which results in a breakeven at $133.10. Any price of NVIDIA below this level at expiration would yield a profit, while prices above it would result in a loss, limited to the initial cost of the option, $490.00. The maximum profit is theoretically significant, as it occurs when the stock price approaches zero, amounting to $13,310. This is the difference between the strike price and zero, adjusted for the premium paid. The maximum loss is capped at $490.00, making this a highly controlled risk strategy for us. The long put strategy is ideal in bearish scenarion where the stock price is expected to decline considerably. Unlike the bear put spread or other multi leg strategies, the long put does not involve selling any additional options, which simplifies the position and eliminates the risk of obligation to the counterparty. However, it is worth noting that the cost of the strategy can be higher compared to spreads, as the premium for the put option must be paid in full upfront. This strategy's profitability heavily depends on the magnitude and timing of the decline in the stock price. If the underlying asset's price remains flat or increases, we will incur a loss equal to the premium paid. Therefore, this strategy is well suited for a market outlook where a sharp and significant downturn is anticipated within the option's time frame.

4. Week analysis and Results

Note: The prices mentioned here are all listed in Appendix A1: Price Tracks.

NVIDIA's stock during the last 14 days of its contract experienced the following fluctuations in price:

$$147 \rightarrow 141 \rightarrow 137 \rightarrow 141 \rightarrow 145 \rightarrow 150 \rightarrow 142 \rightarrow 137 \rightarrow 136 \rightarrow 133.$$

The Long Call Butterfly strategy is ideal when the stock price stays close to the middle strike price at expiration, which in this case was 149. However, NVIDIA's stock experienced significant fluctuations and ultimately ended at 133. This price is far outside the range where the butterfly strategy could generate a profit. As the stock's behavior was not conducive to this strategy's design relying on minimal volatility and a neutral market it resulted in a loss. The significant downward movement during the period made it nearly impossible for the strategy to succeed. In contrast, the Bear Put Spread strategy, which involves buying a higher strike put and selling a lower strike put, is designed to profit from a moderate decline in the stock price. NVIDIA's price consistently declined over the observed period, closing at 133. While the price did not reach the lower strike of the spread which is 121, it fell sufficiently close to generate a profit. The downward trajectory aligned well with the bearish outlook of this strategy, making it effective in this scenario. The Long Put strategy, which profits when the stock price falls below the strike price by more than the premium paid, also performed well. Given that NVIDIA's stock price dropped substantially from 147 to 133, this strategy realized a significant profit. The final price was well below the breakeven point, ensuring that the option's intrinsic value far exceeded the cost of the premium. The steep decline in the stock price perfectly matched the bearish expectations underpinning this approach. However, the Long Call strategy, which benefits from a significant rise in the stock price above the breakeven point, was not as fortunate. Throughout the observed period, the stock price remained far below the breakeven price, never reaching the levels necessary to make the strategy profitable. Even during its brief upward movements, the price failed to climb high enough to offset the premium cost, resulting in a total loss of the premium.

In summary, the bearish strategies Bear Put Spread and Long Put capitalized on the significant downward movement of NVIDIA's stock, generating profits. On the other hand, the neutral to bullish strategies Long Call Butterfly and Long Call suffered losses as they were misaligned with the observed market conditions.

5. Binomial Trees and Black Scholes

> *Note: most of this information comes from the black hull book and translated to our own understanding even tho the concept and calculation are very based on the same as written in the book, Chapter 1 and Chapter 11. (Hull, 2017)*

Binomial trees are indispensable tools for valuing American options, as no analytic solutions exist for these types of derivatives. Unlike European options, which can often be priced using formulas like the Black Scholes Merton model, American options require a more flexible framework to account for their early exercise feature. Binomial trees provide this flexibility, making them the preferred method for pricing American options. The binomial tree approach involves dividing the life of an option into a series of small time intervals, each of length Δt . During each interval, the price of the underlying asset can move to one of two new values: Su, representing an "up" movement, or Sd , representing a "down" movement, where $u > 1$ and $d < 1$. The probability of an up movement is denoted by p , while the probability of a down movement is $1 - p$, as illustrated in **Figure 13** . By constructing this binomial trees allow for the valuation of options through step by step calculations of payoffs at each node. A cornerstone of the binomial tree approach is risk neutral valuation. This principle assumes a hypothetical world where all traded assets have an expected return equal to the risk free interest rate. Under this assumption, the value of an option can be determined by calculating the expected value of its payoffs at each step, discounted back to the present at the risk free rate. This simplifies the pricing process and aligns it with fundamental financial theory. For European options, binomial trees also serve as a numerical method that converges to the Black Scholes Merton price as the number of time intervals increases. However, for American options, which require consideration of early exercise opportunities at each step, binomial trees are uniquely suited to handle their complexity. While some approximations, like the quadratic approach, have been proposed for valuing American options, binomial trees remain the most reliable and widely used method.

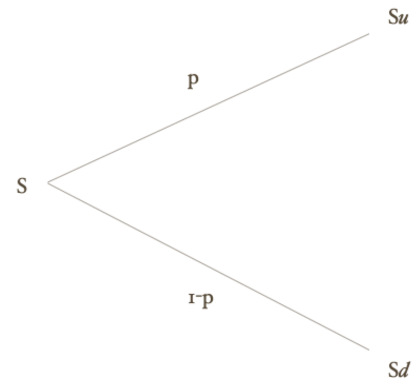


Figure 13 : Asset price movement

But how do we determine the parameters for p, u and d in a binomial tree? To accurately model asset price movements in a binomial tree, the parameters for p, u and d are determined to ensure the correct mean and variance of price changes over a small time interval Δt. These parameters are derived under the assumption of a risk neutral world, where the expected return of the asset equals the risk free interest rate r. Furthermore if the asset provides a yield of q, its expected return in terms of capital gain is r-q. For the mean price to match the risk neutral expectation, the expected value of the asset price at the end of the interval must be equal $Se^{(r\text{-}q)\Delta t}$, where S is the initial price. This gives the condition:

$$Se^{(r-q)\Delta t} = pSu + (1-p)Sd \xrightarrow{\text{equivalently}} e^{(r-q)\Delta t} = pu + (1-p)d$$

Then, the variance of the percentage change in the asset price dduring Δt is required to matche the theoretical value of $\sigma 2$ Δt, where σ is the volatility of the asset. The variance of 1+R, where R is the asset price, is expressed as:

$$pu^2 + (1-p)d^2 - \left[pu + (1-p)d\right]^2 \xrightarrow{\text{substituting} \quad \text{Equation(1)}} pu^2 + (1-p)d^2 - e^{2(r-q)\Delta t} = \sigma^2 \Delta t$$

An additional conditional on p,u and d is also introduced by Cox, Ross and Rubinstein in 1979, which is the following:

$$u = \frac{1}{d}$$

So, Solving equation (1),(2) and (3) while ignoring higher order terms in Δt, lead to the following solutions:

$$a = e^{(r-q)\Delta t}$$

$$u = e^{\sigma\sqrt{\Delta t}}$$

$$d = e^{-\sigma\sqrt{\Delta t}}$$

$$p = \frac{a-d}{u-d}$$

Where equation (4) represents Growth factor, equation (5) Up factor, equation (6) down facto and equation (7) risk free probability of up movement. In the binomial model, the evolution of asset prices is represented as a tree structure. **Figure 2 in Appendix.A4 Figure** illustrates this for a case with four time steps. At the starting point, time 0, the asset price is S_0 , which is known. As time progresses in increments of Δt, the tree branches into possible asset prices. At time Δt, there are two possible prices: S_0u (up movement) and S_0d (down movement). At time 2Δt, there are three possible prices: S_0u_2, S_0 , and S_0d_2, and so on. In general, at time iΔt, there are i+1 possible asset prices, calculated as:

$$S_0 u^j d^{i-j}, \quad j = 0,1,\ldots,i$$

The relationship u=d/1 ensures consistency in the tree. For example, the price after two up movements followed by a down movement (j=2 and i=3) is:

$$S_0 u^2 d = S_0 u$$

This demonstrates the recombining property of the tree, where an up movement followed by a down movement results in the same price as a down movement followed by an up movement. Furthermore The valuation of options in the binomial model begins at the terminal nodes (time T) and proceeds backward through the tree. At timeT, the value of the option is determined by its intrinsic value, for a call option the value is max (S_t-K,0) and for a put option the value is max (K-S_t,0). Where S_t is the asset price at time T, and K is the strike price of the option. At each preceding node, the option value is calculated as the discounted expected value of the option at the next step:

$$\text{Option} \quad \text{Value} = e^{-r\Delta t}\left[p \times (\text{Value} \quad \text{at} \quad \text{Up} \quad \text{Node}) + (1-p) \times (\text{Value} \quad \text{at} \quad \text{Down} \quad \text{Node})\right]$$

Where r is the risk free interest rate, and p is the probability of an up move. Meanwhile For American options, early exercise must be considered at each node. The value at each node is compared to the payoff from immediate exercise, and the higher of the two is selected:

$$\text{Option Value at Node} = \max(\text{Intrinsic Value}, \text{Hold Value})$$

And now, we turn to another critical aspect to consider when valuing options which how the binomial model accounts for dividend paying stocks. Dividends introduce additional complexity, as they impact the stock price when they are paid. A dividend refers to the reduction in the stock price on the ex dividend date caused by the dividend payment. If the dividend yield is known long life stock options are sometimes modeled under the assumption of a continuous dividend yield, denoted by q. In this case, options on a dividend paying stock can be treated similarly to options on a stock index. However, for greater accuracy, dividends are typically assumed to be paid discretely. When there is a single known dividend during the life of the option, and the dividend yield (δ) is known, the binomial tree can be adjusted as follows, before the ex dividend date the stock price at each node in the tree corresponds to:

$$S_0 u^j d^{i-j}, \quad j = 0,1,\ldots,i$$

And efter the dividend date the stock price at each node reflects the dividend yield and is adjusted to:

$$S_0(1-\delta)u^j d^{i-j}, \quad j = 0,1,\ldots,i$$

The resulting tree structure, as shown in **Figure 13** ,ensures that the stock price reductions on ex-dividend dates are accurately represented.

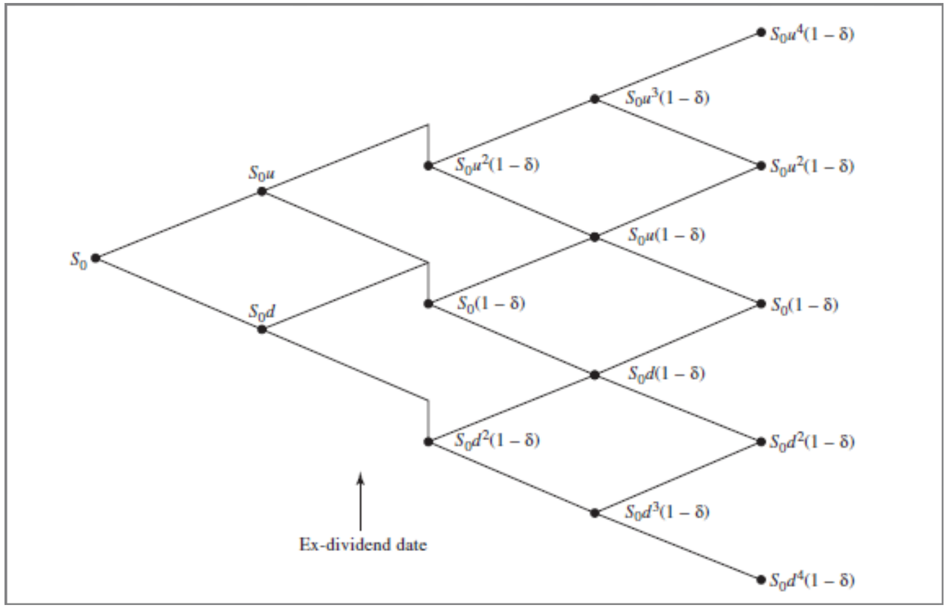


Figure 13 : Option with ex divided date

Note : The following image is imported from Hull's book, Options, Futures, and Other Derivatives (2017)

So, now that we understand how the binomial tree model works and that it operates with a finite number of intervals, what happens when these intervals become smaller and converge to infinity? Building on the foundation of the binomial tree model, we can see how it naturally evolves into the Black-Scholes formula as we move from discrete to continuous time. The binomial tree's core idea is to divide the life of an option into small intervals, during which the price of the underlying asset moves either up or down. As we increase the number of these intervals n and let their length Δt=T:n approach zero, the discrete model transitions into a continuous one. In the limit, the discrete price movements defined in the binomial tree converge to exponential functions of volatility shown in equation (5) and (6). This creates a continuous random walk where the asset price evolves according to a stochastic differential equation:

$$dS = \mu S dt + \sigma S dW$$

This continuous dynamic leads to the Black-Scholes partial differential equation:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0$$

where C(S,t) is the option price as a function of the stock price and time, σ is the volatility, and r is the risk free interest rate. This equation embraces the infinite step version of the backward induction process used in the binomial model.By solving this equation under specific boundary conditions for European options, we arrive at the Black-Scholes formula:

$$C = S_0 N(d_1) - Ke^{-rT} N(d_2) \text{ where;}$$

$$d_1 = \frac{\ln(S_0/K) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \quad \text{and} \quad d_2 = d_1 - \sigma\sqrt{T}$$

The Black Scholes formula is, in essence, the infinite Taylor series of the binomial tree model. As the binomial tree incorporates more and more steps, its discrete calculations converge to the continuous solution provided by Black Scholes. Nonetheless some assumptions has to be made when using the Black Scholes formula. One foundational assumption is that the underlying asset follows a lognormal random walk. This means that the random term in the stochastic equation governing the asset's price is proportional to the asset price itself. While volatility σ may vary with time, it must remain proportional to the asset price. If volatility depends on the asset price, the model becomes significantly more complex, often requiring numerical methods to solve. The drift term μ, which represents the expected return of the asset, does not explicitly appear in the pricing formula because the model operates under a risk neutral framework, where returns are assumed to equal the risk free rate. Another assumption is that the risk free interest rate r is known and constant over the life of the option. While r can be modeled as time dependent if known in advance, in reality, it may be stochastic. Introducing stochastic interest rates adds another layer of complexity to the model. The model also assumes that the underlying asset pays no dividends. While this simplifies the pricing process, the model can be adjusted to incorporate dividend payments when necessary. A central assumption of the Black Scholes model is the absence of arbitrage opportunities. This principle ensures that the pricing is internally consistent and prevents traders from exploiting price differences for risk free profit. However, real markets are not perfectly arbitrage free, particularly when the model's assumptions deviate from actual market conditions.

5.1 Comparison of NVIDIA theoretical and Market Value of a "out of money" call option, $S < K$

Note: All the codes used in this task are included in ***Appendix A5: Code Snippets****. The codes used are named "Binomial Tree for American Call Code," "Black-Scholes Code," and "Graph Plot of Binomial High n Iteration." The corresponding graph prints are provided in* ***Appendix A3: Graphs***

To compare the theoretical and market values of an "out-of-the-money" call option where the strike price, K , is higher than the current asset price, S, we first analyze the provided data and theoretical results. **Table 9** defines the parameters used in the analysis, including a strike price of $K = 141.00$, an underlying asset price of $S = 147.37$, and an implied volatility of 48%. The option type is a call option with a time to expiration of 14 days, an interest rate of 4.75%, and an American exercise style. These parameters establish the basis for comparing the results of the binomial option pricing model and the Black Scholes formula.

Table 9 : Parameters used in Binomial

Parameter	Value
Strike price	141.00
Underlying asset price	147.37
Days to expiration	14
Dividend	0.03
Option type	Call
No. Tree Steps	5
Implied Volatility for Call Options	48.0%
Interest rate	4.75%
Days to ex-dividend	20
Exercise style	American

Table 10 shows the calculated values using different methods and steps. For the binomial model with 5 steps, our code produces a value of 9.02, while an online tool calculates 9.1367. Increasing the number of steps to 10 results in a value of 9.29 from our code and 9.4118 from the online tool. When using 100 steps, the results converge, with our code giving 9.24 and the online tool 9.3563. Similarly, the Black-Scholes model yields 9.23 from our implementation and 9.3475 from the online calculator. These values demonstrate the consistency between different implementations and methods, with small variations likely due to rounding and computational nuances.

Table 10 : Comparison of results, own values and online code

Method	Value	Appendix.A3 Graphs
Binomial 5 (Own code)	9.02	Figure 1
Binomial 5 (Online code)	9.1367	Figure 2
Binomial 10 (Own code)	9.29	Figure 3
Binomial 10 (Online code)	9.4118	Figure 4
Binomial 100 (Own code)	9.24	Figure 5
Binomial 100 (Online code)	9.3563	Figure 6
Black Scholes (Own code)	9.23	
Black Scholes (Online Code)	9.3475	Figure 7

Figure 14 illustrates how the option value stabilizes towards 9.23 as the number of steps in the binomial model increases. Initially, there are minor fluctuations, but beyond 100 steps, the option value plateaus, indicating that the model converges to a stable theoretical price. This convergence affirms the reliability of the binomial model for pricing options with sufficient steps.

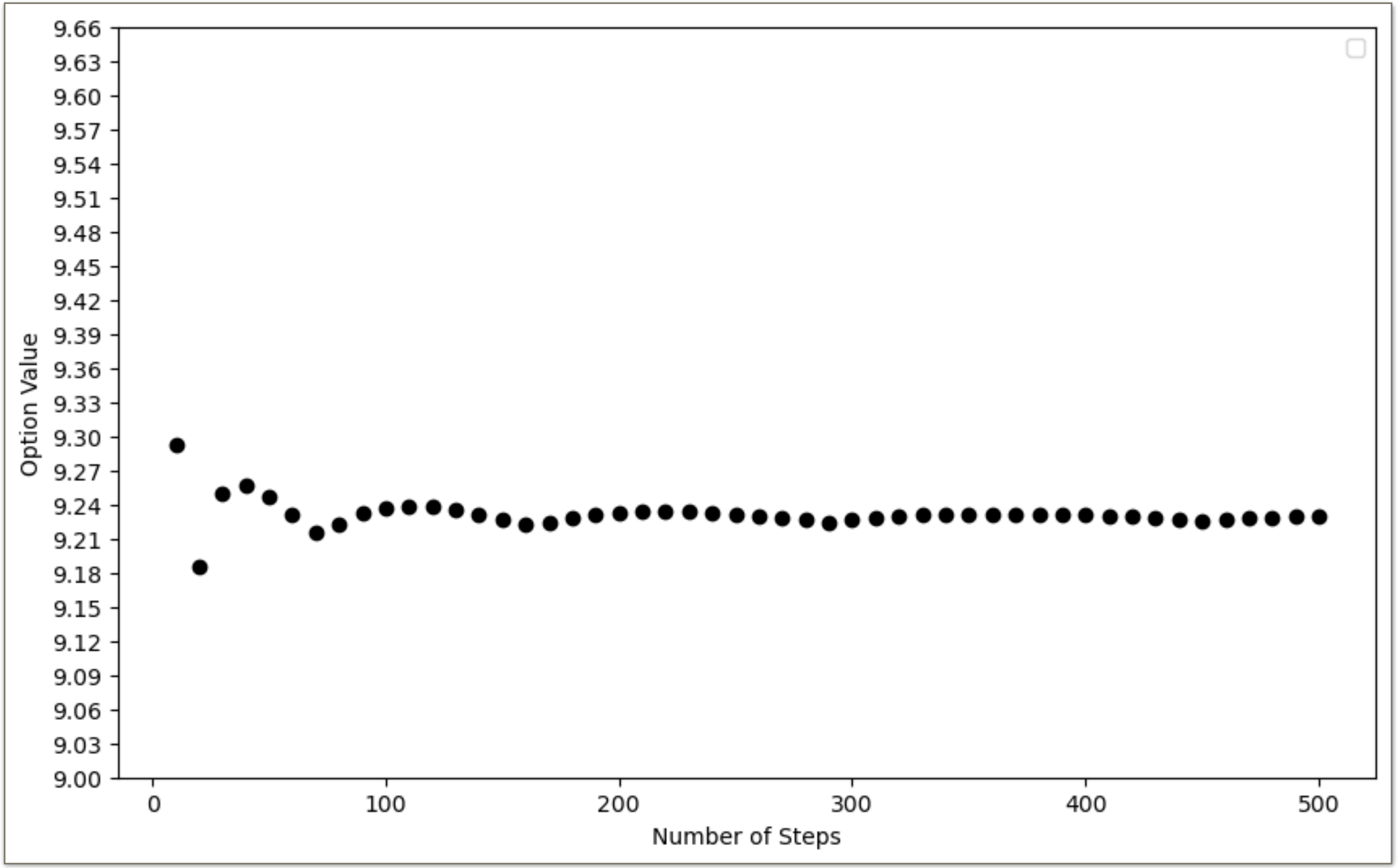


Figure 14 : binomial model increases

Note : The image is a plot generated from the code used, which is included in **Appendix A5: Code Snippets**. The code is titled "Graph Plot of Binomial High n of Iteration.

Table 9 compares the theoretical value at expiration with the real market value at expiration. For this "out-of-the-money" option, the real market value is essentially zero at expiration, as the strike price exceeds the underlying asset price, rendering the option worthless. The theoretical value calculated by both our code and the online tool also reflects this, with a value of 0.08 .

Table 9: Theoretical value at expiration with the real market value at expiration

Method A = own code B = Online code	Reference	Theoretical value at expiration	Real Market Value at expiration	Absolute Difference	Percentage Difference
Binomial 5.A	Bilaga A.1	0	0	0	
Binomial 5.B	Bilaga A.1	0	0,08	0,08	
Binomial 10.A	Bilaga A.1	0	0,08	0,08	
Binomial 10.B	Bilaga A.1	0	0,08	0,08	
Binomial 100.A	Bilaga A.1	0	0,08	0,08	
Binomial 100.B	Bilaga A.1	0	0,08	0,08	
Black Scholes.A	Bilaga A.1	(/)	0,08		
Black Scholes.B	Bilaga A.1	(/)	0,08		

Note : The red dots in the plot indicate a warning, which appears when a value is divided by zero.

This analysis highlights the importance of understanding the option's moneyness when interpreting results. The "out-of-the-money" status explains why the option has minimal value at expiration. Additionally, the minor differences between our code and online tools can be discussed further, the limitations of theoretical models in capturing real world factors such as transaction costs, liquidity, and market sentiment, which might influence observed calculations.

5.2 Comparison of NVIDIA theoretical and Market Value of a "at the money" call option, S = K

The comparison of NVIDIA's theoretical and market value of an "at the money" call option, where the strike price equals the current stock price S = K, provides valuable insights into the dynamics of option pricing and the implications of market changes. Starting with the parameters in **Table 10**, the strike price and underlying asset price are both set at 147.37 USD, reflecting the at the money status of this option. The implied volatility is fixed at 55%, with a 4.75% risk-free rate and a dividend yield of 3%. This setup forms the basis for theoretical calculations using the binomial model and the Black Scholes formula.

Table 10 : Parameters used in Binomial

Parameter	Value
Strike price	147.37
Underlying asset price	147.37
Days to expiration	14
Dividend	0.03
Option type	Call
No. Tree Steps	5
Implied Volatility for Call Options	55.0%
Interest rate	4.75%
Days to ex-dividend	20
Exercise style	American

Table 11 highlights the theoretical values obtained from different models, including the binomial model with 5, 10, and 100 steps, as well as the Black-Scholes model. The values are slightly varied but consistent across the models, with the own code for the binomial 5 step iteration showing a value of 6.69 uUSD, while the corresponding online calculation yields 6.7804 USD. Similarly, for the Black Scholes formula, the own code calculation provides a value of 6.37 USD, and the online tool estimates it at 6.4591 USD. These minor discrepancies stem from differences in computational precision and implementation nuances. It is important to note that these values assume the option remains at the money, which influences both the option price and the comparison to market values.

Table 11 : Comparison of results, own values and online code

Method	Value	Appendix.A3 Graphs
Binomial 5 (Own code)	6.69	Figure 8
Binomial 5 (Online code)	6.7804	Figure 9
Binomial 10 (Own code)	6.21	Figure 10
Binomial 10 (Online code)	6.3032	Figure 11
Binomial 100 (Own code)	6.35	Figure 12
Binomial 100 (Online code)	6.4433	Figure 13
Black Scholes (Own code)	6.37	
Black Scholes (Online Code)	6.4591	Figure 14

Figure 15, which plots the option value against the number of steps, demonstrates the convergence of the binomial model. As the number of steps increases, the calculated option value stabilizes around towards 6.36, , reinforcing the reliability of the model. This convergence underscores the robustness of the binomial tree approach when applied to options of this type.

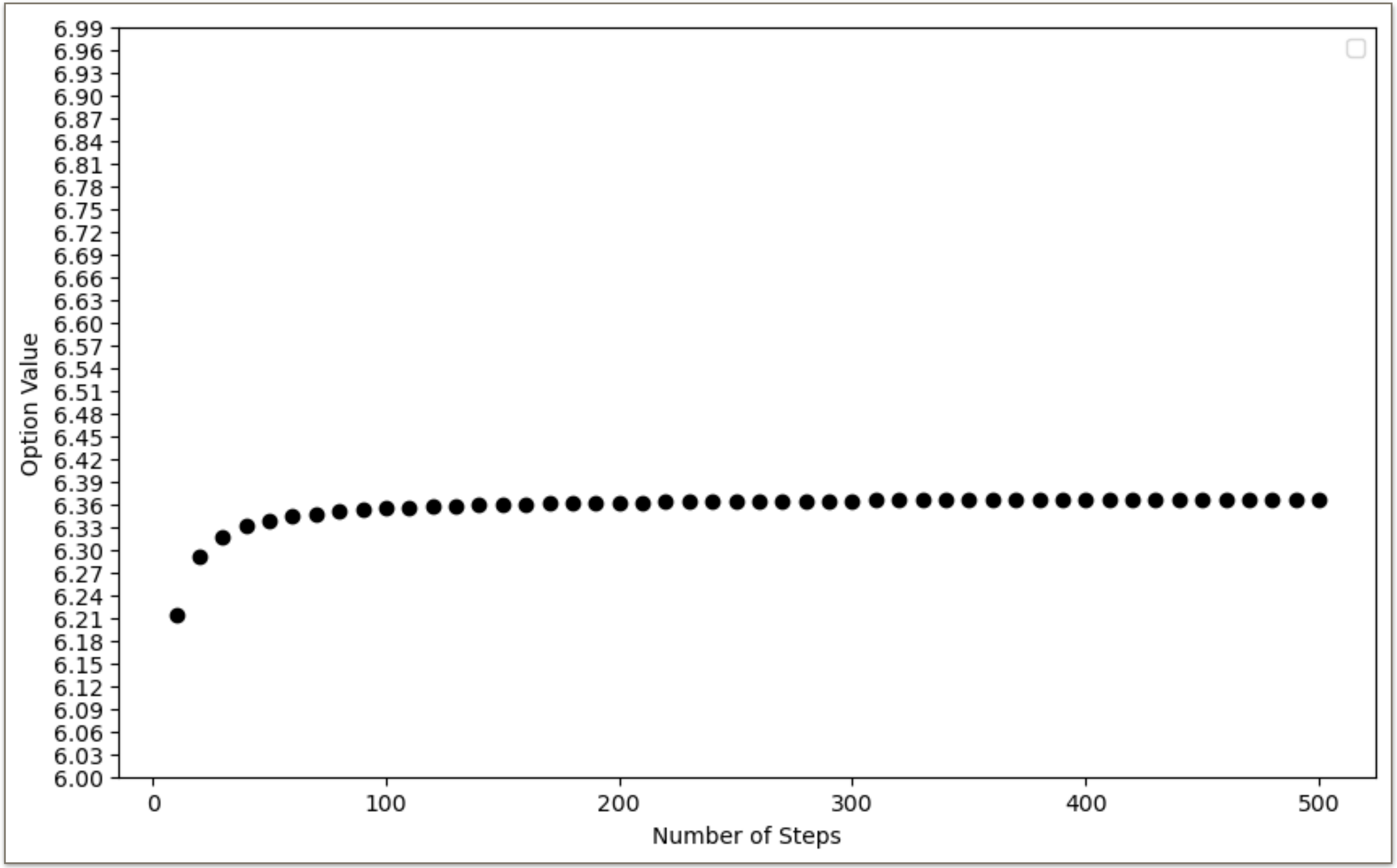


Figure 15: binomial model increases

Note : The image is a plot generated from the code used, which is included in **Appendix A5: Code Snippets**. The code is titled "Graph Plot of Binomial High n of Iteration.

Table 12, however, reveals a critical development: the drastic drop in NVIDIA's stock price led the option to fall out of the money by expiration. At this point, the real market value of the option is zero, as exercising an out of the money call option would yield no profit. This outcome aligns with the fundamental mechanics of options, where at the money options are particularly sensitive to price changes in the underlying asset. When the stock price fell below the strike price at expiration, the intrinsic value became zero, explaining the discrepancy between theoretical calculations at initiation and the observed market reality.

Table 12 : Theoretical value at expiration with the real market value at expiration

Method A = own code B = Online code	Reference	Theoretical value at expiration	Real Market Value at experation	Absolute Difference	Percentage Difference
Binomial 5.A	Bilaga A.1	0	0	0	
Binomial 5.B	Bilaga A.1	0	0	0	
Binomial 10.A	Bilaga A.1	0	0	0	
Binomial 10.B	Bilaga A.1	0	0	0	
Binomial 100.A	Bilaga A.1	0	0	0	
Binomial 100.B	Bilaga A.1	0	0	0	
Black Scholes.A	Bilaga A.1	(/)	0		
Black Scholes.B	Bilaga A.1	(/)	0		

Note : The red dots in the plot indicate a warning, which appears when a value is divided by zero.

The key lesson from this analysis is the importance of accounting for market volatility and directional changes in the underlying asset's price. While the theoretical models provide accurate valuations under stable assumptions, significant market movements, such as the sharp decline in NVIDIA's stock price, can render those valuations irrelevant at expiration. This underscores the dynamic nature of option pricing and the necessity of incorporating scenario analyses and sensitivity assessments to account for potential market shifts. Additionally, the initial implied volatility may not fully capture the risk of such abrupt changes, which emphasizes the importance of ongoing reassessment during the life of the option.

5.3 Comparison of NVIDIA theoretical and Market Value of a ”in the money” call option, S > K

The comparison of NVIDIA’s theoretical and market values for an "in the money" call option, where the underlying stock price exceeds the strike price (S > K), provides insight into the alignment of mathematical models with real-world pricing. Starting with the theoretical framework, we use parameters outlined in the **Table 13**. NVIDIA's underlying asset price is set at 147.37 USD, with a strike price of 141 USD, making the option "in the money." The option has a 14 day expiration, a volatility of 48%, and uses a risk-free interest rate of 4.75%. The dividend yield is set at 3%, and the American style option allows early exercise.

Table 13 : Parameters used in Binomial

Parameter	Value
Strike price	157.5
Underlying asset price	147.37
Days to expiration	14
Dividend	0.03
Option type	Call
No. Tree Steps	5
Volatility	55.0%
Interest rate	4.75%
Days to ex-dividend	20
Exercise style	American

The theoretical calculations for this setup, as displayed in the **Table 14**, show the results from both binomial tree and Black Scholes methods across various step sizes. The binomial tree model with 5 steps yields a theoretical price of 2.71 USD for our code and 2.7572 USD from an online calculator. With 10 steps, the theoretical value decreases slightly to 2.59 USD(our code) and 2.6360 USD (online code). As the steps increase to 100, the values converge, reflecting a price of 2.71 USD (our code) and 2.775 USD (online code). Similarly, the Black Scholes model aligns closely, with our code producing 2.71 USD and the online code yielding 2.7615 USD. These small differences highlight the impact of step size and computational precision in theoretical pricing.

Table 14 : Comparison of results, own values and online code

Method	Value	Appendix.A3 Graphs
Binomial 5 (Own code)	2.71	Figure 15
Binomial 5 (Online code)	2.7572	Figure 16
Binomial 10 (Own code)	2.59	Figure 17
Binomial 10 (Online code)	2.6360	Figure 18
Binomial 100 (Own code)	2.71	Figure 19
Binomial 100 (Online code)	2.7750	Figure 20
Black Scholes (Own code)	2.71	
Black Scholes (Online Code)	2.7615	Figure 21

GFigure 17 illustrates the convergence behavior of the binomial model as the number of steps increases. Initially, there are minor fluctuations in the calculated option value, but the results stabilize towards 2.71 as the steps surpass 50. This convergence is critical, as it demonstrates the robustness of the binomial model when sufficient granularity is introduced. The alignment of the theoretical values between binomial and Black-Scholes models supports their consistency in pricing "in the money" options under the given parameters.

