وَمَا أُوتِيتُمْ مِنَ الْعِلْمِ إِلَّا قَلِيلًا

Analog IC Design

Lecture 02
Review on Circuits Basics

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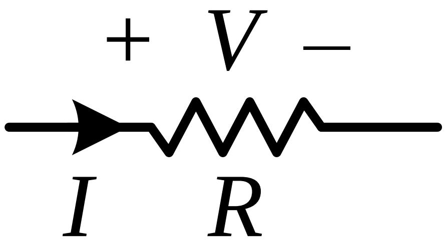
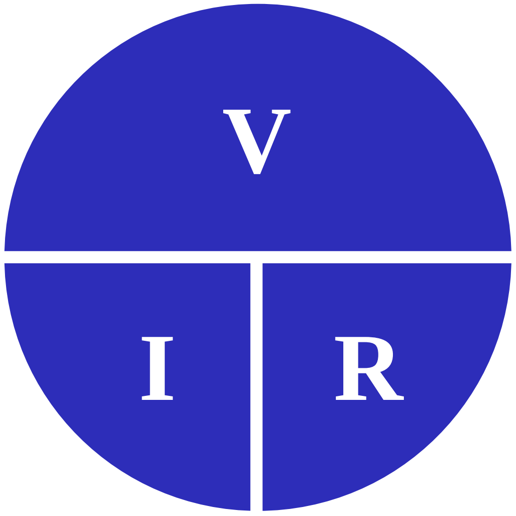
Ohm's Law





$$V = IR$$

$$I = \frac{V}{R}$$

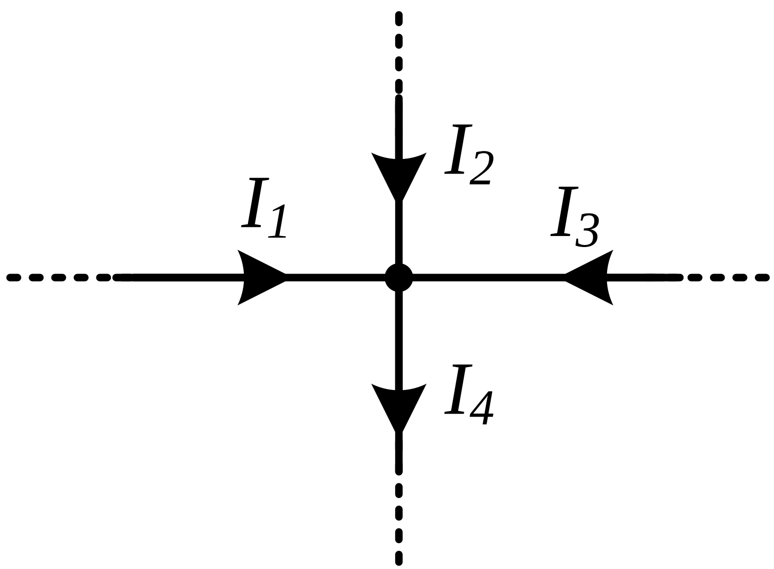
$$R = \frac{V}{I}$$

Kirchhoff’s Current Law (KCL)

- The sum of all currents flowing into a node is zero.

$$\Sigma I = 0$$

$$I_1 + I_2 + I_3 - I_4 = 0$$



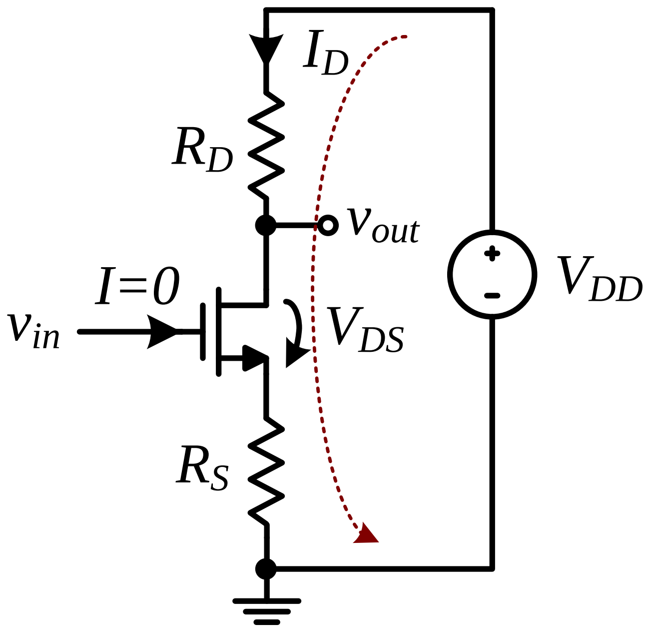
Kirchhoff's Voltage Law (KVL)

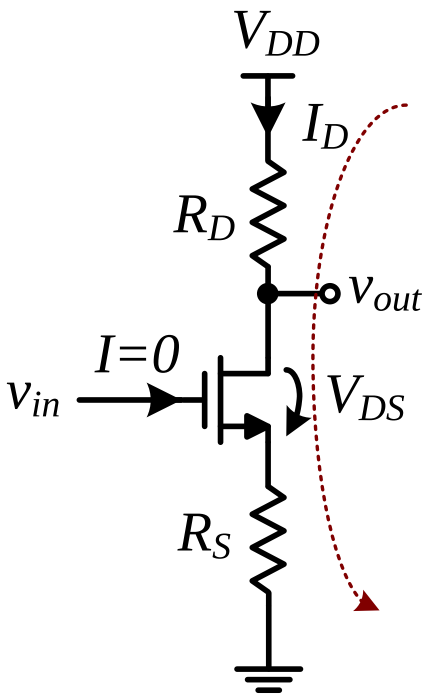
- The sum of all voltage drops around any closed loop is zero

