Hardware & Software Verification

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Lecture 3: More Isabelle
24 October 2024

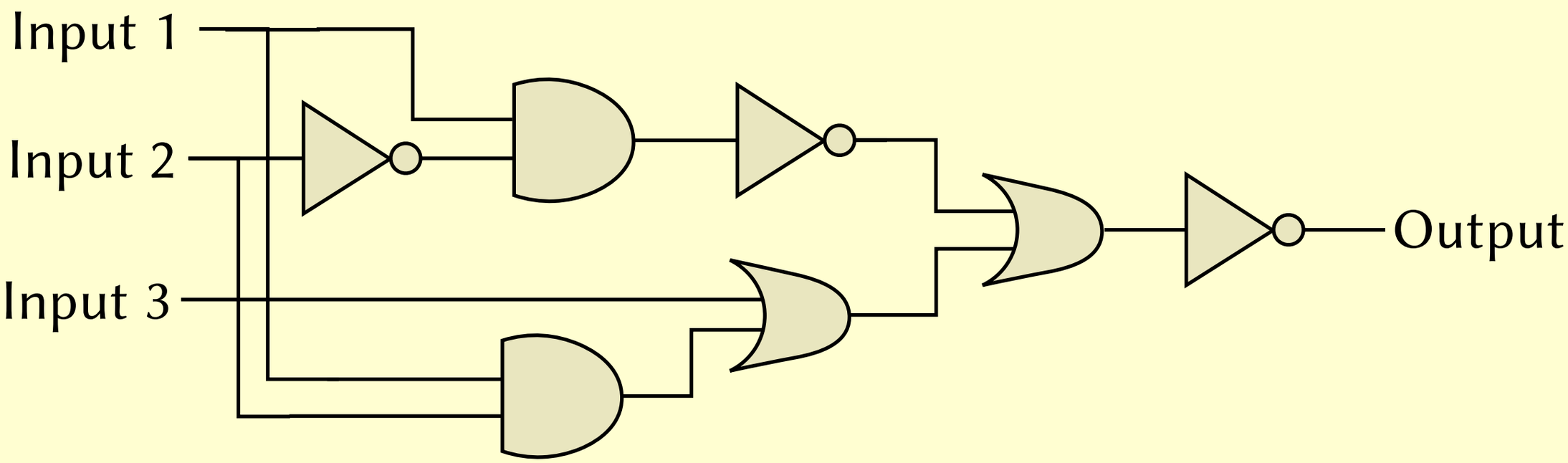
Coursework Admin

- All paired up?
- Coursework deadline is **Friday 13 December**.
- Isabelle and Dafny coursework specifications are available.
- SymbiYosys coursework specification not yet available.

Lecture Outline

- Proving the correctness of a logic synthesiser.

Representing circuits



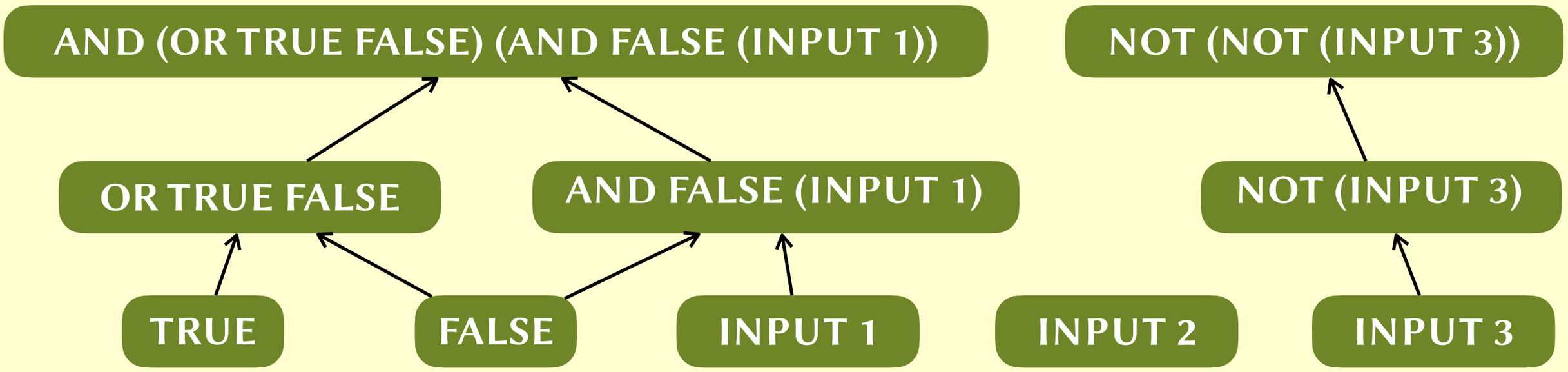

```
NOT
  (OR
    (NOT
      (AND (INPUT 1) (NOT (INPUT 2))))
    (OR
      (INPUT 3)
      (AND (INPUT 1) (INPUT 2))))
```