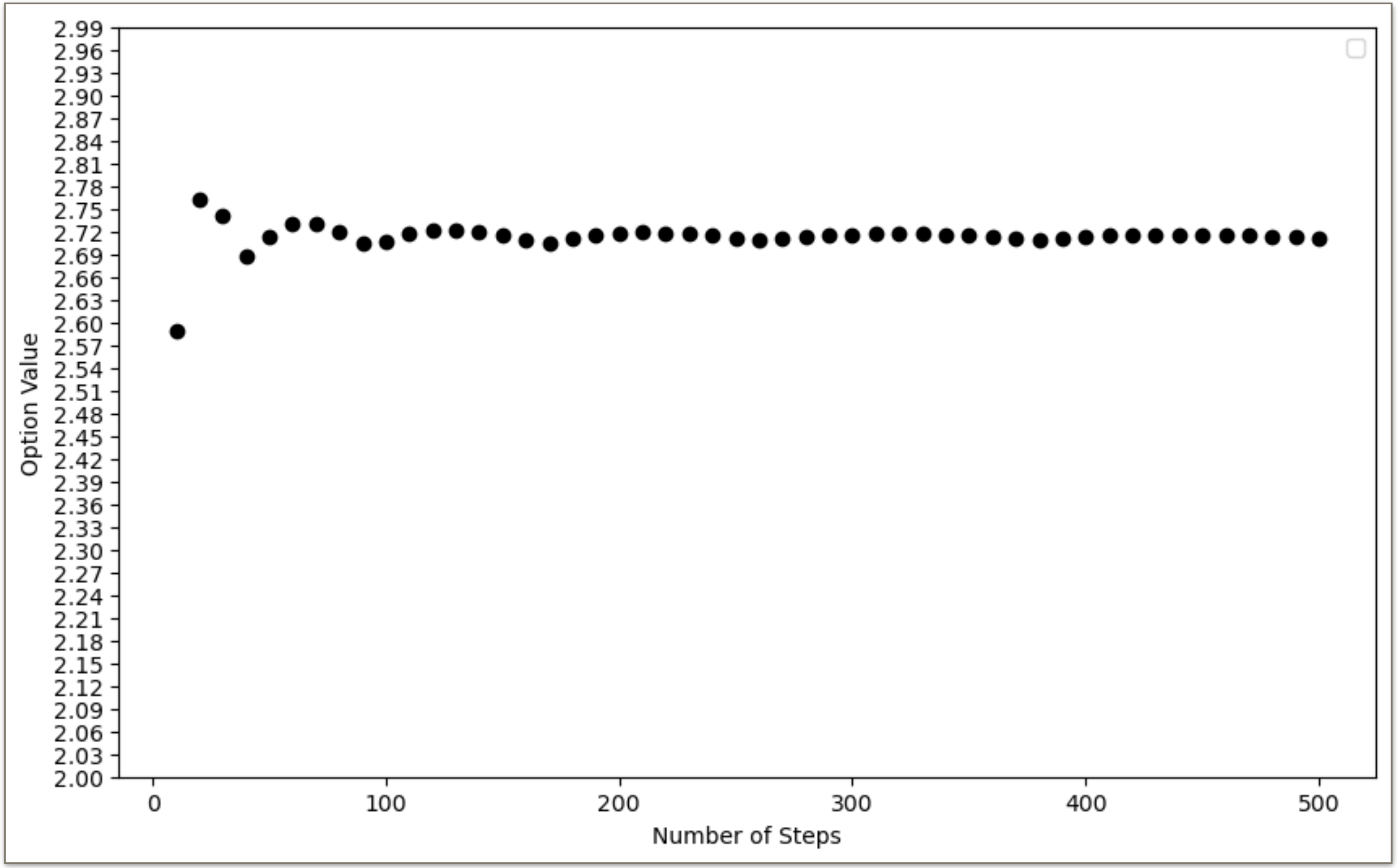


Figure 17 : binomial model increases

Note : The image is a plot generated from the code used, which is included in **Appendix A5: Code Snippets**. The code is titled "Graph Plot of Binomial High n of Iteration.

Finally, the **Table 15** compares the theoretical values with the real market price for the same option, which is 2.21 USD. The absolute differences between the theoretical values and the market price range from 0.38 USD to 0.57 USD, depending on the method and step size. The percentage differences highlight discrepancies: the binomial tree with 5 steps shows a 22.62% difference, while Black-Scholes yields a 24.95% difference. These deviations suggest that theoretical models, while precise in controlled settings, may not capture all market dynamics.

Table 15 : Theoretical value at expiration with the real market value at expiration

Method A = own code B = Online code	Reference	Theoretical value in Nov 21	Real Market Value in Nov 21	Absolute Difference	Percentage Difference
Binomial 5.A	Bilaga A.1	2,71	2,21	0,5	22,62 %
Binomial 5.B	Bilaga A.1	2,7572	2,21	0,5472	24,76 %
Binomial 10.A	Bilaga A.1	2,59	2,21	0,38	17,19 %
Binomial 10.B	Bilaga A.1	2,6360	2,21	0,426	19,28 %
Binomial 100.A	Bilaga A.1	2,71	2,21	0,5	22,62 %
Binomial 100.B	Bilaga A.1	2,7750	2,21	0,565	25,57 %
Black Scholes.A	Bilaga A.1	2,71	2,21	0,5	22,62 %
Black Scholes.B	Bilaga A.1	2,7615	2,21	0,5515	24,95 %

Note : The red dots in the plot indicate a warning, which appears when a value is divided by zero.

Several factors contribute to these discrepancies. Market prices are influenced by supply and demand, liquidity, and external events, which are not incorporated into theoretical models. Furthermore, using the same maturity and strike prices assumes homogeneity in market behavior, but real world contracts may involve slight variations in terms and timing. Additionally, the spread between bid and ask prices could lead to deviations in the reported market price. To bridge these gaps, a sensitivity analysis of volatility, interest rates, and dividend yields could be performed. Adjusting these parameters to better reflect current market conditions might reduce the variance between theoretical and real values.

5. Sensitivity analysis

Note: The sensitivity analysis graph of the results are displayed in ***Appendix A3****: Graphs, specifically in* ***Figure 23*** *and* ***Figure 24****.*

In this task, we calculated the implied volatility for a set of call options with the same expiration date and trade date using the Black Scholes and Newton Raphson model. The real implied volatility was given as 55%, but the results from the code deviated from this value. A sensitivity analysis was performed to evaluate the impact of changing the interest rate on the calculated implied volatilities. Despite these adjustments, the calculated values did not consistently align with the real implied volatility, prompting an investigation into the possible reasons for the discrepancies. The Black Scholes model relies on several key assumptions, including constant volatility and risk free interest rates. In our analysis, implied volatilities were calculated for strike prices ranging from 138 to 158, using market prices provided in the dataset. For example, at a strike price of 145 and a 3% interest rate, the implied volatility was approximately 13%, while at the same strike price with a 13% interest rate, it decrease to 11%. This sensitivity to the interest rate demonstrates that changes in the risk free rate do affect the calculated values, but even at the optimal rate within the given range, the calculated volatilities consistently fell short of the real 55%. One significant factor contributing to the discrepancy is the assumption of constant volatility. The market often exhibits a volatility skew or smile, where implied volatility varies with strike price. This was evident in our results, as the calculated volatilities decreased at higher strike prices, which is typical for options markets but may not align with the actual market dynamics of these specific contracts. For instance, at a strike price of 150, the implied volatility was around 15% across all interest rates, well below the real implied volatility of 55%. Additionally, the analysis uses strike prices from the same contract and time period, which may not capture the broader market dynamics needed to validate the real implied volatility. Comparing strike prices across different contracts or over varying time periods could provide a more comprehensive understanding of volatility behavior. This limitation highlights a possible oversight in the dataset's scope and the need for additional data points to confirm the robustness of the model. Another possible reason for the deviation is related to input precision. The market prices used in the calculations may contain noise or inaccuracies, which can distort the implied volatility. Moreover, the numerical method for calculating implied volatility , Newton Raphson, is sensitive to the initial guess and the relationship between price and volatility, potentially leading to convergence errors or suboptimal results. (Wilmott, 2006, pp. 130–132)

To add a layer of granularity we also compared delta with implied volatility. The graph labeled "Implied Volatility vs. Delta by Interest Rates" visualizes the relationship between implied volatility and delta for various interest rates, ranging from 3% to 13%. Delta, as a measure of an option's sensitivity to changes in the underlying asset's price, provides a dynamic perspective on how the option behaves across different market conditions. Unlike strike prices, which are fixed inputs, delta accounts for the intrinsic and extrinsic factors affecting an option's price, making it a more comprehensive metric for comparison. The reason for including delta in the analysis is that it reflects how "in the money" or "out of the money" an option is, which directly influences its implied volatility. For example, options that are deep in the money or far out of the money often exhibit lower implied volatility compared to those at the money. This relationship is crucial to understanding the pricing dynamics, as the graph clearly illustrates how implied volatility decreases as delta increases, regardless of the interest rate. This trend aligns with market behavior, where at the money options tend to have the highest implied volatility due to greater uncertainty. By comparing implied volatility across delta values, we can observe how the model's sensitivity to different interest rates varies. For instance, at lower deltas (e.g., 0.2), the implied volatilities calculated by the model are higher, but they converge as delta approaches 0.8. This pattern highlights that the interest rate's impact on implied volatility diminishes for options deeper in the money, providing insights into the robustness of the model under varying market conditions. In contrast, the strike price analysis only considers a fixed input variable and does not capture the dynamic pricing effects influenced by money. While strike price is essential for determining implied volatility, delta adds a layer of granularity by incorporating the option's sensitivity to price movements, enhancing our understanding of the implied volatility surface. (Wilmott, 2006, pp. 834–839)

6. Real Option

Real options represent a strategic approach to investment decision making by incorporating flexibility, risk, and uncertainty into the evaluation of large projects. Unlike traditional financial options, real options apply similar principles in a long term and strategic context. Real options rely on the binomial model to assess the net present value (NPV) of investments, factoring in risk and volatility to create more comprehensive financial projections. Real options differ from financial options in several critical ways. Financial options are short term, market driven instruments based on financial assets such as stocks, commodities, or currencies, with their prices dictated by the market. These options are typically small in value and have a minimal relationship with business decisions. Conversely, real options focus on strategic, long term projects where cash flows are influenced by market demand, competition, and internal company decisions. Unlike financial options, real options are not traded and lack a standardized marketplace. The outcomes of real options are closely tied to corporate strategies, often reflecting decisions about expansion, contraction, or the abandonment of projects. The methodology for evaluating real options shares similarities with financial options but is adapted for the complexities of strategic investment. Real options use the binomial model, often supplemented by simulations, to estimate potential outcomes. Volatility is a critical component in this model, often derived from cash flow data or estimated subjectively. Additionally, the risk free interest rate is applied to reflect the opportunity cost of capital. Key types of real options include the option to abandon, expand, contract, or choose between multiple projects, allowing companies to dynamically adjust their strategies in response to changing circumstances. The flexibility offered by real options is particularly advantageous when dealing with uncertainty. For instance, changing volatility levels can be used to simulate the sensitivity of outcomes, providing a broader perspective on potential risks and rewards. This approach allows businesses to adjust dynamically to market conditions, a feature that traditional NPV calculations lack. (Leifs Lecture Presentation) (Hull,2017)

6.1 Company can sell its drug patent for 60% of the present value (Assignment 2.a)

To adapt the original binomial option pricing model for evaluating the real option of the pharmaceutical company, we made several modifications for the unique characteristics of real options. Unlike financial options, real options involve strategic decisions that allow the company to adapt its course of action over time. In this case, the company has the flexibility to sell the drug patent at any point during the 10 year period for 60% of its value. This abandonment option introduces an additional layer of decision-making at every node of the binomial tree, requiring the model to compare the value of continuing the project with the value of selling the patent. The primary modification to the code was the introduction of the abandonment decision. During the backward induction process, the model evaluates two scenarios at each node: continuing with manufacturing or exercising the option to sell the patent. The value at each node is then determined as the maximum of these two values. Mathematically, this process is represented by comparing the continuation value, derived from the risk neutral probability of future cash flows, with the immediate exercise value, which is 60% of the patent's value at that point in time. The binomial tree was also adjusted to reflect the longer time horizon of 10 years, consistent with the pharmaceutical context. The number of steps in the tree was increased to provide sufficient granularity, and the time step for each interval was recalculated accordingly. This extension ensures that the model accurately represents the evolving value of the patent over the decade. Additionally, the initial value of the drug patent was set to 60% of Nvidia's market capitalization, equivalent to 21 641 400 million SEK. This reflects the present value of the asset under consideration. Another significant component of the model is the risk free rate, which remains at 5% annually. This rate is used to discount future cash flows and determine the risk neutral probabilities necessary for the binomial tree calculations. While the risk free rate remained consistent with the original code, its application was integrated into the backward induction process alongside the abandonment option to ensure a cohesive evaluation framework. The updated model produces a binomial tree that not only calculates the continuation value at each node but also incorporates the exercise value of the abandonment option. At each step, the model determines the optimal decision whether to continue manufacturing or sell the patent by comparing the two values. The final output of the model provides the real option value equivalent to at the root of the tree, representing the optimal decision under current conditions and expectations.

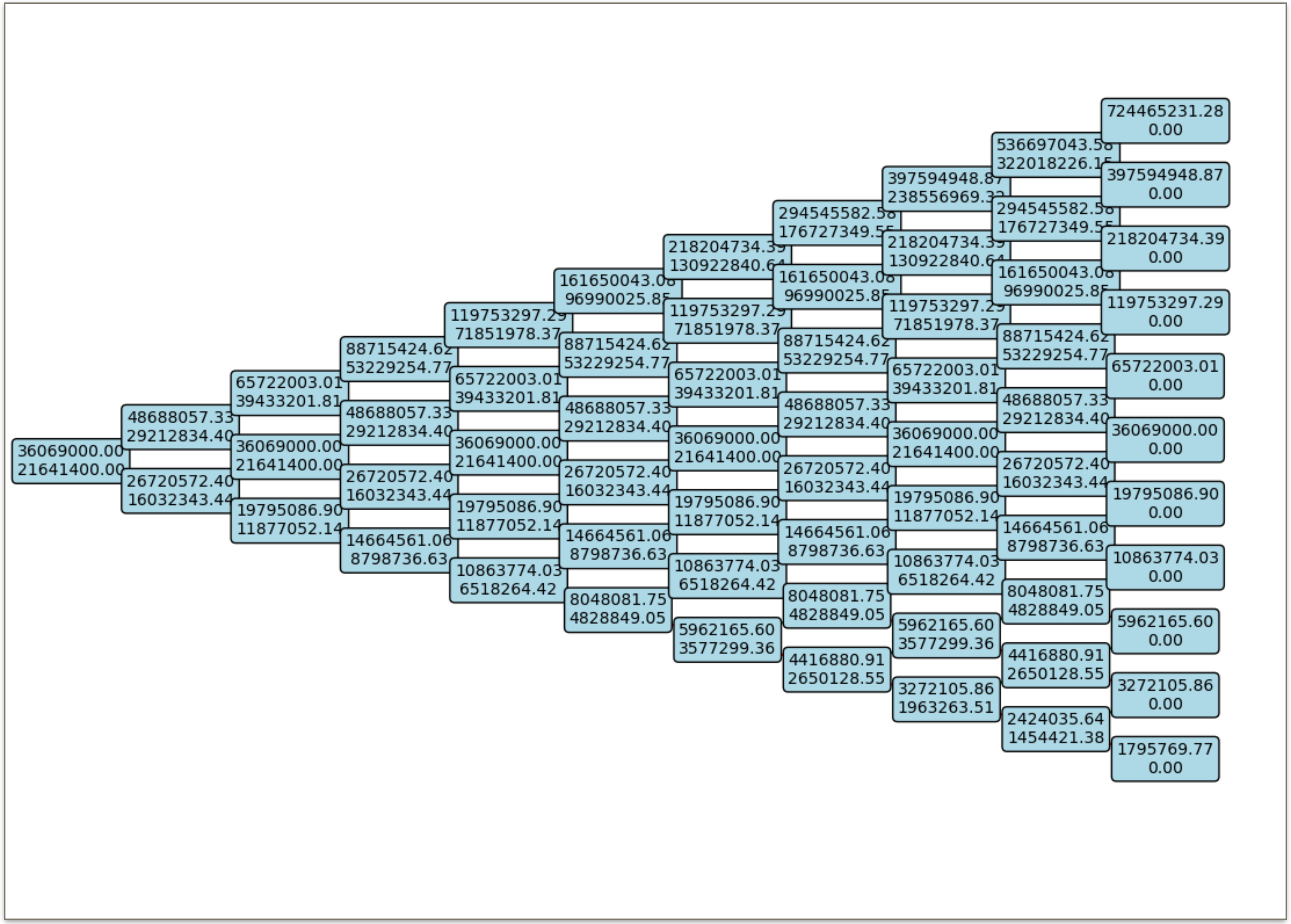


Figure 18 : Tree used to value real option

Note : The code used to generate this graph is included in Appendix A5: Code Snippets. The code is titled "Real Option Code."

6.2 Option to buy a competitor for 20% of the present value (Assignment 2.b)

To evaluate the company's real options of buying a competitor and reducing operations, we adapted the binomial model to include these strategic choices at every decision point in a 10 year period. Each option represents a different type of managerial flexibility, providing insights into the financial impact of expansion or contraction strategies. The first option involves the opportunity to buy a competitor, representing a growth option. This option allows the company to expand its market share and profitability by acquiring a competitor at a cost of 20% of the current patent value. Exercising this option results in a 25% increase in profits. The growth option captures the potential upside of expansion, enabling the company to capitalize on favorable market conditions. The model evaluates the value of this option by calculating the profit increase at each node in the binomial tree while deducting the acquisition cost. The value of the growth option, when calculated individually, was determined to be 40,507,417.97 million SEK.

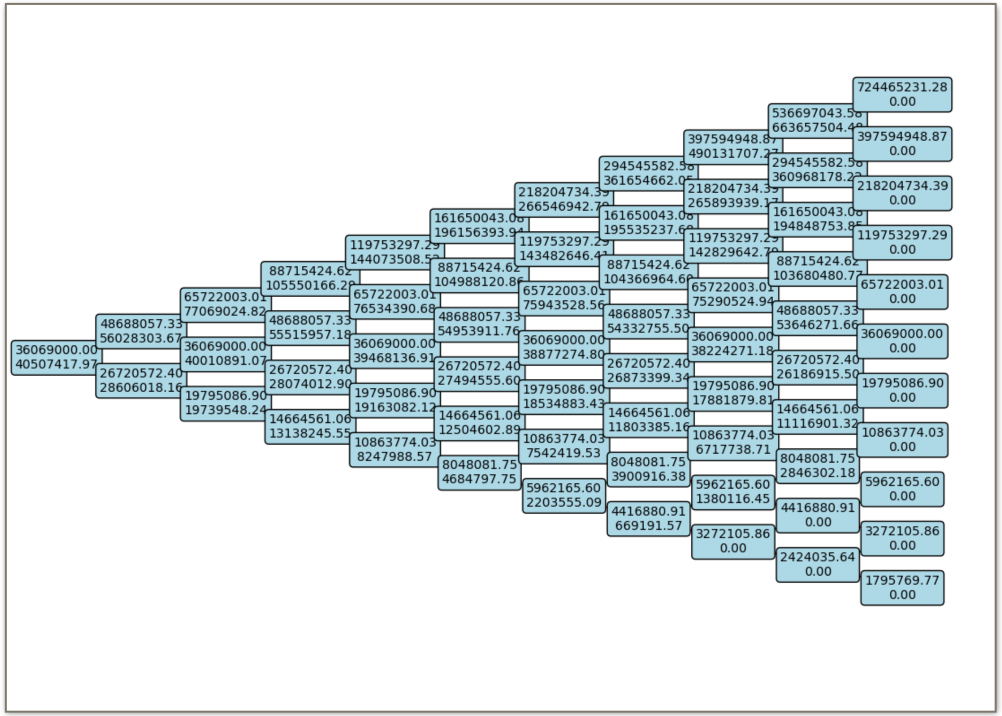


Figure 19 : Tree used to value increase of real option

Note : The code used to generate this graph is included in ***Appendix A5: Code Snippets***. The code is titled "Value of Growth and Reduction of Real Option Code."

The second option enables the company to reduce its operations by 20%, representing a contraction option. This option is designed to provide financial relief in adverse circumstances by saving 30% of the costs associated with the reduced operations. The model evaluates this option by calculating the cost savings at each node and comparing it to the continuation value of the full operation. The contraction option reflects a risk mitigation strategy, allowing the company to remain flexible and adapt to unfavorable market dynamics. The value of the contraction option was calculated to be 33,904,860.00 million SEK

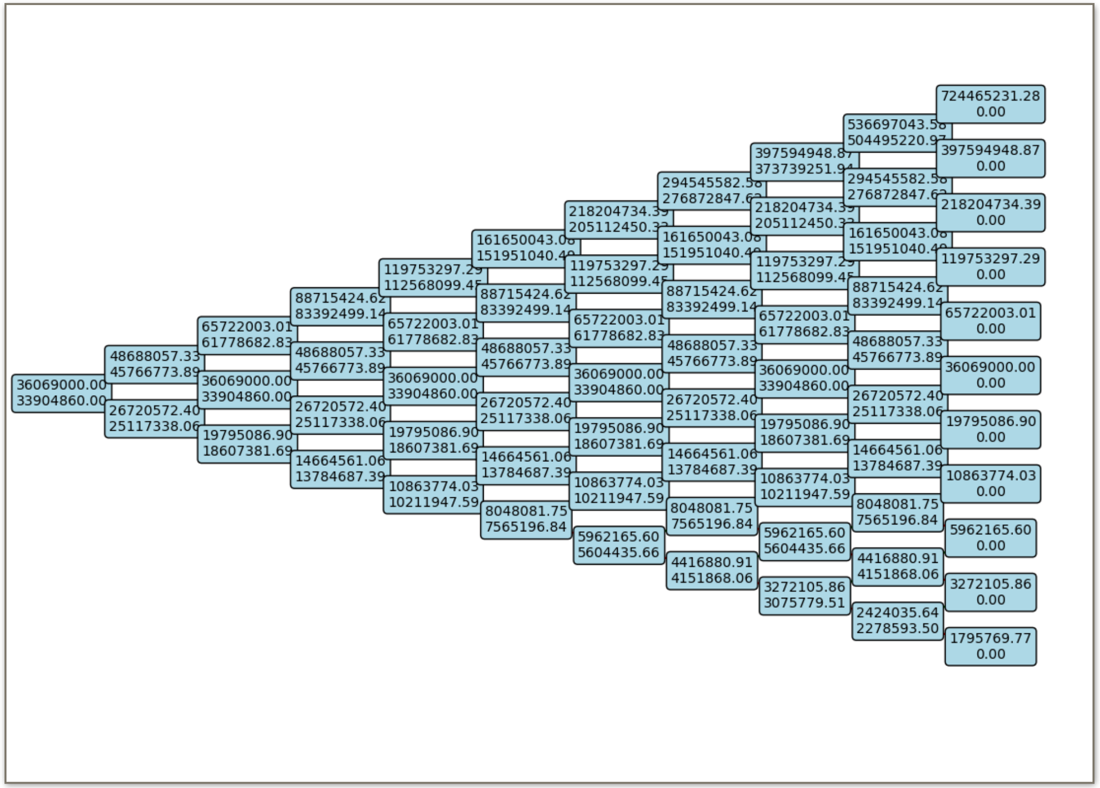


Figure 20 : Tree used to value decrease of real option

Note : The code used to generate this graph is included in ***Appendix A5: Code Snippets***. The code is titled "Value of Growth and Reduction of Real Option Code."

Both options were evaluated using a modified binomial model that incorporated the flexibility to exercise these choices at any point in the 10-year period. At each node, the model determined the optimal decision by comparing the continuation value, the value of exercising the growth option, and the value of exercising the contraction option. The final option values were derived through backward induction, ensuring that the strategic flexibility of these real options was fully captured. The growth option, valued higher, emphasizes the potential upside of expanding operations and increasing profitability. Conversely, the contraction option, while providing significant cost savings, is more old school, focusing on risk reduction rather than profit maximization. Together, these options illustrate the importance of real options in navigating uncertainty and maximizing the company's strategic planning.

References

[1] CFI Team. (2024). Volatility. Corporatefinance.com. https://corporatefinanceinstitute.com/resources/career-map/sell-side/capital-markets/volatility-vol/

[2] CNBC. (2024, June 2). Nvidia dominates the AI chip market, but there's rising competition. CNBC. https://www.cnbc.com/2024/06/02/nvidia-dominates-the-ai-chip-market-but-theres-rising-competition-.html

[3] CSIMarket. (n.d.). Industry growth rates. CSIMarket. https://csimarket.com/Industry/industry_growth_rates.php?rev&sp5

[4] Damodaran, A. (2012). Investment valuation: Tools and techniques for determining the value of any asset (3rd ed.). Wiley.

[5] Damodaran, A. (2020). The dark side of valuation: Valuing young, distressed, and complex businesses (3rd ed.). FT Press.

[6] DQYDJ. (n.d.). S&P 500 profit margin. DQYDJ. https://dqydj.com/sp-500-profit-margin/

[7] Forbes. (n.d.). Goldman Sachs Group. Forbes. https://www.forbes.com/companies/goldman-sachs-group/

[8] FourWeekMBA. (2023, October). Who owns Nvidia? FourWeekMBA. https://fourweekmba.com/wp-content/uploads/2023/10/Who-Owns-NVIDIA.png

[9] Friesen, G. (2024, May 14). Top 5 takeaways from S&P 500 Q1 earnings season. Forbes. https://www.forbes.com/sites/garthfriesen/2024/05/14/top-5-takeaways-from-sp-500-q1-earnings-season/

[10] Hasselblad, A. (2019). Data Envelopment Analysis (DEA). Mid Sweden University, Department of Information Systems and Technology.

[11] Hull, J. C. (2017). Options, futures, and other derivatives (10th ed.). Pearson.

[12] Lipper Alpha Insight. (2024, June). S&P 500 2024 Q1 earnings review: A good start to the year. Refinitiv. https://lipperalpha.refinitiv.com/2024/06/sp-500-2024-q1-earnings-review-a-good-start-to-the-year/

[13] MarketScreener. (n.d.). Nvidia Corporation: Company shareholders. MarketScreener. https://www.marketscreener.com/quote/stock/NVIDIA-CORPORATION-57355629/company-shareholders/

[14] Mišić, M. J., Đurđević, Đ. M., & Tomašević, M. V. (2012). Evolution and trends in GPU computing. In 2012 Proceedings of the 35th International Convention MIPRO (pp. 289–294). IEEE.

[15] Murphy, J. J. (1999). Technical analysis of the financial markets: A comprehensive guide to trading methods and applications. New York Institute of Finance.

[16] Nasdaq. (2024). Escalating geopolitical tensions may impact Nvidia and tech stocks. Nasdaq. https://www.nasdaq.com/articles/escalating-geopolitical-tensions-may-impact-nvidia-and-tech-stocks

[17] Nikolopoulou, K. (2022). What is criterion validity? | Definition & examples. Scribbr. https://www.scribbr.com/methodology/criterion-validity/

[18] NVIDIA. (2024a). FY2024 Nvidia corporate sustainability report. NVIDIA. https://images.nvidia.com/aem-dam/Solutions/documents/FY2024-NVIDIA-Corporate-Sustainability-Report.pdf

[19] NVIDIA. (2024b, November). Q3 FY2024 Nvidia company overview. NVIDIA. https://s201.q4cdn.com/141608511/files/doc_presentations/2024/Nov/Q3-NVDA-Company-Overview-FINAL.pdf

[20] NVIDIA Investor Relations. (n.d.). Investor relations. NVIDIA. https://investor.nvidia.com/home/default.aspx

[21] Oaktree Capital. (n.d.). How to think about risk with Howard Marks. Oaktree Capital. https://www.oaktreecapital.com/insights/insight-video/education/how-to-think-about-risk-with-howard-marks

[22] OneSafe. (2023). Nvidia's AI dominance: Geopolitical tensions and fintech disruption. OneSafe. https://www.onesafe.io/blog/nvidia-ai-dominance-geopolitical-tensions-fintech-disruption

[23] Reddit. (2013). Blackwell is not delayed and it will surprise the market. Reddit. https://www.reddit.com/r/NVDA_Stock/comments/1f4xr10/blackwell_is_not_delayed_and_it_will_surprise_the/

[24] Research-Methodology.net. (n.d.). Nvidia McKinsey 7S model. Research-Methodology.net. https://research-methodology.net/nvidia-mckinsey-7s-model/

[25] S&P Global Market Intelligence. (2024). S&P 500 Q1 2024 sector earnings & revenue data. S&P Global. https://www.spglobal.com/market-intelligence/en/news-insights/research/sp-500-q1-2024-sector-earnings-revenue-data

[26] Tastytrade. (n.d.). How to trade options. Tastytrade. https://tastytrade.com/options/how-to-trade-options/

[27] The Information. (n.d.). Nvidia org chart. The Information. https://www.theinformation.com/org-charts/nvidia

[28] TipRanks. (n.d.). Nvidia (NVDA) earnings. TipRanks. https://www.tipranks.com/stocks/nvda/earnings

[29] Tom's Hardware. (2024). Nvidia to reportedly triple output of compute GPUs in 2024: Up to 2 million H100s.Tom's Hardware. https://www.tomshardware.com/news/nvidia-to-reportedly-triple-output-of-compute-gpus-in-2024-up-to-2-million-h100s

[30] TrendForce. (2024, July 2). Nvidia's H200 order delivered from Q3, boosting server supply chain with strong demand. TrendForce. https://www.trendforce.com/news/2024/07/02/news-nvidias-h200-order-delivered-from-q3-boosting-server-supply-chain-with-strong-demand/

[31] TweakTown. (2024). Nvidia H100 AI GPU lead times improve: 4-month wait is now 2-3. TweakTown. https://www.tweaktown.com/news/97478/nvidia-h100-ai-gpu-lead-times-improve-4-month-wait-is-now-2-3/index.html

[32] Untaylored. (2024). Exploring Nvidia's business model and revenue streams. Untaylored. https://www.untaylored.com/post/exploring-nvidia-s-business-model-and-revenue-streams

[33] Vestinda. (2024). What is Nvidia (NVDA) and how does it work? Vestinda. https://www.vestinda.com/blog/stocks/what-is-nvidia-nvda-and-how-does-it-work#Partnerships_and_Collaboration

[34] Wilmott, P. (2006). Paul Wilmott on quantitative finance (2nd ed.). Wiley.

[35] Yahoo Finance. (2024). Nvidia Corporation (NASDAQ: NVDA) is largely controlled by institutional investors.Yahoo. https://finance.yahoo.com/news/nvidia-corporation-nasdaq-nvda-largely-120024903.html

[36] Yahoo News. (2023, December 22). Nvidia holds world's most valuable chipmaker title as valuation exceeds $1 trillion. Yahoo. https://www.yahoo.com/news/news/m/f8dcb6f5-af37-3b08-80f2-b88aae5f7ba9/nvidia-holds-world-s-most.html

[37] TradingView. (n.d.). *NVIDIA Corporation (NASDAQ: NVDA) stock chart*. TradingView. https://www.tradingview.com/chart/?symbol=NASDAQ%3ANVDA

[38] AlphaQuery. (n.d.). Nvidia (NVDA) 30-day implied volatility: Call options. AlphaQuery. https://www.alphaquery.com/stock/NVDA/volatility-option-statistics/30-day/iv-call

[39] Trading Economics. (n.d.). United States interest rate. Trading Economics. https://tradingeconomics.com/united-states/interest-rate

Appendix.A1 Price Tracks

Note: This appendix contains price tracks for Nvidia options expiring on 29 November, tracked from 14 November. Data includes both OTM and ITM call and put options for 10 strikes above and below the ATM strike.