$$\Sigma V = 0$$

$$-V_{DD} + I_D R_D + V_{DS} + I_D R_S = 0$$

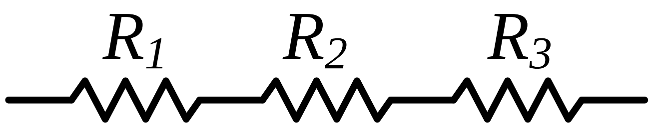
$$V_{DD} = I_D(R_D + R_S) + V_{DS}$$





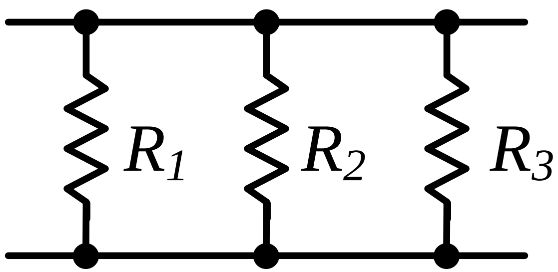
Resistor Combinations

- Resistors in series: Largest resistor dominates



$$R_{eq} = R_1 + R_2 + R_3$$

- Resistors in parallel: Smallest resistor dominates



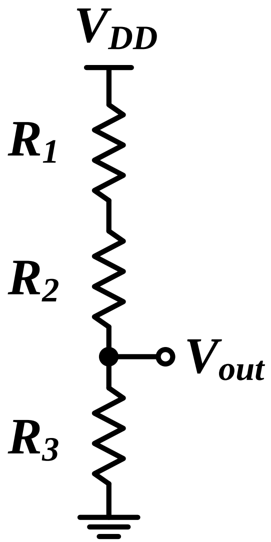
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

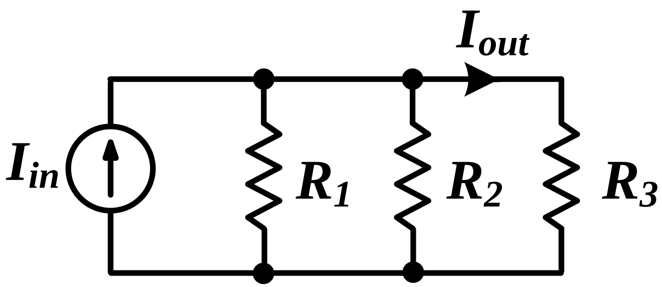
Voltage and Current Dividers

- Voltage divider → the largest resistor takes most of the voltage
- Current divider → the smallest resistor (largest conductance) takes most of the current
  - Remember that current flows in the least resistance path

$$V_{out} = V_{DD} \cdot \frac{R_3}{R_1 + R_2 + R_3}$$

$$I_{out} = I_{in} \cdot \frac{G_3}{G_1 + G_2 + G_3}$$



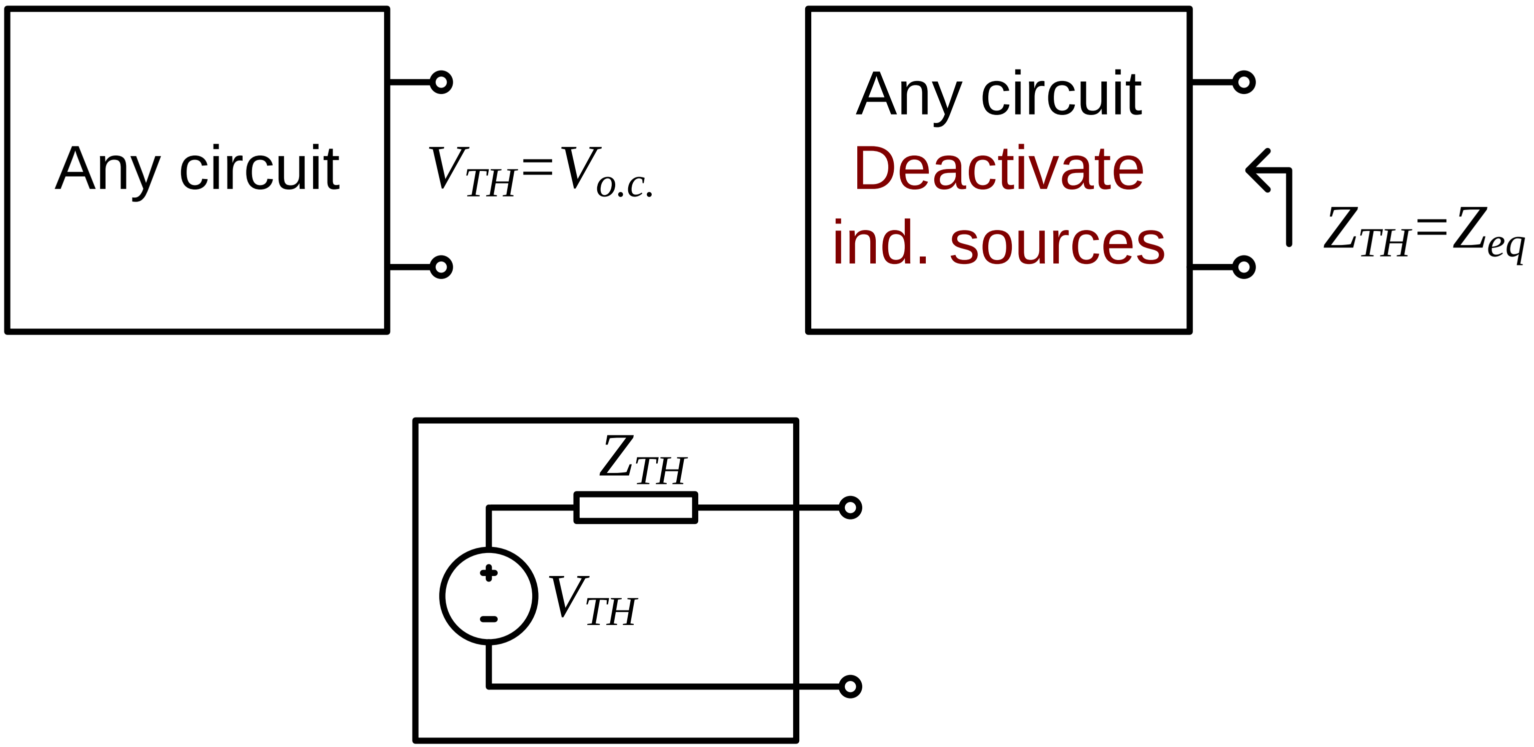


Thevenin Equivalent Circuit

- Any one port circuit can be replaced by a voltage source and a series impedance

