LAFDS Session 3,4&5 Homework

Full Name: ______________________________

Group No.: ____

Lecturer Name: _____________________

Submission date: _/_/__

Please write down all the steps not the final answer only

Questions:

1. Use the determinant to find the values of m for which u1, u2 and u3 are linearly dependent

$$\mathbf{u}_1 = \begin{bmatrix} m \\ 4 \\ 0 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ 8 \end{bmatrix} \text{ and } \mathbf{u}_3 = \begin{bmatrix} 0 \\ -1 \\ m \end{bmatrix}$$

2.

Are the vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 5 \\ 5 \\ 0 \\ 0 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \end{bmatrix}$ linearly dependent or independent?

3. What is the inverse of the transformation $F:R^2 \rightarrow R^2$ given by F(x,y)=(x+3y , x+5y) ?

(Explanation: Before transformation vector F = (x , y), after transformation F = (x+3y , x+5y). Find the transformation matrix and its inverse)

4. Find a linear transformation $R^2 \rightarrow R^2$ that maps (1,1) to (−1,4) and (−1,3) to (−7,0)

5. Which of the following matrices are in row echelon form? Which are in reduced row echelon form?

$$\begin{bmatrix} 1 & 3 & 5 \\ 2 & 3 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

6.

Let $A = \begin{bmatrix} 1 & 4 & -1 \\ 1 & 5 & 0 \\ 0 & 3 & 3 \end{bmatrix}$.

(a) Is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in the span of the columns of A? What about $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$?

(b) Is $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ a linear combination of the columns of A? What about $\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$?

7. Solve the following systems of equations:

a)
$$\begin{array}{rcrcrcr} 2x_1 & & & - & 6x_3 & = & -8 \\ & & x_2 & + & 2x_3 & = & 3 \\ 3x_1 & + & 6x_2 & - & 2x_3 & = & -4 \end{array}$$

b)
$$\begin{array}{rcrcrcr} & & x_2 & + & 5x_3 & = & -4 \\ x_1 & + & 4x_2 & + & 3x_3 & = & -2 \\ 2x_1 & + & 7x_2 & + & x_3 & = & -2 \end{array}$$

8.

Determine the dimension of, and a basis for, the row space of the matrix $B = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 0 & 1 \\ 0 & 2 & -1 \\ 1 & 1 & 4 \end{bmatrix}$

9.

For what value of b is the vector $\mathbf{b} = (1, 2, 3, b)^T$ in the column space of the following matrix?

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & -4 & -5 \\ 6 & 3 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

10.

Find all values of 'a' which will prove that A has eigenvalues 0, 3, and −3.

$$A = \begin{bmatrix} -2 & 0 & 1 \\ -5 & 3 & a \\ 4 & -2 & -1 \end{bmatrix}$$

11. Find a formula for $\begin{bmatrix} 1 & -6 \\ 2 & -6 \end{bmatrix}^k$ by diagonalizing the matrix.

Practice with Code:

Question 1

(MAKE SURE YOU ADD COMMENTS EXPLAINING YOUR CODE)

1. Implement the function S = IsSquare(A) that checks the dimension of input matrix A, and returns 1 if the input matrix is square and 0 otherwise

2. Implement the function N = Props(A) that checks the properties of the input matrix A:

   2.1 checks the input matrix is square,

   2.2 computes the matrix rank, prints the rank value (hint: use numpy method)

   2.3 computes the matrix determinant (hint: use numpy method) and prints its value

   2.4 returns 1 if the input matrix has rank = no. of rows in matrix A and 0 otherwise

3. Compose the equations corresponding to the two word-problems below and put them in matrix-vector form AX = B. Call "Props" to find more about their properties.