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
8.6	-0.15	8.55	8.65	89	14,462	138.0	0.11	-0.03	0.10	0.11	420	15375
7.8	0.25	7.55	7.65	41	8,481	139,0	0.13	-0.03	0.12	0.13	254	14580
6.6	-0.16	6.60	6.70	867	70398	140,0	0.17	-0.04	0.16	0.17	3417	51989
5.7	-0.15	5.65	5.75	1782	17,715	141,0	0.21	-0.08	0.21	0.22	1390	15797
4.76	-0.14	4.70	4.80	270	21,842	142,0	0.30	-0.09	0.29	0.30	1289	23010
3.86	-0.14	3.90	4.00	403	13479	143,0	0.43	-0.10	0.43	0.44	2483	19613
3.1	-0.20	3.05	3.15	934	12584	144,0	0.64	-0.17	0.63	0.64	3405	22470
2.37	-0.18	2.35	2.37	7273	61247	145,0	0.92	-0.17	0.92	0.94	6765	37342
1.79	-0.14	1.76	1.78	10124	24317	146,0	1.30	-0.17	1.30	1.32	12345	18878
1.25	-0.25	1.23	1.25	21483	43893	147,0	1.81	-0.22	1.80	1.83	8261	21077
		Nov 14 , 2024 9.59am				***147.37***						
0.86	-1.87	1.66	1.68	15990	6129	148,0	2.37	-0.19	2.37	2.40	3248	19033
0.57	-1.61	1.13	1.15	29237	16414	149,0	3.15	-0.15	3.05	3.15	1132	10669
0.37	-1.41	0.74	0.76	9652	15574	150,0	4.00	-0.10	3.85	3.95	1838	12826
0.15	-1.12	0.46	0.47	17498	30747	152,0	5.50	-0.5	5.6	5.75	184	3001
0.13	-0.87	0.29	0.20	10491	24877	152,5	6.10	0.05	6.10	6.20	30	1705
0.07	-0.61	0.17	0.18	8095	23939	154,0	7.65	-0.15	7.55	7.65	47	658
0.04	-0.43	0.11	0.12	30358	76946	155,0	8.65	-0.25	8.55	8.65	24	3060
0.03	-0.29	0.08	0.09	5298	25985	156,0	9.55	0.10	9.55	9.65	67	355

0.02	-0.18	0.06	0.07	10,797	41.194	157,5	11.25	0.60	11.05	11.15	1	82
0.02	-0.12	0.04	0.05	9,279	39.915	158,0	11.15	-0.75	11.55	11.65	38	136

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
8.25	-6.16	8.35	8.45	12152	22444	124.0	0.02	-0.01	0.01	0.02	2406	14957
7.30	-6.52	7.35	7.45	337	2913	125.0	0.01	-0.02	0.01	0.02	1121	10383
6.45	-6.15	6.35	6.45	4592	8821	126.0	0.01	-0.02	0.01	0.02	1233	15110
5.25	-6.71	5.35	5.45	4133	84944	127.0	0.01	-0.03	0.01	0.02	6209	29548
4.20	-6.74	4.35	4.45	739	15985	128.0	0.01	-0.04	0.01	0.02	1993	22598
3.23	-6.30	3.35	3.45	984	8476	129.0	0.01	-0.03	0.01	0.02	3064	11520
2.37	-6.23	2.22	2.43	3138	14244	130.0	0.02	-0.04	0.02	0.03	14047	15285
1.46	-6.49	1.41	1.45	18634	8245	131.0	0.05	-0.01	0.05	0.06	26667	14016
0.51	-6.49	0.58	0.60	73532	64957	132.0	0.2	0.13	0.2	0.21	160061	51602
0.15	-5.59	0.14	0.15	98661	17426	133.0	0.77	0.68	0.74	0.78	122220	18796

Nov 15 , 2024 2:48 pm

141.37

0.04	-4.99	0.04	0.05	129.135	19973	134.0	1.61	1.51	1.64	1.69	131209	25443
0.02	-4.01	0.02	0.03	151.399	14264	135.0	2.61	2.46	2.62	2.67	108839	20546
0.01	-3.08	0.01	0.02	126.235	11780	136.0	3.6	3.35	3.55	3.7	80919	23450
0.01	–2,30	0	0.01	240101	58115	137.0	4.62	4.2	4.55	4.65	57885	41104
0.01	-1.58	0	0.01	55502	25082	138.0	5.62	4.91	5.55	5.65	20288	26563
0.01	-1.03	0	0.01	47770	35529	139.0	6.7	5.57	6.55	6.65	19214	25978
0.01	-0.61	0	0.01	132982	134535	140.0	7.56	5.85	7.55	7.65	6688	21595
0.01	-0.35	0	0.01	22491	37292	141.0	8.6	6.1	8.55	8.65	1994	13301
0.01	-0.18	0	0.01	60529	133279	142.0	9.6	6.34	9.55	9.65	7021	12241

0.01	-0.05	0	0.01	14849	56151	143.0	11.65	6.25	11.55	11.65	681	973

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
13.23	-3.05	13.35	13.45	71	1383	128.0	2.51	0.47	2.51	2.53	856	8243
12.7	-2.8	12.6	12.7	31	1740	129.0	2.79	0.51	2.77	2.80	703	5254
11.84	-2.86	11.9	12.0	339	10320	130.0	3.05	0.56	3.05	3.15	2248	31030
11.2	-2.7	11.1	11.25	16	1243	131.0	3.4	0.68	3.35	3.45	379	3917
10.55	-2.6	10.55	10.65	86	5005	132.0	3.77	0.82	3.75	3.80	697	13663
9.81	-2.54	9.9	10.0	34	1495	133.0	4.15	0.9	4.1	4.15	320	7746
9.4	-2.35	9.3	9.4	83	2596	134.0	4.51	0.96	4.45	4.55	299	14023
8.7	-2.28	8.7	8.8	414	10711	135.0	4.95	1.0	4.9	4.96	1481	31059
8.2	-2.3	8.15	8.25	151	2347	136.0	5.38	1.13	5.35	5.40	394	14401
7.67	-2.13	7.65	7.75	204	3682	137.0	5.9	1.28	5.75	5.85	1233	6223

Nov 18, 2024 9:50 am

137.80

7.15	-2.0	7.15	7.2	756	4876	138.0	6.4	1.35	6.3	6.40	1146	7956
6.65	-2.05	6.6	6.7	1305	3132	139.0	6.88	1.38	6.85	6.9	1254	7186
6.15	-1.85	6.15	6.2	3354	28822	140.0	7.44	1.54	7.35	7.45	2487	28924
5.73	-1.82	5.7	5.8	1537	5550	141.0	7.93	1.55	7.9	8.	233	4994
5.3	-1.65	5.35	5.4	1710	13782	142.0	8.6	1.6	8.45	8.55	234	10253
5.0	-1.57	4.95	5.0	1400	7448	143.0	9.25	1.85	9.05	9.15	252	6260
4.59	-1.56	4.55	4.6	917	8070	144.0	9.8	1.8	9.7	9.8	119	5768
4.25	-1.4	4.2	4.3	2377	51776	145.0	10.51	2.0	10.35	10.45	2519	24039
3.92	-1.38	3.9	4.0	816	9169	146.0	11.35	2.27	11.05	1115	63	5535
3.6	-1.38	3.6	3.65	755	11019	147.0	11.88	2.18	11.7	11.85	44	7008

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
11.13	-2.02	11.1	11.55	1309	5116	132.0	2.91	-0.04	2.86	2.88	8236	16105
10.45	-1.9	10.4	10.55	1834	1830	133.0	3.2	-0.05	3.15	3.20	3547	8998
9.8	-1.95	9.75	9.85	1896	3053	134.0	3.55	0.0	3.45	3.55	5167	15101
9.17	-1.81	9.1	9.25	10057	11190	135.0	3.9	-0.05	3.85	3.95	27132	36282
8.5	-2.0	8.5	8.65	3170	3388	136.0	4.3	0.05	4.25	4.30	9907	18771
7.9	-1.9	7.95	8.05	4882	4771	137.0	4.75	0.13	4.65	4.75	10630	7949
7.45	-1.7	7.4	7.5	24773	19065	138.0	5.17	0.12	5.15	5.20	12617	9085
6.85	-1.85	6.85	7.0	24328	13591	139.0	5.7	0.2	5.6	5.65	12012	7954
6.4	-1.6	6.35	6.5	76880	44736	140.0	6.23	0.33	6.1	6.15	30406	34855
5.95	-1.6	5.95	6.0	23261	13235	141.0	6.7	0.32	6.6	6.70	7377	5887
		Nov 19 , 2024 9:31 am				***141.61***						
5.45	-1.5	5.5	5.55	17837	16793	142.0	7.25	0.25	7.15	7.30	4537	10344
5.0	-1.57	5.05	5.15	8536	8663	143.0	7.85	0.45	5.9	7.90	1578	5931
4.69	-1.46	4.65	4.75	8250	9255	144.0	8.45	0.45	8.3	8.50	2545	5280
4.3	-1.35	4.3	4.35	46539	56159	145.0	9.09	0.58	8.85	9.15	8770	23372
3.95	-1.35	3.95	4.0	6767	11485	146.0	9.85	0.77	9.6	9.80	1092	5176
3.63	-1.35	3.6	3.7	7103	12091	147.0	10.47	0.77	10.25	10.50	1355	6392
3.32	-1.21	3.3	3.4	17409	27593	148.0	11.0	0.69	11.0	11.20	658	6220
3.03	-1.17	3.0	3.1	7496	14072	149.0	11.55	0.58	11.6	12	614	2049
2.78	-1.12	2.77	2.79	91741	180898	150.0	12.64	0.94	12.4	12.65	2532	10730
2.2	-0.85	2.22	2.22	15995	17465	152.5	13.66	0.19	14.3	14.65	524	1130

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
11.35	-1.54	11.35	11.4	1056	4117	136.0	2.64	0.68	2.63	2.65	2429	20323
10.7	-1.48	10.65	10.75	492	4440	137.0	2.93	0.73	2.94	2.97	3373	9282
10.08	-1.37	10.0	10.1	630	23639	138.0	3.25	0.8	3.25	3.35	2625	9209
9.4	-1.4	9.35	9.45	773	13466	139.0	3.65	0.91	3.65	3.70	3011	7695
8.75	-1.3	8.75	8.8	5209	41664	140.0	4.05	1.0	4.05	4.10	22797	37276
8.2	-1.2	8.2	8.25	6278	11659	141.0	4.5	1.11	4.45	4.50	1616	6633
7.68	-1.12	7.6	7.7	2651	16236	142.0	4.92	1.17	4.9	4.95	5611	14664
7.1	-1.1	7.1	7.15	5444	10176	143.0	5.41	1.21	5.35	5.45	4516	9343
6.55	-1.1	6.6	6.65	7750	9863	144.0	5.9	1.3	5.85	5.95	7096	6207
6.1	-1.0	6.1	6.15	24203	53927	145.0	6.45	1.4	6.4	6.45	16443	25117
		Nov 20 , 2024 10:50 am				***145.64***						
5.65	-0.94	5.65	5.7	11500	12387	146.0	7.0	1.5	6.95	7.00	3769	6172
5.25	-0.79	5.2	5.25	6470	15481	147.0	7.55	1.55	7.5	7.55	3893	7767
4.85	-0.75	4.85	4.85	6377	28743	148.0	8.05	1.5	8.05	8.15	709	6415
4.45	-0.7	4.4	4.5	4173	15430	149.0	8.65	1.45	8.65	8.75	675	2288
4.06	-0.64	4.05	4.1	41464	189544	150.0	9.37	1.7	9.3	9.40	2613	10645
3.25	-0.48	3.3	3.3	8375	22326	152.5	11.0	1.62	11.0	11.10	284	1126
2.55	-0.38	2.56	2.56	21043	161517	153.0	12.7	1.7	12.9	12.90	537	2506
1.97	-0.32	1.99	1.99	13582	18351	155.0	14.8	2.93	14.6	14.80	155	427
1.51	-0.24	1.53	1.53	31770	90928	157.5	16.85	2.05	16.9	16.90	568	3248
1.17	-0.14	1.16	1.17	6981	109573	160.0	20.35	3.55	19.1	19.10	33	197

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
9.05	-0.35	8.90	9.10	6,608	16,369	141.00	4.10	0.71	4.00	4.20	11,416	7,827
7.52	-1.28	8.30	8.50	9,307	14,581	142.00	4.60	0.85	4.40	4.60	24,371	16,502
7.90	-0.30	7.75	8.10	15,490	11,289	143.00	4.99	0.79	4.40	5.05	13,718	11,201
7.35	-0.30	7.15	7.45	24,640	12,882	144.00	5.45	0.85	5.20	5.50	23,096	12,944
6.80	-0.30	6.75	6.85	99,654	67,886	145.00	5.60	0.75	5.80	5.95	44,049	39,157
6.30	-0.29	6.20	6.35	35,302	19,074	146.00	6.35	0.85	6.05	6.45	12,112	7,329
5.85	-0.19	5.80	5.90	20,113	17,205	147.00	7.02	1.02	6.75	7.00	8,780	7,272
5.40	-0.20	5.30	5.45	20,704	20,772	148.00	7.47	0.92	7.10	7.60	1,811	6,514
4.85	-0.30	4.85	5.00	13,919	16,496	149.00	8.06	0.86	7.70	6.15	1,825	2,405
4.55	-0.15	4.50	4.60	128,112	200,673	150.00	8.64	0.97	6.45	8.70	7,334	13,256
		Nov 21 , 2024 9:30 am				**150.23**						
3.65	-0.08	3.55	3.65	30,151	29,982	152.50	10.40	1.02	9.45	10.35	1,316	1,456
2.85	-0.08	2.62	2.86	86,091	174,065	155.00	12.13	1.13	11.00	12.25	3,242	2,649
2.21	-0.08	2.17	2.25	40,860	33,759	157.50	14.06	1.19	12.45	14.60	845	867
1.71	-0.04	1.69	1.74	113,334	121,966	160.00	15.95	1.15	15.60	15.90	2,340	4,592
1.31	0.00	1.28	1.32	31,001	115,727	162.50	18.20	1.40	16.70	20.00	276	294
1.00	0.01	0.97	1.01	66,118	64,301	165.00	20.20	0.88	18.85	22.00	332	845
0.77	0.03	0.76	0.77	19,488	20,094	167.50	22.30	0.84	21.00	22.90	27	152
0.57	0.01	0.57	0.58	47,756	59,966	170.00	24.95	1.25	23.60	26.95	258	729
0.44	0.02	0.40	0.45	14,021	16,143	172.50	26.86	0.71	26.05	29.40	9	148
0.33	0.01	0.32	0.34	23,152	32,581	175.00	29.55	1.05	28.50	31.70	56	354

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
9.15	-4.75	9.05	9.15	521	1,775	133.00	0.01	-0.04	0.01	0.02	2,724	8,819
8.10	-4.75	8.05	8.15	692	2,367	134.00	0.01	-0.05	0.01	0.02	2,401	15,526
7.10	-4.76	7.05	7.15	3,394	11,720	135.00	0.02	-0.03	0.02	0.03	8,474	35,129
6.00	-4.85	6.05	6.15	608	3,802	136.00	0.02	-0.04	0.02	0.03	3,562	14,669
5.20	-4.75	5.05	5.15	989	3,397	137.00	0.02	-0.04	0.02	0.03	4,428	8,804
4.15	-4.85	4.10	4.15	2,238	23,682	138.00	0.03	-0.05	0.02	0.03	7,111	10,357
3.17	-4.83	3.10	3.15	1,578	12,336	139.00	0.04	-0.06	0.03	0.04	11,847	12,596
2.18	-4.84	2.14	2.17	37,267	35,105	140.00	0.07	-0.06	0.06	0.07	139,781	38,512
1.28	-4.65	1.27	1.29	27,034	15,470	141.00	0.18	-0.01	0.18	0.19	58,769	16,720
0.64	-4.56	0.63	0.65	75,321	13,390	142.00	0.54	0.26	0.53	0.55	80,886	16,988
		Nov 22 , 2024 1:32 pm				**142.13**						
0.27	-3.92	0.27	0.28	135,236	11,616	143.00	1.17	0.76	1.17	1.19	90,260	22,845
0.11	-3.24	0.11	0.12	138,126	13,951	144.00	2.00	1.40	2.01	2.04	55,794	12,669
0.06	-2.57	0.06	0.07	208,875	54,400	145.00	2.97	2.09	2.95	2.99	49,498	34,673
0.04	-1.96	0.04	0.05	89,050	20,381	146.00	3.90	2.65	3.90	4.00	18,776	10,525
0.03	-1.44	0.03	0.04	115,999	17,880	147.00	4.95	3.22	4.90	5.00	12,371	10,153
0.02	-1.03	0.02	0.03	49,569	26,579	148.00	6.01	3.70	5.90	6.00	4,095	6,894
0.03	-0.67	0.02	0.03	20,863	18,356	149.00	6.85	3.98	6.90	6.95	2,398	5,504
0.02	-0.46	0.01	0.02	204,187	206,639	150.00	7.91	4.21	7.90	7.95	3,844	13,035
0.01	-0.15	0.01	0.02	35,030	58,008	152.50	10.35	4.55	10.40	10.45	699	1,655
0.01	-0.08	0.00	0.01	44,263	113,440	155.00	13.00	4.65	12.85	12.95	770	2,222

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
10.30	-3.57	10.15	10.35	110	1,052	126.00	0.19	0.04	0.18	0.19	2,998	4,605
9.35	-3.73	9.30	9.40	85	1,111	129.00	0.24	0.06	0.24	0.25	1,592	6,473
8.45	-3.92	8.30	8.45	1,319	6,528	130.00	0.30	0.09	0.30	0.31	13,336	20,444
7.50	-4.00	7.45	7.55	282	2,451	131.00	0.40	0.16	0.40	0.41	4,587	5,941
6.63	-3.77	6.55	6.65	214	1,773	132.00	0.50	0.18	0.50	0.52	6,394	4,370
5.85	-3.78	5.70	5.80	552	2,446	133.00	0.65	0.26	0.65	0.66	10,194	7,788
4.95	-3.55	4.95	5.00	634	3,377	134.00	0.85	0.35	0.85	0.86	12,027	6,301
4.20	-3.55	4.20	4.25	6,242	11,239	135.00	1.09	0.47	1.10	1.11	29,230	23,305
3.50	-3.30	3.50	3.60	3,472	5,552	136.00	1.43	0.67	1.42	1.44	11,122	11,475
2.97	-3.03	2.95	2.97	13,704	2,516	137.00	1.79	0.83	1.80	1.79	17,037	9,621
		Nov 25, 2024 10:19 am				***137,35***						
2.43	-2.82	2.41	2.43	16,511	3,476	136.00	2.24	1.04	2.26	2.29	25,139	18,051
1.97	-2.58	1.94	1.96	16,430	3,633	139.00	2.61	1.31	2.84	2.87	22,700	9,148
1.57	-2.33	1.56	1.58	45,103	19,011	140.00	3.40	1.57	3.40	3.45	25,996	26,558
1.24	-2.06	1.23	1.25	24,331	7,206	141.00	4.15	1.94	4.10	4.20	8,105	6,659
0.98	-1.75	0.96	0.97	30,146	21,711	142.00	4.95	2.29	4.85	4.95	10,274	9,346
0.76	-1.48	0.76	0.77	16,031	29,243	143.00	5.65	2.45	5.45	5.75	2,526	8,850
0.61	-1.25	0.60	0.61	16,259	23,877	144.00	6.60	2.83	6.50	6.60	2,170	7,553
0.49	-1.01	0.48	0.49	49,339	72,388	145.00	7.48	3.06	7.40	7.50	2,348	13,786
0.39	-0.83	0.39	0.40	9,190	60,491	146.00	8.40	3.26	6.30	8.40	454	4,719
0.33	-0.65	0.33	0.34	9,077	16,237	147.00	9.26	3.33	9.20	9.20	284	5,142

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
10.45	1.00	10.20	10.30	189	1,174	127.00	0.12	-0.07	0.11	0.12	1,862	5,980
9.15	0.70	9.20	9.35	219	1,137	126.00	0.14	-0.11	0.14	0.15	1,078	6,251
8.25	0.70	8.25	8.40	241	1,234	129.00	0.19	-0.15	0.18	0.19	1,220	8,683
7.40	0.95	7.30	7.40	1,249	7,878	130.00	0.25	-0.20	0.24	0.25	10,636	26,003
6.43	0.83	6.40	6.50	554	2,862	131.00	0.34	-0.26	0.32	0.33	2,788	18,296
5.64	0.84	5.50	5.60	1,781	3,164	132.00	0.42	-0.36	0.43	0.44	4,317	9,630
4.70	0.62	4.70	4.75	1,781	3,723	133.00	0.59	-0.45	0.56	0.59	15,919	11,669
3.92	0.42	3.90	3.95	1,882	5,178	134.00	0.79	-0.55	0.79	0.80	12,803	12,600
3.25	0.39	3.15	3.25	10,106	14,651	135.00	1.07	-0.45	1.06	1.08	26,830	31,704
2.54	0.22	2.55	2.57	12,738	14,311	136.00	1.43	-0.79	1.42	1.44	23,256	15,737
		Nov 26 , 2024 10:47 am				***136.77***						
2.00	0.14	1.99	2.01	34,550	21,139	137.00	1.86	-0.94	1.86	1.89	30,995	18,482
1.56	0.06	1.52	1.54	37,016	18,489	138.00	2.40	-0.95	2.40	2.41	24,398	19,962
1.14	-0.03	1.14	1.16	29,727	17,837	139.00	2.98	-1.17	3.00	3.05	9,070	11,923
0.86	-0.09	0.85	0.86	57,923	59,105	140.00	3.73	-1.12	3.70	3.75	7,075	29,923
0.63	-0.14	0.62	0.63	39,451	59,923	141.00	4.55	-1.20	4.45	4.55	2,054	7,345
0.47	-0.15	0.46	0.47	18,428	40,886	142.00	5.35	-1.15	5.25	5.40	1,762	9,721
0.34	-0.16	0.33	0.34	12,328	35,899	143.00	6.05	-1.38	6.15	6.20	687	8,155
0.26	-0.15	0.25	0.26	52,060	66,849	144.00	7.20	-0.98	7.10	7.20	443	5,840
0.19	-0.15	0.19	0.20	32,557	61,350	145.00	6.10	-1.05	6.05	6.15	779	13,199
0.17	-0.12	0.16	0.17	7,481	62,789	146.00	9.15	-1.15	9.00	9.15	263	4,672

Last	CHG	BID	ASK	Vol	Open Int	Strike	Last	CHG	BID	ASK	VOL	Open Int
10.45	-2.35	10.75	10.85	3	338	124.0	0.04	0.0	0.03	0.04	40	4,739
9.15	-2.29	9.75	9.85	245	1578	125.0	0.04	-0.01	0.04	0.05	1578	13,478
8.25	-2.47	8.75	8.85	35	578	126.0	0.05	-0.01	0.04	0.05	523	8,352
7.40	-2.35	7.75	7.90	29	1237	127.0	0.07	0.0	0.06	0.07	979	6,275
6.43	-2.31	6.80	6.90	163	1245	128.0	0.09	0.01	0.06	0.09	1260	10,735
5.64	-2.3	5.80	5.95	252	1228	129.0	0.11	0.01	0.1	0.11	1517	10,251
4.70	-2.3	4.85	4.95	4074	7815	130.0	0.16	0.03	0.15	0.16	13195	28,264
3.92	-2.28	3.95	4.05	632	3122	131.0	0.24	0.05	0.24	0.25	3491	19,125
3.25	-2.15	3.10	3.20	2528	3071	132.0	0.39	0.13	0.36	0.39	9472	10,912
2.54	-2.01	2.35	2.28	7844	3559	133.0	0.61	0.24	0.62	0.63	22126	15,72
		Nov 27 , 2024 10:02 am				***133.52***						
1.68	-1.87	1.66	1.68	15990	6129	134.0	0.94	0.38	0.94	0.96	21647	51,755
1.16	-1.61	1.13	1.15	29237	16414	135.0	1.41	0.61	1.41	1.43	15197	37,625
0.75	-1.41	0.74	0.76	9652	15574	136.0	2.02	0.86	2.02	2.05	8379	19,967
0.47	-1.12	0.46	0.47	17498	30747	137.0	2.75	1.15	2.75	2.79	3726	24,205
0.29	-0.87	0.29	0.20	10491	24877	138.0	3.55	1.4	3.45	3.6	1297	20,608
0.18	-0.61	0.17	0.18	8095	23939	139.0	4.4	1.6	4.45	4.5	1868	11,51
0.11	-0.43	0.11	0.12	30358	76946	140.0	5.41	1.86	5.35	5.45	5343	28,529
0.08	-0.29	0.08	0.09	5298	25985	141.0	6.3	1.9	6.35	6.5	650	7,952
0.07	-0.18	0.06	0.07	10,797	41.194	142.0	7.43	2.23	7.25	7.45	514	9.975
0.05	-0.12	0.04	0.05	9,279	39.915	143.0	9.27	2.07	9.35	9.40	202	7.439

Appendix.A2 Technical Analysis

Note: This appendix presents an analysis of Nvidia's chart and volume behavior during the release of its last six quarterly earnings report

Figure 6 : Represent Nvidia's Q1 stock price on May 24, 2023, with a fluctuation ranging from 30.51 to 38.41

Note : The following picture is a screenshot taken from TradingView, showing Nvidia stock.(TradingView)

Figure 6 : Represent Nvidia's Q2 stock price on Aug 23, 2023, with a fluctuation ranging from 47.10 to 47.14 accompanied by positive volume momentum.

Note : The following picture is a screenshot taken from TradingView, showing Nvidia stock.(TradingView)

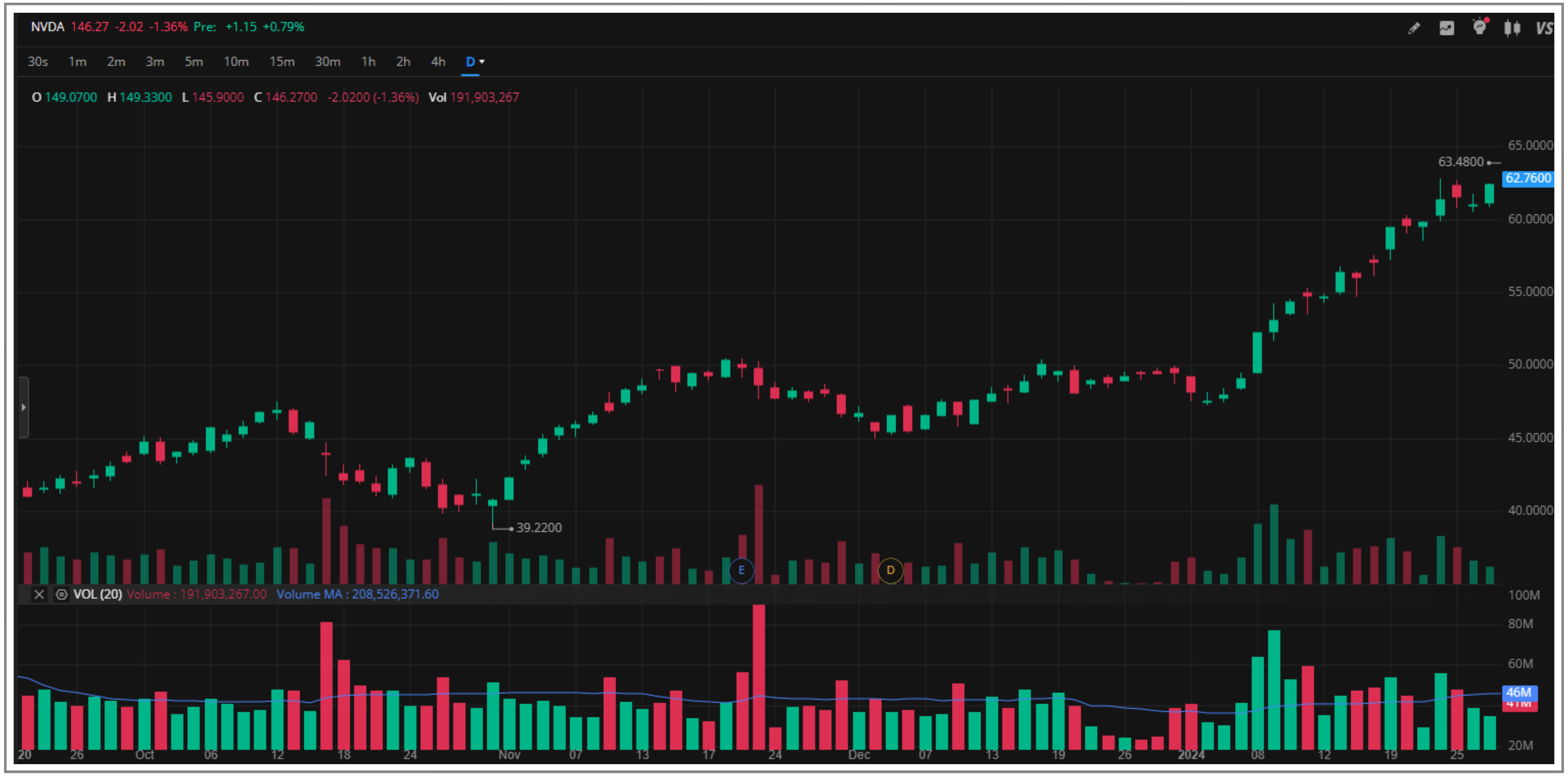


Figure 6 : Represent Nvidia's Q3 stock price on Nov 21, 2023, with a fluctuation ranging from 49.93 to 48.7 accompanied by negative volume momentum.

Note : The following picture is a screenshot taken from TradingView, showing Nvidia stock.(TradingView)

Figure 6 : Represent Nvidia's Q4 stock price on Feb 21 2024, with a fluctuation ranging from 67.46.10 to 78.52 accompanied by negative volume momentum

Note : The following picture is a screenshot taken from TradingView, showing Nvidia stock.(TradingView)

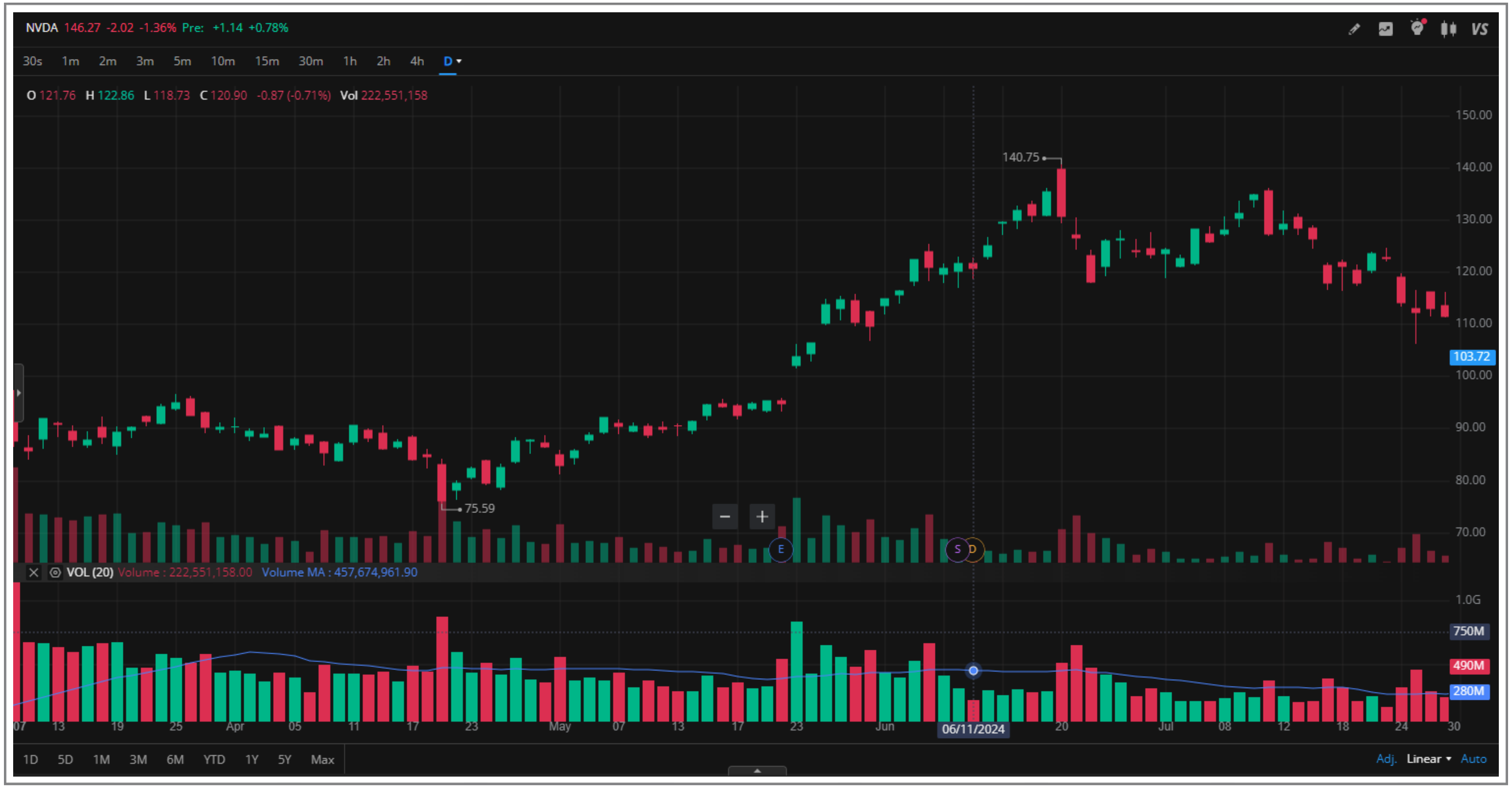


Figure 6 : Represent Nvidia's Q1 stock price on May 22, 2024, with a fluctuation ranging from 94.93 to 103.78 accompanied by negative volume momentum

Note : The following picture is a screenshot taken from TradingView, showing Nvidia stock.(TradingView)

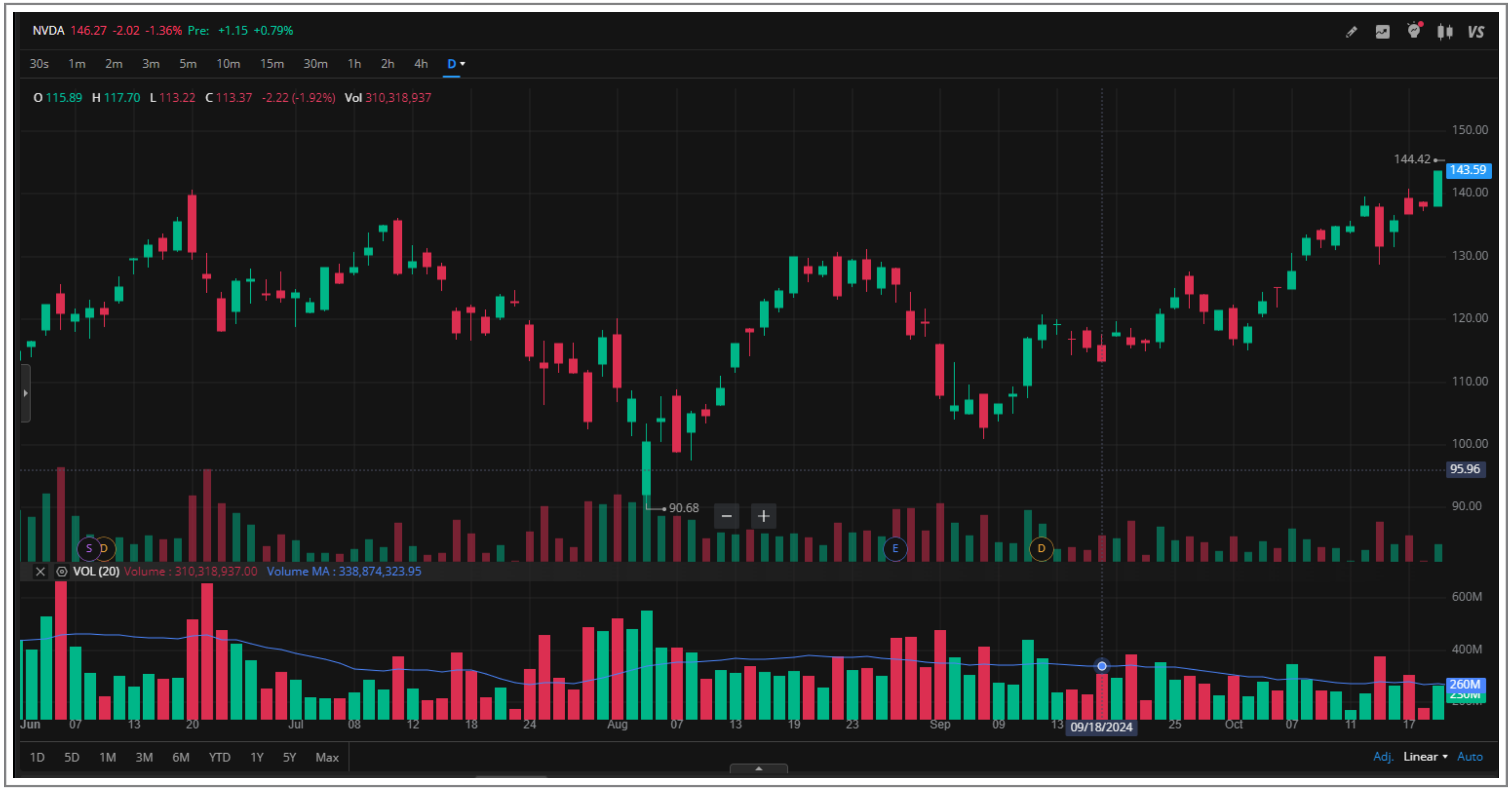


Figure 6 : Represent Nvidia's Q2 stock price on Aug 28, 2024, with a fluctuation ranging from 125.6 to 117.68 accompanied by negative volume momentum

Note : The following picture is a screenshot taken from TradingView, showing Nvidia stock.(TradingView)

Appendix.A3 Graphs

Note This appendix includes graphs generated using custom code and publicly available scripts. These graphs visualize key data and insights relevant to the report. Each graph is labeled, with captions providing context and linking the visualizations to their respective analyses in the main text.