$$V_{TH} = V_{o.c.}$$

$$Z_{TH} = Z_{eq} \text{ (turn OFF all independent sources)}$$



Norton Equivalent Circuit

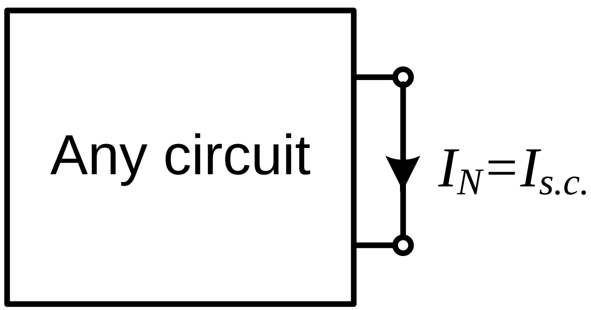
- Any one port circuit can be replaced by a current source and a parallel impedance

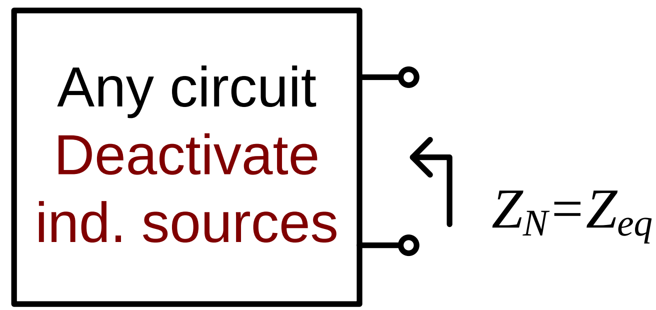
$$I_N = I_{s.c.}$$

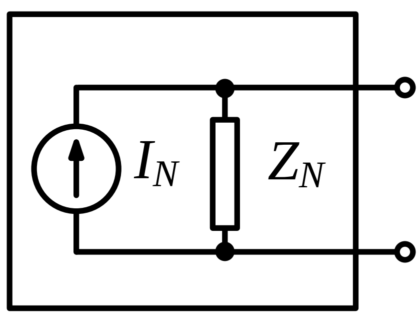
$$Z_N = Z_{eq} \text{ (turn OFF all independent sources)}$$

$$\boldsymbol{Z_N = Z_{TH}}$$

$$\boldsymbol{V_{TH} = V_{o.c.} = I_N \times Z_N}$$







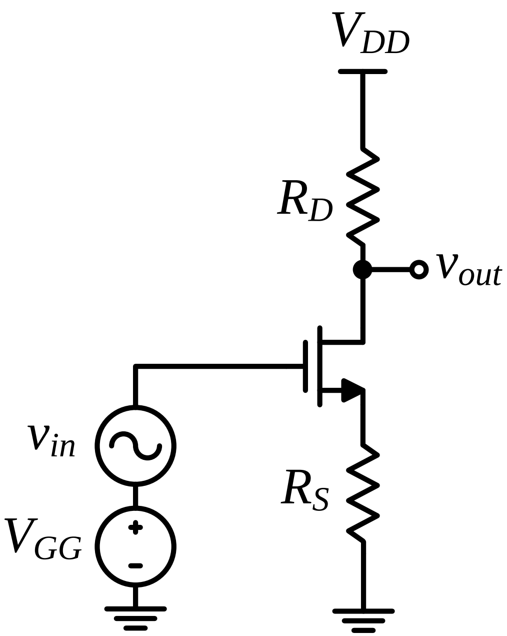
Superposition Theorem

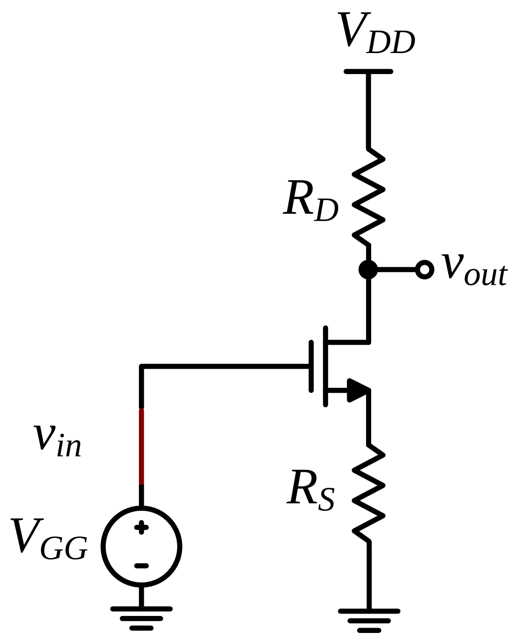
- Deactivate all independent sources except one
  - Independent voltage source → short circuit (s.c.)
  - Independent current source → open circuit (o.c.)
  - Do NOT deactivate dependent sources
- Solve the circuit
- Repeat the previous two steps for every source
- Algebraically add all the results

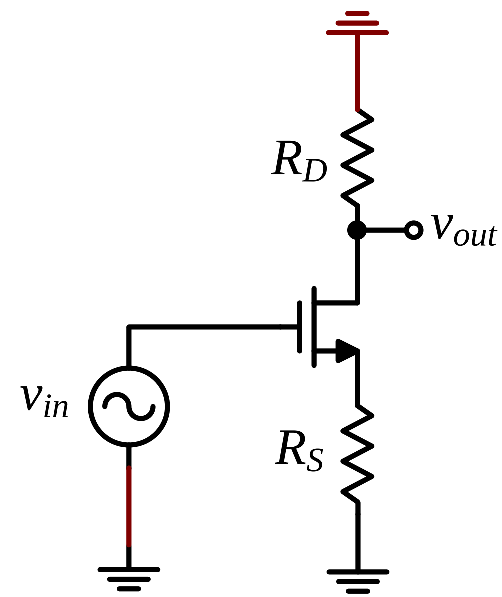
We use this frequently to separate AC and DC solutions