DEMO

Recursive data structures

```
datatype "circuit" =
  NOT "circuit"
| AND "circuit" "circuit"
| OR "circuit" "circuit"
| TRUE
| FALSE
| INPUT "int"
```

circuit ::= NOT *circuit*
| AND *circuit circuit*
| OR *circuit circuit*
| TRUE
| FALSE
| INPUT *int*



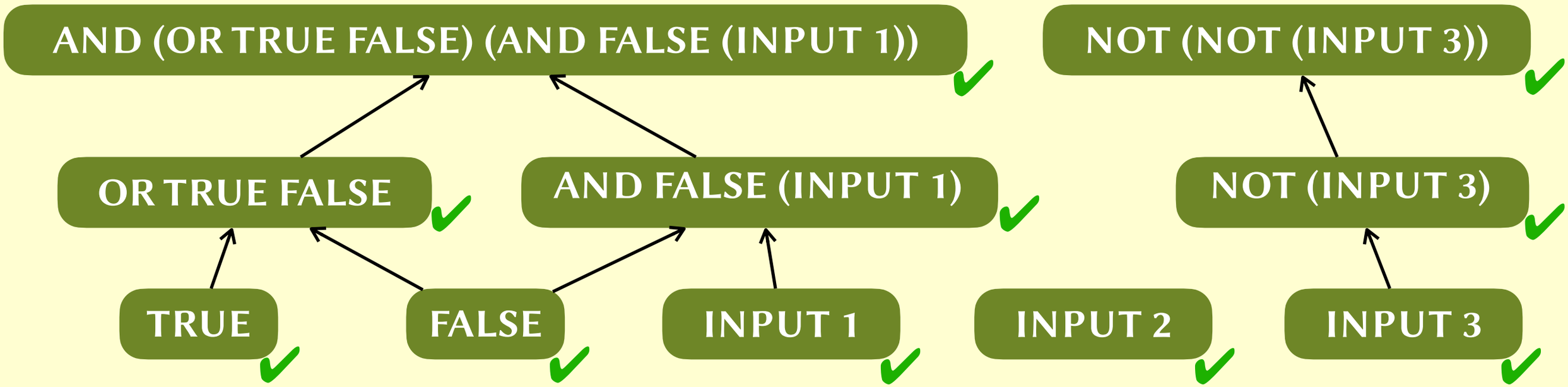

DEMO

Structural induction

- Suppose we want to show that property P holds for all circuits.

- It suffices to show that each constructor preserves P.

1. $\forall c.\ P(c) \Longrightarrow P(\texttt{NOT}\ c)$
2. $\forall c_1, c_2.\ (P(c_1) \wedge P(c_2)) \Longrightarrow P(\texttt{AND}\ c_1\ c_2)$
3. $\forall c_1, c_2.\ (P(c_1) \wedge P(c_2)) \Longrightarrow P(\texttt{OR}\ c_1\ c_2)$
4. $P(\texttt{TRUE})$
5. $P(\texttt{FALSE})$
6. $\forall i.\ P(\texttt{INPUT}\ i)$