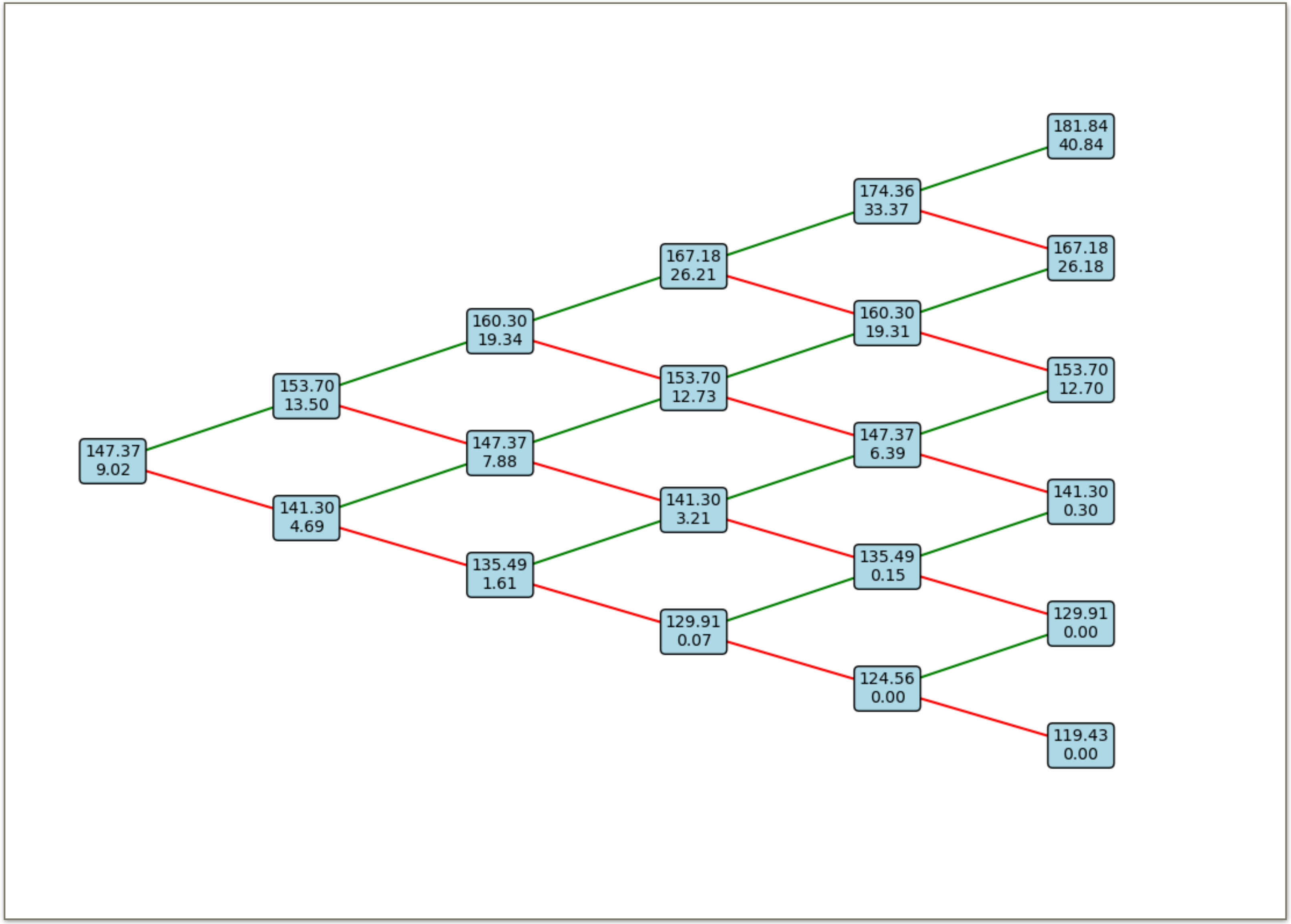


Figure 1 : The graph shows the binomial tree with 5 steps of the OTM Call option made from personal code.

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Binomial Tree for American Call."

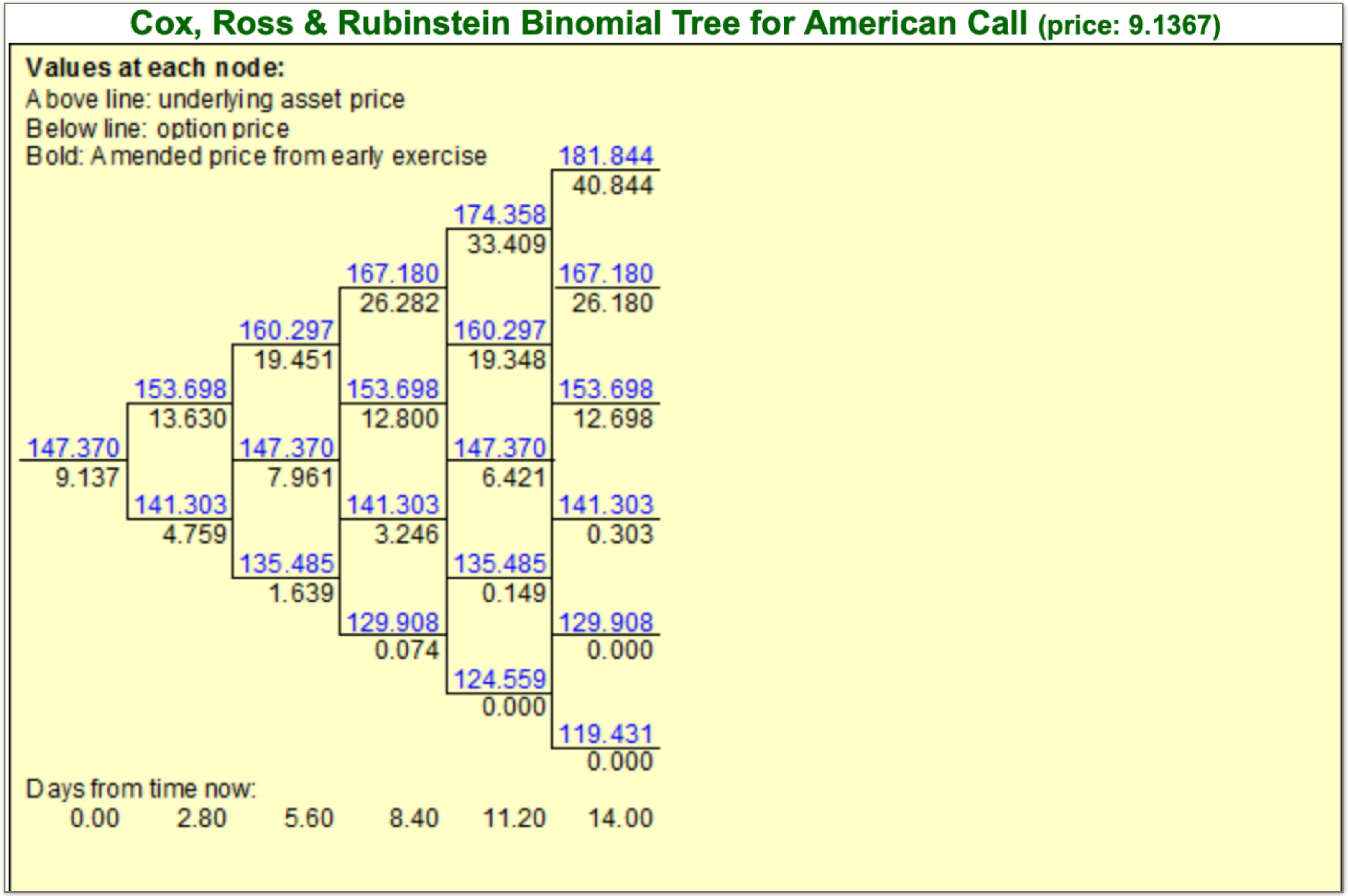


Figure 2 : The graph shows the binomial tree with 5 steps of the OTM Call option made from option calculator.

Note : The graph has been created using the Options Calculator linked in the assignment.

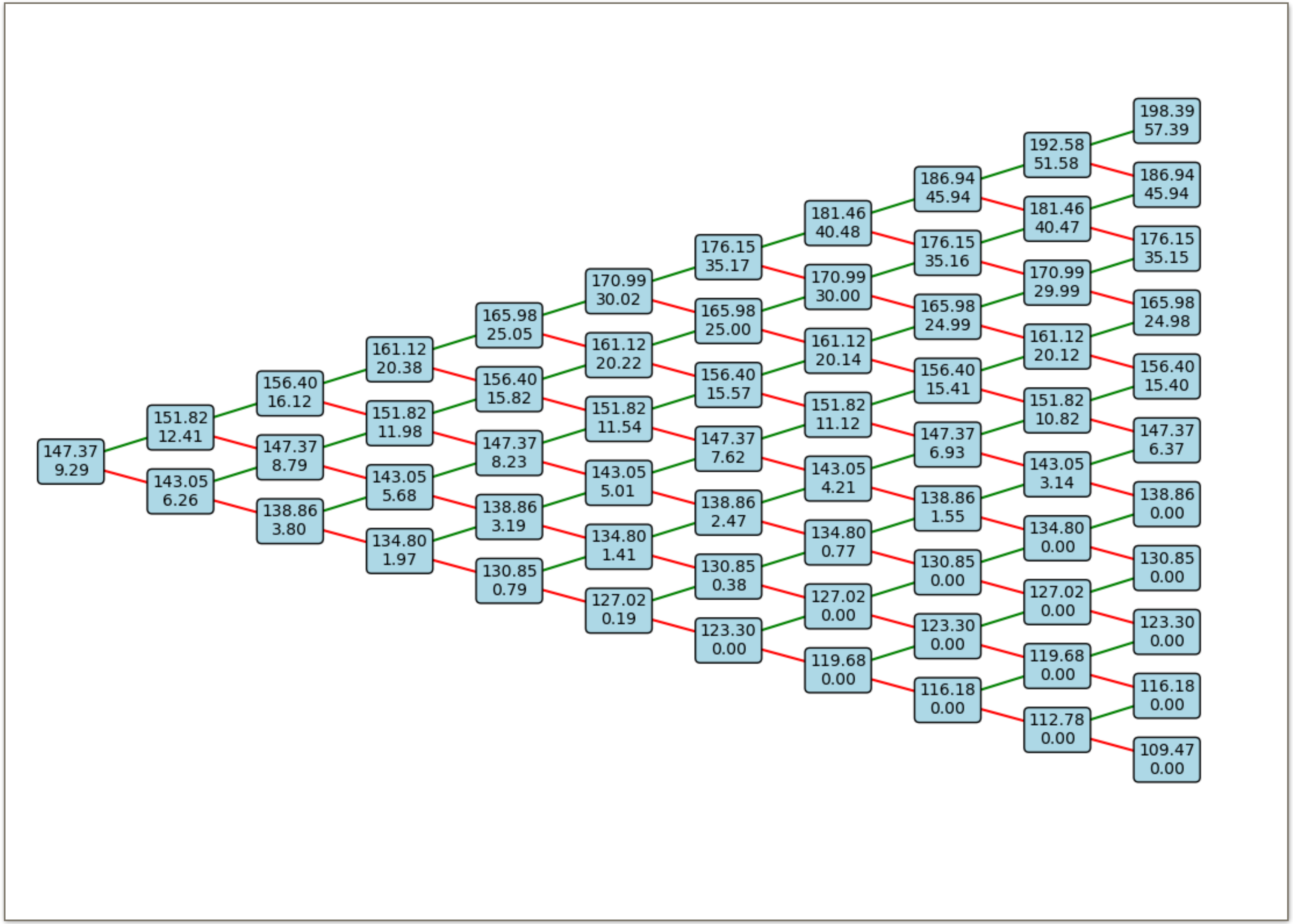


Figure 3 : The graph shows the binomial tree with 10 steps of the OTM Call option made from personal code.

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Binomial Tree for American Call."

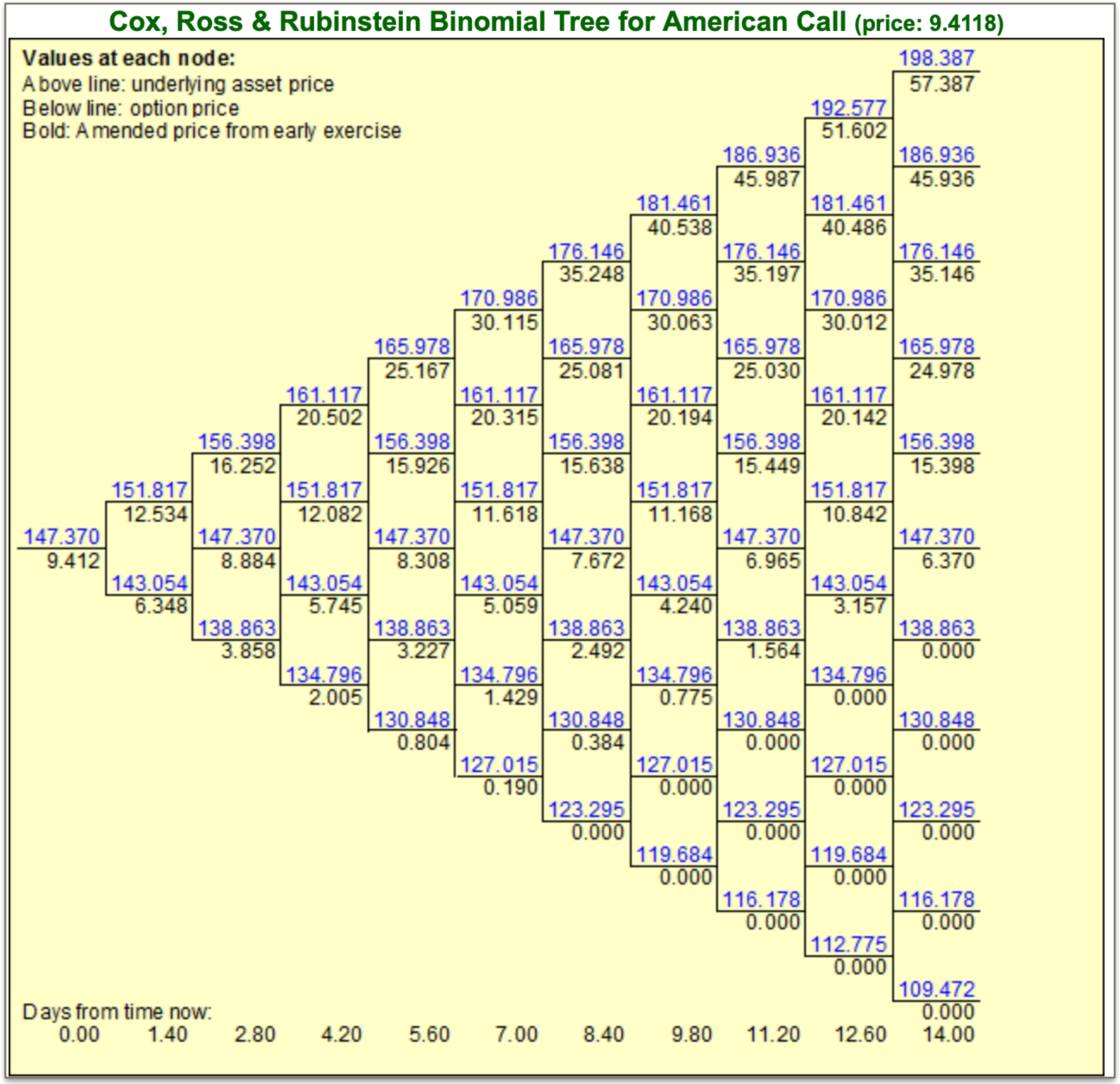


Figure 4 : The graph shows the binomial tree with 10 steps of the OTM Call option made from option calculator online.

Note : The graph has been created using the Options Calculator linked in the assignment.

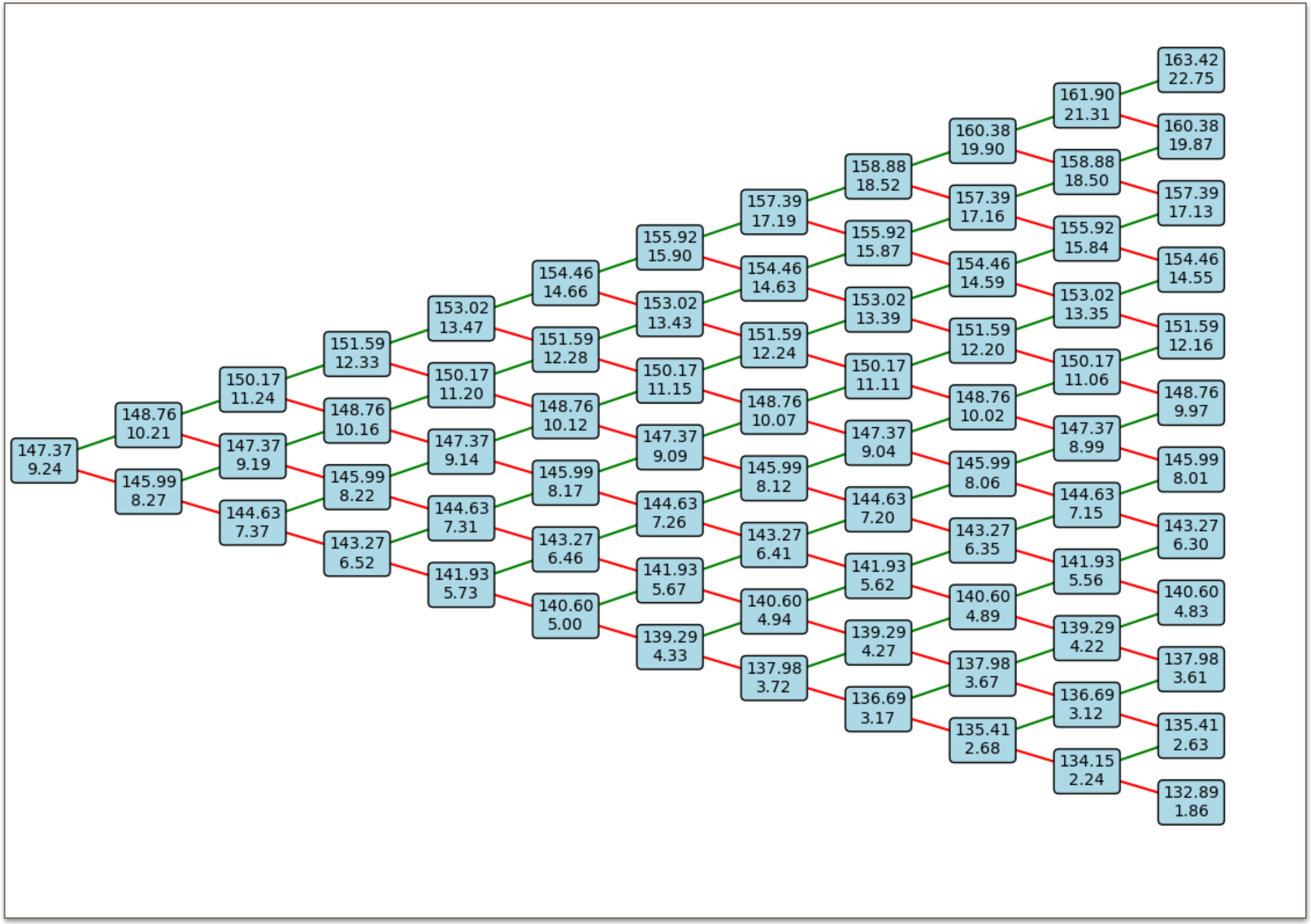


Figure 5 : The graph shows the binomial tree with 100 steps of the OTM Call option made from personal code.

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Binomial Tree for American Call."

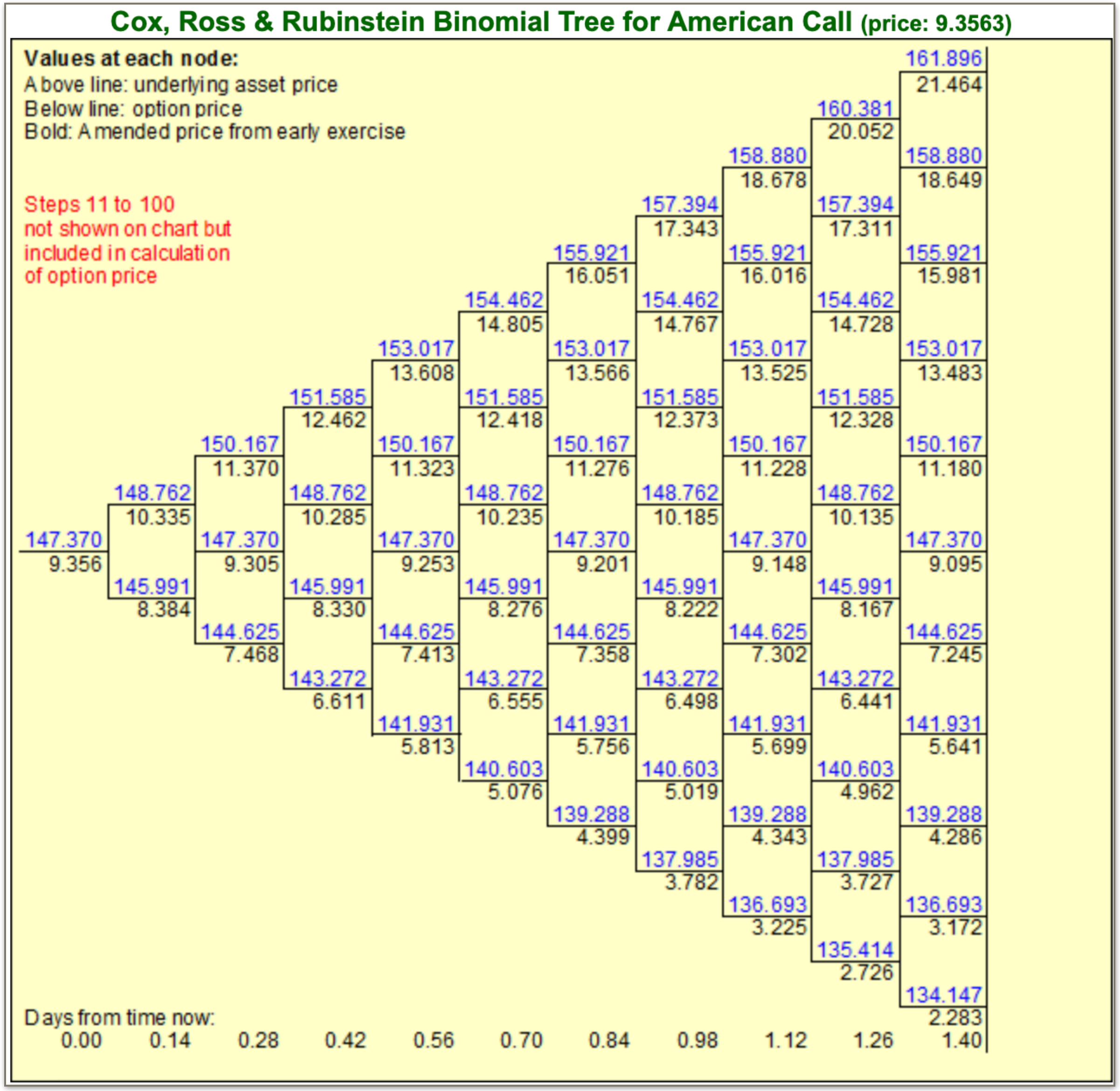


Figure6 : The graph shows the binomial tree with 100 steps of the OTM Call option made from online option calculator.

Note : The graph has been created using the Options Calculator linked in the assignment.

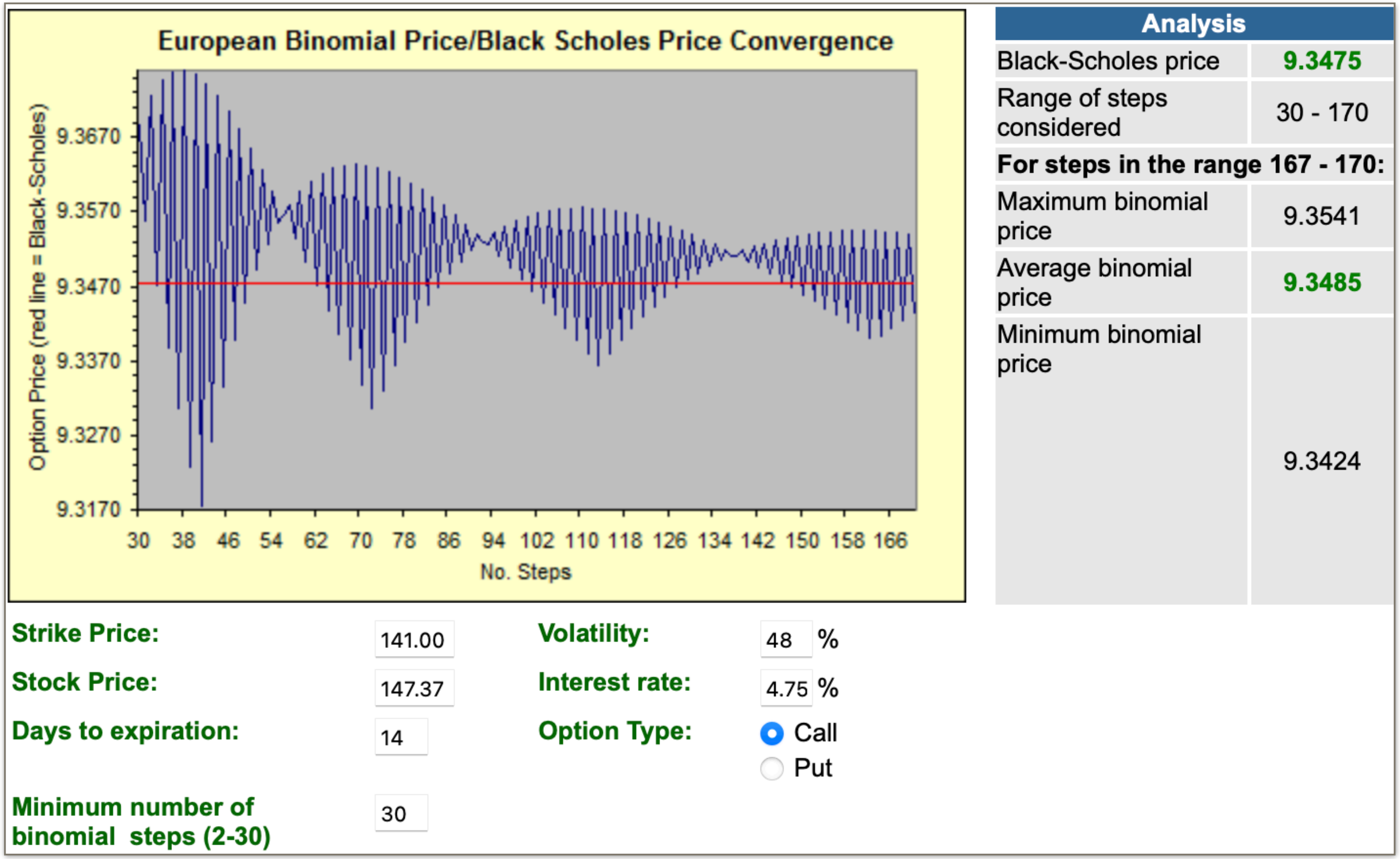


Figure 7 : The graph shows the Black Scholes price convergence calculated on option calculator.

Note : The graph has been created using the Options Calculator linked in the assignment.

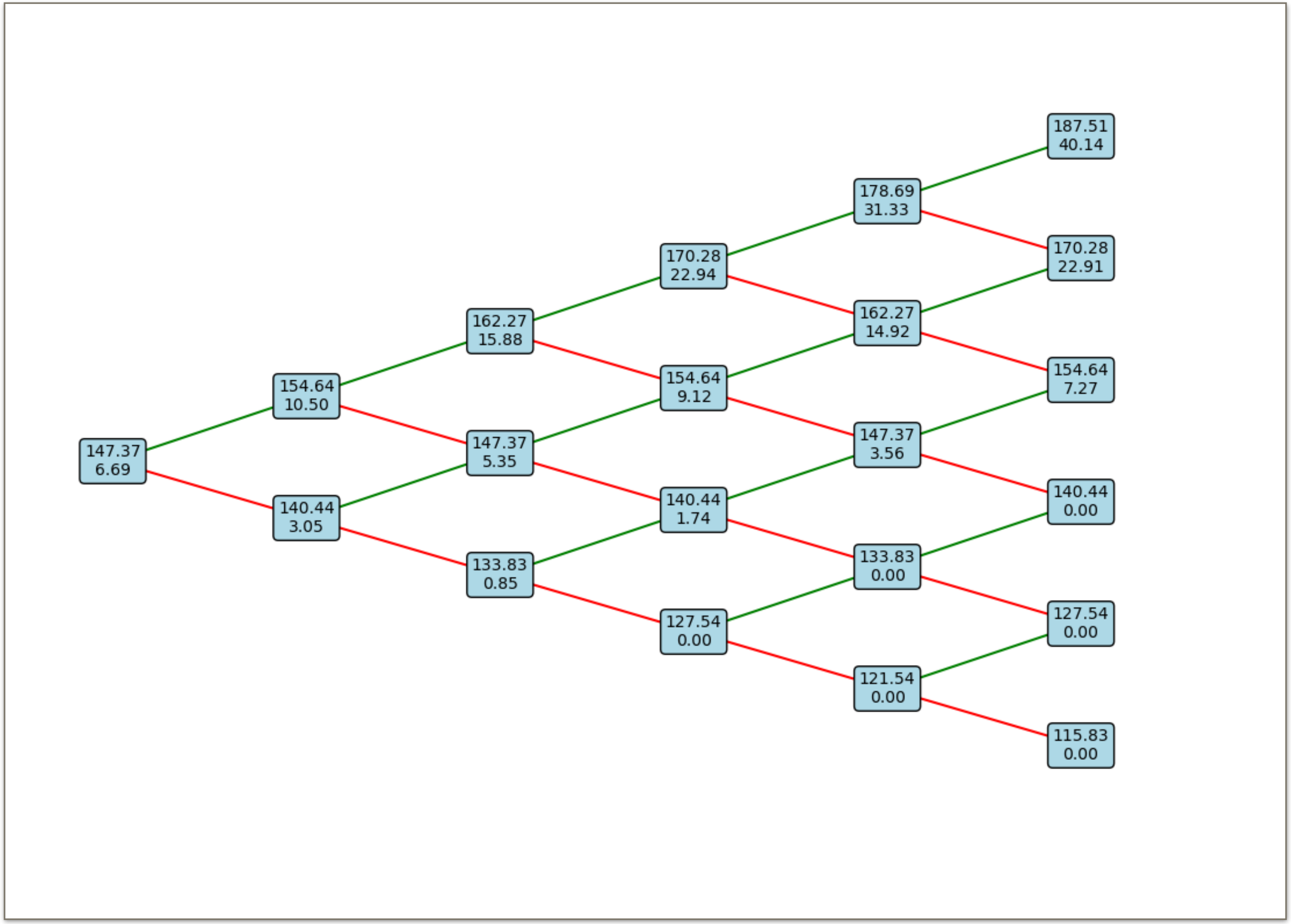


Figure 8 : The graph shows the binomial tree with 5 steps of the ATM Call option made from personal code.

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Binomial Tree for American Call."