Superposition Theorem

DC + AC = DC + AC







Capacitance

$$Q = CV$$

$$i = \frac{dQ}{dt} = C\,\frac{dV}{dt}$$

$$V = V_o \cos\omega t = V_o \cdot Re\left\{e^{j\omega t}\right\} \;\Rightarrow\; V_o e^{j\omega t}$$

$$i = C\,\frac{dV}{dt} = j\omega C\left(V_o e^{j\omega t}\right) = j\omega C \cdot V$$

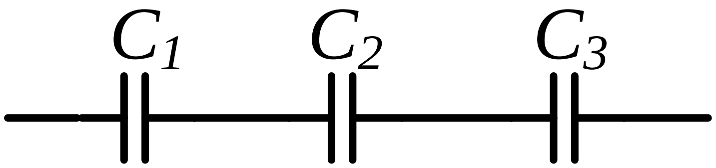
$$Z_C = \frac{V}{i} = \frac{1}{j\omega C} = \frac{1}{sC} \;\Rightarrow\; X_C = \frac{1}{\omega C}$$

$$\omega \uparrow\uparrow \;\Rightarrow\; X_C \to 0 \;\Rightarrow\; s.c.$$

$$\omega \downarrow\downarrow \;\Rightarrow\; X_C \to \infty \;\Rightarrow\; o.c.$$

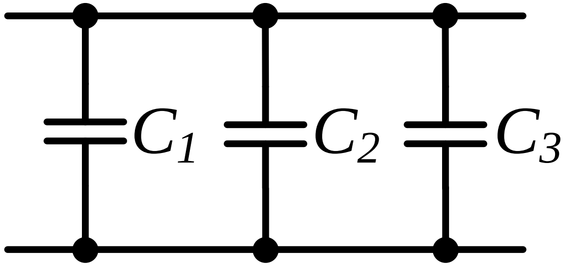
Capacitance Combinations

- Capacitors in series: Smallest capacitor dominates



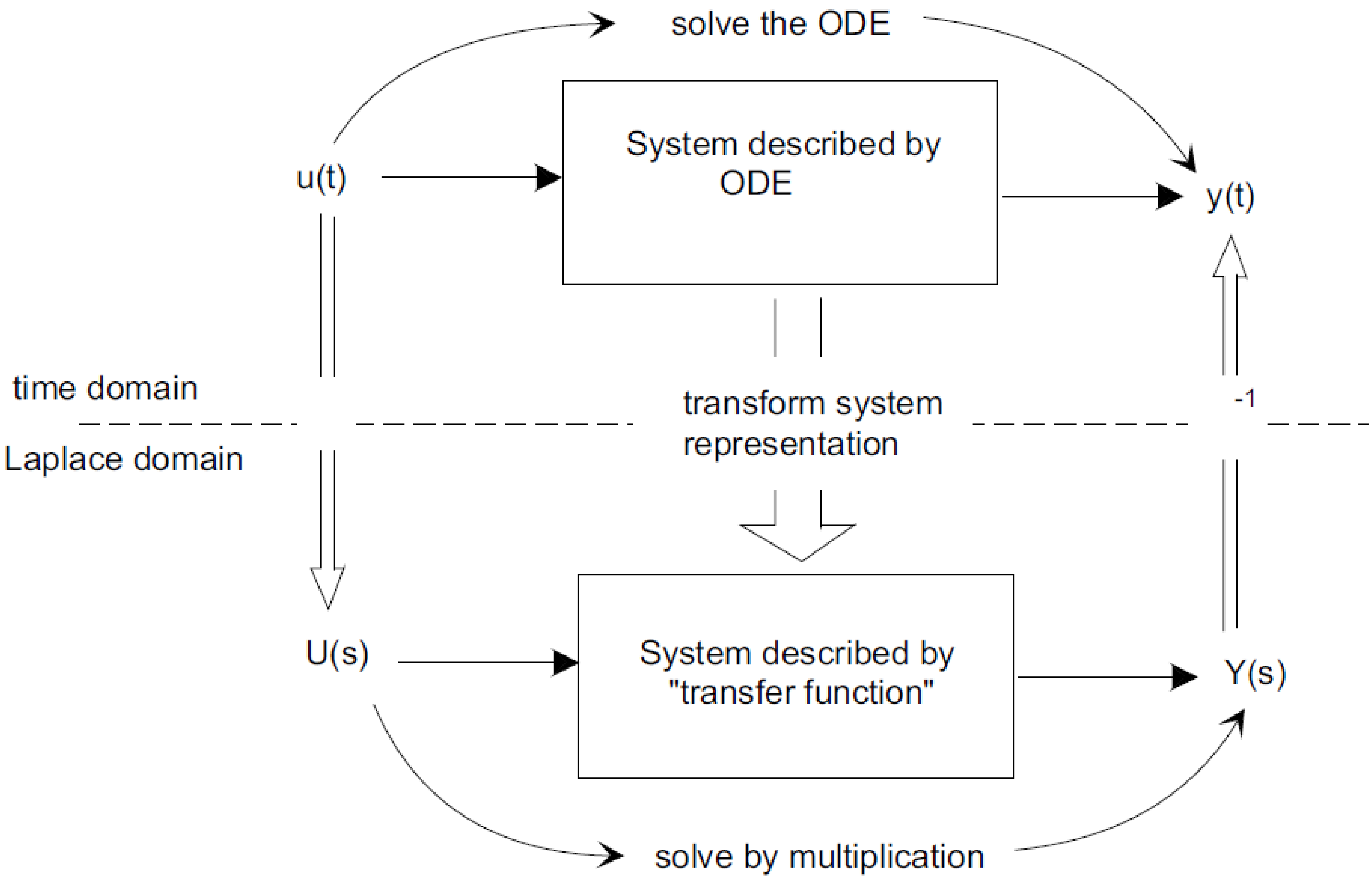
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- Capacitors in parallel: Largest capacitor dominates



$$C_{eq} = C_1 + C_2 + C_3$$

Laplace Transform (LT)



[D. Rowell, MIT 2.004, 2008]