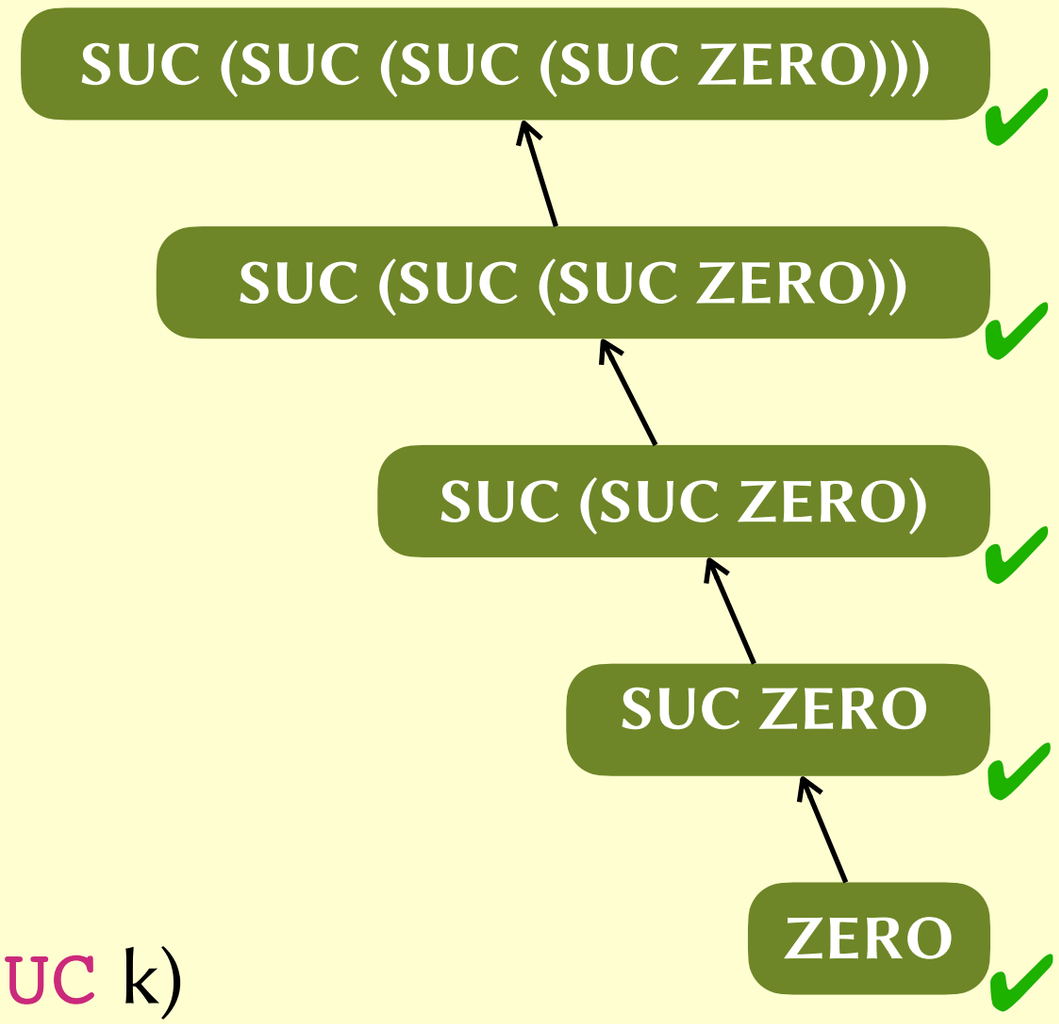


Mathematical induction

```
datatype "nat" =
   ZERO
 | SUC "nat"
```

nat ::= ZERO
| SUC *nat*

1. P(ZERO)
2. ∀k. P(k) ⟹ P(SUC k)



Proof by structural induction

Theorem. simulate (mirror c) ρ = simulate c ρ.

Proof by structural induction

Define P(c) = (∀ρ. simulate (mirror c) ρ = simulate c ρ).

Theorem. P(c) holds for all c.

Proof. We proceed by structural induction on c.

Proof by structural induction

Define P(c) = (∀ρ. simulate (mirror c) ρ = simulate c ρ).

Theorem. P(c) holds for all c.

Proof. We proceed by induction on the structure of c.

Case NOT. Fix arbitrary c, and assume P(c) as induction hypothesis.

simulate (mirror (NOT c)) ρ

= simulate (NOT (mirror c)) ρ *[by defn of mirror]*

= ¬ simulate (mirror c) ρ *[by defn of simulate]*

= ¬ simulate c ρ *[by induction hypothesis]*

= simulate (NOT c) ρ *[by defn of simulate]*

Thus P(NOT c).


DEMO

Rule induction

```
fun f where
  "f✔(Suc (Suc n)) = f✔n + f✔(Suc n)"
| "f✔(Suc 0) = 1"
| "f✔0 = 1"
```

$$\frac{f(n) = A \quad f(Suc\ n) = B}{f(Suc\ (Suc\ n)) = A + B} \qquad \frac{}{f(Suc\ 0) = 1} \qquad \frac{}{f(0) = 1}$$

$$\frac{P(n) \qquad P(Suc\ n)}{P(Suc\ (Suc\ n))} \qquad \frac{}{P(Suc\ 0)} \qquad \frac{}{P(0)}$$

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1. $\forall n.\ (P(n) \wedge P(\texttt{Suc}\ n)) \Longrightarrow P(\texttt{Suc}\ (\texttt{Suc}\ n))$
2. $P(\texttt{Suc}\ 0)$
3. $P(0)$

Summary

- Recursive data structures
- Recursive functions
- Structural induction
- Rule induction