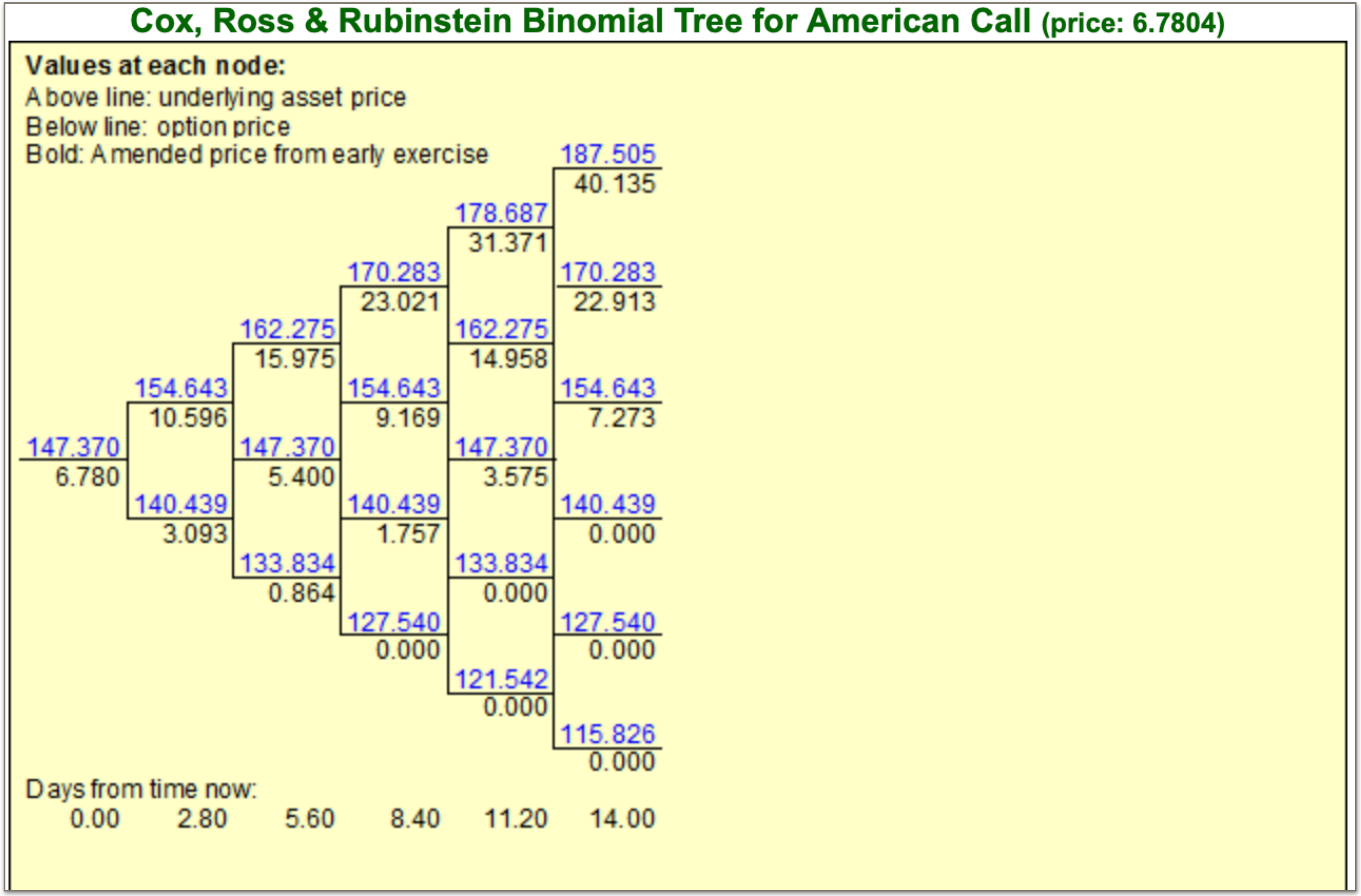


Figure 9 : The graph shows the binomial tree with 5 steps of the ATM Call option made from online code.

Note : The graph has been created using the Options Calculator linked in the assignment.

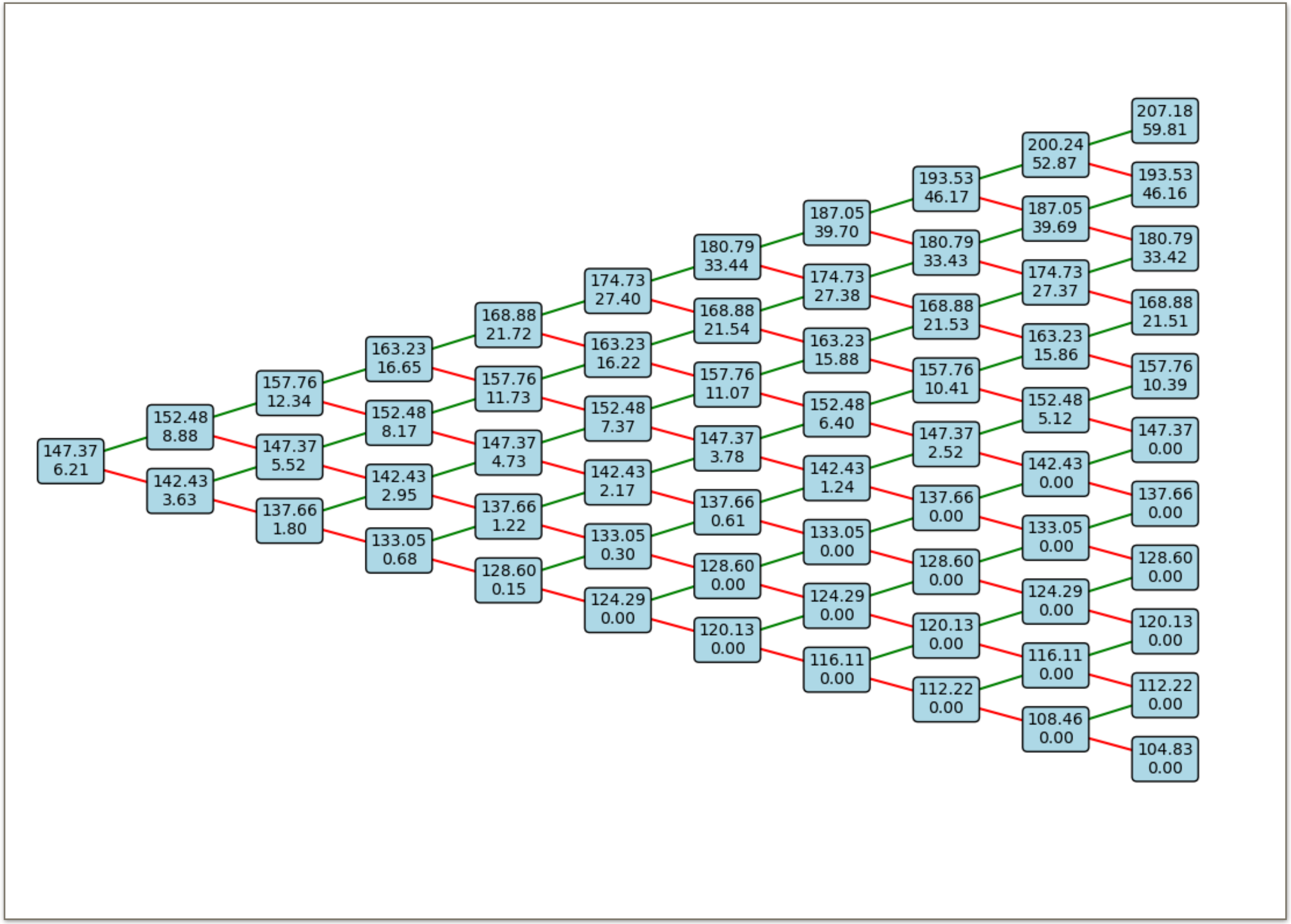


Figure 10 : The graph shows the binomial tree with 10 steps of the ATM Call option made from personal code.

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Binomial Tree for American Call."

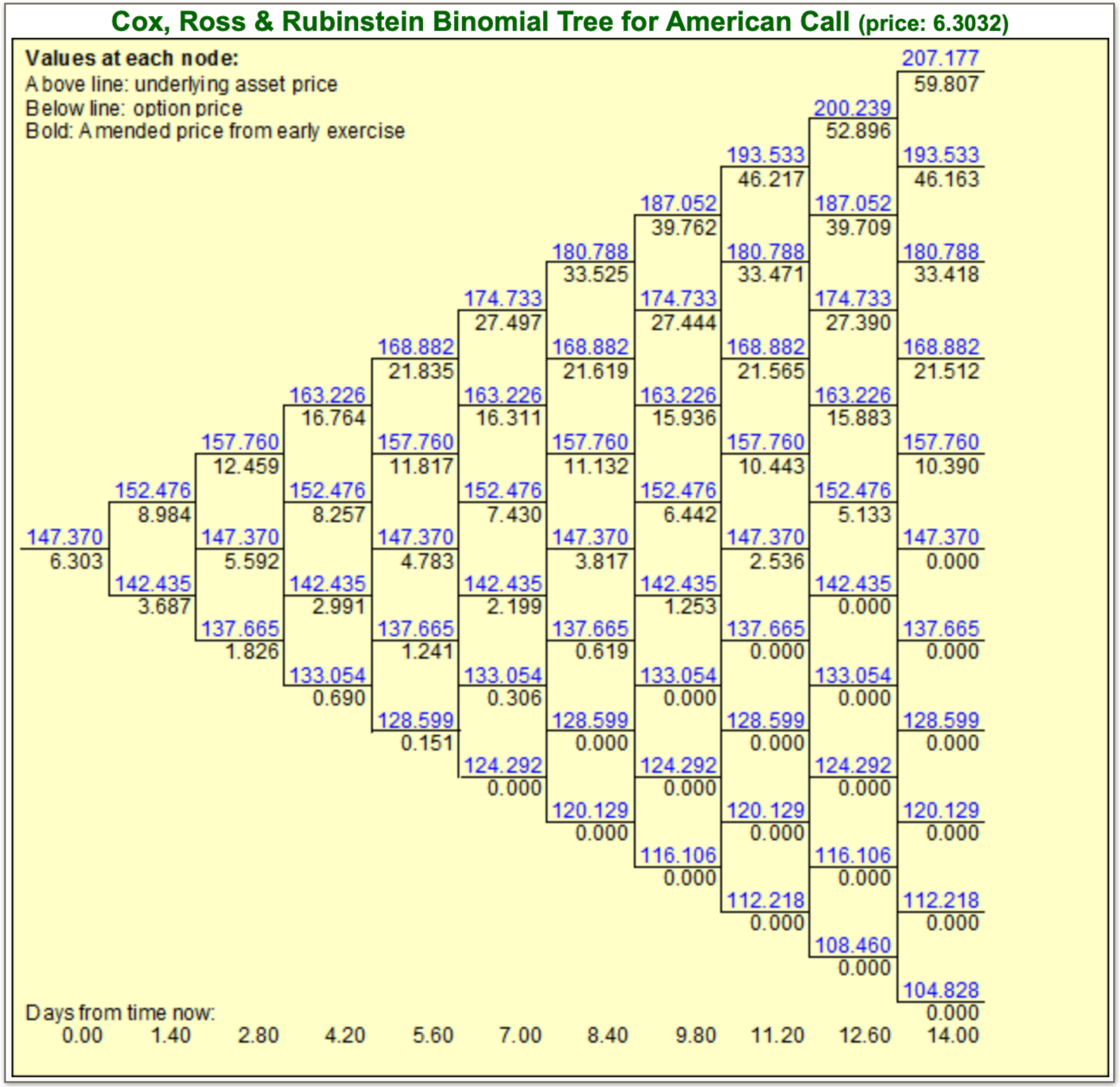


Figure 11 : The graph shows the binomial tree with 10 steps of the ATM Call option made from online code.

Note : The graph has been created using the Options Calculator linked in the assignment.

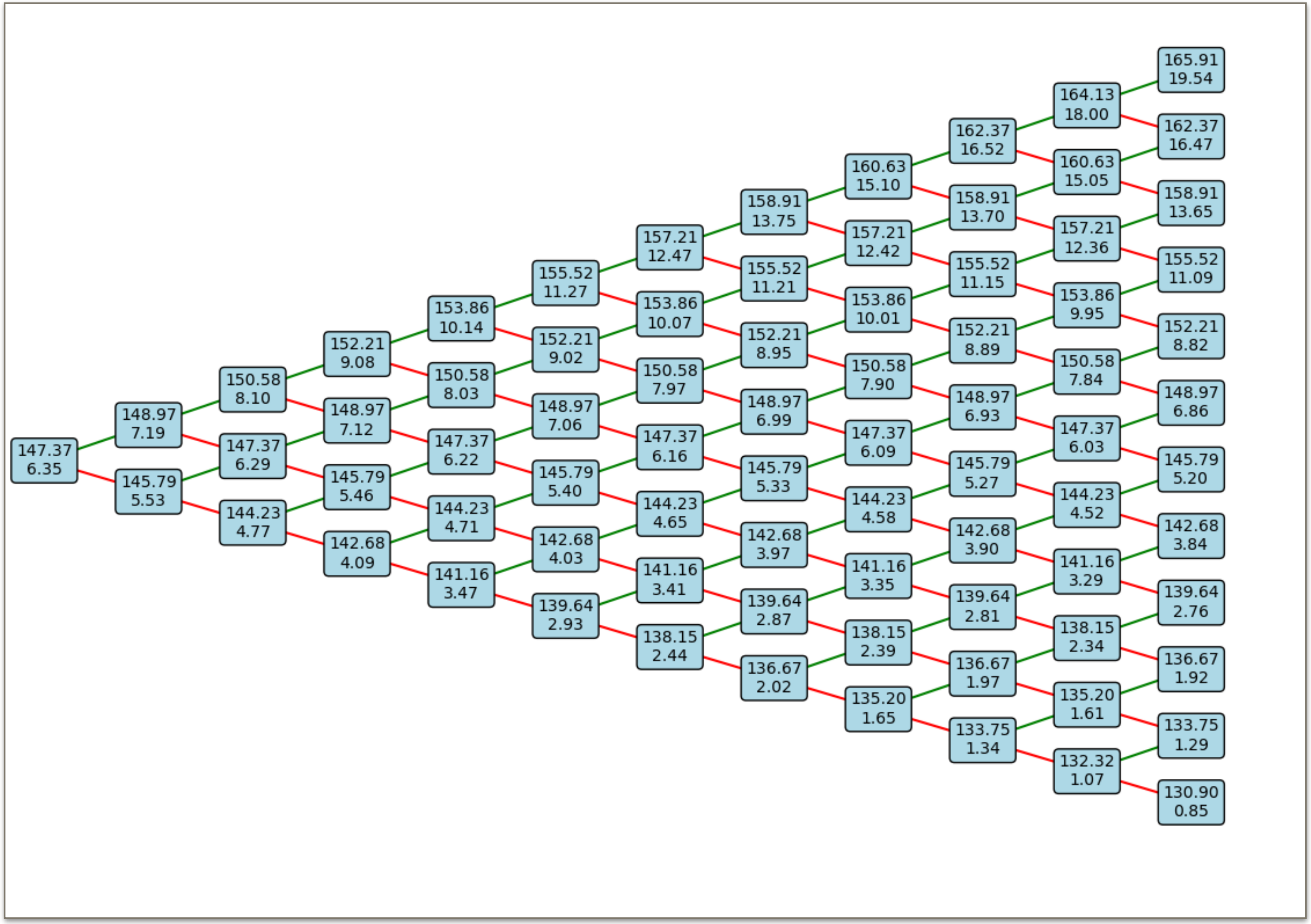


Figure 12 : TThe graph shows the binomial tree with 100 steps of the ATM Call option made from personal code.

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Binomial Tree for American Call."

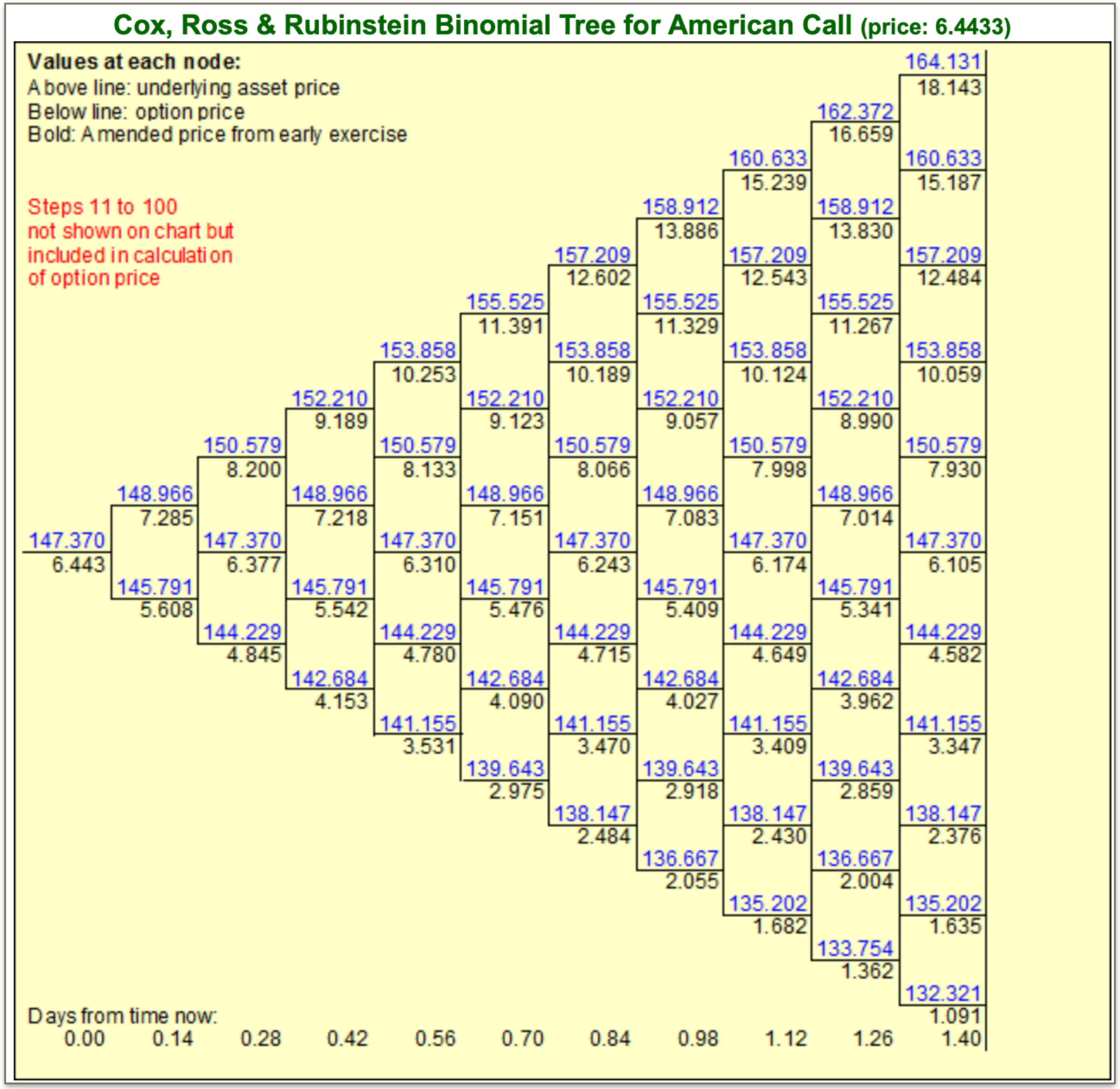
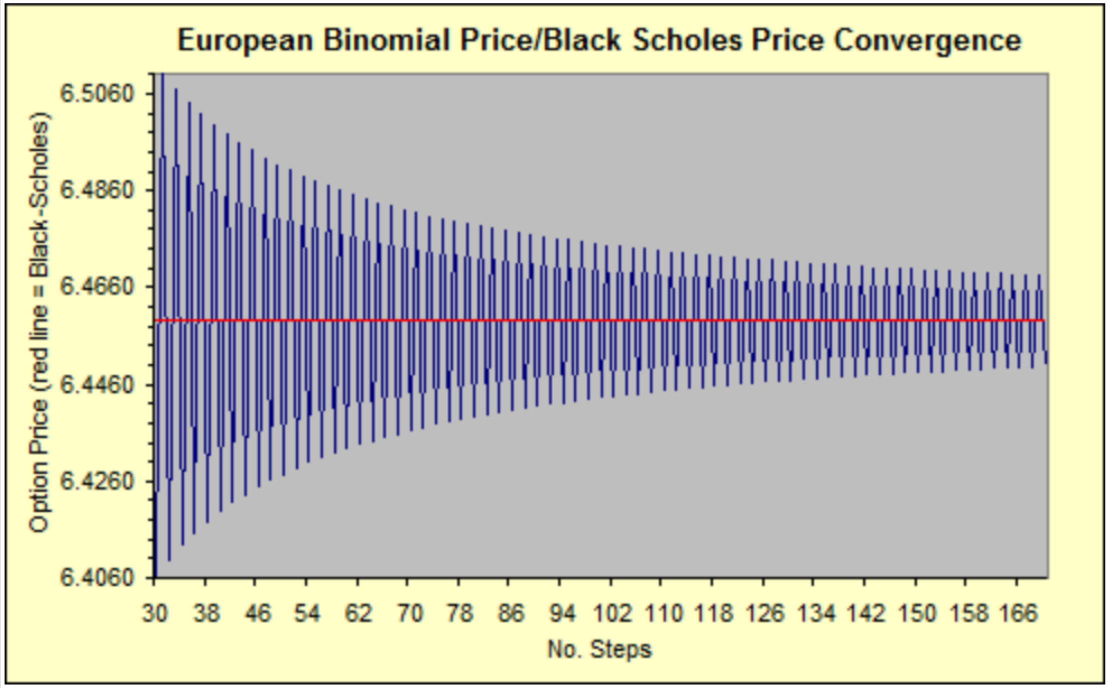


Figure 13: The graph shows the binomial tree with 100 steps of the ATM Call option made from personal code.

Note : The graph has been created using the Options Calculator linked in the assignment.



Analysis	
Black-Scholes price	6.4591
Range of steps considered	30 - 170
For steps in the range 167 - 170:	
Maximum binomial price	6.4685
Average binomial price	6.4591
Minimum binomial price	6.4497

Strike Price: 147.37

Stock Price: 147.37

Days to expiration: 14

Minimum number of binomial steps (2-30) 30

Volatility: 55 %

Interest rate: 4.75 %

Option Type: Call (selected) / Put

Figure 14 : The graph shows the Black Scholes price convergence calculated on option calculator.

Note : The graph has been created using the Options Calculator linked in the assignment.

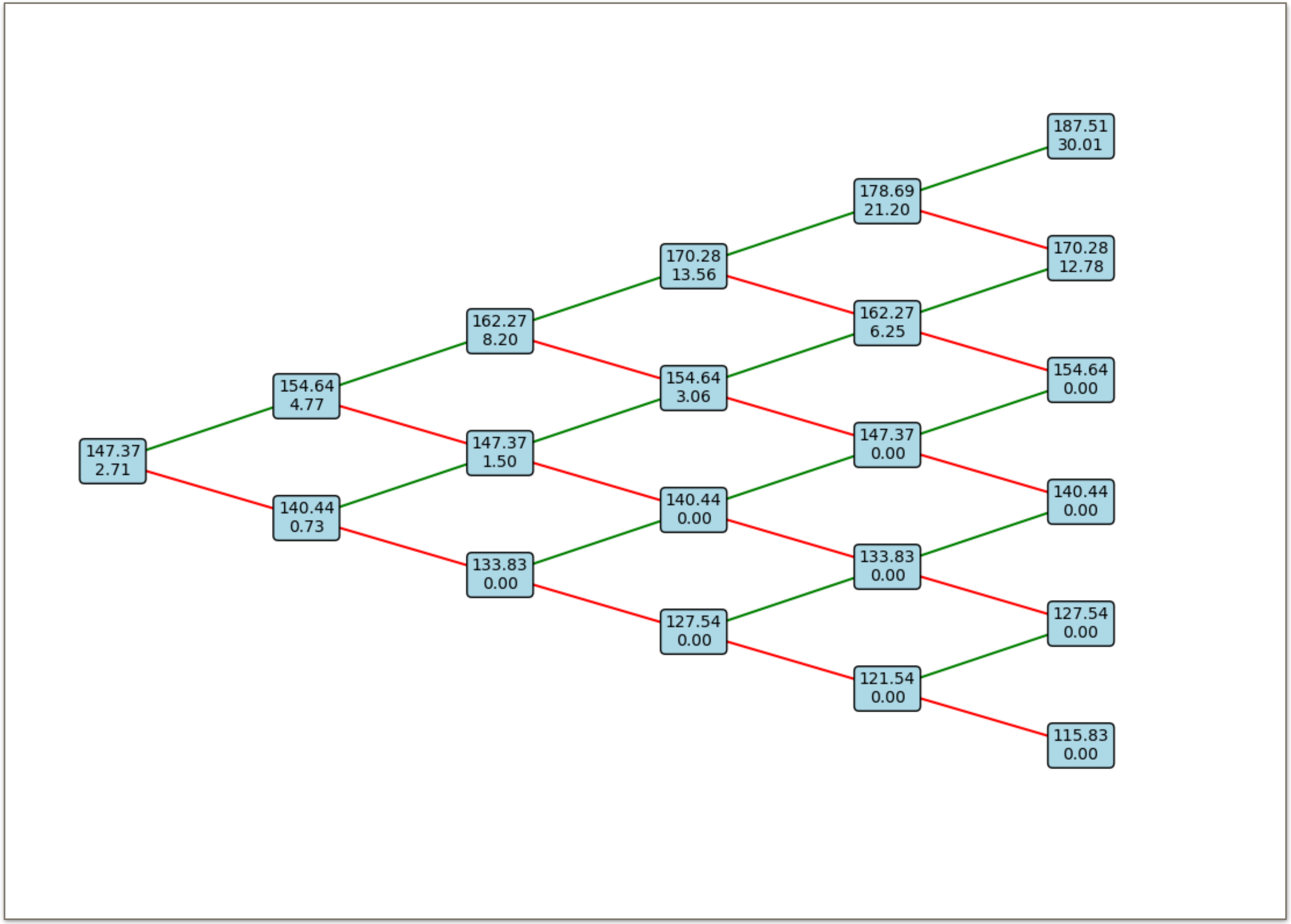


Figure 15 : The graph shows the binomial tree with 5 steps of the OTM Call option made from personal code

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Binomial Tree for American Call."

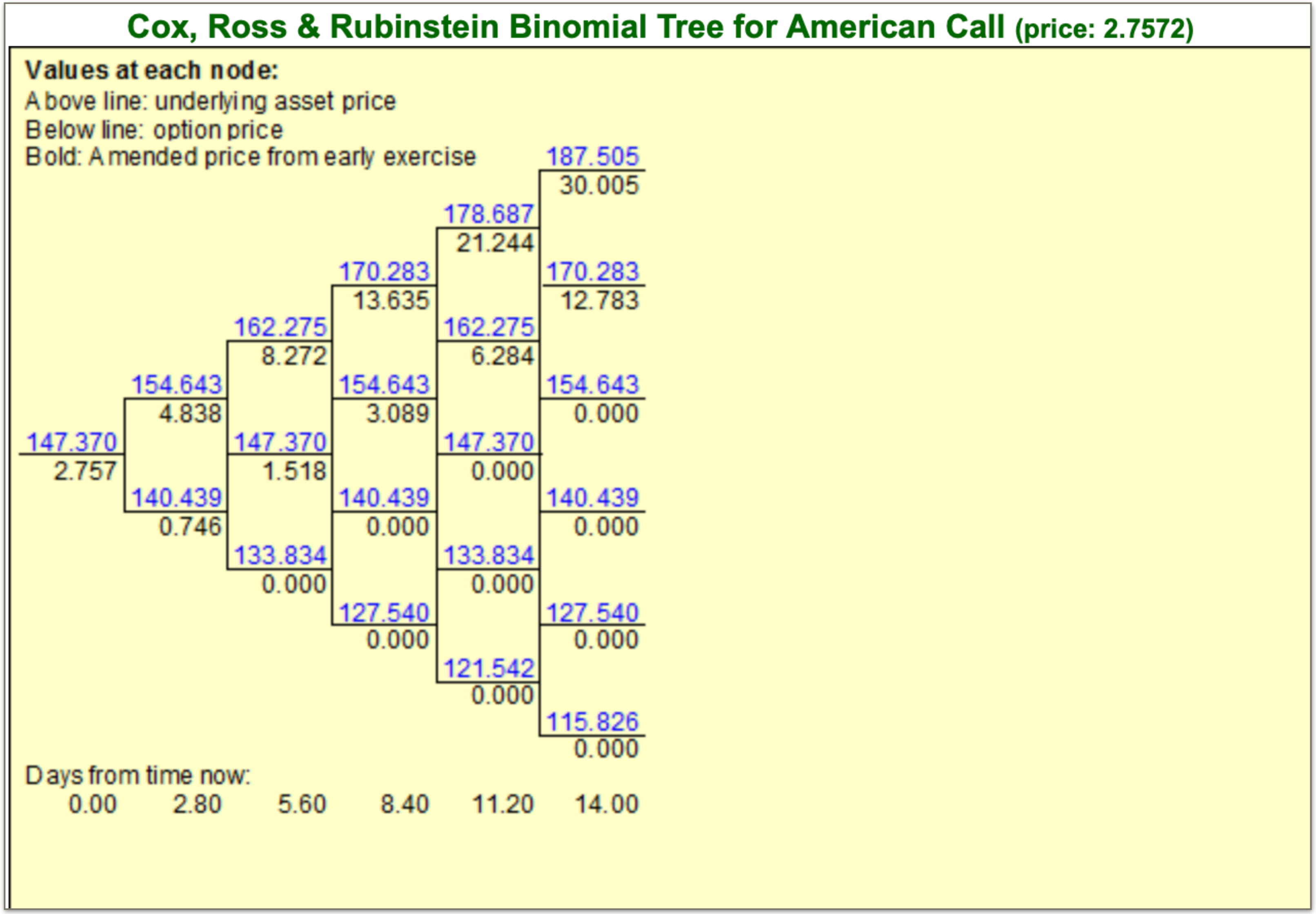


Figure 16 : The graph shows the binomial tree with 5 steps of the OTM Call option made from online code.

Note : The graph has been created using the Options Calculator linked in the assignment.

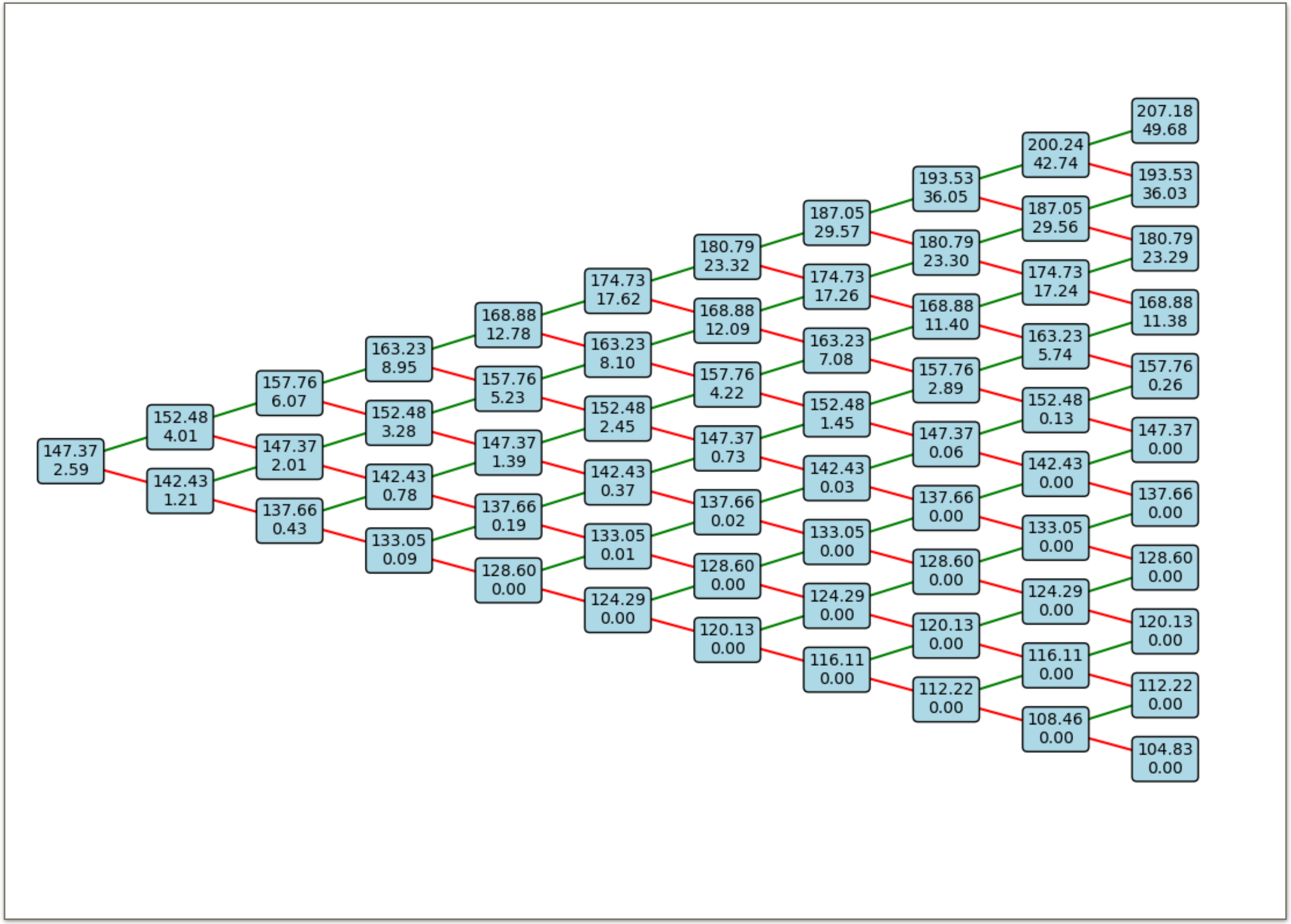


Figure 17 : TThe graph shows the binomial tree with 10 steps of the OTM Call option made from personal code.

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Binomial Tree for American Call."