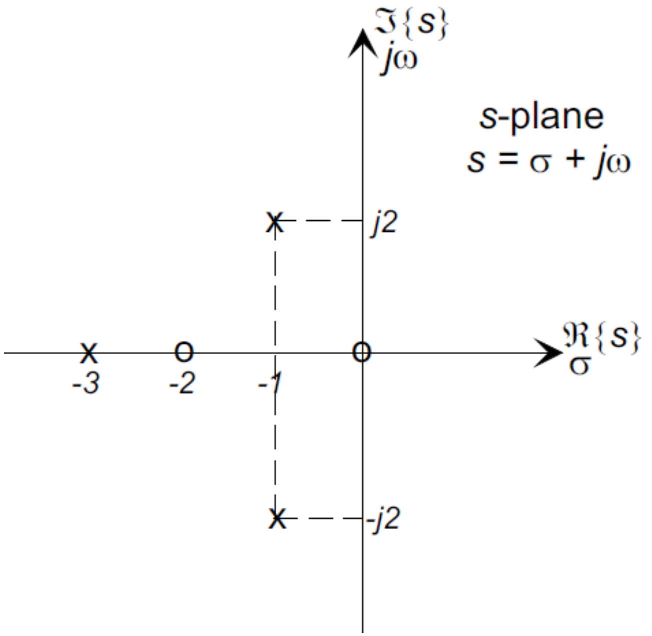
Poles and Zeros

- Transfer function

$$H(s) = \frac{N(s)}{D(s)}$$

- Zeros: roots of the numerator ➔ $N(s) = 0$
- Poles: roots of the denominator (characteristic eq.) ➔ $D(s) = 0$
- For physical systems, poles & zeros are real or complex conjugate
- Example:

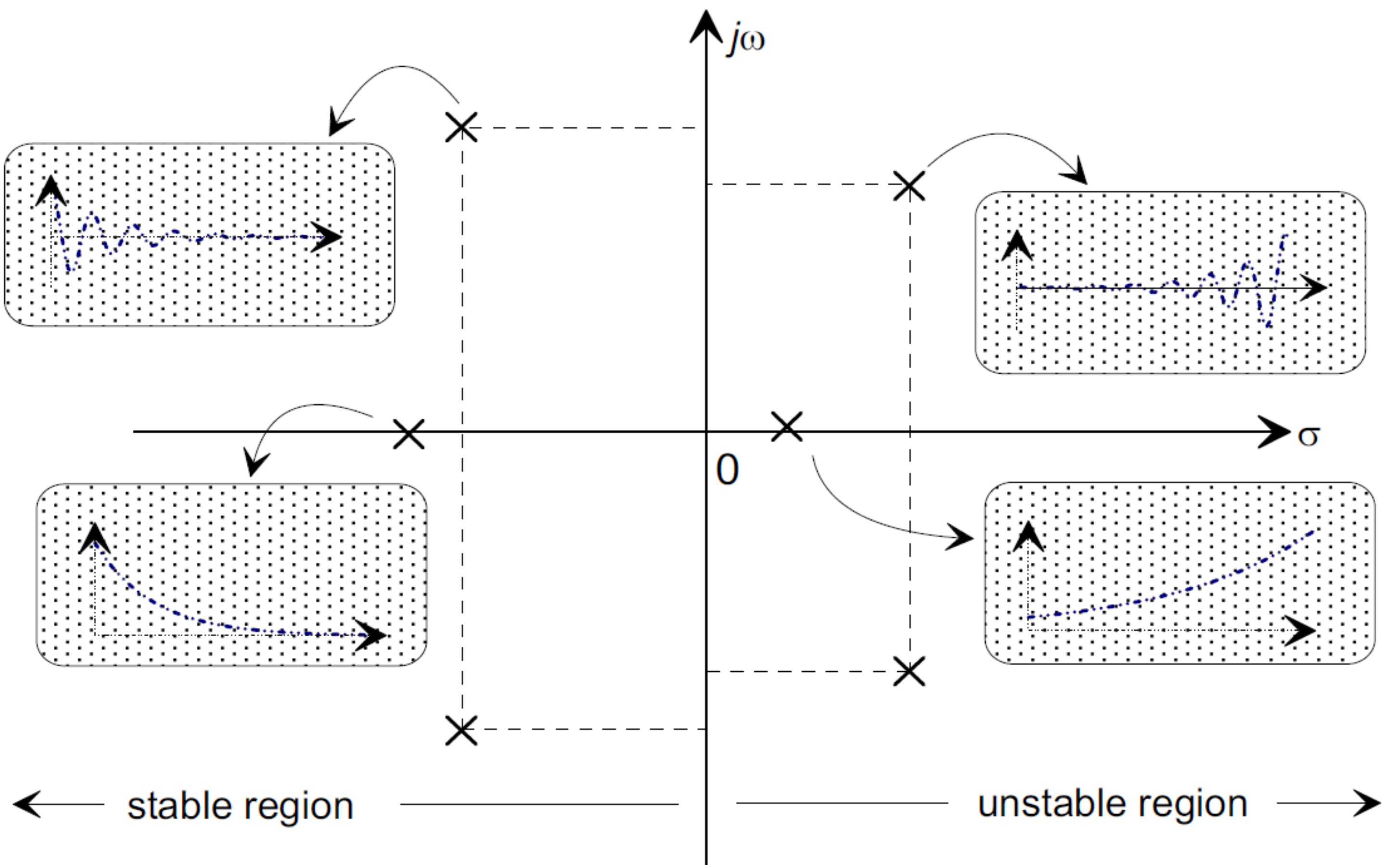
$$\begin{aligned} G(s) &= \frac{5s^2+10s}{s^3+5s^2+11s+5} \\ &= \frac{5s(s+2)}{(s+3)(s^2+2s+5)} \\ &= \frac{5s(s+2)}{(s+3)(s+(1+j2))(s+(1-j2))} \end{aligned}$$



[D. Rowell, MIT 2.004, 2008]

Pole-Zero Plot

- Poles in LHP: Decaying exponential ➔ Stable system
  - BIBO: Bounded input bounded output
- Poles in RHP: Growing exponential ➔ Unstable system



 [D. Rowell, MIT 2.004, 2008]

