LAFDS Sessions 1&2 Homework

Full Name: ______________________________

Group No.: ____

Lecturer Name: ____________________

Submission date: _/_/__

Please write down all the steps not the final answer only

Questions:

Question set 1:

Given the matrices:

$$A = \begin{bmatrix} -1 & 23 & 10 \\ 0 & -2 & -11 \end{bmatrix}, \quad B = \begin{bmatrix} -6 & 2 & 10 \\ 3 & -3 & 4 \\ -5 & -11 & 9 \\ 1 & -1 & 9 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 9 & -5 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix}, \quad E = [3], \quad F = \begin{bmatrix} 3 \\ 5 \\ -11 \\ 7 \end{bmatrix}, \quad G = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}$$

a) What is the dimension of each matrix?

b) Which matrices are square?

c) Which matrices are symmetric?

d) Which matrix has the entry at row 3 and column 2 equal to -11?

e) Which matrices has the entry at row 1 and column 3 equal to 10?

f) Which are column matrices?

g) Which are row matrices?

h) Find A^T, C^T, E^T, G^T.

Questions set 2:

Given the matrices:

$$A = \begin{bmatrix} 23 & 10 \\ 0 & -11 \end{bmatrix}, \quad B = \begin{bmatrix} -6 & 0 & 0 \\ -1 & -3 & 0 \\ -5 & 3 & -9 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$, \quad D = \begin{bmatrix} -7 & 3 & 2 \\ 0 & 2 & 4 \\ 0 & 0 & 9 \end{bmatrix}, \quad E = \begin{bmatrix} 12 & 0 & 0 \\ 0 & 23 & 0 \\ 0 & 0 & -19 \end{bmatrix}$$

a) Which of the above matrices are diagonal?

b) Which of the above matrices are lower triangular?

c) Which of the above matrices are upper triangular?

Questions set 3:

A, B, C, D and E are matrices given by:

$$A = \begin{bmatrix} -1 & 1 & -2 \\ 0 & -2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} -1 & 2 & 0 \\ 0 & -3 & 4 \\ -1 & -2 & 3 \end{bmatrix}, \quad C = [-3 \quad 2 \quad 9 \quad -5 \quad 7$$

$$D = \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix}, \quad E = \begin{bmatrix} 3 \\ 5 \\ -11 \end{bmatrix}, \quad F = \begin{bmatrix} -1 & 0 & 2 \\ -2 & -3 & 4 \\ 1 & 4 & -3 \end{bmatrix}$$

Find if possible:

a) AB
b) BC
c) AD
d) EF
e) FE

Question set 4:

Find x and y if possible

A) $\begin{bmatrix} 2x+y & 3 & 10 \\ y+1 & -2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 3 & 10 \\ 3 & -2 & 0 \end{bmatrix}$

B) $[6 \quad -4 \quad -6 \quad x-y] = -2[-3 \quad 2 \quad 2x+2y \quad 13]$

C) $\begin{bmatrix} x+y & -2 \\ x-y & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 12 & -8 \end{bmatrix}$

Question set 5:

i. Vector spaces are closed on addition and scalar multiplication. Find the unknown vectors (u) and scalars (r and s) in the following equations:

a) $2\mathbf{u} = 2\begin{bmatrix} 4 \\ 6 \\ 0 \end{bmatrix} - 3\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

b) $-3\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} - \mathbf{u} = -2\begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}$

c) $r\begin{bmatrix} 1 \\ 0 \end{bmatrix} + s\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

d) $r\begin{bmatrix} 1 \\ 2 \end{bmatrix} + s\begin{bmatrix} 2 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

ii.

Show that the vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{u}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ span the vector space $\mathbb{R}^3$

iii. Show that in the space R^3 the vectors x = (1, 1, 0), y = (0, 1, 2), and z = (3, 1, −4) are linearly dependent by finding scalars α and β such that αx + βy + z = 0.

iv. u , v and w are vectors in R^n such that $w = v - 4\alpha u$ where α is a real number.

Find α if u and w are orthogonal, the norm of u is equal to 5 and $u^T v = 3$.

v. Find all values of Θ such that the vector $u = \begin{bmatrix} \sin(\theta) \\ \cos(\theta) \end{bmatrix}$ and $v = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ are orthogonal

vi.

a) Show that the vectors $u = K\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$, $v = L\begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$, and $w = M\begin{bmatrix} -2 \\ 10 \\ 1 \end{bmatrix}$, where K, L and M are real constants, are orthogonal.
b) Find positive values for the constants K, L and M so that the vectors u, v and w form an orthonormal basis

Practice with Code:

1. Write a NumPy code line(s) to get and print your numpy library version
2. Write a NumPy code line(s) to get help on the "add" function.
3. Write a NumPy code line(s) to test whether any of the elements of an input array is non-zero
4. Write a NumPy code line(s) to compute the x and y coordinates for points on a sine curve and plot the points using matplotlib.
5. Write a NumPy code line(s) to extract all numbers which are less and greater than a specified integer in an input array
6. Write a NumPy code line(s) to find the missing (hint: undefined) data in an input array

Reading homework:

- Numpy vs. Scipy: https://bit.ly/3vURVkl
- Numpy documentation: https://numpy.org/doc/stable/user/quickstart.html

- "Python for Data Analysis by Wes McKinney" Chapter 4: https://www.oreilly.com/library/view/python-for-data/9781449323592/ch04.html
- Scalars and vectors: https://www.mathsisfun.com/algebra/scalar-vector-matrix.html
- Vectors and matrices: https://www.statlect.com/matrix-algebra/vectors-and-matrices
- Dot product: https://www.mathsisfun.com/algebra/vectors-dot-product.html
- Operations on matrices: https://medium.com/linear-algebra/part-2-operations-on-matrices-3caab542aebd
- Applications of matrices: https://www.vedantu.com/maths/application-of-matrices
- Linear Transformation and matrices: http://amsi.org.au/ESA_Senior_Years/SeniorTopic8/8a/8a_2content_3.html
- Linear combination, span, linear independence: https://medium.com/linear-algebra-basics/vector-span-f90b989d712d