[[IMPORTANT: Create all your numpy arrays as float64 data type arrays]]

4. Implement the function reducerow(C,pivotrow,targetrow,pivot)

4.1 Takes an input matrix C, index of a pivot row vector, index of a target row vector, index of a pivot element. It returns nothing but overrides the matrix C

4.2 Checks if the pivot element = 1 otherwise perform vector operation to make it = 1

4.3 Uses vector operations using the two vectors “pivotrow” & “targetrow” to make the element corresponding to the pivot element in the targetrow vector = 0

5. Implement the function f = SolveLinearSystem (A,B)

5.1 Takes as an input coefficient matrix A and target vector B

5.2 Adjoins A and B then perform row reduction operations using “reducerow” trying to convert it A’s columns into an identity matrix

5.3 returns f = the summation of A’s columns elements after row reduction

Hint: print your overridden matrix to check the changes

6. Call “SolveLinearSystem” to solve the systems 1 and 2 you previously created in 3

7. Elaborate your solution (write commented text)

7.1 If f<3 explain the reason and propose one possible solution (if any) to the equation system

7.2 otherwise write down the system solution

8. Type these code lines and check your answers for systems 1 and 2

```
x = np.linalg.solve(A, B) # Solves a full-rank system of linear equations ax = b.
print(x)
np.allclose(np.dot(A, x), B) # Returns True if two arrays are element-wise equal within a tolerance.
```

9. Call the same function “SolveLinearSystem” to get inv(A) for systems 1 and 2

10. Check your solution if possible using np.linalg.inv() function

(MAKE SURE YOU ADD COMMENTS EXPLAINING YOUR CODE)

System 1:

A wedding planner ordered 200 flowers for tables decorations. He ordered sunflowers at $1.50 each, roses at $5.75 each and daisies at $2.60 each. He ordered mostly sunflowers and 20 fewer roses than daisies. The total order came to $589.5. How many of each type of flower was ordered?

System 2:

A food-cart owner regularly buys oil to fry the chicken and potatoes he sells. Sometimes he doesn't sell well but he needs to keep buying oil anyways. In three consecutive days he made zero profit. In the first day, he sold 2 packs of potatoes and 1 pack of chicken but had to buy 3 bottles of oil. In the second day, he sold 4 packs of potatoes and 2 pack of chicken but had to buy 6 bottles of oil. In the third day, he sold 1 pack of potatoes and a friend gave him 1 bottle of oil and took 1 pack of chicken instead. What were the prices of the chicken, potatoes and oil?

Question 2 (Self-study – no submission required)

- **Lab 5 notebook:** Review carefully, check the provided links and re-run the code yourself for computing Eigenvalues and eigen vectors, diagonalization, PCA and svd using numpy
- Scipy and Sympy sections are optional

Reading homework:

- **Determinants:** https://www.youtube.com/watch?v=Ip3X9LOh2dk (video 3blue1brown)
  - https://medium.com/sho-jp/linear-algebra-101-part-5-determinants-b54f990782cc
  - https://www.mathsisfun.com/algebra/matrix-determinant.html
  - https://medium.com/linear-algebra/part-20-determinants-e4b2fbcce883
  - https://medium.com/linear-algebra/part-21-properties-of-determinants-1af8a231fd2b?source=---------0-----------------------

- **Inverse of a matrix**
  - https://www.mathsisfun.com/algebra/matrix-inverse.html
  - https://www.mathsisfun.com/algebra/matrix-inverse-minors-cofactors-adjugate.html
- **Some Applications of the Eigenvalues and Eigenvectors of a square matrix:**
  - A note by Michael Nasab (Lecturer at California State Polytechnic University) https://www.cpp.edu/~manasab/eigenvalue.pdf
- **What are Eigenvalues and Eigenvectors?**
  - https://medium.com/fintechexplained/what-are-eigenvalues-and-eigenvectors-a-must-know-concept-for-machine-learning-80d0fd330e47
  - https://medium.com/sho-jp/linear-algebra-part-6-eigenvalues-and-eigenvectors-35365dc4365a
- **Diagonalization**
  - https://textbooks.math.gatech.edu/ila/diagonalization.html
  - https://machinelearningmastery.com/introduction-to-eigendecomposition-eigenvalues-and-eigenvectors/
- **Principal component analysis (PCA)**
  - https://towardsdatascience.com/pca-eigenvectors-and-eigenvalues-1f968bc6777a
  - https://machinelearningmastery.com/calculate-principal-component-analysis-scratch-python/
- **Singular Value Decomposition (SVD)**
  - https://www.geeksforgeeks.org/singular-value-decomposition-svd/
  - https://www.geeksforgeeks.org/singular-value-decomposition/?ref=rp
  - https://machinelearningmastery.com/singular-value-decomposition-for-machine-learning/