Frequency Response

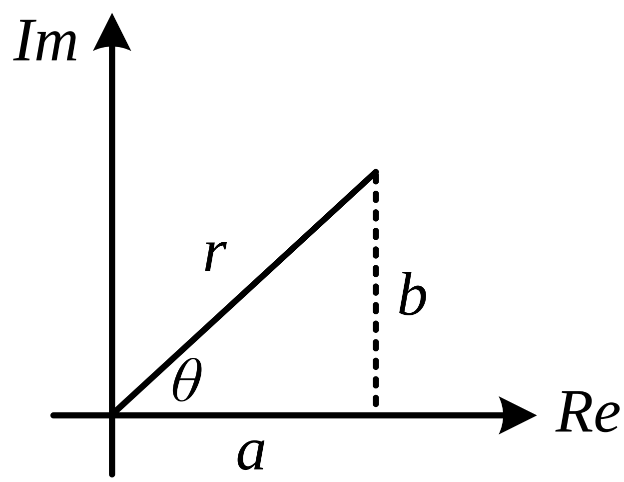
- Transfer function

$$H(s) = \frac{N(s)}{D(s)}$$

- Fourier Transform is a special case of Laplace Transform: $s \Rightarrow j\omega$
  - $\sigma = 0$ ➔ Steady state response for sinusoidal input
- Transfer function ➔ Frequency response: $s \Rightarrow j\omega$

$$H(j\omega) = \frac{V_{out}(j\omega)}{V_{in}(j\omega)} = |H(j\omega)|e^{j\phi}$$

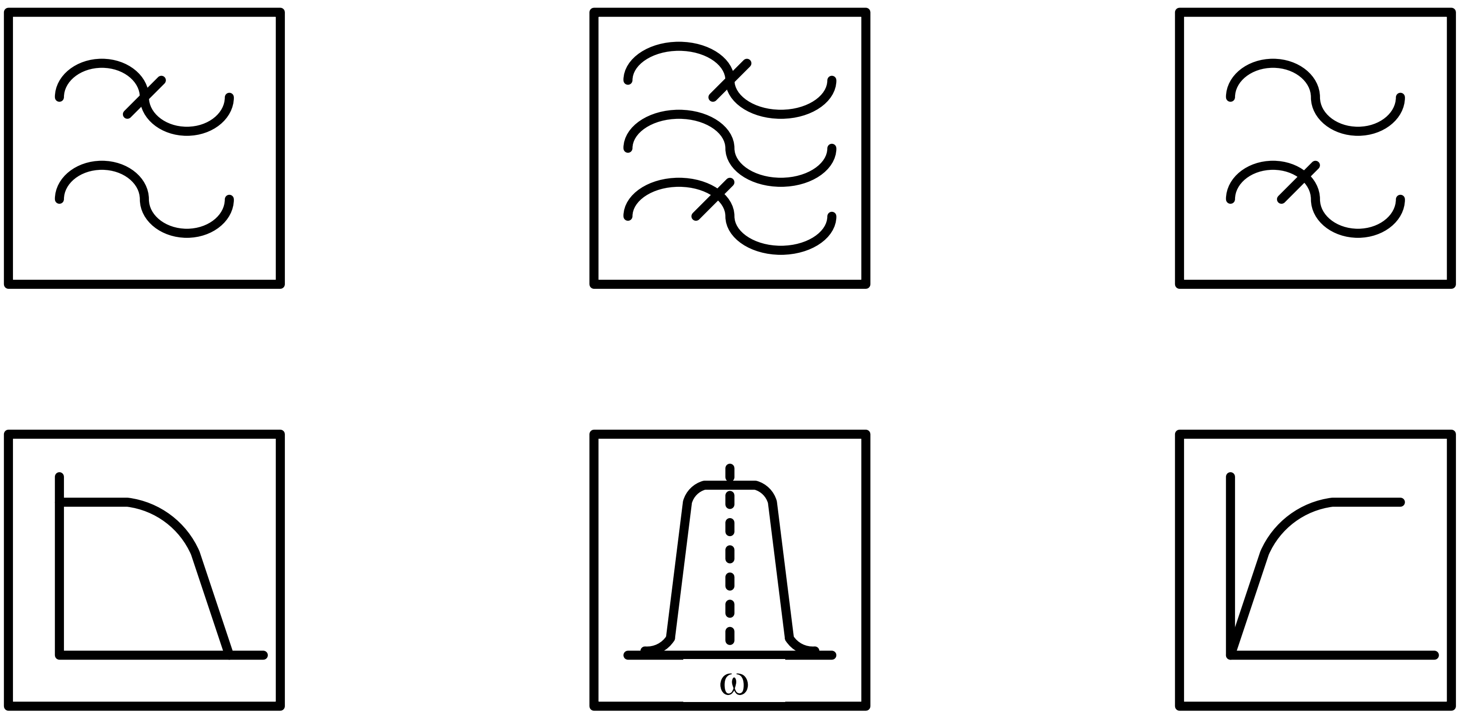
- $a + jb = re^{j\theta}$
- $r = \text{Magnitude}(a + jb) = \sqrt{a^2 + b^2}$
- $\theta = \text{Phase}(a + jb) = \tan^{-1}\frac{b}{a}$



Frequency Response

- Y-axis: magnitude of frequency response, x-axis: frequency

LPF | BPF | HPF

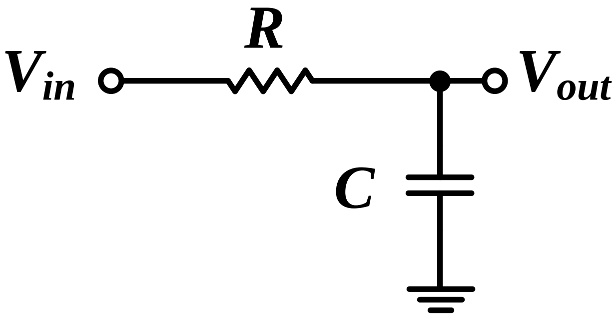


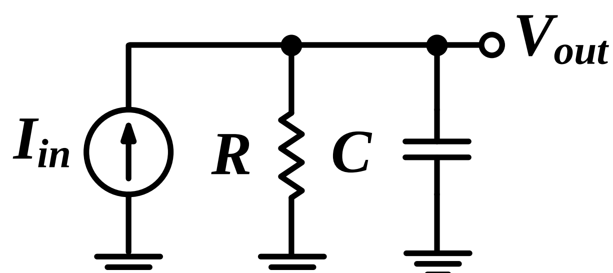
1st Order LPF

$$H(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{1/sC}{R + 1/sC} = \frac{1}{1 + sRC} = \frac{1}{1 + s\tau}$$

$$H(j\omega) = \frac{v_{out}(j\omega)}{v_{in}(j\omega)} = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + \frac{j\omega}{\omega_c}}$$