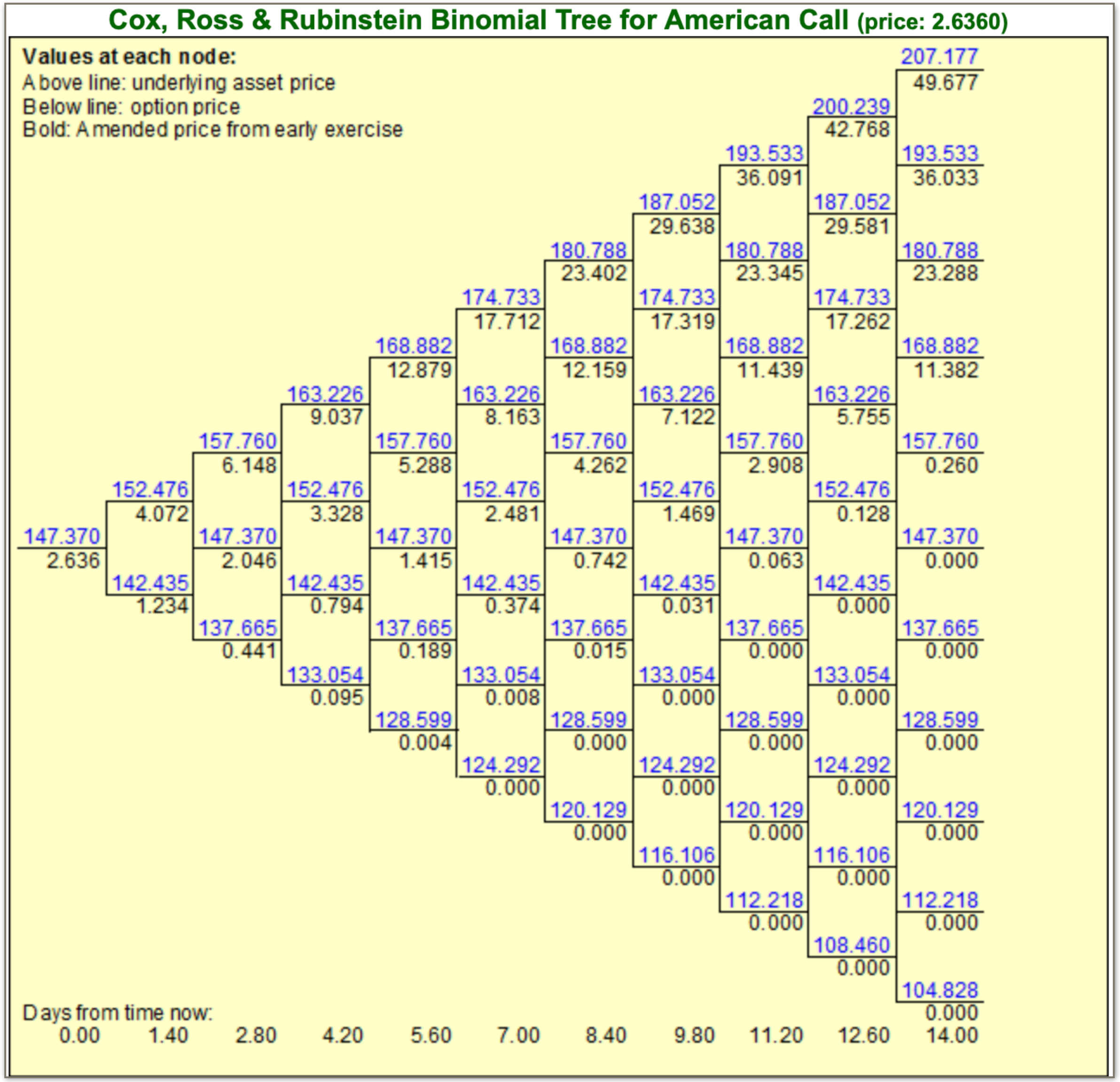


Figure 18 : The graph shows the binomial tree with 10 steps of the OTM Call option made from online code.

Note : The graph has been created using the Options Calculator linked in the assignment.

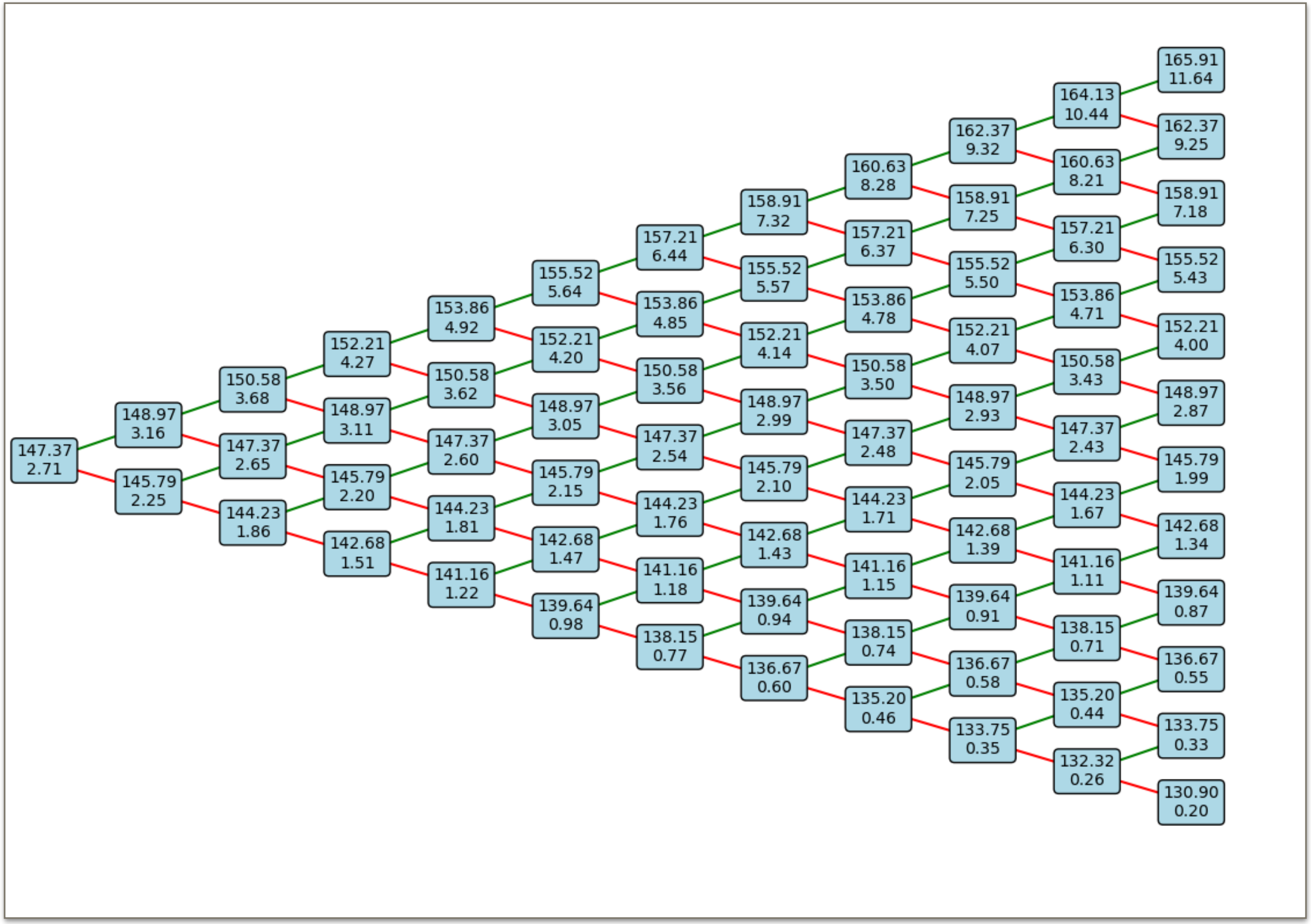


Figure 19 : The graph shows the binomial tree with 100 steps of the OTM Call option made from personal code.

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Binomial Tree for American Call."

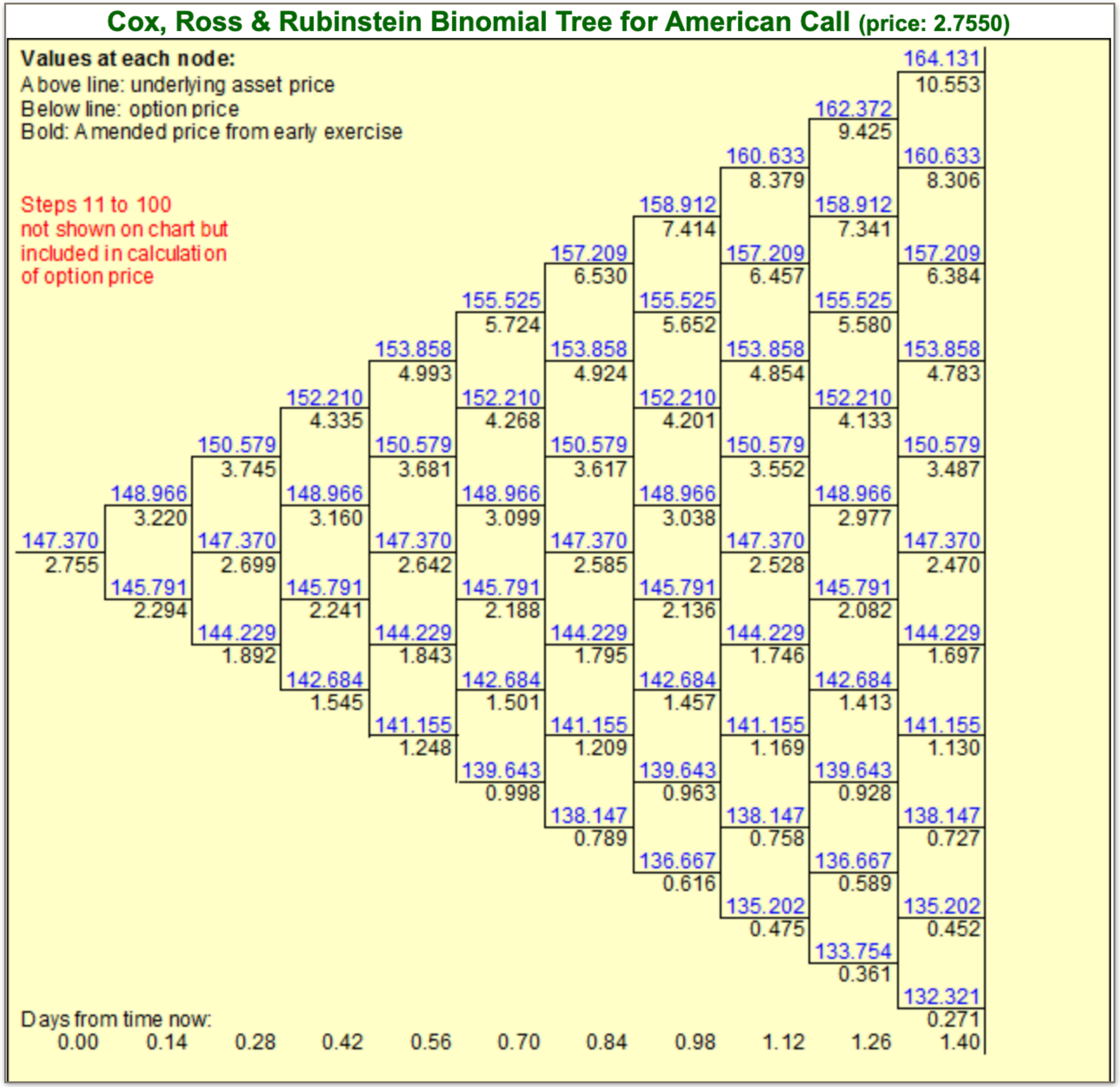
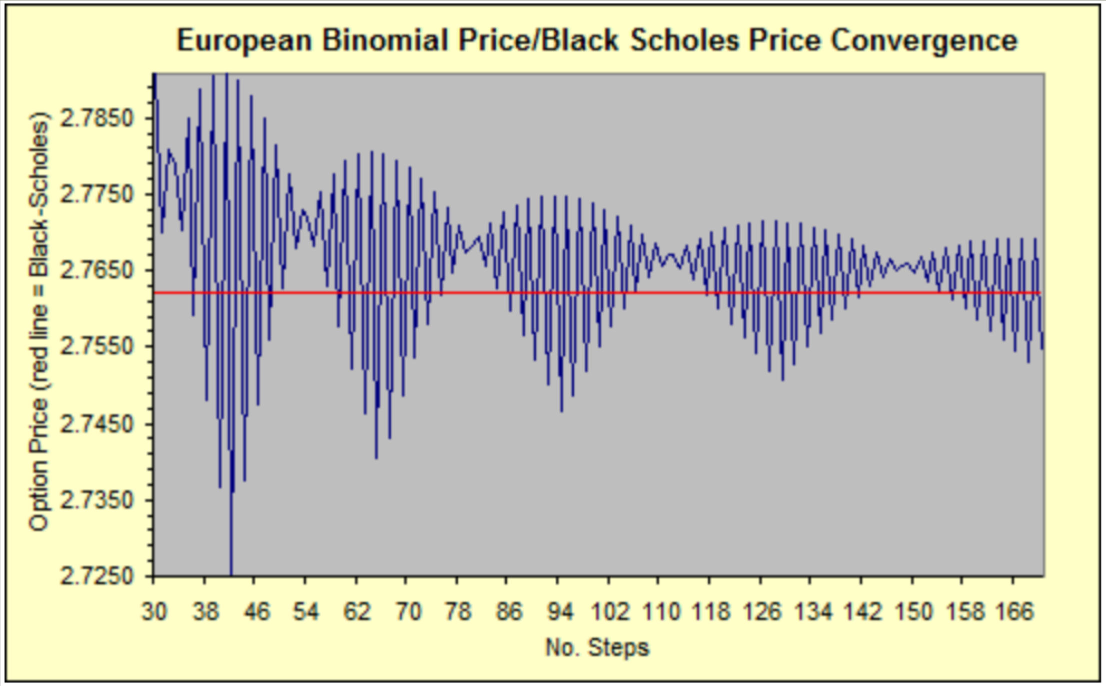


Figure 20 : The graph shows the binomial tree with 100 steps of the OTM Call option made from online code.

Note : The graph has been created using the Options Calculator linked in the assignment.



Analysis	
Black-Scholes price	**2.7622**
Range of steps considered	30 - 170
For steps in the range 167 - 170:	
Maximum binomial price	2.7692
Average binomial price	**2.7615**
Minimum binomial price	2.7530

Strike Price: 157.50

Stock Price: 147.37

Days to expiration: 14

Minimum number of binomial steps (2-30) 30

Volatility: 55 %

Interest rate: 4.75 %

Option Type: ◉ Call ○ Put

Figure 21: The graph shows the Black Scholes price convergence calculated on option calculator.

Note : The graph has been created using the Options Calculator linked in the assignment.

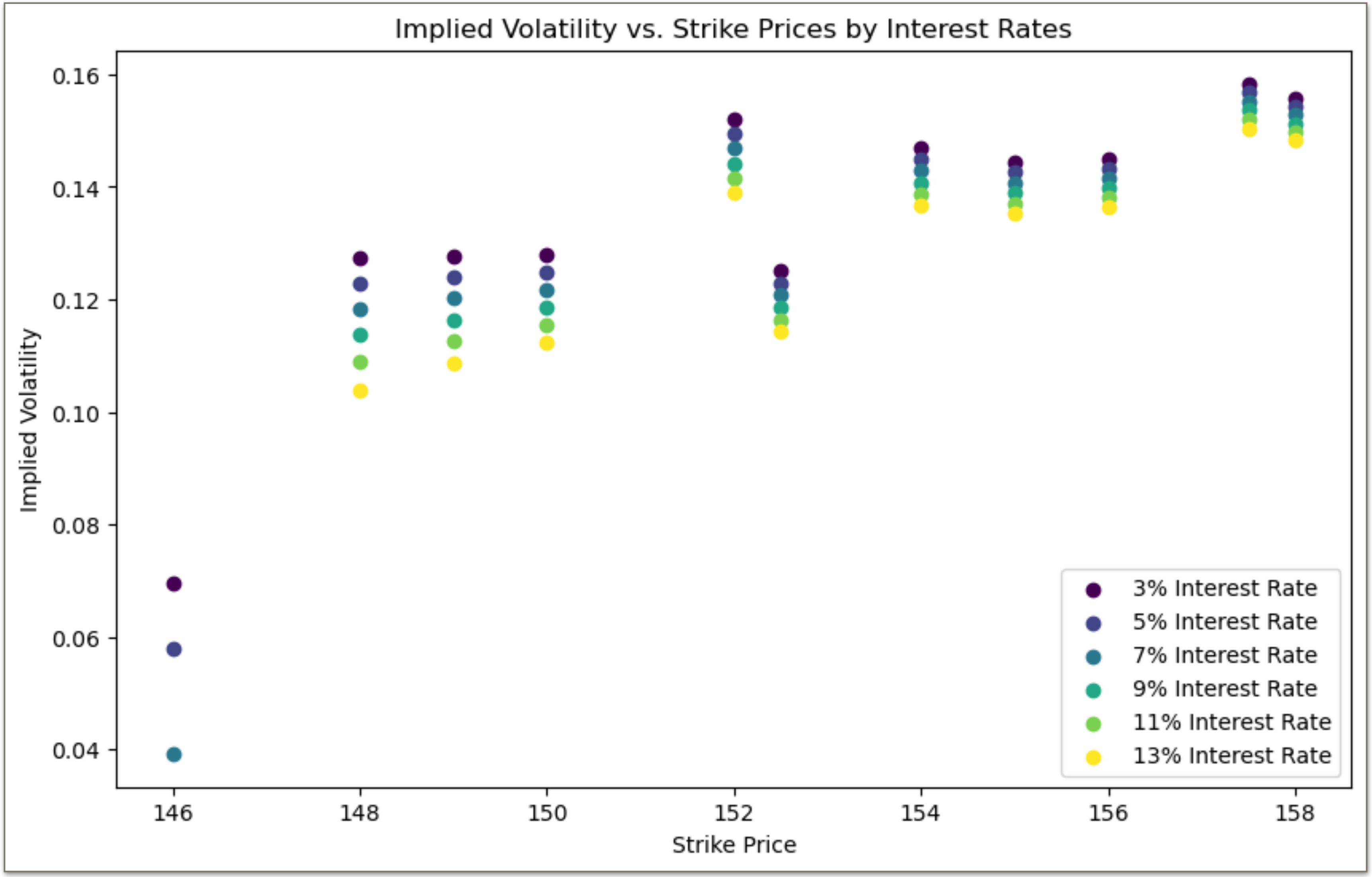


Figure 23 : Represent Implied Volatility vs strike prices by interest Rates

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Sensitivity Analysis for Volatility Calculation, Implied Volatility vs Strike Price vs Vega Code."

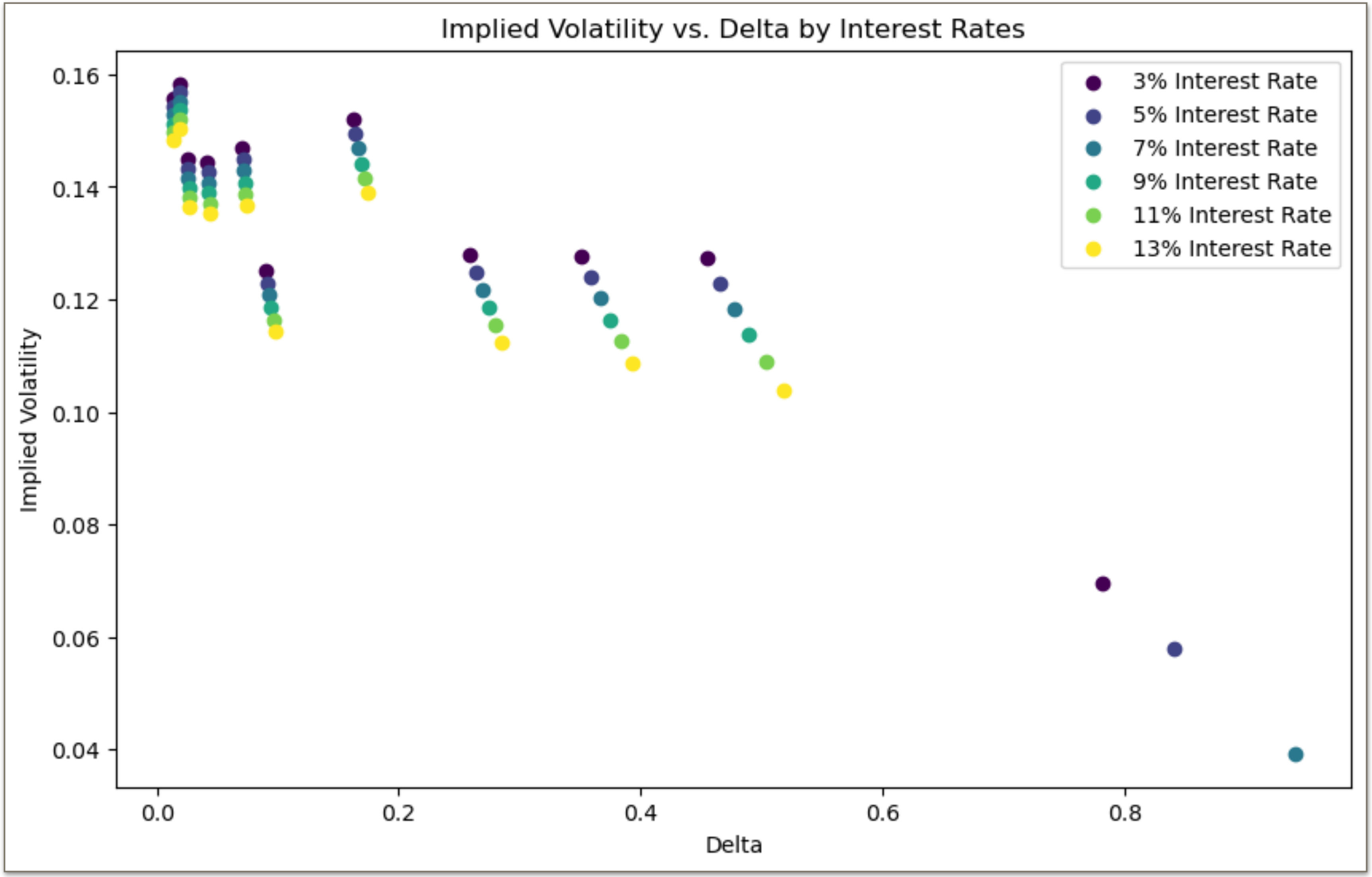


Figure 23 : Represent Implied Volatility vs delta prices by interest Rates

Note : The code can be found in Appendix A5: Code Snippets. The name of the code is "Sensitivity Analysis for Volatility Calculation, Implied Volatility vs Strike Price vs Vega Code."

Appendix.A4 Figure

Note: This appendix contains figures that required additional space for clarity and detail. These include enlarged images, charts, and visual data that support the main text but could not be accommodated within the body of the report. Each figure is labeled and referenced for ease of navigation.

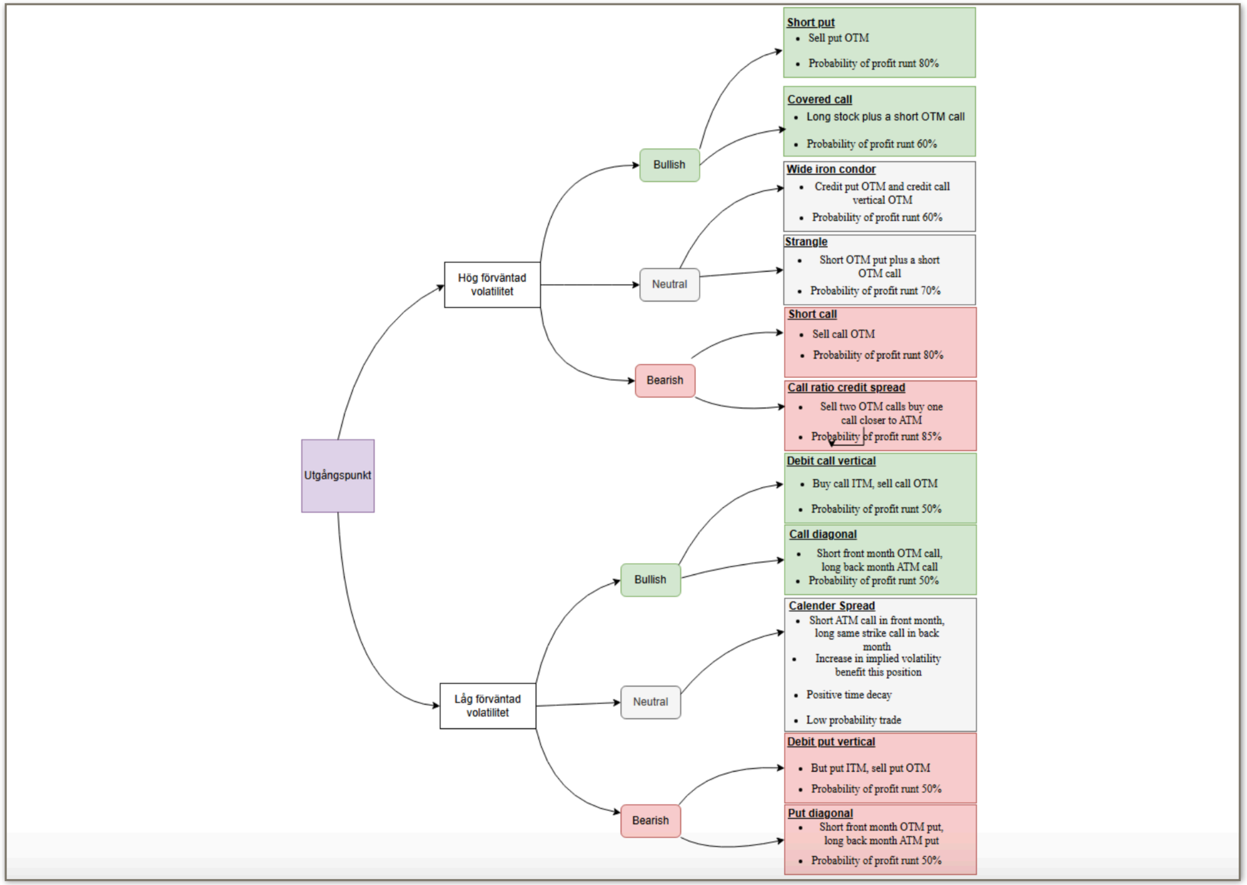


Figure 1 : Tree used to value an option

Note : Figure is a translation of Tastytrade Tree. (Tastytrade n.d)

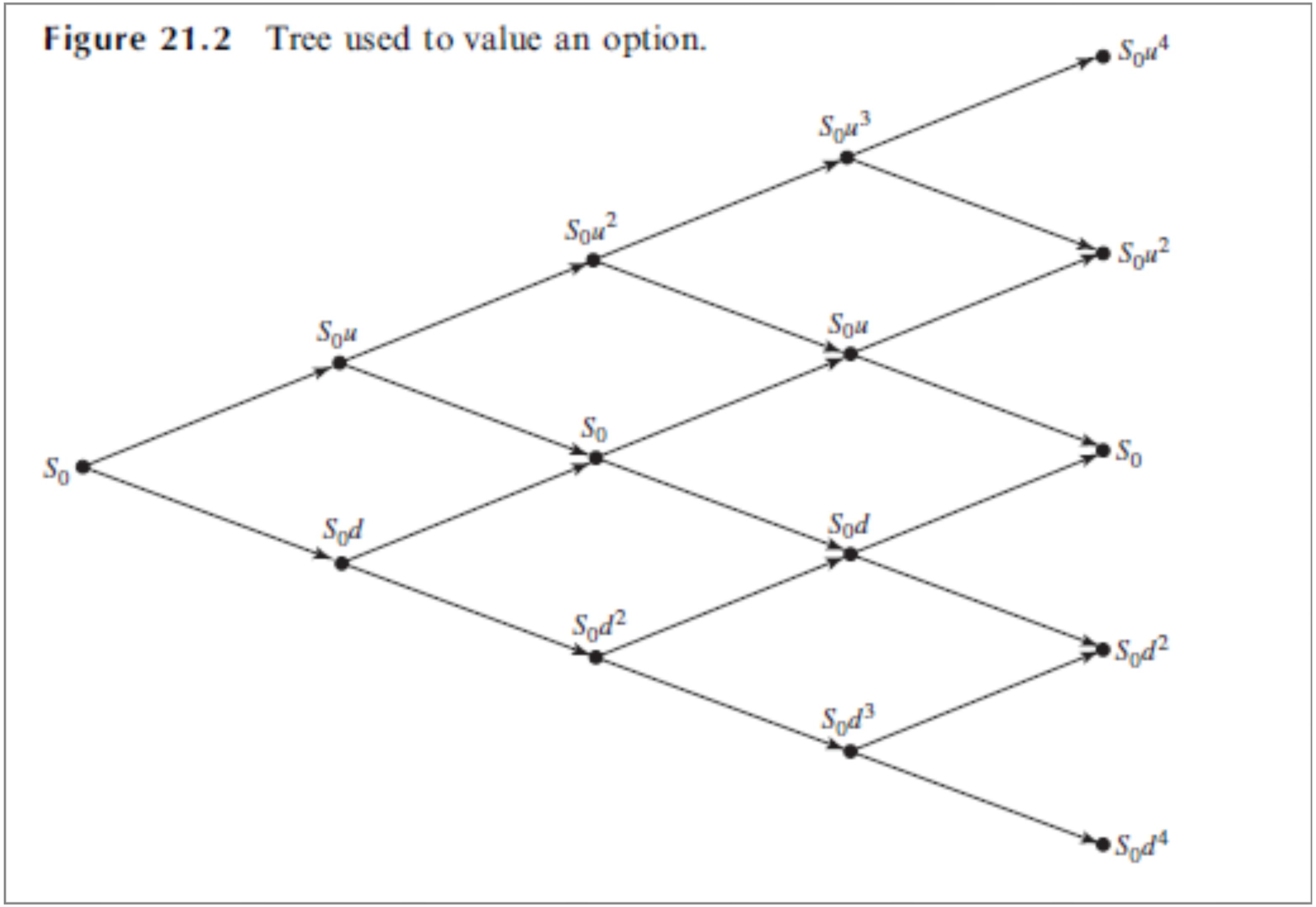


Figure 2: Tree used to value an option

Note : The following image is imported from Hull's book, Options, Futures, and Other Derivatives (2017).

Appendix.A5 Code Snippets

Note: This appendix contains code snippets used to complete various tasks in this assignment. The included code provides solutions to data processing, analysis, and visualization challenges discussed in the report. Each snippet is labeled and includes brief explanations where necessary for clarity.