- $\tau = RC$: time constant
- $\omega_c = \frac{1}{\tau} = \frac{1}{RC}$: cutoff/corner frequency
- Poles: $s_p = -\frac{1}{\tau} = -\omega_c$
- Zeros: ?
- $|H(j\omega)| = \frac{1}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
- $P\big(H(j\omega)\big) = -\tan^{-1}\frac{\omega}{\omega_c}$





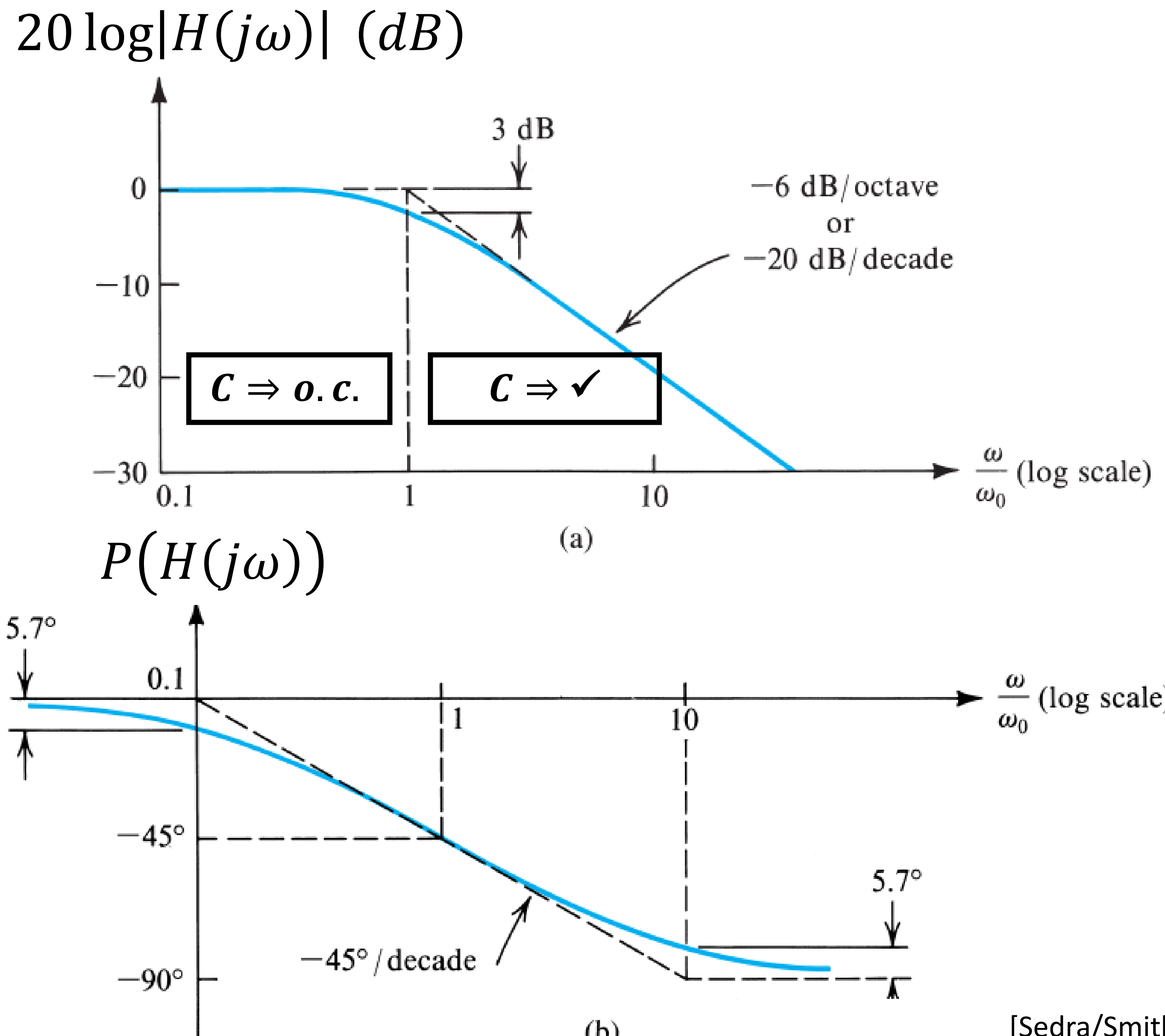
Bode Plot Rules

	Pole	Zero
Magnitude	-20 dB/decade Actual Mag @ pole: -3 dB	+20 dB/decade Actual Mag @ zero: +3 dB
Phase	-90^o Actual Phase @ pole: -45^o	LHP zero: $+90^o$ Actual Phase @ zero: $+45^o$
		RHP zero: -90^o Actual Phase @ zero: -45^o

→ RHP: Right-half plane ($Re\{s\} > 0$)

→ LHP: Left-half plane ($Re\{s\} < 0$)

1^{st} Order LPF Bode Plot



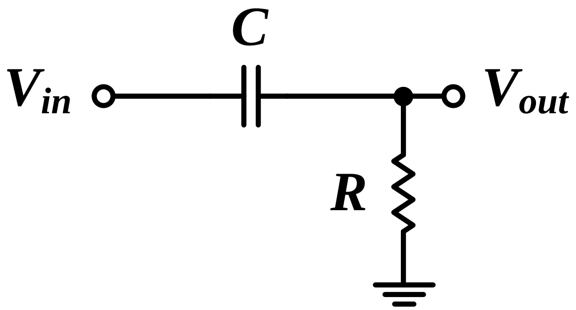
[Sedra/Smith, 2015]

1st Order HPF

$$H(s) = \frac{v_{out}(s)}{v_{in}(s)} = \frac{R}{R + 1/sC} = \frac{sRC}{1 + sRC} = \frac{s\tau}{1 + s\tau}$$

$$H(j\omega) = \frac{v_{out}(j\omega)}{v_{in}(j\omega)} = \frac{R}{R + 1/j\omega C} = \frac{j\omega RC}{1 + j\omega RC} = \frac{\frac{j\omega}{\omega_c}}{1 + \frac{j\omega}{\omega_c}}$$

- Poles: $s_p = -\frac{1}{\tau} = -\omega_c$
- Zeros: $s_z = 0$



- $|H(j\omega)| = \frac{\frac{\omega}{\omega_c}}{\sqrt{1+\left(\frac{\omega}{\omega_c}\right)^2}}$
- $P\big(H(j\omega)\big) = 90^o - \tan^{-1}\frac{\omega}{\omega_c}$

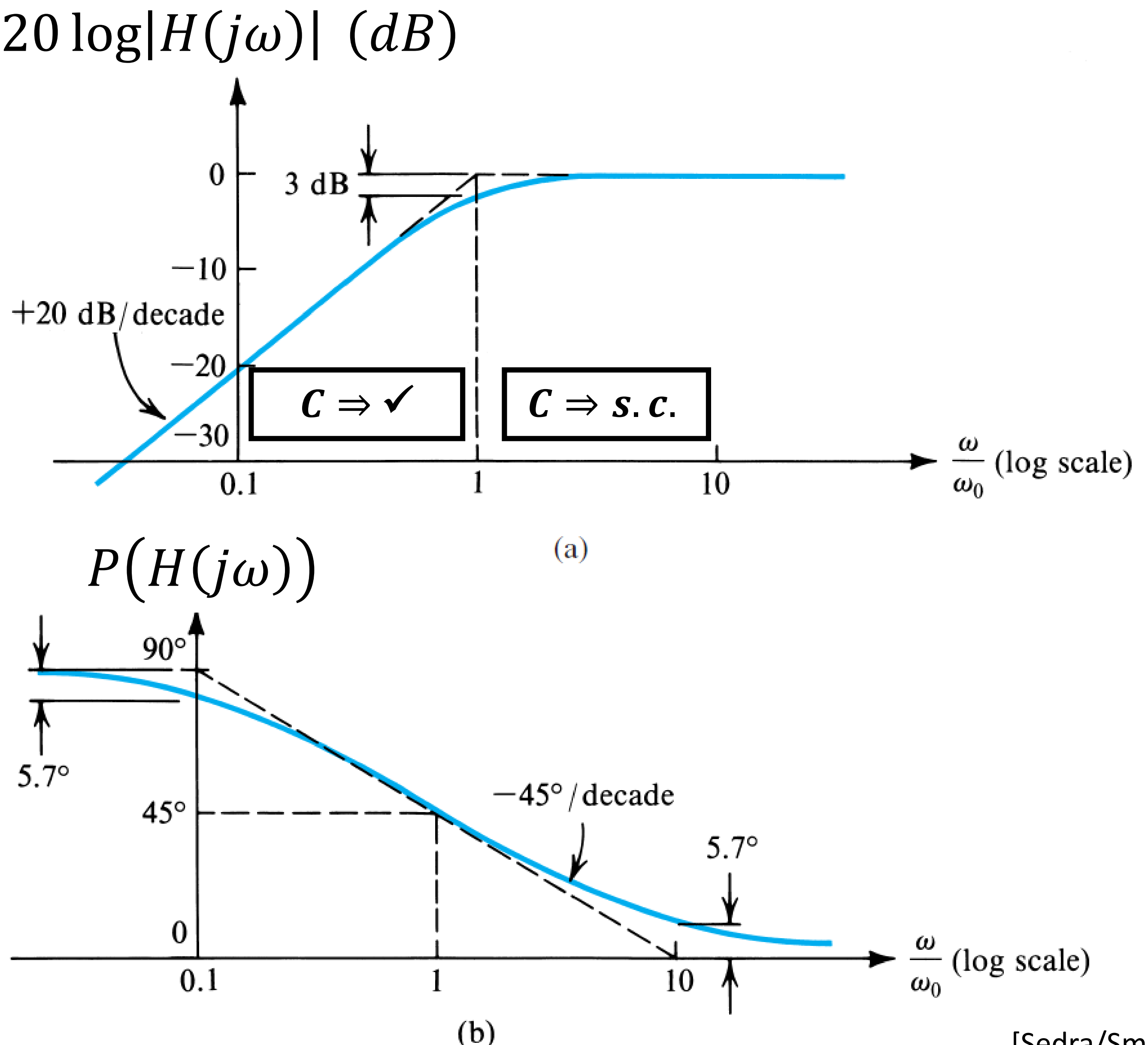
Bode Plot Rules

<table>
<tr><th></th><th>Pole</th><th>Zero</th></tr>
<tr><td>Magnitude</td><td>-20 dB/decade
Actual Mag @ pole: -3 dB</td><td>+20 dB/decade
Actual Mag @ zero: +3 dB</td></tr>
<tr><td rowspan="2">Phase</td><td rowspan="2">-90^{o}
Actual Phase @ pole: -45^{o}</td><td>LHP zero:
$+90^{o}$
Actual Phase @ zero: $+45^{o}$</td></tr>
<tr><td>RHP zero:
-90^{o}
Actual Phase @ zero: -45^{o}</td></tr>
</table>

→ RHP: Right-half plane ($Re\{s\} > 0$)

→ LHP: Left-half plane ($Re\{s\} < 0$)

1st Order HPF Bode Plot



[Sedra/Smith, 2015]

References

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- A. Sedra and K. Smith, "Microelectronic circuits," Oxford University Press, 7th ed., 2015
- B. Razavi, "Fundamentals of microelectronics," 2nd ed., Wiley, 2014

Thank you!