Binomial Tree for American Call Code:

```python
import numpy as np
import matplotlib.pyplot as plt

Tabnine | Edit | Test | Explain | Document | Ask
def crr_binomial_tree_american_option(S, K, T, r, q, sigma, steps, option_type):
    dt = T / steps  # Time step
    u = np.exp(sigma * np.sqrt(dt))  # Up factor (swapped)
    d = 1 / u  # Down factor (swapped)
    p = (np.exp((r - q) * dt) - d) / (u - d)  # Risk-neutral probability

    # Initialize asset price tree
    asset_tree = np.zeros((steps + 1, steps + 1))
    for i in range(steps + 1):
        for j in range(i + 1):
            asset_tree[j, i] = S * (u ** j) * (d ** (i - j))

    # Initialize option price tree
    option_tree = np.zeros((steps + 1, steps + 1))
    for j in range(steps + 1):
        if option_type == 'call':
            option_tree[j, steps] = max(0, asset_tree[j, steps] - K)
        elif option_type == 'put':
            option_tree[j, steps] = max(0, K - asset_tree[j, steps])
```

```python
    # Backward induction to calculate option price
    for i in range(steps - 1, -1, -1):
        for j in range(i + 1):
            if option_type == 'call':
                early_exercise = asset_tree[j, i] - K
            elif option_type == 'put':
                early_exercise = K - asset_tree[j, i]
            hold_value = np.exp(-r * dt) * (p * option_tree[j + 1, i + 1] + (1 - p) * option_tree[j, i + 1])
            option_tree[j, i] = max(early_exercise, hold_value)

    return asset_tree, option_tree, option_tree[0, 0]  # Return the root option price

Tabnine | Edit | Test | Explain | Document | Ask
def plot_tree_christmas_style(asset_tree, option_tree, steps):
    fig, ax = plt.subplots(figsize=(14, 10))

    x_offsets = np.arange(0, steps + 1) * 2  # Horizontal spacing
    center_y = 0  # Center vertically for the root
    y_offsets = [center_y + np.arange(0, i + 1) * 15 - (i * 7) for i in range(steps + 1)]  # Adjust positions for triangular shap

    for i in range(steps + 1):
        for j in range(i + 1):
            x_pos = x_offsets[i]
            y_pos = y_offsets[i][j]
            ax.text(x_pos, y_pos, f"{asset_tree[j, i]:.2f}\n{option_tree[j, i]:.2f}",
                    ha='center', va='center', fontsize=10,
                    bbox=dict(boxstyle='round,pad=0.3', edgecolor='black', facecolor='lightblue'))
```

```python
    # Draw tree-like structure
    for i in range(steps):
        for j in range(i + 1):
            x_start = x_offsets[i]
            x_end = x_offsets[i + 1]
            y_start = y_offsets[i][j]
            y_end_up = y_offsets[i + 1][j + 1]  # Connect upward
            y_end_down = y_offsets[i + 1][j]  # Connect downward

            ax.plot([x_start, x_end], [y_start, y_end_up], color='green', linewidth=1.5)
            ax.plot([x_start, x_end], [y_start, y_end_down], color='red', linewidth=1.5)

    ax.set_xlim(-1, steps * 2 + 2)
    ax.set_ylim(-steps * 10 - 5, steps * 10 + 5)
    ax.axis('off')
    plt.show()

# Input Parameters
S = 147.37  # Underlying asset price
K = 141.00  # Strike price
T = 14 / 365  # Time to expiration in years
r = 4.75 / 100  # Risk-free interest rate
q = 0.03  # Dividend yield
sigma = 48 / 100  # Volatility
steps = 5  # Number of steps
option_type = 'call'

asset_tree, option_tree, option_price = crr_binomial_tree_american_option(S, K, T, r, q, sigma, steps, option_type)


print(f"The calculated price of the {option_type} option is: {option_price:.2f}")
```

Black Scholes Code:

```python
from scipy.stats import norm
import numpy as np

Tabnine | Edit | Test | Explain | Document | Ask
def black_scholes(S, K, T, r, q, sigma, option_type):

    d1 = (np.log(S / K) + (r - q + 0.5 * sigma ** 2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)

    if option_type == 'call':
        price = S * np.exp(-q * T) * norm.cdf(d1) - K * np.exp(-r * T) * norm.cdf(d2)
    elif option_type == 'put':
        price = K * np.exp(-r * T) * norm.cdf(-d2) - S * np.exp(-q * T) * norm.cdf(-d1)
    else:
        raise ValueError("Invalid option type. Use 'call' or 'put'.")
    return price

# Input Parameters
S = 147.37  # Underlying asset price
K = 157.5  # Strike price
T = 14 / 365  # Time to expiration in years
r = 4.75 / 100  # Risk-free interest rate
q = 0.03  # Dividend yield
sigma = 55 / 100  # Volatility
option_type = 'call'  # Choose 'call' or 'put'


bs_price = black_scholes(S, K, T, r, q, sigma, option_type)


print(f"Black-Scholes Price ({option_type} option): {bs_price:.2f}")
```

Graph plot of Binomial high N of iteration:

```python
import numpy as np
import matplotlib.pyplot as plt

Tabnine | Edit | Test | Explain | Document | Ask
def crr_binomial_tree_american_option(S, K, T, r, q, sigma, steps, option_type):
    dt = T / steps  # Time step
    u = np.exp(sigma * np.sqrt(dt))  # Up factor
    d = 1 / u  # Down factor
    p = (np.exp((r - q) * dt) - d) / (u - d)  # Risk-neutral probability

    # Initialize asset price tree
    asset_tree = np.zeros((steps + 1, steps + 1))
    for i in range(steps + 1):
        for j in range(i + 1):
            asset_tree[j, i] = S * (u ** j) * (d ** (i - j))

    # Initialize option price tree
    option_tree = np.zeros((steps + 1, steps + 1))
    for j in range(steps + 1):
        if option_type == 'call':
            option_tree[j, steps] = max(0, asset_tree[j, steps] - K)
        elif option_type == 'put':
            option_tree[j, steps] = max(0, K - asset_tree[j, steps])

    # Backward induction to calculate option price
    for i in range(steps - 1, -1, -1):
        for j in range(i + 1):
            if option_type == 'call':
                early_exercise = asset_tree[j, i] - K
            elif option_type == 'put':
                early_exercise = K - asset_tree[j, i]
            hold_value = np.exp(-r * dt) * (p * option_tree[j + 1, i + 1] + (1 - p) * option_tree[j, i + 1])
            option_tree[j, i] = max(early_exercise, hold_value)
```

```python
    return option_tree[0, 0]  # Return root option price
```

Tabnine | Edit | Test | Explain | Document | Ask

```python
def plot_option_value_vs_steps(S, K, T, r, q, sigma, option_type):
    steps_range = range(10, 501, 10)  # Steps from 10 to 500, increment by 10
    option_values = []

    for steps in steps_range:
        option_price = crr_binomial_tree_american_option(S, K, T, r, q, sigma, steps, option_type)
        option_values.append(option_price)

    # Plot the graph
    plt.figure(figsize=(10, 6))
    plt.scatter(steps_range, option_values, color='black')  # Use scatter plot with black dots
    plt.xlabel('Number of Steps')
    plt.ylabel('Option Value')
    plt.ylim(2,2.99)  # Set y-axis range from 12 to 13
    plt.yticks(np.arange(2,2.99, 0.03))  # Y-axis ticks with 0.03 changes
    plt.legend()
    plt.show()

# Input Parameters
S = 147.37  # Underlying asset price
K = 157.5  # Strike price
T = 14 / 365  # Time to expiration in years
r = 4.75 / 100  # Risk-free interest rate
q = 0.03  # Dividend yield
sigma = 55 / 100  # Volatility
option_type = 'call'  # Choose 'call' or 'put'

# Plot the graph
plot_option_value_vs_steps(S, K, T, r, q, sigma, option_type)
```

Sensitivity analysis for Volatility calculation, Implied Volatility vs Strike price vs Vega code:

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from scipy.stats import norm
from scipy.optimize import brentq

# Black-Scholes formula
Tabnine | Edit | Test | Explain | Document | Ask
def black_scholes_call_price(S, K, T, r, sigma):
    d1 = (np.log(S / K) + (r + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    return S * norm.cdf(d1) - K * np.exp(-r * T) * norm.cdf(d2)

# Vega calculation (partial derivative of price with respect to volatility)
Tabnine | Edit | Test | Explain | Document | Ask
def vega(S, K, T, r, sigma):
    d1 = (np.log(S / K) + (r + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
    return S * np.sqrt(T) * norm.pdf(d1)

# Implied volatility using Newton-Raphson
Tabnine | Edit | Test | Explain | Document | Ask
def implied_volatility(market_price, S, K, T, r, tol=1e-5, max_iter=100):
    sigma = 0.5  # Initial guess
    for i in range(max_iter):
        price = black_scholes_call_price(S, K, T, r, sigma)
        v = vega(S, K, T, r, sigma)
        price_diff = market_price - price

        if abs(price_diff) < tol:
            return sigma

        sigma += price_diff / v
```

```python
    return np.nan  # Return NaN if no solution is found

# Delta calculation
Tabnine | Edit | Test | Explain | Document | Ask
def calculate_delta(S, K, T, r, sigma):
    d1 = (np.log(S / K) + (r + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
    return norm.cdf(d1)

# Provided data
data = pd.DataFrame({
    'Strike': [138, 139, 140, 141, 142, 143, 144, 145, 146, 148, 149, 150, 152, 152.5, 154, 155, 156, 157.5, 158],
    'MarketPrice': [8.6, 7.8, 6.6, 5.7, 4.76, 3.86, 3.1, 2.37, 1.79, 1.25, 0.86, 0.57, 0.37, 0.15, 0.13, 0.07, 0.04, 0.03, 0.02]
})

expiry = 14 / 365  # Time to expiry in years
asset_price = 147.37  # Current asset price

# Interest rates from 3% to 13% with step of 2%
interest_rates = np.arange(0.03, 0.14, 0.02)

# Colors for plotting
dot_colors = plt.cm.viridis(np.linspace(0, 1, len(interest_rates)))

# Plotting Implied Volatility vs. Strike Prices
plt.figure(figsize=(10, 6))
for rate, color in zip(interest_rates, dot_colors):
    temp_data = data.copy()
    temp_data['ImpliedVolatility'] = temp_data.apply(
        lambda row: implied_volatility(row['MarketPrice'], asset_price, row['Strike'], expiry, rate), axis=1
    )
```

```python
    plt.scatter(temp_data['Strike'], temp_data['ImpliedVolatility'], color=color, label=f'{rate*100:.0f}% Interest Rate')

plt.title('Implied Volatility vs. Strike Prices by Interest Rates')
plt.xlabel('Strike Price')
plt.ylabel('Implied Volatility')
plt.legend()
plt.show()

# Plotting Implied Volatility vs. Delta
plt.figure(figsize=(10, 6))
for rate, color in zip(interest_rates, dot_colors):
    temp_data = data.copy()
    temp_data['ImpliedVolatility'] = temp_data.apply(
        lambda row: implied_volatility(row['MarketPrice'], asset_price, row['Strike'], expiry, rate), axis=1
    )
    temp_data['Delta'] = temp_data.apply(
        lambda row: calculate_delta(asset_price, row['Strike'], expiry, rate, row['ImpliedVolatility']), axis=1
    )

    plt.scatter(temp_data['Delta'], temp_data['ImpliedVolatility'], color=color, label=f'{rate*100:.0f}% Interest Rate')

plt.title('Implied Volatility vs. Delta by Interest Rates')
plt.xlabel('Delta')
plt.ylabel('Implied Volatility')
plt.legend()
plt.show()
```

Real option Code:

```python
import numpy as np
import matplotlib.pyplot as plt
```

Tabnine | Edit | Test | Explain | Document | Ask

```python
def crr_binomial_tree_real_option(S, T, r, sigma, steps, abandonment_rate):
    dt = T / steps  # Time step
    u = np.exp(sigma * np.sqrt(dt))  # Up factor
    d = 1 / u  # Down factor
    p = (np.exp(r * dt) - d) / (u - d)  # Risk-neutral probability

    # Initialize asset price tree
    asset_tree = np.zeros((steps + 1, steps + 1))
    for i in range(steps + 1):
        for j in range(i + 1):
            asset_tree[j, i] = S * (u ** j) * (d ** (i - j))

    # Initialize option price tree
    option_tree = np.zeros((steps + 1, steps + 1))

    # Backward induction with abandonment option
    for i in range(steps - 1, -1, -1):
        for j in range(i + 1):
            continue_value = np.exp(-r * dt) * (p * option_tree[j + 1, i + 1] + (1 - p) * option_tree[j, i + 1])
            abandon_value = abandonment_rate * asset_tree[j, i]
            option_tree[j, i] = max(continue_value, abandon_value)

    return asset_tree, option_tree, option_tree[0, 0]  # Return the root option price
```

Tabnine | Edit | Test | Explain | Document | Ask

```python
def plot_tree(asset_tree, option_tree, steps):
    fig, ax = plt.subplots(figsize=(14, 10))
```

```python
    x_offsets = np.arange(0, steps + 1) * 2  # Horizontal spacing
    y_offsets = [np.arange(0, i + 1) * 15 - (i * 7) for i in range(steps + 1)]  # Adjust positions for triangular shape

    for i in range(steps + 1):
        for j in range(i + 1):
            x_pos = x_offsets[i]
            y_pos = y_offsets[i][j]
            ax.text(x_pos, y_pos, f"{asset_tree[j, i]:.2f}\n{option_tree[j, i]:.2f}",
                    ha='center', va='center', fontsize=10,
                    bbox=dict(boxstyle='round,pad=0.3', edgecolor='black', facecolor='lightblue'))

    for i in range(steps):
        for j in range(i + 1):
            x_start = x_offsets[i]
            x_end = x_offsets[i + 1]
            y_start = y_offsets[i][j]
            y_end_up = y_offsets[i + 1][j + 1]  # Connect upward
            y_end_down = y_offsets[i + 1][j]  # Connect downward

            ax.plot([x_start, x_end], [y_start, y_end_up], color='green', linewidth=1.5)
            ax.plot([x_start, x_end], [y_start, y_end_down], color='red', linewidth=1.5)

    ax.set_xlim(-1, steps * 2 + 2)
    ax.set_ylim(-steps * 10 - 5, steps * 10 + 5)
    ax.axis('off')
    plt.show()
```

```python
# Input Parameters
S = 36069000  # Present value of the drug patent in Million SEK
T = 10  # Time horizon in years
r = 5 / 100  # Risk-free interest rateä-sigma = 30 / 100  # Volatility
steps = 10  # Number of steps in the binomial tree
abandonment_rate = 0.6  # Sell the patent for 60% of its value

# Calculate the Real Option Value
asset_tree, option_tree, real_option_value = crr_binomial_tree_real_option(S, T, r, sigma, steps, abandonment_rate)

# Print the Real Option Value
print(f"The value of the real option is: {real_option_value:.2f} Million SEK")

# Plot the Binomial Tree
plot_tree(asset_tree, option_tree, steps)
```

Value of Growth and Reduction of Real option Code:

```
import numpy as np
import matplotlib.pyplot as plt

Tabnine | Edit | Test | Explain | Document | Ask
def crr_binomial_tree_real_option(S, T, r, sigma, steps, growth_rate=None, growth_cost=None, reduction_rate=None, cost_saving=None):
    dt = T / steps  # Time step
    u = np.exp(sigma * np.sqrt(dt))  # Up factor
    d = 1 / u  # Down factor
    p = (np.exp(r * dt) - d) / (u - d)  # Risk-neutral probability

    # Initialize asset price tree
    asset_tree = np.zeros((steps + 1, steps + 1))
    for i in range(steps + 1):
        for j in range(i + 1):
            asset_tree[j, i] = S * (u ** j) * (d ** (i - j))

    # Initialize option price tree
    option_tree = np.zeros((steps + 1, steps + 1))

    # Backward induction with growth and reduction options
    for i in range(steps - 1, -1, -1):
        for j in range(i + 1):
            continue_value = np.exp(-r * dt) * (p * option_tree[j + 1, i + 1] + (1 - p) * option_tree[j, i + 1])

            growth_value = -np.inf
            if growth_rate and growth_cost:
                growth_value = (1 + growth_rate) * asset_tree[j, i] - growth_cost * S

            reduction_value = -np.inf
            if reduction_rate and cost_saving:
                reduction_value = asset_tree[j, i] - cost_saving * (reduction_rate * asset_tree[j, i])
```

```python
            option_tree[j, i] = max(continue_value, growth_value, reduction_value)

    return asset_tree, option_tree, option_tree[0, 0]  # Return the root option price
```

Tabnine | Edit | Test | Explain | Document | Ask

```python
def plot_tree(asset_tree, option_tree, steps):
    fig, ax = plt.subplots(figsize=(14, 10))

    x_offsets = np.arange(0, steps + 1) * 2  # Horizontal spacing
    y_offsets = [np.arange(0, i + 1) * 15 - (i * 7) for i in range(steps + 1)]  # Adjust positions for triangular shape

    for i in range(steps + 1):
        for j in range(i + 1):
            x_pos = x_offsets[i]
            y_pos = y_offsets[i][j]
            ax.text(x_pos, y_pos, f"{asset_tree[j, i]:.2f}\n{option_tree[j, i]:.2f}",
                    ha='center', va='center', fontsize=10,
                    bbox=dict(boxstyle='round,pad=0.3', edgecolor='black', facecolor='lightblue'))

    for i in range(steps):
        for j in range(i + 1):
            x_start = x_offsets[i]
            x_end = x_offsets[i + 1]
            y_start = y_offsets[i][j]
            y_end_up = y_offsets[i + 1][j + 1]  # Connect upward
            y_end_down = y_offsets[i + 1][j]  # Connect downward

            ax.plot([x_start, x_end], [y_start, y_end_up], color='green', linewidth=1.5)
            ax.plot([x_start, x_end], [y_start, y_end_down], color='red', linewidth=1.5)
```

```python
    ax.set_xlim(-1, steps * 2 + 2)
    ax.set_ylim(-steps * 10 - 5, steps * 10 + 5)
    ax.axis('off')
    plt.show()

# Input Parameters
S = 36069000  # Present value of the drug patent in Million SEK
T = 10  # Time horizon in years
r = 5 / 100  # Risk-free interest rate
sigma = 30 / 100  # Volatility
steps = 10  # Number of steps in the binomial tree

# Case 1: Option to Buy a Competitor
growth_rate = 0.25  # Increase in profit (25%)
growth_cost = 0.2  # Cost to acquire competitor (20% of present value)
asset_tree, option_tree, growth_option_value = crr_binomial_tree_real_option(S, T, r, sigma, steps, growth_rate=growth_rate, growth_
print(f"The value of the growth option is: {growth_option_value:.2f} Million SEK")
plot_tree(asset_tree, option_tree, steps)

# Case 2: Option to Reduce Operations
reduction_rate = 0.2  # Reduction in operations (20%)
cost_saving = 0.3  # Cost saving (30% of reduced operations)
asset_tree, option_tree, reduction_option_value = crr_binomial_tree_real_option(S, T, r, sigma, steps, reduction_rate=reduction_rate
print(f"The value of the reduction option is: {reduction_option_value:.2f} Million SEK")
plot_tree(asset_tree, option_tree, steps)
```

PART B: PORTFOLIO THEORY

YASSEN TITROUQ & ISAK KRONEKVIST HJELM

MID SWEDEN UNIVERSITY

1. Markowitz Portfolio Theory

Portfolio optimization, especially modern portfolio theory, is a key concept in financial analysis aimed at selecting assets that provide the highest possible return for the lowest risk. The goal is to construct an efficient portfolio, balancing risk and return using mathematical models and statistical tools. The theory operates on the premise that diversification, when executed thoughtfully, can optimize a portfolio's performance. This involves strategically allocating investments across assets with varying levels of risk and return to achieve an "efficient portfolio." An efficient portfolio either maximizes return for a given level of risk or minimizes risk for a given level of return. This trade off between risk and return is the cornerstone of Markowitz's model. The theory starts with a set of assumptions. It assumes that returns on assets are normally distributed over a single period, enabling predictions about their behavior. Furthermore, the expected return of an asset, often called its drift, and its volatility (measured as standard deviation) become the primary metrics for decision making. The correlations between asset returns also play a crucial role, as diversification benefits arise when assets with low or negative correlations are combined. The mathematical framework of Markowitz's model revolves around several key equations. For a portfolio consisting of N assets, the portfolio value is expressed as the weighted sum of individual asset values. The weights, W_i, represent the proportion of the portfolio invested in each asset and must sum to one. The portfolio's expected return, u_{Π}, is calculated as the weighted average of the individual expected returns, emphasizing the proportional contribution of each asset. Similarly, the portfolio's risk, represented as the standard deviation σ_{Π} is derived from the covariance matrix, which measures dependencies between asset returns. The variance of the portfolio, the square of the standard deviation, is given by:

$$\sigma_{\Pi}^2 = \sum_{i=1}^{N}\sum_{j=1}^{N} W_i W_j \rho_{ij} \sigma_i \sigma_j,$$

where p_{ij} represents the correlation coefficient between assets i and j, and σ_i and σ_j are their respective standard deviations. This equation illustrates the importance of correlation: when assets are uncorrelated or negatively correlated, the portfolio's risk can be reduced even as the number of assets increases. A critical concept within the Markowitz framework is the efficient frontier, which is a graphical representation of the set of optimal portfolios. These portfolios offer the highest return for a given risk or the lowest risk for a given return. The efficient frontier is derived by varying the weights of assets within the portfolio while adding to constraints, such as ensuring the weights sum to one. The result is a curve that illustrates the trade offs between risk and return. Portfolios below the frontier are suboptimal, while those on the frontier represent the best possible outcomes for investors with specific risk tolerances. Markowitz's theory also integrates the notion of individual risk preferences. While the efficient frontier objectively identifies optimal portfolios, the selection of a specific portfolio depends on an investor's subjective trade off between risk and reward. This decision can be further refined using utility functions or indifference curves, which represent an investor's preferences for various risk return combinations. The application of Markowitz's theory extends to scenarios involving risk free assets and market indices. For instance, the Capital Asset Pricing Model (CAPM) builds on the principles of portfolio theory by introducing the concept of beta, a measure of an asset's sensitivity to market movements. The CAPM explains how the risk and return of an asset are related to its contribution to the overall market portfolio's risk. Similarly, extensions of the model, such as the multi index model, incorporate additional factors like bond or currency indices to better capture the complexities of real world markets.

Despite its use, the practical implementation of Markowitz's model presents challenges. Estimating the expected returns, volatilities, and correlations for a large number of assets can be computationally intensive and prone to errors. Additionally, the assumptions of the model, such as normally distributed returns and the absence of transaction costs, may not hold in real world scenarios. However, the simplicity and robustness of the model make it a cornerstone of financial theory, providing valuable insights into portfolio management. (Wilmott, 2006, pp.317-330)

1.1 Iteration process

In order to solve the portfolio optimization problem iteratively, we utilize LINGO software, leveraging a model based on concepts from Chapter 13 of the LINGO manual. The goal of this optimization is to minimize the variance of the portfolio's end-of-period value while adhering to specific constraints. The LINGO code is designed to model the balance between risk and return, which are central components of Markowitz's portfolio theory. The code defines an objective function to minimize the variance of the portfolio, a measure of risk. The variance is expressed as a quadratic function that incorporates both the individual variances of the assets and their covariances. The covariance represents the relationship between the returns of two assets, and it plays a crucial role in determining the portfolio's overall risk. In our model, the covariance values, were calculated externally using an Excel file. This file included the historical return data of the assets, which was used to compute the correlation coefficients p_{ij} and the standard deviations σ. These computed values were then used to populate the variance covariance matrix, represented in **table x,** which forms the basis of the quadratic terms in the LINGO code. The LINGO model also incorporates two constraints to ensure practical feasibility. First, the budget constraint ensures that 100% of the portfolio budget is allocated across the two assets, represented as $\text{Asset}(1) + \text{Asset}(2) = 1$. Second, a return constraint guarantees that the portfolio achieves a minimum required of 25% end of period return, as expressed in section **1.1 Iteration 1, Gold and Nvidia** a $1.05678 \cdot N + 1.01178354 \cdot G \geq 1.025$. These constraints reflect the trade offs between risk and return that are central to portfolio management.

Table x : Covariance Table

	Nvidia	Guld	Apple	S&P 500	BTC
Nvidia	0.0269	0.01428672	0.016557	0.015129	0.028499
Guld	0.0143	0.0010498	0.003327	0.001787	0.015793
Apple	0.0166	0.00332692	0.005701	0.004081	0.018073
S&P 500	0.0151	0.00178657	0.004081	0.002568	0.016593
BTC	0.0285	0.01579264	0.018073	0.016593	0.030706

Note : Near the columns labeled "*Growth (%)*" green boxes indicate positive growth, signifying favorable performance, while red boxes indicate negative growth, representing unfavorable performance

1.1 Iteration 1, Gold and Nvidia

In the first iteration of the portfolio optimization process, the LINGO model was utilized to minimize the variance of the portfolio's end-of-period value while meeting specific constraints. The objective function aimed to minimize risk, expressed through the variance formula, which incorporates the variances and covariances of the assets Nvidia and gold. Additionally, the model included two constraints: the budget constraint, ensuring that 100% of the portfolio budget is allocated $N + G = 1$, and the return constraint, guaranteeing a minimum required return of 25% at the end of the period

$$1.05678 \cdot N + 1.01178354 \cdot G \geq 1.025$$

The results of the iteration, as shown in the output, determined that the optimal allocation consisted of approximately 29.4% invested in Nvidia and 70.6% in gold. This allocation successfully minimizes the portfolio's risk while meeting the required minimum annual return of 25%. Nvidia, with its higher expected return, contributes to achieving the return target, while gold, characterized by lower variance, significantly reduces the overall portfolio risk.

Code:

```
MODEL:
!Minimize end-of-period variance in portfolio
value;
[VAR] MIN = 0.0269 * N*N + 0.01428672
*N*G
+ 0.01428672 *N*G + 0.0010498*G*G;

! Use exactly 100% of the starting budget;
 [BUD] N + G = 1;

! Required wealth at end of period;
[RET] 1.05678* N + 1.01178354*G >= 1.025;
END
```

Output

Variable	Value	Reduced Cost
N	0.2937167	0.000000
G	0.7062836	0.000000
Row	**Slack or Surplus**	**Dual Price**
VAR	0.8771809E-02	-1.000000
BUD	-0.2469136E-06	0.5771738
RET	0.000000	-0.5802123

1.2 Iteration 2, Gold ,Nvidia and Apple

Code:

```
MODEL:
!Minimize end-of-period variance in portfolio value;
[VAR] MIN = 0.0269 * N*N + 0.01428672 *N*G +
0.016557 * A *N
+ 0.01428672 *N*G + 0.0010498*G*G + 0.003327 *
A*G
+ 0.016557 * A *N + 0.003327 * A*G + 0.005701*A*A;
! Use exactly 100% of the starting budget;
 [BUD] N + G + A = 1;

        0.5 >= G ;

! Required wealth at end of period;
[RET] 1.05678* N + 1.01178354*G + 1.01273*A>=
1.025;
END
```

Output:

```
Variable           Value        Reduced Cost
       N       0.2937167            0.000000
       G       0.7062836            0.000000

     Row    Slack or Surplus      Dual Price
     VAR       0.8771809E-02       -1.000000
     BUD      -0.2469136E-06       0.5771738
     RET            0.000000      -0.5802123
```

In the second iteration of the portfolio optimization process, the LINGO model was expanded to include a third asset, Apple, alongside Nvidia and gold. The objective remained the same: to minimize the variance of the portfolio's end of period value while meeting specific constraints. The objective function now accounted for the variances of all three assets and their covariances, reflecting the additional diversification opportunity introduced by Apple. The constraints in this iteration included the budget constraint, which ensures that the total allocation across Nvidia, gold, and Apple equals 100%, $N + G + A = 1$. Additionally, a new constraint limited the allocation to gold to a maximum of 50%, $G \leq 0.5$. This restriction was introduced to diversify the portfolio further. The return constraint ensured that the portfolio achieved a minimum required return of 25% at the end of the period

$$1.05678 \cdot N + 1.01178354 \cdot G + 1.01273 \cdot A \geq 1.025$$

The results of this iteration indicated an optimal allocation of approximately 28.9% to Nvidia, 50.0% to gold, and 21.1% to Apple. This allocation continued to minimize the portfolio's risk while satisfying the required return constraint. Nvidia remains a key contributor to meeting the return target, while the inclusion of Apple, with its unique risk return profile, enhances diversification and reduces overall variance. Gold still plays a dominant role in lowering the portfolio's total risk, although its share was capped to allow room for the new asset.

1.3 Iteration 3, Gold ,Nvidia, Apple and S&P500

Code:

```
MODEL:
!Minimize end-of-period variance in portfolio value;
[VAR] MIN = 0.0269 * N*N + 0.01428672 *N*G +
0.016557 * A *N + 0.015129*N*S
+ 0.01428672 *N*G + 0.0010498*G*G + 0.003327 *
A*G+ 0.001787*G*S
+ 0.016557 * A *N + 0.003327 * A*G + 0.005701*A*A
+ 0.004081*A*S
+ 0.015129*N*S + 0.001787*G*S + 0.004081*A*S +
0.002568*S*S;
! Use exactly 100% of the starting budget;
 [BUD] N + G + A + s = 1;

        0.3 >= N ;
        0.3 >= G ;
        0.3 >= A ;
        0.3 >= S ;

! Required wealth at end of period;
[RET] 1.05678* N + 1.01178354*G + 1.01273*A +
1.00782*S>= 1.025;
END
```

Output:

Variable	Value	Reduced Cost
N	0.3000000	0.000000
G	0.3000000	0.000000
A	0.2651737	0.000000
S	0.1348273	0.000000

Row	Slack or Surplus	Dual Price
VAR	0.1035854E-01	-1.000000
BUD	-0.9281051E-06	0.6126117
3	0.000000	-0.5826417E-02
4	0.000000	-0.4018592E-02
5	0.3482635E-01	0.000000
6	0.1651727	0.000000
RET	0.000000	-0.6207637

In the fourth iteration of the portfolio optimization process, the LINGO model was further expanded to include a fourth asset, the S&P 500 index, alongside Nvidia, gold, and Apple. The primary objective remained to minimize the portfolio's variance, with additional constraints designed to limit the allocation to each individual asset, ensuring no asset exceeded 30% of the total portfolio. The updated variance function incorporated the variances and covariances of all four assets, reflecting their individual and pairwise contributions to the portfolio's risk. The budget constraint ensured that the total allocation of the portfolio remained at 100%, expressed as $N + G + A + S = 1$. Additionally, the model enforced a cap of 30% on the weight of each asset to enhance diversification across the portfolio. The return constraint required that the portfolio achieve a minimum annual return of 25%, captured in the equation

$$1.05678 \cdot N + 1.01178354 \cdot G + 1.01273 \cdot A + 1.00782 \cdot S \geq 1.025$$

The results from this iteration provided an optimal allocation of approximately 30% to Nvidia, 30% to gold, 26.5% to Apple, and 13.5% to the S&P 500. This allocation demonstrates a balanced diversification strategy, effectively minimizing the portfolio's risk while achieving the required return. Nvidia and gold maintain significant roles due to their ability to complement each other's risk return characteristics. Apple contributes further diversification and growth potential, while the S&P 500 provides broad market exposure, ensuring a stable and diversified portfolio.

1.4 Iteration 4, Gold ,Nvidia, Apple, S&P500 and Bitcoin

Code:

```
MODEL:
!Minimize end-of-period variance in portfolio value;
[VAR] MIN = 0.0269 * N*N + 0.01428672 *N*G +
0.016557 * A *N + 0.015129*N*S + 0.028499*N*B
+ 0.01428672 *N*G + 0.0010498*G*G + 0.003327 *
A*G+ 0.001787*G*S + 0.015793*G*B
+ 0.016557 * A *N + 0.003327 * A*G + 0.005701*A*A
+ 0.004081*A*S + 0.018073*A*B
+ 0.015129*N*S + 0.001787*G*S + 0.004081*A*S +
0.002568*S*S + 0.016593*S*B
+ 0.028499*N*B + 0.015793*G*B+ 0.018073*A*B+
0.016593*S*B+ 0.030706*B*B;
! Use exactly 100% of the starting budget;
 [BUD] N + G + A + S + B = 1;

        0.25 >= N ;
        0.25 >= G ;
        0.25 >= A ;
        0.25 >= S ;

! Required wealth at end of period;
[RET] 1.05678* N + 1.01178354*G + 1.01273*A +
1.00782*S + 1.035697* B >= 1.025;
END
```

Output:

Variable	Value	Reduced Cost
N	0.2500000	0.000000
G	0.2500000	0.000000
A	0.2500000	0.000000
S	0.1523706	0.000000
B	0.9762941E-01	0.000000

In the final iteration of the portfolio optimization process, the LINGO model was extended to include Bitcoin as the fifth asset, alongside Nvidia, gold, Apple, and the S&P 500 index. The objective function was updated to minimize the variance of the portfolio, incorporating the variances and covariances of all five assets. This addition aimed to explore the diversification potential and the high growth rate of Bitcoin, while maintaining the constraints of the previous iterations. The budget constraint ensured that the sum of all asset weights equaled 100%, expressed as $N + G + A + S + B = 1$. Additional constraints limited the allocation of each individual asset to a maximum of 25%, reflecting a balanced diversification strategy. The return constraint required the portfolio to achieve a minimum annual return of 25%, represented by

$$1.05678 \cdot N + 1.01178354 \cdot t\ G + 1.01273 \cdot A + 1.00782 \cdot S + 1.035697 \cdot B \geq 1.025$$

The results indicated an optimal allocation of 25% each to Nvidia, gold, and Apple, 15.2% to the S&P 500, and 9.8% to Bitcoin. This allocation highlights a highly diversified portfolio, with Bitcoin's smaller share reflecting its higher volatility. Nvidia, gold, and Apple continue to form the core of the portfolio, contributing stability and consistent returns. The S&P 500 provides broad market exposure, while Bitcoin, despite its smaller allocation, enhances the portfolio's growth potential.

1.2 Explanation for the choice of assets

Note: Beta value of each asset have been calculated externally using an Excel file

The portfolio construction in this project follows an iterative process, with each step designed to optimize the risk and return profile in accordance with Modern Portfolio Theory (MPT). Each iteration aims to improve diversification, reduce the portfolio's overall volatility, and create an optimal balance between risk and return.

The first iteration includes only two assets: gold and Nvidia. The selection of these assets is motivated by their fundamentally different roles within a portfolio. Gold, an asset with a negative beta, often close to or below zero, tends to increase in value when the stock market declines, making it an effective hedge against market volatility. Additionally, gold is known for its low volatility compared to equities. Nvidia, on the other hand, is a high beta asset, exceeding 1, meaning it moves more sharply than the market. Nvidia is a growth stock with significant potential but also high volatility. The combination of gold and Nvidia provides a balance between defensive and aggressive exposure. However, this iteration resulted in a variance of 0.008772, which, although acceptable for the initial step, highlights the room for improvement in diversification and risk management.

In the second iteration, Apple is introduced to complement Nvidia within the technology sector while offering lower volatility and beta compared to Nvidia. Apple is a stable blue chip stock with a high market share and consistent returns. Historically, it has demonstrated a beta close to 1.24, indicating it moves largely in line with the market. Adding Apple enhances diversification within the technology sector and reduces the overall risk of the portfolio. This improvement is reflected in the portfolio's variance, which increased slightly to 0.009619. This minor increase can be attributed to the addition of Apple, which provides balanced growth with reduced risk exposure compared to Nvidia.

In the third iteration, the S&P 500 Index is included to provide broad exposure to the entire U.S. stock market. This step reduces idiosyncratic risk, which is the risk associated with individual stocks such as Nvidia or Apple. The S&P 500 has a beta close to 1, reflecting the average movements of the market. Since the index comprises 500 different companies across multiple sectors, it lowers correlation with the existing portfolio assets and improves overall diversification. This iteration resulted in a variance of 0.010359, which, while slightly higher than the previous iteration, is justified by the broader market exposure and reduced dependency on individual stock performance.

In the fourth iteration, Bitcoin is incorporated to align the portfolio's performance more closely with Nvidia's exceptional growth while introducing an alternative asset class. Bitcoin was selected for its ability to match Nvidia's high growth and volatility profile. This iteration resulted in a variance of 0.011864, the highest among the portfolios. The inclusion of Bitcoin adds significant potential for returns but comes with the trade-off of increased risk. However, for investors with a higher risk appetite, this iteration is particularly attractive as it balances the potential for high returns with a diverse set of assets.

In reflecting on the variance changes across these iterations, it becomes evident that as the portfolio diversified, the variance increased slightly with each additional asset. This is because the assets added in each iteration Apple, the S&P 500, and Bitcoin were chosen to enhance potential returns, often at the cost of slightly increased risk. Despite this, the overall portfolio remains well balanced, offering diversified exposure to defensive, growth, and alternative assets. It is important to highlight that earlier drafts considered Coca Cola and Celsius as potential additions to the portfolio. Coca Cola, with its low beta, would have further reduced variance but was excluded in favor of more growth oriented assets. Celsius, while offering higher growth potential, presented risks that were ultimately outweighed by the dominance of Nvidia in the portfolio's optimization model.

1.3 Risk free asset with a yearly return of 4%

In this iteration of the portfolio optimization, we introduced a risk free asset with an annual return of 4%. This addition altered the optimization framework, shifting the goal from minimizing variance to maximizing the Sharpe ratio. The Sharpe ratio is a widely used measure in portfolio management that calculates the excess return (above the risk free rate) per unit of risk. This iteration allowed us to assess the impact of including a riskfree asset on the portfolio composition and performance. The LINGO code for this optimization reflects these changes. The objective function was redefined to maximize the Sharpe ratio:

$$\text{Sharpe Ratio} = \frac{\text{Portfolio Return} - \text{Risk-Free Rate}}{\text{Portfolio Standard Deviation}}$$

The risk free rate was calculated as 1.0032737, representing the monthly equivalent of an annual 4% return. The portfolio return, composed of Nvidia, gold, Apple, the S&P 500, and Bitcoin, was computed as a weighted sum of the individual asset returns. Similarly, the portfolio variance was calculated as a quadratic function of the individual variances and covariances of the assets. The square root of this variance gave the portfolio's standard deviation, a measure of risk. The constraints included first Each asset weigh Nvidia(N), gold (G), Apple (A), S&P 500 (S), Bitcoin (B) was capped at 25% to ensure a diversified portfolio. Then, the total allocation across all assets was required to equal 100%, $N + G + A + S + B = 1$. After running the optimization, the resulting portfolio consisted of 25% allocations to Nvidia, gold, Apple, and Bitcoin, while the S&P 500 received no allocation. The calculated Sharpe ratio for this portfolio was 0.2047716.

In the previous iteration, which did not include the risk free asset, in section **1.4 Iteration 4, Gold ,Nvidia, Apple, S&P500 and Bitcoin** ,the portfolio was optimized to minimize variance while achieving a minimum annual return of 25%. That portfolio had allocation of 25% each to Nvidia, gold, and Apple, 15.2% to the S&P 500, and 9.8% to Bitcoin. The key difference between the two iterations lies in the exclusion of the S&P 500 in the current portfolio. While the S&P 500 contributed to diversification in the variance minimization model, it was excluded in this iteration as it was deemed less efficient in terms of return to risk trade offs when the risk free asset was included. The introduction of the risk free asset fundamentally changes the optimization objective. Previously, the goal was to minimize variance, which heavily favors diversification to reduce risk. As a result, all five assets were included to spread risk as broadly as possible. However, with the inclusion of a risk free asset, the Sharpe ratio becomes the new benchmark. The Sharpe ratio evaluates the performance of a portfolio relative to its risk, emphasizing the excess return over the risk free rate. Assets with a lower contribution to the portfolio's risk adjusted return, such as the S&P 500 in this case, are deprioritized or excluded. Instead, the optimization favors assets that offer higher returns relative to their risks, resulting in equal allocations to Nvidia, gold, Apple, and Bitcoin. Another factor is the impact of the 25% cap on individual asset weights. Without this constraint, the model might have allocated disproportionately to Bitcoin or Nvidia due to its higher return, but the cap ensured a balanced portfolio. The S&P 500's exclusion, despite its diversification benefits, demonstrates that its return was insufficiently high to justify its inclusion under the Sharpe ratio framework.

Code:

```
MODEL:
MAX =
(1.05678* N + 1.01178354*G + 1.01273*A + 1.00782*S
+ 1.035697* B- 1.0032737)/
((0.0269 * N*N + 0.01428672 *N*G + 0.016557 * A *N
+ 0.015129*N*S + 0.028499*N*B
+ 0.01428672 *N*G + 0.0010498*G*G + 0.003327 *
A*G+ 0.001787*G*S + 0.015793*G*B
+ 0.016557 * A *N + 0.003327 * A*G + 0.005701*A*A
+ 0.004081*A*S + 0.018073*A*B
+ 0.015129*N*S + 0.001787*G*S + 0.004081*A*S +
0.002568*S*S + 0.016593*S*B
+ 0.028499*N*B + 0.015793*G*B+ 0.018073*A*B+
0.016593*S*B+ 0.030706*B*B)^.5);

        0.25 >= N ;
        0.25 >= G ;
        0.25 >= A ;
        0.25 >= S ;
        0.25 >= B;

! Use exactly 100% of the starting budget;
[BUD] N + G + A + S + B = 1;
END
```

Output:

Variable	Value	Reduced Cost
N	0.2500000	0.000000
G	0.2500000	0.000000
A	0.2500000	0.000000
S	0.000000	0.000000
B	0.2500000	0.000000

0.2047716

1.4 Dual optimization model for maximize return

In this iteration, we implemented a dual optimization model aimed at maximizing portfolio return while adhering to a predefined acceptable level of risk. Unlike the previous iteration, which focused on minimizing variance while meeting a set return target, this model shifted the focus to maximizing return while capping the variance at 0.0025. The LINGO code used for this optimization defines the objective function as the maximization of the portfolio's weighted return, considering the allocations to Nvidia (N), gold (G), Apple (A), the S&P 500 (S), and Bitcoin (B). The variance constraint ensures that risk remains within acceptable limits, while additional constraints enforce full budget utilization and diversification by capping individual allocations at 25%. The output of the model resulted in allocations of 25% each to Nvidia, gold, and Apple, 22.83% to the S&P 500, and 21.17% to Bitcoin. The variance was capped precisely at 0.0025, and the portfolio achieved a return of 2.2633%. This result highlights a balanced portfolio with controlled risk while striving for optimal returns within the given constraints.

When compared to the previous iteration, where the objective was to minimize variance for a required return of 2.5%, notable differences arise. The earlier model produced allocations of 25% each to Nvidia, gold, and Apple, 15.23% to the S&P 500, and 9.76% to Bitcoin, with a variance of 0.0118866. While the variance in this dual model is significantly lower, the return is slightly reduced at 2.2633% compared to the 2.5% in the earlier iteration. However, this reduced return is offset by the substantial decrease in variance, indicating a much lower risk level for a return that is still relatively close to the earlier model. The differences between the two approaches highlight the inherent trade offs in portfolio optimization. The previous model prioritized achieving a fixed return, leading to higher risk as reflected in its variance. In contrast, the dual model prioritizes managing risk, resulting in a portfolio that achieves a slightly lower return but with significantly greater stability. For risk averse investors, this approach would be ideal, as it ensures that the portfolio operates within a defined risk tolerance.

From our perspective, as investors who embrace risk for the potential of higher returns, the dual model still provides valuable insights. While the return is slightly lower, the significant reduction in risk makes it an attractive option. This demonstrates that a lower risk portfolio can achieve returns close to a higher risk portfolio under carefully balanced constraints. However, we might still favor the previous approach for its focus on maximizing return, as the higher variance aligns with our willingness to take risks for potentially greater gains. In terms of methodology, the dual model offers clear advantages, including controlled risk and flexibility to adapt to specific investor preferences. The risk constraint ensures a stable portfolio while still allowing for return optimization. However, this comes with trade offs, as the constraint limits the potential for higher returns compared to an unconstrained model. Additionally, the computational complexity of incorporating precise variance and covariance calculations adds a layer of difficulty.

Code:

```
MMODEL:
! Maximize end-of-period portfolio return subject to a risk
limit;
! Objective function: Maximize return;
[MAXRET] MAX = 1.05678 * N + 1.01178354 * G +
1.01273 * A + 1.00782 * S + 1.035697 * B;

! Risk limit (variance constraint);
[RISK] 0.0269 * N * N + 0.01428672 * N * G + 0.016557 * A
* N + 0.015129 * N * S + 0.028499 * N * B
+ 0.01428672 * N * G + 0.0010498 * G * G + 0.003327 * A *
G + 0.001787 * G * S + 0.015793 * G * B
+ 0.016557 * A * N + 0.003327 * A * G + 0.005701 * A * A +
0.004081 * A * S + 0.018073 * A * B
+ 0.015129 * N * S + 0.001787 * G * S + 0.004081 * A * S +
0.002568 * S * S + 0.016593 * S * B
+ 0.028499 * N * B + 0.015793 * G * B + 0.018073 * A * B +
0.016593 * S * B + 0.030706 * B * B <= 0.0025;

! Allocation constraints (ensure full budget utilization and
diversification);
[BUDGET] N + G + A + S + B = 1;
[CAPN] N <= 0.25;
[CAPG] G <= 0.25;
[CAPA] A <= 0.25;
[CAPS] S <= 0.25;
[CAPB] B <= 0.25;
END
```

Output:

```
Objective value:                              1.022633
Infeasibilities:                              0.000000
Total solver iterations:                             5
Elapsed runtime seconds:                          0.16

Model Class:                                        QP

Total variables:                      5
Nonlinear variables:                  5
Integer variables:                    0

Total constraints:                    8
Nonlinear constraints:                1

Total nonzeros:                      20
Nonlinear nonzeros:                  15


                                Variable           Value        Reduced Cost
                                       N       0.2500000            0.000000
                                       G       0.2500000            0.000000
                                       A       0.2500000            0.000000
                                       S       0.2288305            0.000000
                                       C       0.2116945E-01        0.000000
```

2. Data Envelopment Analysis and Efficient Portfolios

Data Envelopment Analysis (DEA) is a method based on an application of linear programming. The original idea behind the method was developed to measure performance. The method assumes that stakeholders with the same type of input and output can be compared relatively to one another (Ramanathan, 2003).

2.1 Method

The first part of Assignment 4 involved conducting a DEA using the specific CCR method. The aim was to distinguish the efficiency of various portfolios and analyze how efficiency differs between each portfolio. CCR refers to the Charnes, Cooper, and Rhodes model, which was developed by the same authors in 1978. The method facilitates the comparison of efficiency between different entities, in this case, four portfolios (Ramanathan, 2003). To perform a CCR analysis, various inputs and outputs are required as variables to determine the efficiency of each portfolio. In this assignment, variance and volatility were used as inputs, while return served as the output. The return was predefined and constant across all portfolios, set at a monthly rate of 2.5%.

2.1.1 Method Part A

2.1.1.1 Variance calculation

In this case, the variance is associated with the predetermined return for each portfolio. It originates from the application of the Variance Portfolio Model, which is based on minimizing variance (risk) to maximize profit (Malkiel & Xu, 1997). The mathematical model is as follows:

Variable & Description
$N =$ **Number of assets**
$w_i =$ **Weight for asset** i
$r_i =$ **Monthly return for asset** i
$r_i =$ **Monthly return for asset** i
$r_i =$ **Monthly return for asset** i

Equations		
Minimizing variance:	$\min p = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \sigma_{ij}$	(1)
Budget restrictions:	$\sum_{i=1}^{N} w_i = 1$	(2)
Return requirement:	$\sum_{i=1}^{N} r_i w_i \geq 1.025$	(3)

Use the value of p from every portfolio to use in Table X.

2.1.1.2 Volatility calculation

Variable & Description
$\overline{P}$ = Mean price
D_i = The difference between each price and the average price
V = Variance
σ = Standard deviation
τ = Volatility in percent

Equations		
Mean price	$\overline{P} = \frac{\sum_{i=1}^{N} P_i}{N}$	(4)
The difference between each price and average price	$D_i = (P_i - \overline{P})$	(5)
Square the difference	$D_i^2 = (P_i - \overline{P})^2$	(6)
Use the sum of the squared difference and divide it with the number of months	$V = \frac{\sum_{i=1}^{N} D_i^2}{N}$	(7)
Standard deviation:	$\sigma = \sqrt{V}$	(8)
Calculate the volatility in percent	$\tau = \frac{\sigma}{\overline{P}} \times 100$	(9)

Then use t in **Table 1** as input for each portfolio. (CFI Team, 2024).

2.1.1.3 Calculation of efficiency

Variable & Description
v_n = **Weights of inputs** n
N = **Total inputs**
u_m = **Weights of outputs** m
M = **Total outputs**
θ_j = **The relative efficiency of DMU**$_j$
h = **The total of the portfolio**

Equations		
Goal function	Maximize: $\theta_j = \dfrac{\sum_{m=1}^{M} y_m u_{mj}}{\sum_{n=1}^{N} x_n v_{nj}}$	(10)
Subject to	$\dfrac{\sum_{r=1}^{M} u_m y_m}{\sum_{n=1}^{N} x_n v_{nk}} \leq 1, \quad k = 1,2,\ldots,h$	(11)
	$u_m \geq 0, \quad m = 1,2,\ldots,M$	(12)
	$v_n \geq 0, \quad n = 1,2,\ldots,N$	(13)

(Martić et al, 2009).

The relative efficiency j can be defined as the ratio between the weighted sum of outputs, also referred to as the "virtual output," and the weighted sum of inputs, also referred to as the "virtual input." The efficiency of portfolio j is calculated using the maximization function (10), under the condition that constraint function (11) holds true, ensuring that 0 < j 1. The weights reflect the relative importance of each input and output. The equation is nonlinear, which makes it challenging to solve. Therefore, it can be transformed into a linear equation to facilitate solving it using linear programming (Martić et al, 2009). To linearize the equation, the denominator must be set to 1, so that only the numerator is maximized:

Variable & Description
v_n = **Weights of inputs** n
N = **Total inputs**
u_m = **Weights of outputs** m
M = **Total outputs**
θ_j = **The relative efficiency of DMU**$_j$
h = **The total of the portfolio**

	Equations	
Goal function	Maximize: $\theta_j = \sum_{m=1}^{M} y_m u_{mj}$	(14)
Subject to	$\sum_{n=1}^{N} x_n v_{nj} = 1$	(15)
	$\sum_{m=1}^{M} y_m u_{mk} - \sum_{n=1}^{N} x_n v_{nk} \leq 0, \quad (k = 1,\ldots, h)$	(16)
	$u_m \geq 0, \quad (k = 1,\ldots, M)$	(17)
	$v_n \geq 0, \quad (k = 1,\ldots, N)$	(18)

(Martić et al, 2009).

After linearizing the equation, it was implemented in Lingo to calculate the efficiency scores for each portfolio. The complete code can be found in **Appendix A**.

2.1.2 Method Part A

2.1.2.1 Content Validity

The concept describes how well a test, model, or instrument measures what it is intended to measure and how comprehensively it reflects its intended purpose. It ensures the quality of the test's components, such as inputs, outputs, and variables, and how these align with the theoretical aspects of the analysis. There are three key aspects of content validity: how the domain is defined, how the domain is represented, and the relevance of the domain (Sireci, 1998).

The concept describes how well a test, model, or instrument measures what it is intended to measure and how comprehensively it reflects its intended purpose. It ensures the quality of the test's components, such as inputs, outputs, and variables, and how these align with the theoretical aspects of the analysis. There are three key aspects of content validity: how the domain is defined, how the domain is represented, and the relevance of the domain (Sireci, 1998).

Definition of the Domain

Before conducting a content validity analysis on the portfolio, it is essential to define how the domain is measured. In this case, when evaluating portfolio efficiency, the domain can be defined as “how effectively the portfolio balances risk with the return it delivers.” By defining the domain in a specific manner, it becomes easier to determine which inputs and outputs should be considered when assessing portfolio efficiency (Sireci, 1998).

Representation of the Domain

Content validity requires that relevant components, such as inputs and outputs, are represented in the model. In this case, inputs could include variance, volatility, or transaction costs, while outputs might consist of return, the Sharpe ratio, or dividends. A model that omits critical components may be considered incomplete and fail to adequately represent the scope of the measurement (Sireci, 1998).

Relevance of the Domain

The variables used for the portfolio must be relevant to what is being measured. Including variables that are not pertinent to the intended measurement of the portfolio can compromise the accuracy of the analysis. For example, if the defined domain of the portfolio is risk and return, incorporating unrelated market variables as inputs would harm the validity of the portfolio. This could result in portfolio efficiency being based on incorrect data that does not accurately represent the true performance of the portfolio (Sireci, 1998).

2.1.2.2 Face validity

The concept pertains to how well a test measures what it is intended to measure. In this case, it concerns the inputs and outputs of the portfolio and whether they are relevant to what is being assessed (Nevo, 1985).

2.1.2.2 Criterion validity

The concept describes how accurately the portfolio measures what it is intended to measure. Often in criterion validity, a "Golden Standard" is used to evaluate how well the test performs. This refers to other tests that are considered accepted and reliable within the field as a benchmark for quality. However, a "Golden Standard" is not always available for comparison, making it impossible to conduct a criterion validity analysis in such cases (Nikolopoulou, 2022).

2.2 Results

2.2.1 Results Part A

Table 1 : Shows the Variables and the efficiency of the portfolios

Scenarios	**Inputs**	**Output**		
	Variance	*Volatility*	*Return*	*Efficiency*
Portfolio 1	0,008772	57,5 %	1,025	1
Portfolio 2	0,009619	35,8 %	1,025	1
Portfolio 3	0,010359	36,7 %	1,025	0,9754768
Portfolio 4	0,01186447	39,7 %	1,025	0,9017632

Table 1 presents the calculated variances and volatilities as inputs, with return classified as the output, chosen based on the required return. It can be observed from **Table 1** that Portfolio 1 and Portfolio 2 are efficient, as they have an efficiency score of 1. However, Portfolio 3 and Portfolio 4 are not efficient, as their efficiency scores are below 1.

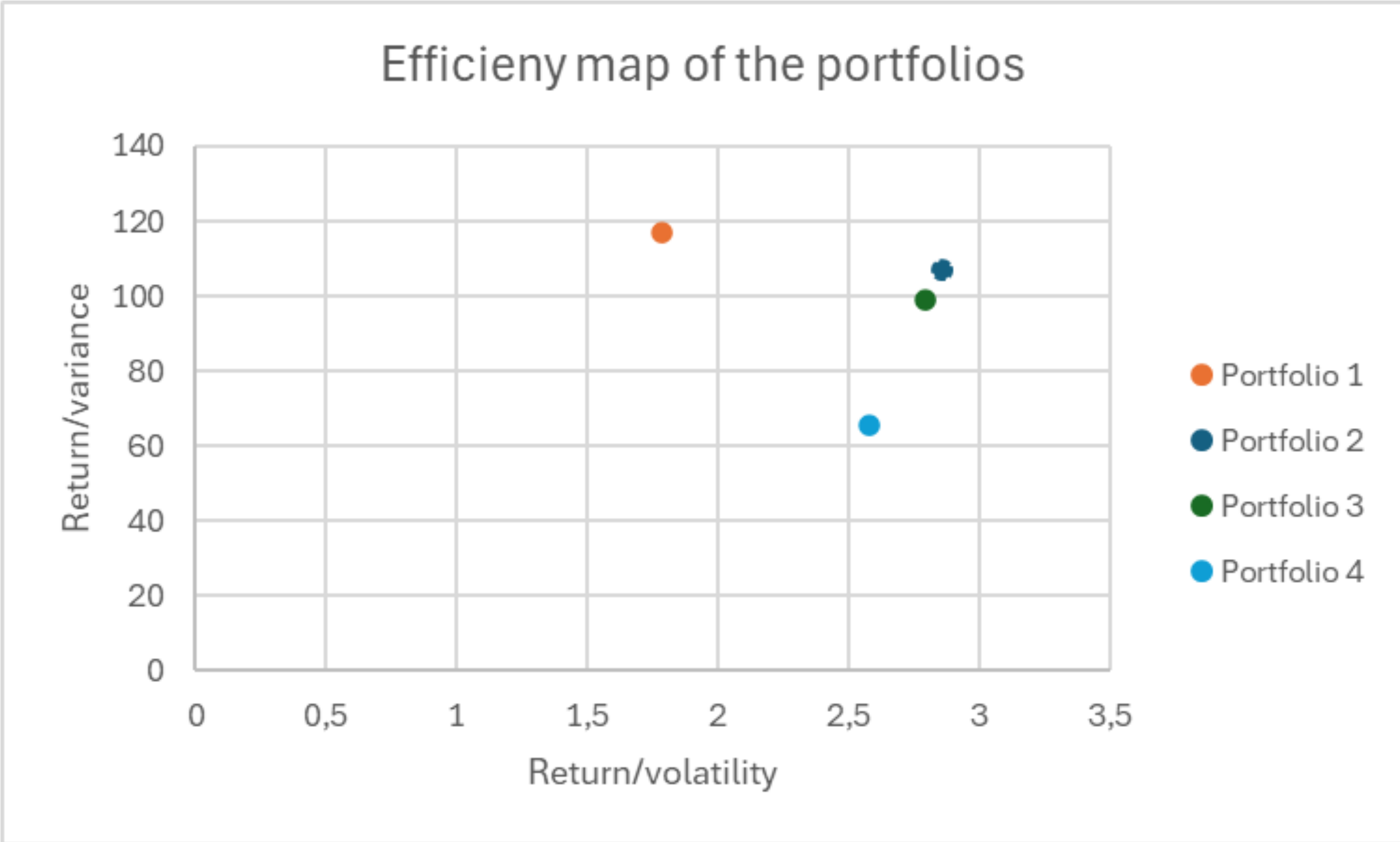


Figure 1 : Efficiency map of the portfolios

Note : The graphs illustrate the data from **Excel FIle**

2.3 Diskussion

2.3.1. Discussion Part A

The results from Part A of the CCR model indicated that Portfolio 1 and Portfolio 2 were the most efficient based on the variables used. This outcome appears reasonable, as both Portfolio 1 and Portfolio 2 had superior inputs in different aspects. For instance, Portfolio 1 had the lowest variance, making it the most efficient in that regard, although it also exhibited very high volatility. At the same time, Portfolio 2 had almost equally low variance but the lowest volatility, which justifies both portfolios being on the efficient frontier.

A comparison of the results from Table 1 and Diagram 1 reveals that Portfolios 1 and 2 are also positioned on the efficient frontier in the chart, aligning well with the table. The chart also visualizes the results for Portfolios 3 and 4. Portfolio 3 has an efficiency score close to 1 but slightly below it, which is reflected in the geographical representation of Diagram 1, where Portfolio 3 is positioned just below Portfolio 2. Finally, Portfolio 4 is located further away geographically, as was evident in Table 1, where its efficiency score showed a larger deviation.

If the CCR model were to be reconstructed, it would be interesting to include additional outputs. As described by (Hasselblad, 2019), a correctly executed CCR model should include more outputs than inputs, which was not the case here. However, in this specific scenario, this limitation likely does not have a significant impact, as the results appear reasonable. Potential outputs could include metrics such as the Sharpe ratio or dividends for each asset. However, the model would need to be completely revised, as none of these assets currently provide dividends.

2.3.2. Discussion Part B

2.3.2.1 Content Validation

Definition of the Domain

The construction of portfolio efficiency is clearly defined in the report as the ability to balance risk (inputs) with respect to high returns (output). This definition aligns with traditional portfolio theory, where minimizing risk while maximizing returns is a primary focus (Malkiel & Xu, 1997). By defining the domain, a clear framework is established for identifying the inputs and outputs that reflect portfolio efficiency. This is a crucial step in ensuring content validity, as it clarifies the objective of what the portfolio is measured against and excludes irrelevant variables (Sireci, 1998).

At the same time, the domain has been defined solely around variance and volatility to measure risk. While these are not poorly chosen inputs for the intended purpose, it must be acknowledged that additional inputs could strengthen the measurement of efficiency in the same context. These inputs could include factors such as transaction costs or diversification. The use of only two inputs may result in a limited representation of portfolio efficiency, particularly in real-world scenarios where other factors can play a significant role.

Representation of the domain

Content validity requires that the inputs and outputs of the portfolio are relevant to what is being measured.

Inputs:

Variance is a well known measure of risk, capturing the dispersion of returns for each asset. Its use is theoretically justified and aligns with Markowitz's Modern Portfolio Theory. However, variance assumes a normal distribution of returns, which does not always reflect reality, particularly for portfolios exposed to black-swan events or other extreme market conditions. This limitation could reduce the model's applicability in volatile or non-normal market environments. Such conditions might be relevant in the composition of the portfolio under analysis, given its heavy weighting in high-risk assets, with gold as the exception. Over the past two years, markets have experienced significant gains, with the S&P 500 increasing by more than 25% annually an abnormal scenario for the market. However, since all portfolios in this analysis contain similar assets, the impact of this limitation is reduced in this specific case.

Volatility represents risk in percentage terms, offering an intuitive and practical measure of market uncertainty. Its derivation from variance strengthens the model by providing an alternative but related perspective on risk. However, like variance, volatility risks oversimplifying the situation by ignoring deviations from a normal distribution.

Outputs:

The return is fixed at 2.5% across all portfolios, simplifying comparisons between them. While a constant return facilitates efficiency evaluations, it does not account for risk-free rates, which can influence the true efficiency of a portfolio. A risk-adjusted measure, such as the Sharpe ratio, could have been included to provide a more nuanced understanding of portfolio performance. Additionally, a constant return does not reflect variations in performance across portfolios, limiting the model's ability to differentiate between high-risk, high-return portfolios and low-risk, low-return portfolios, as all portfolios are assigned the same return.

Finally, the model does not include other relevant variables such as transaction costs, diversification levels, or liquidity, which are often critical in determining the efficiency of real-world portfolios. For instance, if transaction costs differ between portfolios but are excluded from the model, the results may be misleading, as a portfolio with high transaction costs might incorrectly appear to be the most efficient. However, since all portfolios in this analysis are structured similarly, the risk of missing relevant variables is reduced, as differences between portfolios are minimized.

In conclusion the model's selected inputs and outputs are theoretically relevant, the exclusion of additional critical factors limits its ability to fully represent portfolio efficiency in a real-world context. Nevertheless, given the uniformity of the portfolios under consideration, these limitations have less impact on the results in this specific case.

The relevance of the domain

Both volatility and variance are directly relevant to the portfolio's content, as they each describe a type of risk. Their use is consistent with financial theory, aligning with the objective of minimizing risk. Return is also a useful measure of how well a portfolio performs and is relevant as an output. However, assigning the same return to all portfolios simplifies the complexity of a real portfolio's performance.

In conclusion, the content validity of the model is relatively high, as the portfolio is clearly defined, and the representation of variables is appropriate, though limited. While the model is simplified by the assumption of a constant return across all portfolios, the overall content validity remains robust. However, it could be further improved by adopting a more complex model that provides a more nuanced focus on the interplay between risk and return.

2.3.2.2 Face Validity

Face validity refers to how well the content of the model appears to be appropriate for evaluating portfolio efficiency. As previously mentioned, the model has a clear definition of what the portfolios are being evaluated against, namely risk relative to return. Therefore, all inputs and outputs in the model are suitable for assessing portfolio efficiency. However, as noted earlier, adding more inputs and outputs could provide a more comprehensive analysis of portfolio efficiency, as additional variables might offer a more nuanced perspective. While the content has strong relevance to efficiency, the face validity of the model can be considered relatively high but not perfect, as some variables are missing that could deepen the analysis.

2.3.2.3 Criterion validity

As previously mentioned, there are limitations to using only three variables to measure efficiency. However, overall, the results accurately measure what the model was intended to evaluate. The model demonstrates that portfolios with the lowest risk in terms of variance and volatility achieve the highest efficiency. Similarly, the efficiency of Portfolio 3, which has values close to those of Portfolio 2, is reflected in its efficiency score, with Portfolio 3 achieving nearly an efficiency of 1. Nevertheless, as noted earlier, additional variables could have been included to provide a more comprehensive assessment of portfolio efficiency regarding risk and return.

Finally, there is no "golden standard" value available for comparison to truly assess the strength of the criterion validity. However, this is perhaps unsurprising given the significant variation in portfolio structures and the absence of a single "correct" way to construct a portfolio. If the analysis were to be conducted again, it would be valuable to experiment with additional variables as inputs and outputs to observe how the results might change. Overall, we believe that the criterion validity of the efficiency model is at a reasonable level.

2.3.2.4 Fuzzy DEA

Transforming a standard DEA into a Fuzzy DEA fundamentally changes the framework of the model. A Fuzzy DEA can address real-world challenges, specifically when input and output data are less precise or uncertain. However, such a model introduces greater complexity in its construction and can be more challenging to work with (Hasselblad, 2019).

One advantage of Fuzzy DEA is its ability to handle imprecise and uncertain data, which is common in financial analysis and portfolio management. In portfolio management, inputs such as variance and volatility can fluctuate significantly due to the inherent uncertainty of highly dynamic markets (Malik, 2003). Similarly, outputs such as returns can vary depending on the time horizon used and are often based on the portfolio's future performance, which is difficult to predict.

Fuzzy DEA increases the robustness of efficiency scores. Since it can accommodate fluctuating data, the model can generate efficiency intervals instead of specific values. This enables us to understand how portfolios might vary in their efficiency, allowing for more nuanced decision-making (Omrani et al, 2022). For example, an inefficient portfolio might become efficient under certain conditions, depending on how the uncertainty in the variables changes.

However, there are also disadvantages to using Fuzzy DEA. First and foremost, the complexity of using the model increases significantly. Understanding and managing uncertainties, where results may change depending on the range of variables, requires a certain level of expertise and experience (Saati & Hatami-Marbini, 2012), which we lack since we have only conducted a simple standard DEA. Additionally, fuzzy data can reduce the interpretability of the model. Since a Fuzzy DEA generates intervals for efficiency scores, it may be more challenging to draw clear conclusions, as it does not provide a definitive answer on which portfolio is the most efficient.

In conclusion, extending a standard DEA to a Fuzzy DEA offers both advantages and disadvantages. The advantages lie in the model's ability to handle uncertain data and provide more realistic efficiency measures, reflecting the reality that data is often imprecise. A Fuzzy DEA would be particularly useful in this context, as portfolio data tends to vary significantly. However, a Fuzzy DEA is considerably more challenging to manage, introducing risks of subjectivity and requiring more advanced computational efforts.

References

[1] CFI Team. (2024). Volatility. Corporatefinance.com. https://corporatefinanceinstitute.com/resources/career-map/sell-side/capital-markets/volatility-vol/

[2] Hasselblad, A. (2019). Data Envelopment Analysis (DEA). Mid Sweden University, Department of Information Systems and Technology.

[3] Malik, F. (2003). Sudden changes in variance and volatility persistence in foreign exchange markets. Journal of Multinational Financial Management, 13(3), 217–230. https://www.sciencedirect.com/science/article/abs/pii/S1042444X0200052X

[4] Malkiel, B. G., & Xu, Y. (1997). Risk and return revisited. Journal of Portfolio Management, 23(3), 9.

[5] Nevo, B. (1985). Face validity revisited. Journal of Educational Measurement, 22(4), 287–293. https://onlinelibrary.wiley.com/doi/abs/10.1111/j.1745-3984.1985.tb01065.x

[6] Nikolopoulou, K. (2022). What is criterion validity? | Definition & examples. Scribbr. https://www.scribbr.com/methodology/criterion-validity/

[7] Omrani, H., Alizadeh, A., Emrouznejad, A., & Teplova, T. (2022). A robust credibility DEA model with fuzzy perturbation degree: An application to hospitals performance. Expert Systems with Applications, 189, 116021. https://www.sciencedirect.com/science/article/abs/pii/S0957417421013683

[8] Ramanathan, R. (2003). An introduction to data envelopment analysis; A tool for performance measurement. Sage Publications India Pvt Ltd.

[9] Saati, S., & Hatami-Marbini, A. (2012). Improving the computational complexity and weights dispersion in fuzzy DEA. In Uncertainty Modeling in Knowledge Engineering and Decision Making (pp. 1070–1075).

[10] Sireci, S. G. (1998). The construct of content validity. Social Indicators Research, 45, 83–117.

[11] Wilmott, P. (2006). Paul Wilmott on quantitative finance (2nd ed.). Wiley.

Appendix

Appendix A

```
MODEL:
MAX = P1 + p2 +p3 + p4;

p1 = 1.025*v1;

1.025*v1-(0.008772*u1+0.575*u2)<=0; !Port 1;
1.025*v1-(0.009619*u1+0.358*u2)<=0; !Port 2;
1.025*v1-(0.010359*u1+0.367*u2)<=0; !Port 3;
1.025*v1-(0.01186447*u1+0.397*u2)<=0; !Port 4;
0.008772*u1+0.575*u2=1;

p2 = 1.025*v12;

1.025*v12-(0.008772*u12+0.575*u22)<=0; !Port 1;
1.025*v12-(0.009619*u12+0.358*u22)<=0; !Port 2;
1.025*v12-(0.010359*u12+0.367*u22)<=0; !Port 3;
1.025*v12-(0.01186447*u12+0.397*u22)<=0; !Port 4;
0.009619*u12+0.358*u22=1;

p3 = 1.025*v13;

1.025*v13-(0.008772*u13+0.575*u23)<=0; !Port 1;
1.025*v13-(0.009619*u13+0.358*u23)<=0; !Port 2;
1.025*v13-(0.010359*u13+0.367*u23)<=0; !Port 3;
1.025*v13-(0.01186447*u13+0.397*u23)<=0; !Port 4;
0.010359*u13+0.367*u23=1;

p4 = 1.025*v14;

1.025*v14-(0.008772*u14+0.575*u24)<=0; !Port 1;
1.025*v14-(0.009619*u14+0.358*u24)<=0; !Port 2;
1.025*v14-(0.010359*u14+0.367*u24)<=0;!Port 3;
1.025*v14-(0.01186447*u14 + 0.397*u24)<=0;!Port 4;
0.01186447*u14 + 0.397*u24=1;
```